

# $r$ -process nucleosynthesis: An Introduction

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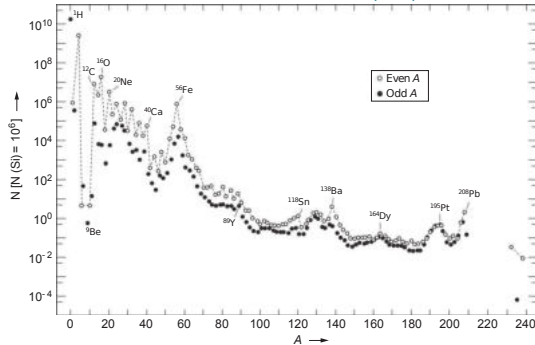
March 3 – 9, 2024

Rußbach am Paß Gschütt (Austria)

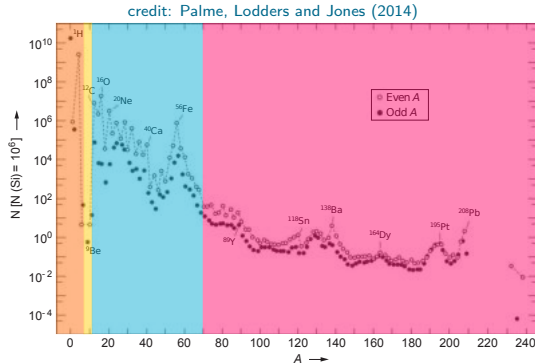


# Solar system elements: where do they come from?

credit: Palme, Lodders and Jones (2014)



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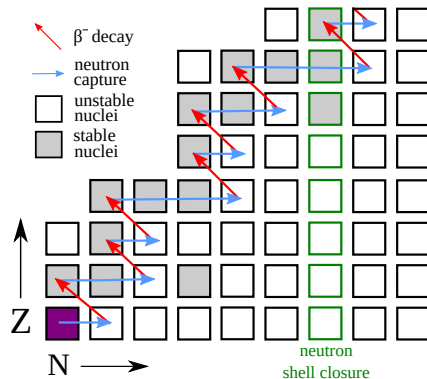


- **H, He and (some) Li:** Big-Bang nucleosynthesis.
- **(some) Li, Be and B:** Galactic cosmic rays on interstellar medium.
- **Nuclei up to Fe peak:** Stellar evolution and explosions.
- **Beyond Fe peak:**  $p$  nuclei, and ( $s$ - and  $r$ -)neutron capture processes.

## The $s$ process

B<sup>2</sup>FH, Rev. Mod. Phys. 29, 547 (1957) ; A. Cameron, Report CRL-41 (1957)

$s$ (low neutron capture) process:  $\tau_{(n,\gamma)} > \tau_{\beta^-}$ ,  $n_n = 10^{10-12} \text{ cm}^{-3}$

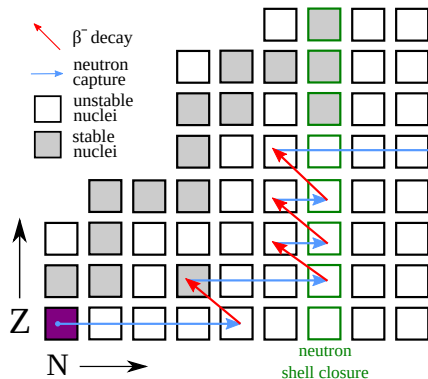


- The path to heavier nuclei stays **close to stability**.
- Astrophysical site:  $0.8-8 M_{\odot}$  stars.

## The $r$ process

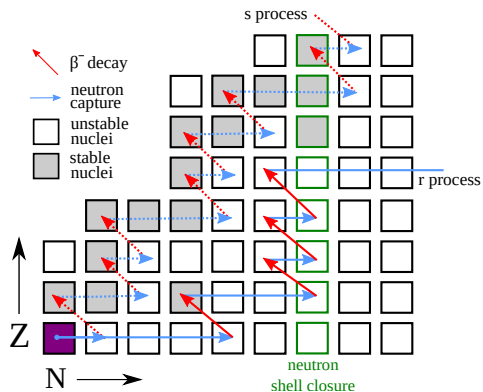
$B^2FH$ , Rev. Mod. Phys. 29, 547 (1957) ; A. Cameron, Report CRL-41 (1957)

$r$ (apid neutron capture) process:  $\tau_{(n,\gamma)} \ll \tau_{\beta^-}$ ,  $n_n > 10^{26} \text{ cm}^{-3}$



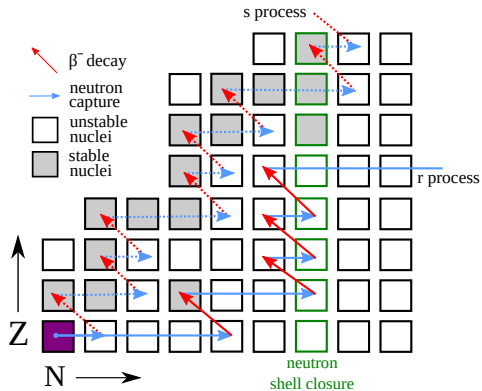
- The path to heavier nuclei goes through **neutron-rich nuclei**.
- Astrophysical site with **high neutron fluxes**  $\rightarrow$  **transient object**.

## Solar system abundances of heavy elements

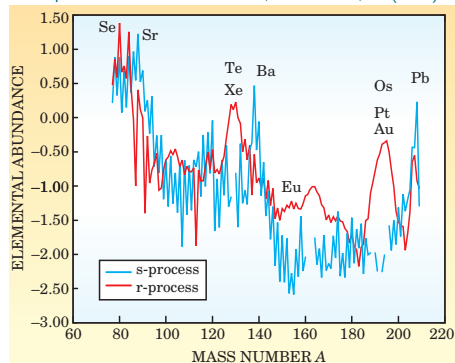


- Accumulation of material at **shell closures**  $\rightarrow$  **peak structure**.
- Roughly equal contributions by  $s$  and  $r$  process.

## Solar system abundances of heavy elements

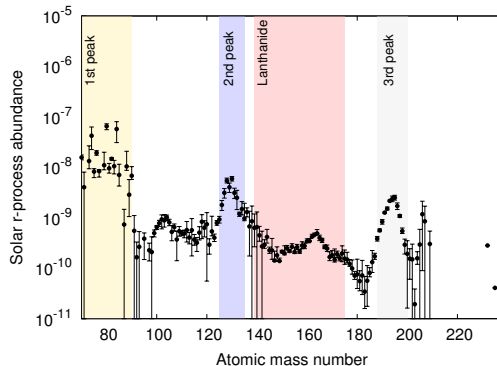


Adapted from Sneden and Cowan, Science 299, 70 (2003)



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## Modeling $r$ -process abundances



K. Hotokezaka *et al.*, *Int. J. Mod. Phys. D* 27, 1842005 (2018)

### Astrophysical site

Sets thermodynamic conditions

### Nuclear physics

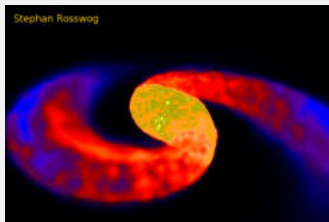
Shapes abundances distribution



## $r$ -process: astrophysical sites

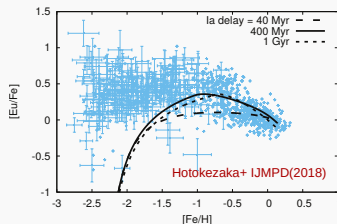
see poster of Ryota Hatami (#4) and Jan Kuske (#5)

### Neutron star mergers



- GW170817 and kilonova:  
lanthanides are produced!
- From simulations:
  - $\sim 10^{-2} M_{\odot}$  ejected mass.
  - Neutron rich material.
  - Synthesis of nuclei with  $A > 90$ .

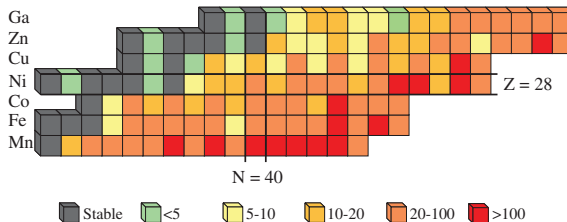
### Other sources



- Other scenarios:
  - CCSne (produce first peak).
  - Collapsars
  - Magnetorotational supernovae
  - ...
- Dominating source?

## $r$ -process: nuclear physics input

- $r$  process requires the knowledge of the properties of **neutron-rich nuclei**:
  - Nuclear masses,  $\beta$ -decay rates, neutron-capture rates, fission rates and yields, ...
- Rely on theoretical predictions: impact of uncertainties?

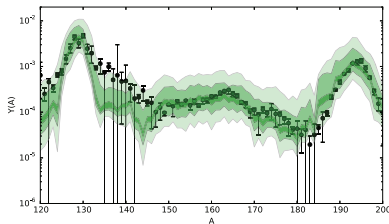


S. N. Liddick *et al.*, PRL **116**, 242502 (2016)

- Nuclei with **longest lifetimes** have largest impact.

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## Composition change

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- **Decays:**  $a \rightarrow b + c$

$$\frac{dn_a}{dt} = -\lambda_a n_a \quad \text{with} \quad \lambda_a = \ln(2)/t_{1/2}(a)$$

The decay rate  $\lambda_a$  can depend on temperature and density.

In order to distinguish between changes in the density (hydrodynamic) from changes in the composition (nuclear reactions), one introduces the **abundance**:

$$Y_i = \frac{n_i}{n} \quad \text{with} \quad n \approx \frac{\rho}{m_u} \equiv \text{number density of nucleons}$$

$$\frac{dY_a}{dt} = -\lambda_a Y_a$$

E.g.:  $\beta$  decay, spontaneous fission

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$$\frac{dn_a}{dt} = -n_a n_b \langle \sigma v \rangle_{ab}$$
$$\frac{dY_a}{dt} = -Y_a Y_b \frac{\rho}{m_u} \langle \sigma v \rangle_{ab} \equiv -\lambda_a Y_a$$

being  $\lambda_a = (Y_b \rho / m_u) \langle \sigma v \rangle_{ab}$  the destruction rate of target nuclei  $a$  by reaction with  $b$ .

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- **Photodissociations:**  $a + \gamma \rightarrow b + c$

$$\frac{dn_a}{dt} = -n_a n_\gamma \langle \sigma c \rangle_{a\gamma}$$

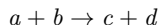
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being  $\lambda_a = n_\gamma \langle \sigma c \rangle_{a\gamma}$  the photodissociation rate of target nuclei  $a$ .



## Stellar reaction rates

General reaction between  $n_a$  and  $n_b$  particles per volume of targets  $a$  and projectiles  $b$ :



The number of reactions per unit of time, volume and pair of reactants:

$$r_{ab} = \frac{n_a(v_a)n_b(v_b)}{(1 + \delta_{ab})} \sigma(v)v, \quad \text{with} \quad v = |v_a - v_b|$$

Nuclei in stellar environments follow a Maxwell-Boltzmann thermal distribution:

$$P(v)dv = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{mv^2}{2kT} \right) dv$$

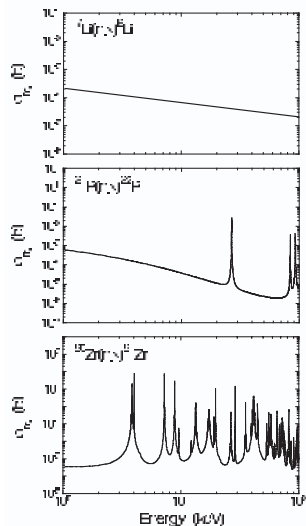
leading to the MB averaged cross section

$$\langle \sigma v \rangle = \int_0^\infty \sigma P(v) v dv = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp \left( -\frac{E}{k_B T} \right) dE \quad \text{with} \quad \mu = \frac{m_a m_b}{m_a + m_b}; \quad E = \frac{\mu v^2}{2}.$$

The reaction rate  $r_{ab} = n_a n_b \langle \sigma v \rangle$  depends critically on the cross section  $\sigma(E)$ .

# Cross sections

C. Iliadis, *Nuclear Physics of Stars*



- Neutron-induced reactions cross sections can be computed within the Hauser-Feshbach statistical theory: statistical average over many resonances.
- For a  $i(n, \gamma)m$  reaction proceeding from the target nucleus  $i$  in the state  $\mu(J_i^\mu, \pi_i^\mu)$  to a final nucleus  $m$  in a state  $\nu(J_m^\nu, \pi_m^\nu)$  through the compound state  $J^\pi$  with energy  $E$ :

$$\sigma_{n, \gamma}^{\mu \nu}(E_{i, n}) \propto \frac{T_n^\mu T_\gamma^\nu}{T_{\text{tot}}}$$

- Inverse  $c(\gamma, b)a$  rates can be computed from detailed balance:

$$\lambda_\gamma = \left( \frac{m_u kT}{2\pi \hbar^2} \right)^{3/2} \frac{G_a G_b}{G_c} \left( \frac{A_a A_b}{A_c} \right)^{3/2} \exp(-Q/kT) \frac{\langle \sigma v \rangle_{ab}}{1 + \delta_{ab}}$$

being  $G(T) = \sum_i (2J_i + 1) e^{-E_i/(kT)}$ .

## $\beta$ decays

Beta-decay rates can be computed from Fermi's Golden rule:

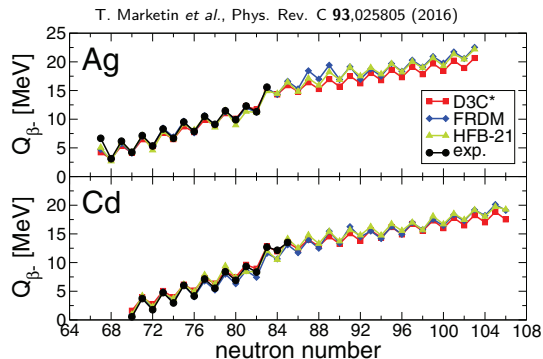
$$\lambda_{\beta} = \frac{\ln(2)}{t_{1/2}} = \frac{g^2 m_e^5 c^4 |M_{fi}|^2}{f(Q_{\beta}) 2\pi^3 \hbar^7}$$

being  $|M_{fi}|$  the (GT and F) nuclear matrix elements, and  $f(Q_{\beta})$  the Fermi integral.

Two main ingredients:

- $Q_{\beta}$ -values: large for neutron-rich nuclei.
- $\beta$ -decay strength: determines the possible emission of delayed neutrons.

For heavy nuclei, forbidden transition ( $l_{e\nu} > 0$ ) become relevant for the estimation of the rates.



## Network equations

The time derivative of the abundances is modeled through a set of differential equations (reaction network equations):

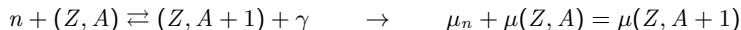
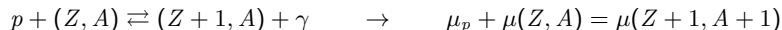
$$\frac{dY_i}{dt} = \sum_j P_j^i \lambda_j Y_j + \sum_{jk} P_{jk}^i \frac{\rho}{m_u} \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{jkl} P_{jkl}^i \left( \frac{\rho}{m_u} \right)^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l$$

$P^i$  describe how often the nucleus  $i$  is created or destroyed, and whether two or more identical nuclei are involved in the reactions.

- Stiff system: timescales change by several orders of magnitude.
- Decoupled from hydrodynamic.
- Tremendous simplifications at high  $T$  and  $\rho$ .

## Nuclear statistical equilibrium (NSE)

- At high  $T$  and  $\rho$ , fusion reactions and photodissociation rates are large, leading to chemical equilibria



- Nuclear statistical equilibrium (NSE): chemical equilibria across the whole nuclear chart

$$Z\mu_p + N\mu_n = \mu(Z, A)$$

- At NSE, nuclear abundances depend on nuclear properties (binding energy  $B_i$ ) and environment conditions ( $T$ ,  $\rho$ , and  $Y_e = \sum_i Z_i Y_i$ ), but not the individual rates!

$$Y_i(Z_i, N_i) = Y_n^{N_i} Y_p^{Z_i} \frac{G_i(T) A_i^{3/2}}{2_i^A} \underbrace{\left( \frac{\rho}{m_u} \right)^{(A_i-1)}}_{\substack{\text{high } \rho \\ \text{favor heavy nuclei}}} \overbrace{\left( \frac{2\pi\hbar^2}{m_u kT} \right)^{3(A_i-1)/2}}^{\substack{\text{high } T \\ \text{favor light nuclei}}} \underbrace{\exp\left(\frac{B_i}{kT}\right)}_{\substack{\text{intermediate:} \\ \text{tightly bound nuclei}}}$$

- Weak reactions?

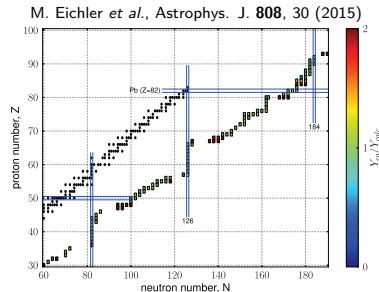
## Quasi statistical equilibrium (QSE)

For astrophysical conditions with large  $n_n$  and  $T \sim 1$  GK:

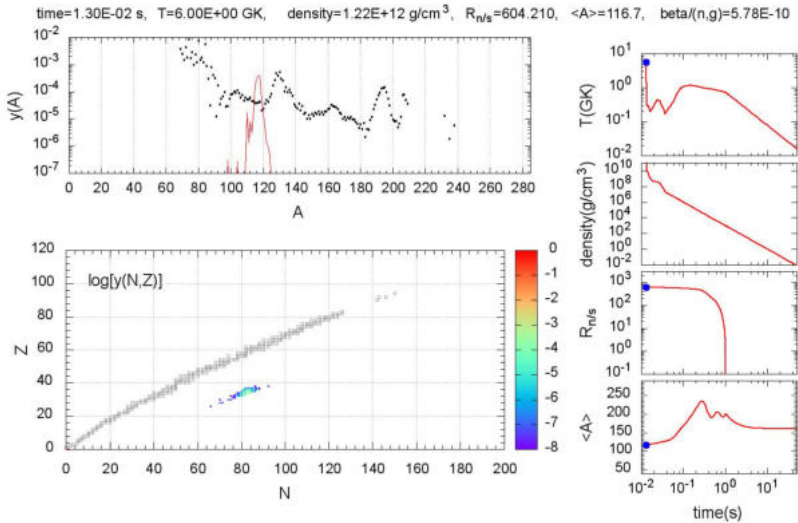
- Neutron-rich nuclei ( $S_n \lesssim 2$  MeV) can be produced.
- Charge-particle reactions are frozen: isotopic chains are only connected through  $\beta$  decays.
- But chemical equilibrium between  $(n, \gamma)$  and  $(\gamma, n)$  reactions still holds.
- This produces a quasiequilibrium clusters along isotopic chains:  $(\gamma, n) \rightleftharpoons (n, \gamma)$  equilibrium. Abundance ratios in each isotopic chain independent of reaction rates:

$$\frac{Y(Z, A+1)}{Y(Z, A)} = n_n \frac{G(Z, A+1)}{2G(Z, A)} \left( \frac{A+1}{A} \right)^{3/2} \left( \frac{2\pi\hbar^2}{m_u kT} \right)^{3/2} \exp \left( \frac{S_n(A+1)}{kT} \right)$$

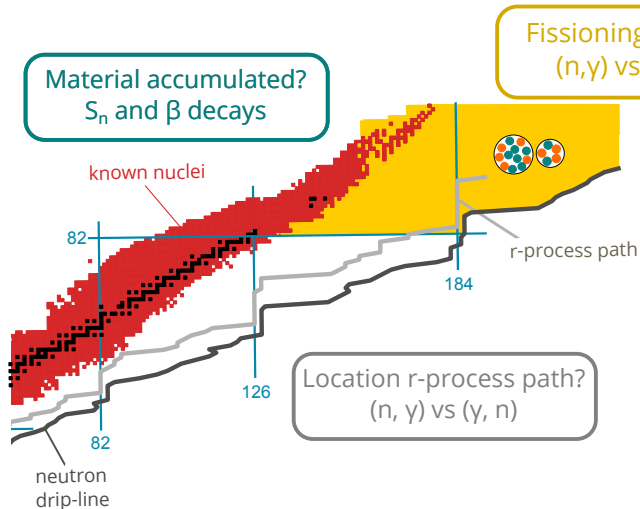
- For a given  $n_n$  and  $T$ , abundance maximum in all isotopic chains have the same  $S_n$  ( $\sim 2 - 3$  MeV).



## $r$ -process nucleosynthesis in NSM



## Nuclear inputs



The  $r$ -process requires the knowledge of nuclear properties of **neutron-rich nuclei**:

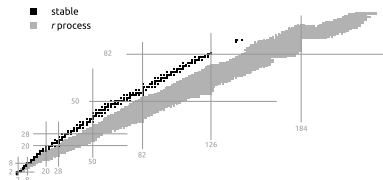
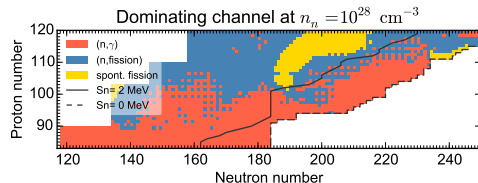
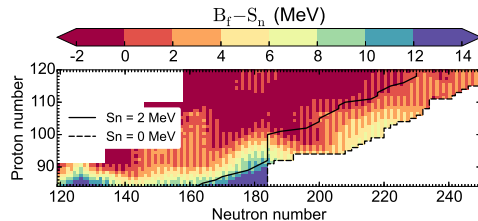
- nuclear masses;
- $\beta$ -decay rates;
- neutron capture rates;
- fission rates and yields.



1) Compute **nuclear properties** using EDF.

2) Calculate **stellar reaction rates** from Hauser-Feshbach theory.

3) Obtain  **$r$ -process abundances and light curve** using nuclear network calculations.



## The Hartree-Fock-Bogolyubov (HFB) formalism

The ground-state wavefunction is obtained by minimizing the total energy:

$$\delta E[|\Psi\rangle] = 0,$$

where  $|\Psi\rangle$  is a quasiparticle ( $\beta$ ) vacuum:

$$|\Psi\rangle = \prod_{\mu} \beta_{\mu} |0\rangle \quad \Rightarrow \quad \beta_{\mu} |\Psi\rangle = 0.$$

The energy density functionals (EDF) provide a phenomenological ansatz of the effective nucleon-nucleon interaction:

- Gogny, Skyrme, relativistic EDF, BCPM, ...

Sophisticated many-body methods can be built on top: QRPA ( $\beta$ -decays and excited states), PGCM (excited states), TDDFT and TDGCM (fission)...

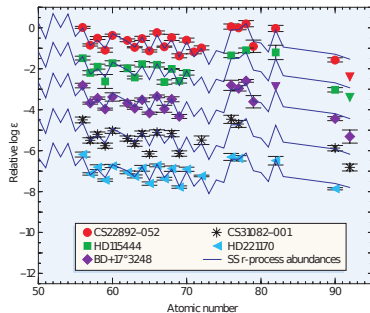
# Fission and $r$ process

Thielemann+(1983), Panov+(2005), Martinez-Pinedo+(2007), Beun+(2008), Petermann+(2012),

Eichler+(2015), Goriely(2015), Mumpower+(2018), Giuliani+(2018,2020), Zhu+(2019), Wu+(2019), Vassh+(2019,2020), ...

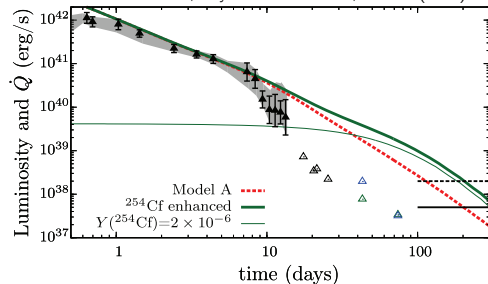
## Abundances

J. J. Cowan and C. Sneden, Nature 440 1151 (2006)



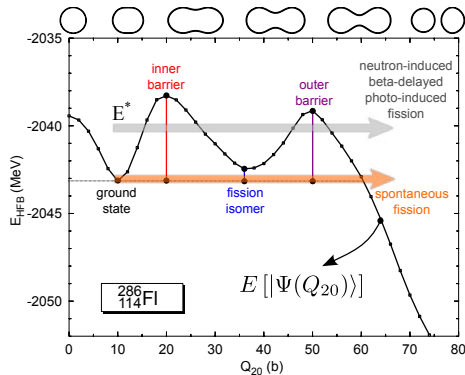
## Kilonova

M.-R. Wu et al., Phys. Rev. Lett. 122, 062701 (2019)



Fission shapes abundances and kilonova light curve, providing a mechanism for a **robust  $r$ -process nucleosynthesis**.

# The fission process



## Potential Energy Surface $\mathcal{V}$

Energy evolution from the initial state to the scission point.

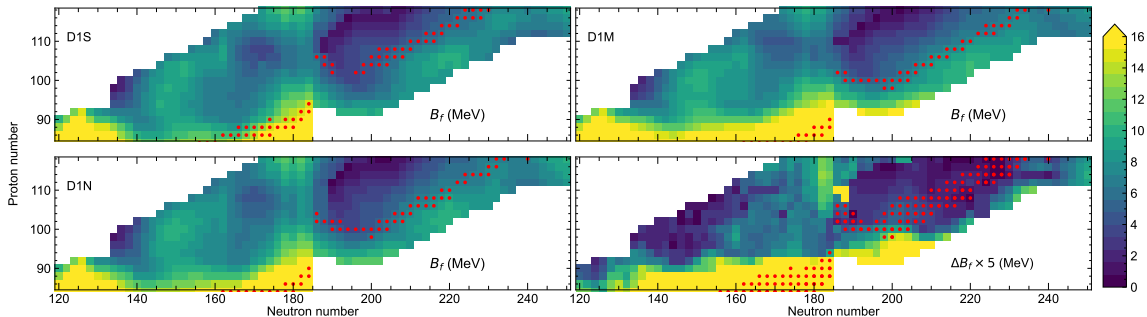
## Collective inertias $\mathcal{M}$

Resistance of the nucleus against the deformation forces.

$$P \propto 1/\exp(2S[L]) \rightarrow S[L(Q_{20})] = \int_a^b dQ_{20} \sqrt{\mathcal{M}(Q_{20})[\mathcal{V}(Q_{20}) - E_0]}$$

## Systematic of fission barriers $B_f$

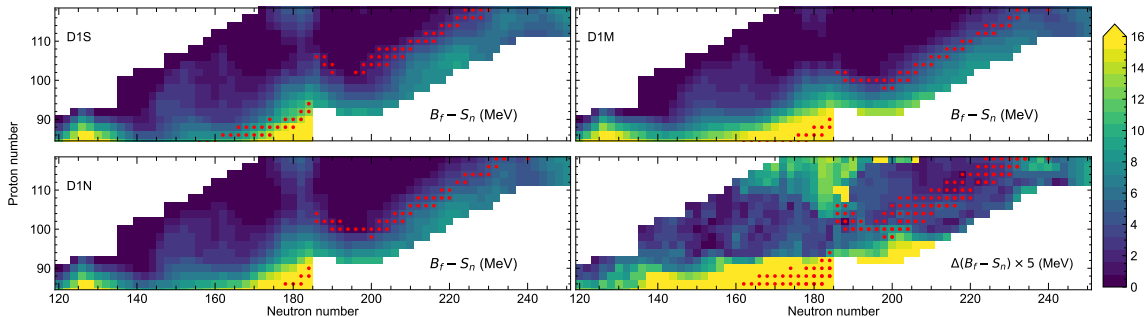
- $B_f \rightarrow$  stability against fission.



- $B_f$  trends qualitatively similar across different Gogny interactions:
  - $Z > 94$ : systematic deviations  $\sim 1$ – $2$  MeV.
  - $Z \leq 94$ : deviations up to 10 MeV (but very high  $B_f$ ).
- Location of the  $r$  process differ above  $N = 184$ .

## Systematic of $B_f - S_n$

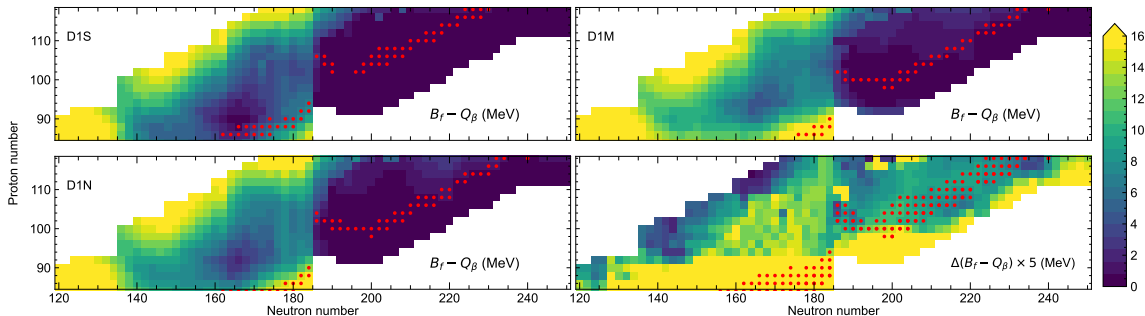
- For  $B_f - S_n \lesssim 2 \text{ MeV}$  ( $n, \text{fis}$ ) dominates over ( $n, \gamma$ ).



- Production of (super)heavy nuclei requires the **overcoming of neutron shell closure** at  $N = 184$ .
- $r$ -process path pushed into a region of low  $B_f - S_n$ .

## Systematic of $B_f - Q_\beta$

- $B_f - Q_\beta \rightarrow$  competition between fission and  $\beta$ -decay.

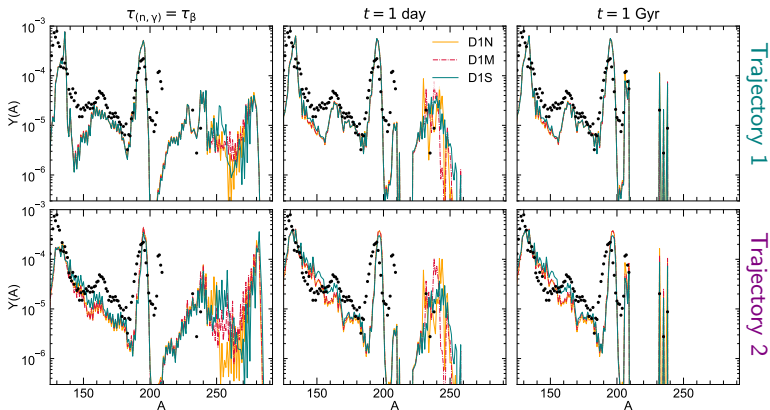


- Path towards stability interrupted by region of [low fission barriers](#).

## Impact of fission on the $r$ -process: abundances $Y$

Dynamical ejecta from neutron star merger (C. E. Collins *et al.*, MNRAS 101093 (2023)):

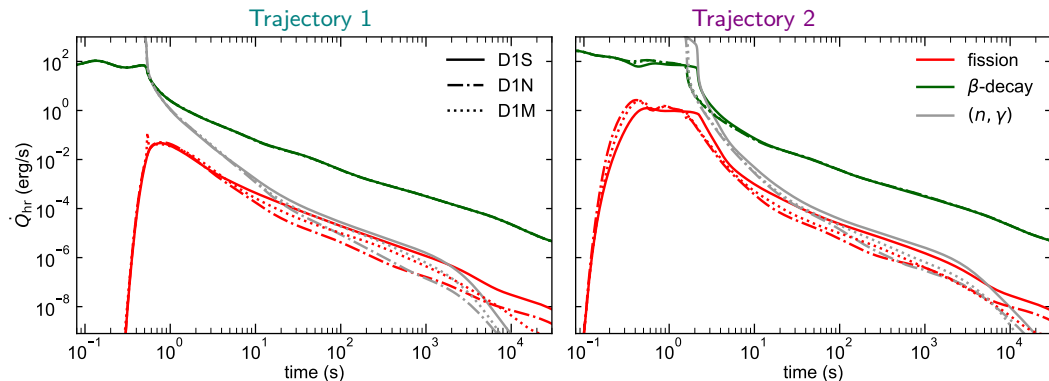
- **Trajectory 1:**  $Y_e = 0.151$ ;  $n/s = 105$ ;
- **Trajectory 2:**  $Y_e = 0.027$ ;  $n/s = 1100$ .





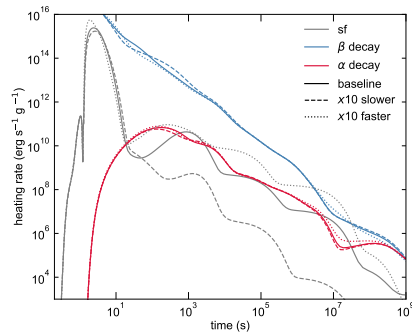
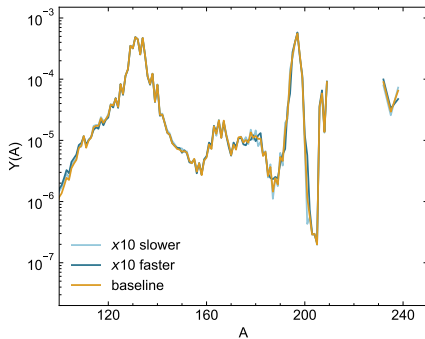
## Impact of fission on the $r$ -process: heating rates

- **Trajectory 1:**  $Y_e \lesssim 0.05$ ;  $n/s \approx 600$
- **Trajectory 2:**  $Y_e \lesssim 0.15$ ;  $n/s \approx 120$



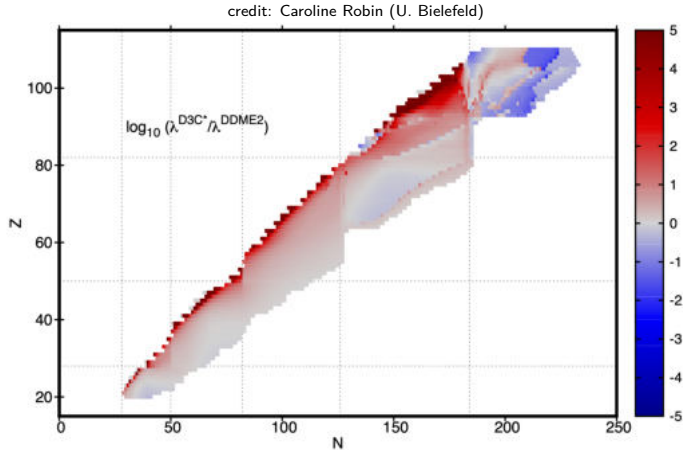
## Impact of $\beta$ -decay rates on fission

- Impact of  $\beta$ -decay half-lives varies with the observable.
- We modified  $t_{1/2}^{\beta}$  (FRDM)  $\geq 3$  s and study the impact on abundances and heating rates.



Fission heating rate sensitive to “slow”  $\beta$ -decay rates  
( $\tau_{\beta} \gtrsim$  few seconds)

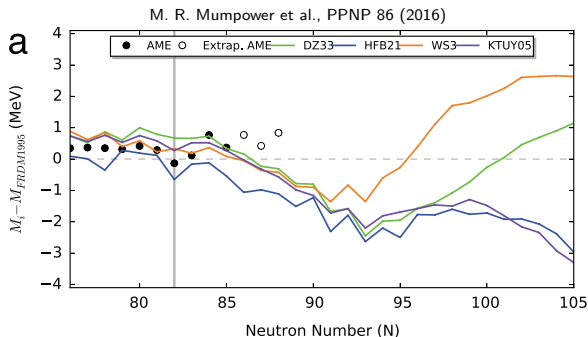
## Systematic of $\beta$ -decay rates



- $\beta$ -decay rates closer to stability show larger uncertainties  $\rightarrow$  more systematic studies are required (see also E. M. Ney *et al.*, Phys. Rev. C **102**, 034326 (2020)).

## Nuclear masses - Global and local changes

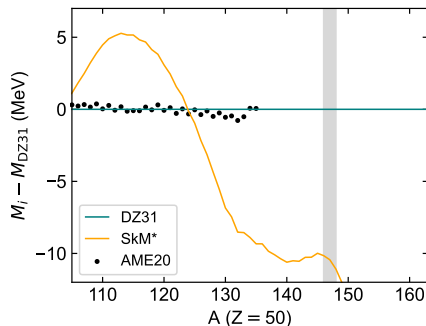
Nuclear masses determine thresholds energy for  $n$  captures,  $\beta$  decays and fission, and the location of  $r$ -process path in  $(n, \gamma) \rightleftharpoons (\gamma, n)$  equilibrium.



Very different predictions far from stability: are these **differences relevant** for the  $r$ -process abundances?

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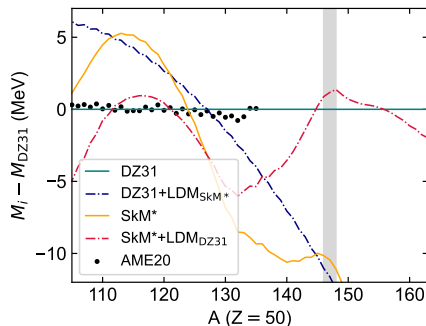


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Masses = homogeneous part (global, LDM) + quantum shell-correction (local)

## Nuclear masses - Global and local changes

Nuclear masses determine thresholds energy for  $n$  captures,  $\beta$  decays and fission, and the location of  $r$ -process path in  $(n, \gamma) \rightleftharpoons (\gamma, n)$  equilibrium.

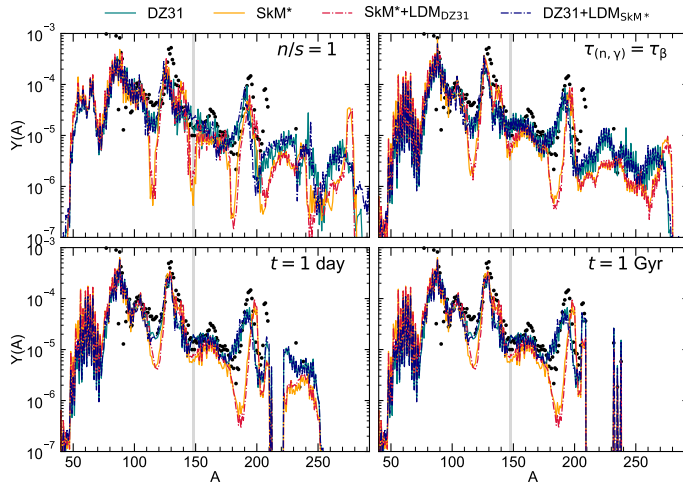


Very different predictions far from stability: are these **differences relevant** for the  $r$ -process abundances?

Masses = homogeneous part (global, LDM) + quantum shell-correction (local)

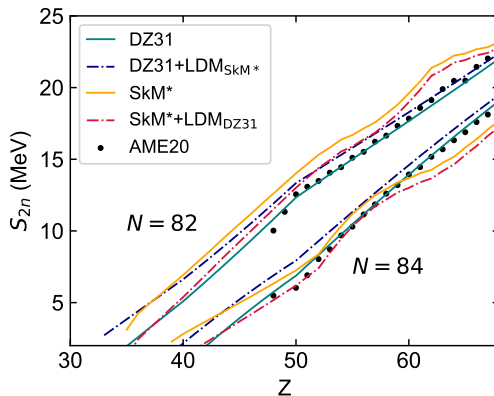
# Symmetry energy and $r$ process

NSM trajectories from Collins *et al.*, MNRAS 101093 (2023)



Abundances **insensitive to global changes in masses** (e.g., symmetry energy).

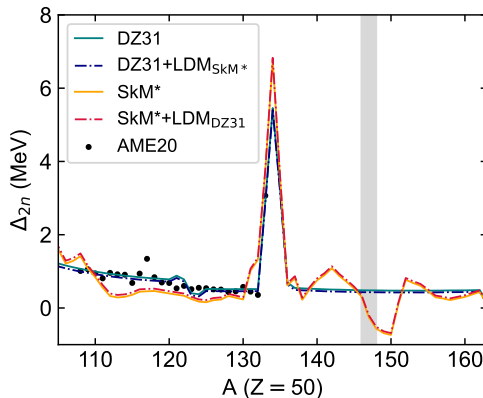
## Nuclear masses - Global and local changes



Abundance mostly related to **local changes** on  $S_{2n}$  (rather than bulk properties of masses).



## Nuclear masses - Global and local changes



Abundance mostly related to **local changes** on  $S_{2n}$  (rather than bulk properties of masses).

## Collaborators

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## Funding



## Postdoc position

Postdoc position in Nuclear Physics Group at Universidad Autónoma de Madrid.

Starting date: mid-end 2024

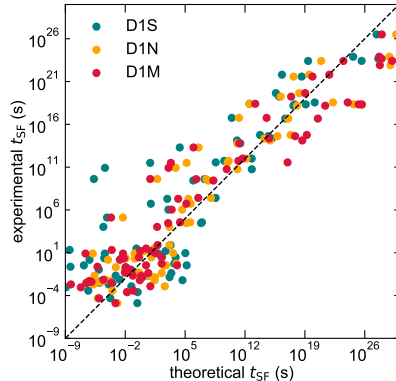
Contact: [samuel.giuliani@uam.es](mailto:samuel.giuliani@uam.es)

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Backup

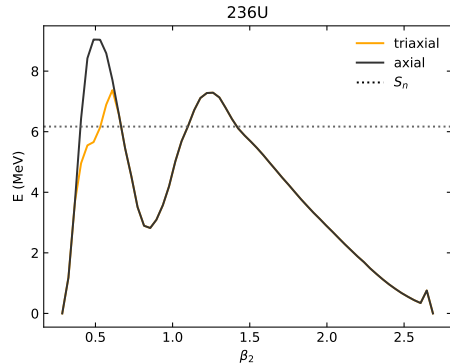
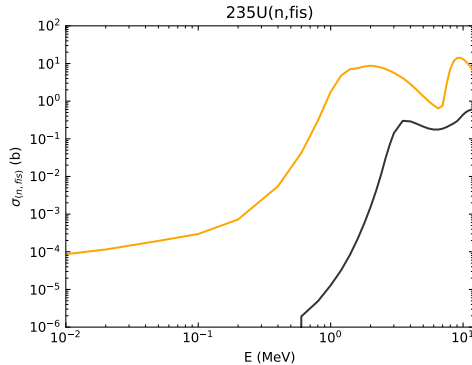
## $t_{1/2}^{\text{sf}}$ : theory vs experiment

- D1S, D1N and D1M  $t_{1/2}^{\text{sf}}$  are systematically **overpredicted**:
  - inclusion of pairing as dof (minimum action path vs minimum energy path), collective inertias, triaxiality...
- Collective inertias are renormalized in order to reproduce experimental  $t_{1/2}^{\text{sf}}$ .



## Global calculation of fission nuclear properties

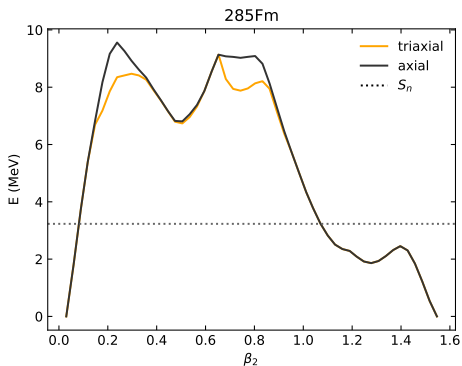
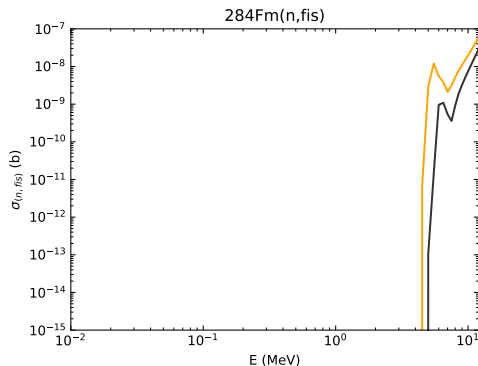
- Model assumptions  $\rightarrow$  systematic uncertainties  $\rightarrow$  impact on nucleosynthesis?
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- Systematic deviations close to stability could be compensated in exotic nuclei (and vice versa)  $\rightarrow$  global calculations from different models are required.

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