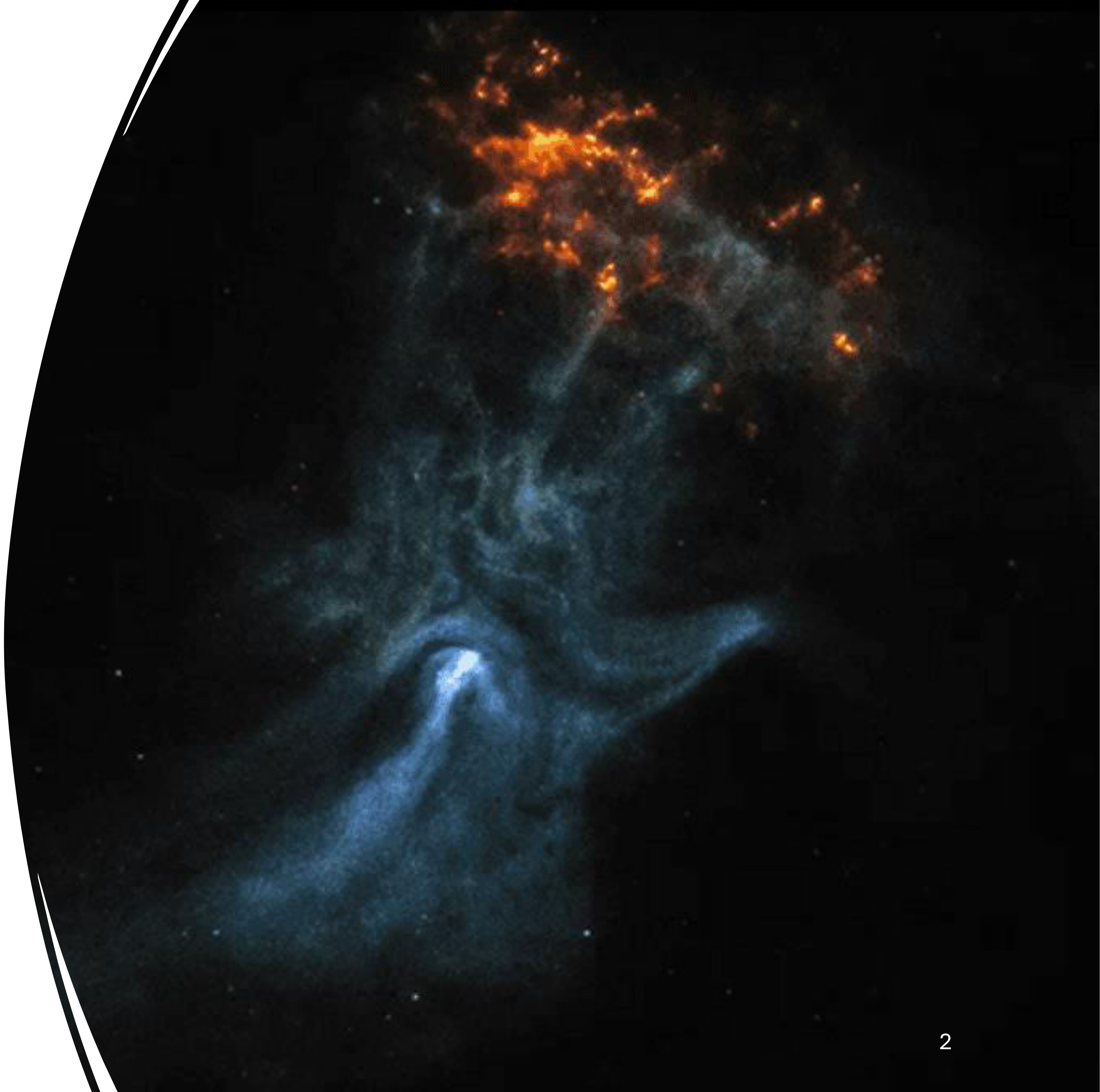
# Fast Emulation of Neutron Star Properties

Brendan Reed

# Overview

- What are emulators?
- Why do we want to emulate?
- Application to Neutron Stars



# What are emulators?

- Ansatz: Video game emulation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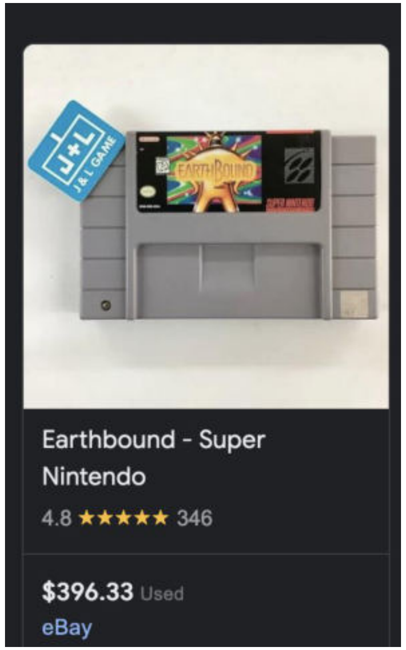


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# Alternative

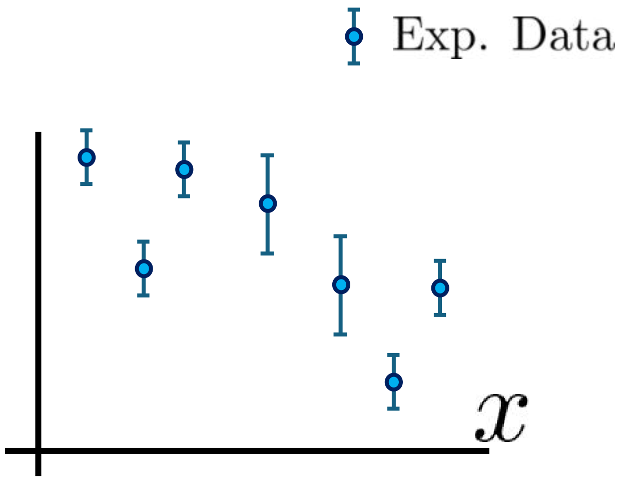
- Use emulators to play the game instead

# What Happened

- Took the true thing (SNES)
- Created a program (or model) which needs input (game file)
- Provided input then outputs (emulates) playing the real game

- Provides much easier method of playing than using the real thing
- With less input, can still get desired result

# Back to Physics



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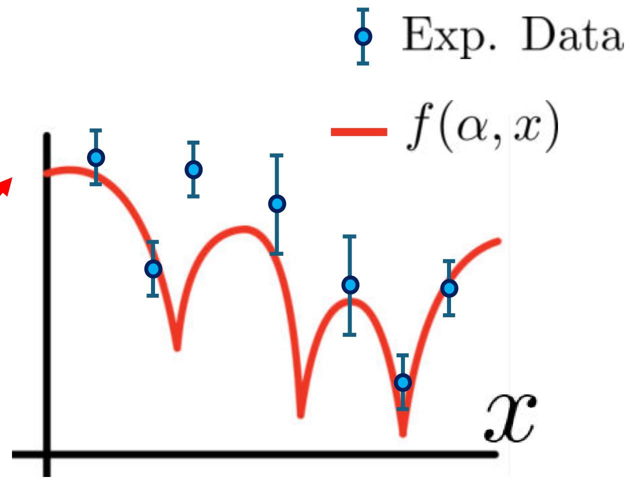
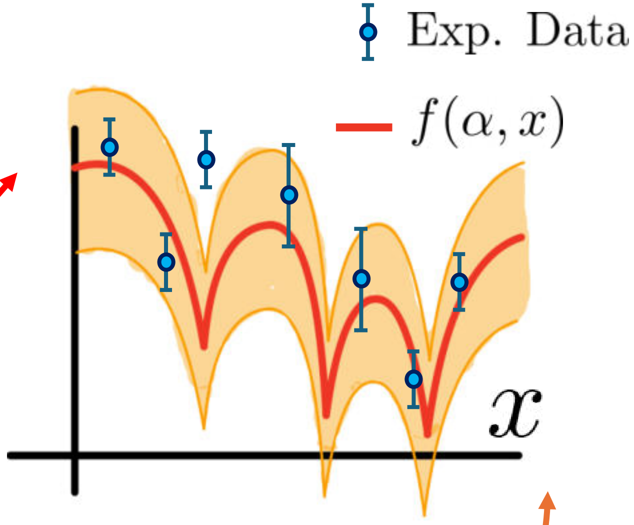



Parameters

Physical model $f(\alpha, x)$

# Back to Physics




Parameters

Physical model $f(\alpha, x)$

Posterior distribution $P(\alpha|\mathbf{Y}) = \dfrac{P(\mathbf{Y}|\alpha)P(\alpha)}{P(\mathbf{Y})}$

# Back to Physics




**Posterior distribution**

(what we want)

$$\boxed{P(\alpha|\mathbf{Y})} = \frac{\boxed{P(\mathbf{Y}|\alpha)}\,\boxed{P(\alpha)}}{\boxed{P(\mathbf{Y})}}$$

**Likelihood**

(what we think happened)

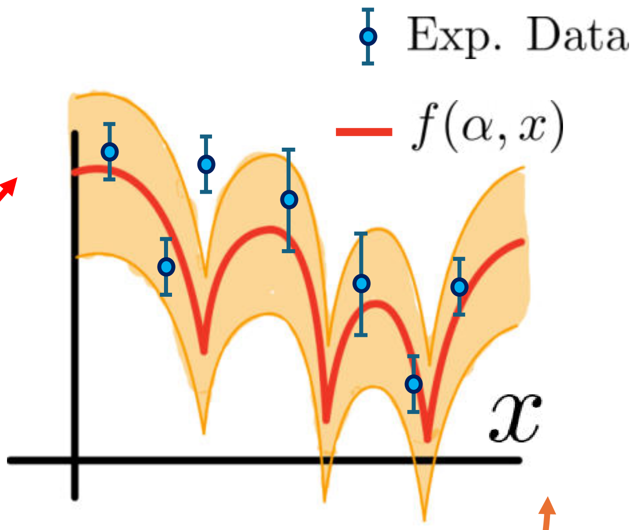
**Prior**

(what we think we know)

**Evidence**

(how much sense all makes)

# Back to Physics




Parameters

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# Back to Physics

“High Fidelity”

→

“Emulated”


Physical model $f(\alpha, x) \longrightarrow \hat{f}(\alpha, x)$

Posterior distribution $P(\alpha|\boldsymbol{Y}) \longrightarrow \hat{P}(\alpha|\boldsymbol{Y})$

# Back to Physics

“High Fidelity” → “Emulated”


Physical model: $f(\alpha, x) \rightarrow \hat{f}(\alpha, x)$

Posterior distribution: $P(\alpha|\mathbf{Y}) \rightarrow \hat{P}(\alpha|\mathbf{Y})$

Model Order Reduction:

- Reduced basis methods (~eigenvector continuation)
- Dynamic Mode Decomposition
- SINDy and NIF
- ...

Gaussian Process

Neural Networks

...

Normalizing flows

Chaos expansion

Variational Bayes

...

# Back to Physics

"High Fidelity"

"Emulated"


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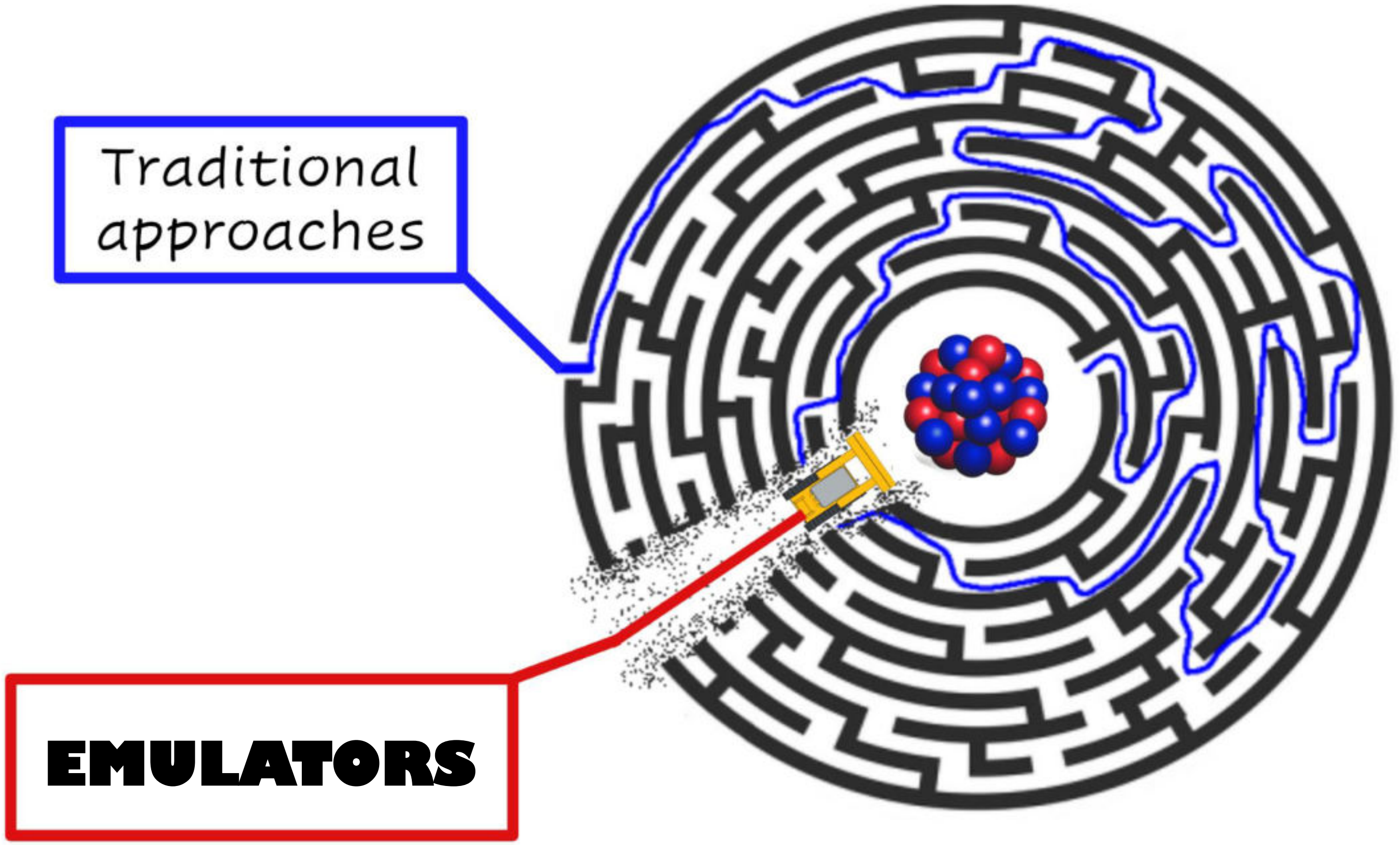
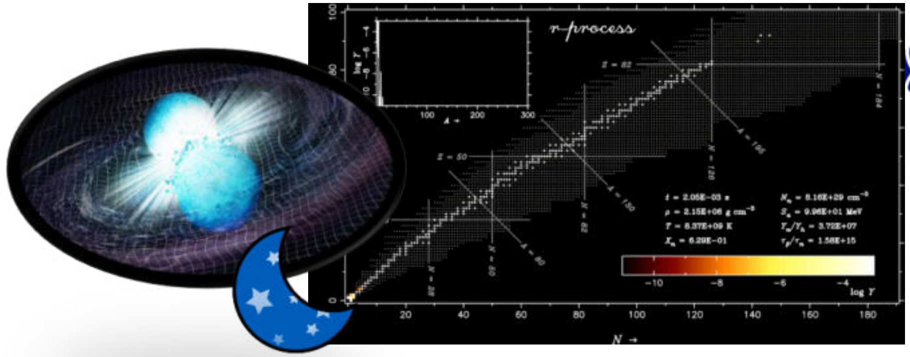
These methods can be data hungry

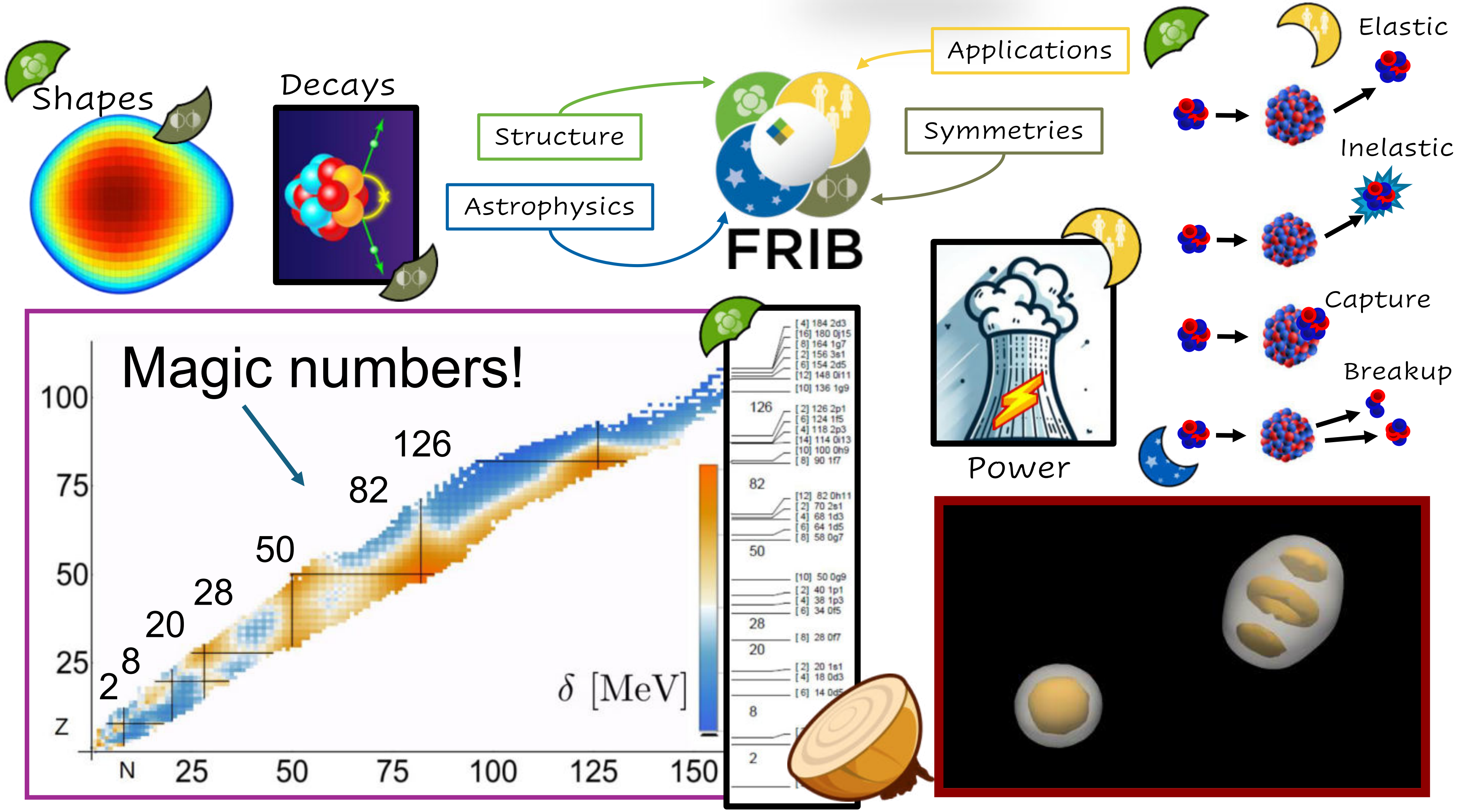
Opportunity for high performance computing to shine

Why do we want to emulate things?



# Big questions



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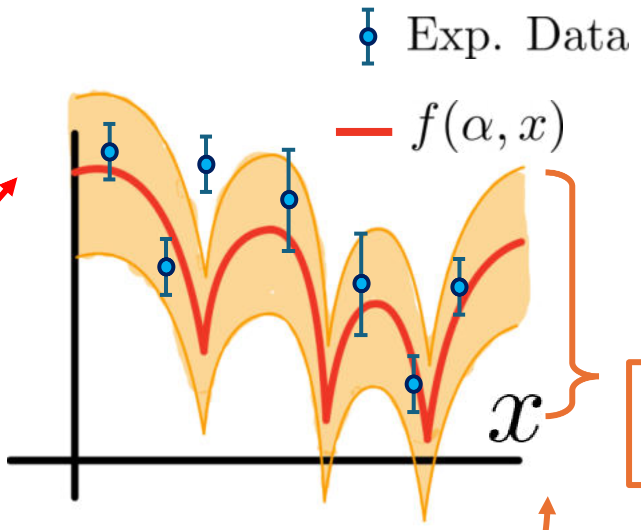
# Dimensionality Reduction




5,000,000 samples

Parameters

Physical model $f(\alpha, x)$

Posterior distribution $$P(\alpha|\mathbf{Y}) = \frac{P(\mathbf{Y}|\alpha)P(\alpha)}{P(\mathbf{Y})}$$

$f(\alpha, x)$

1 second

2 months

1 hour

1/2 century

# One Application: Neutron Stars

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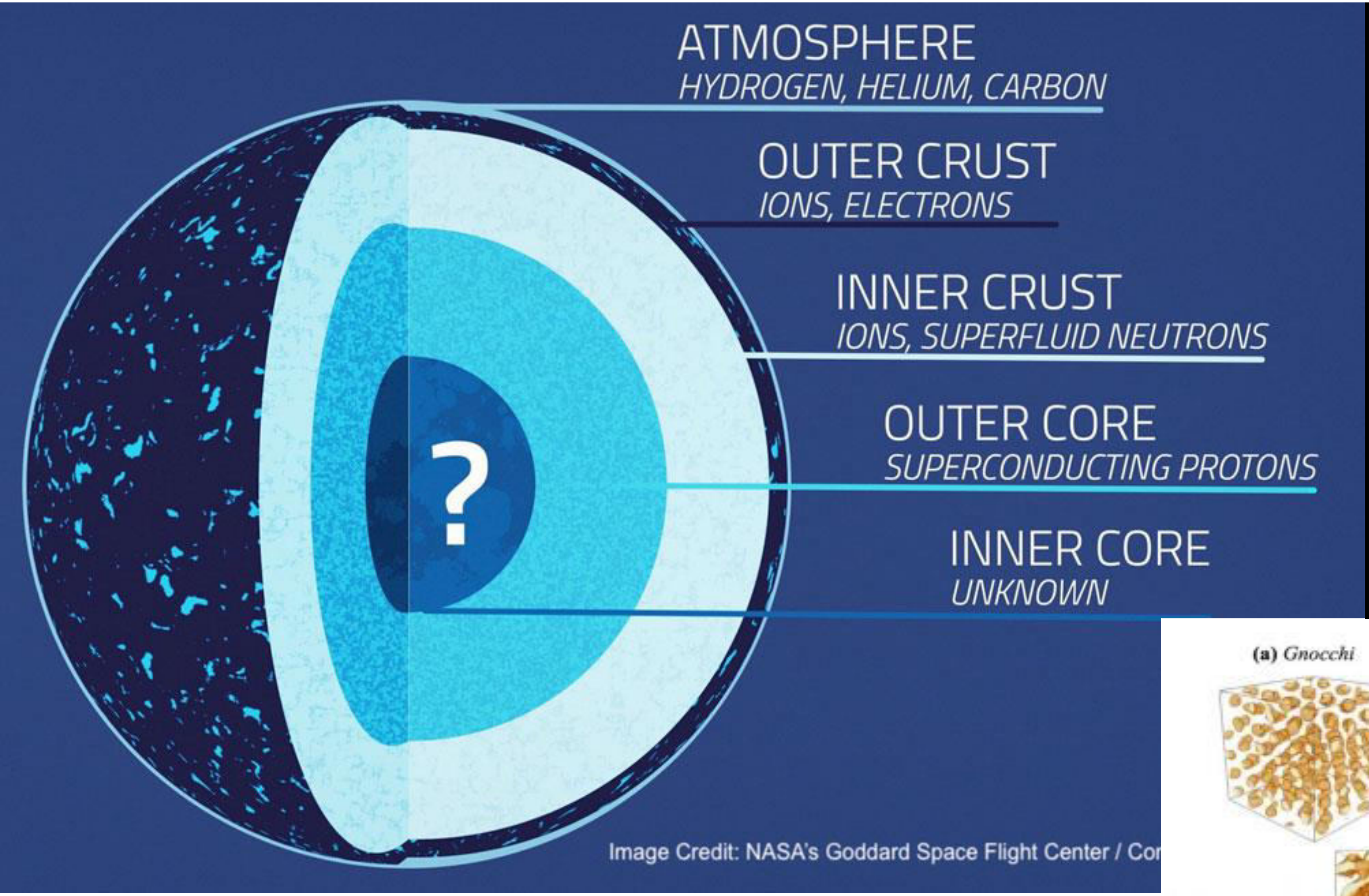
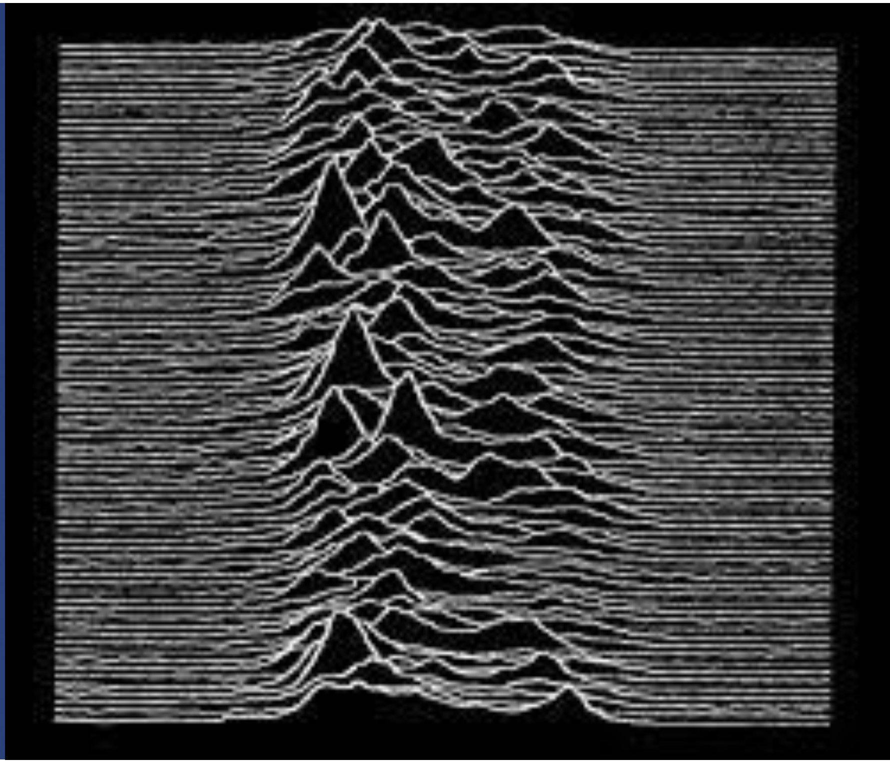
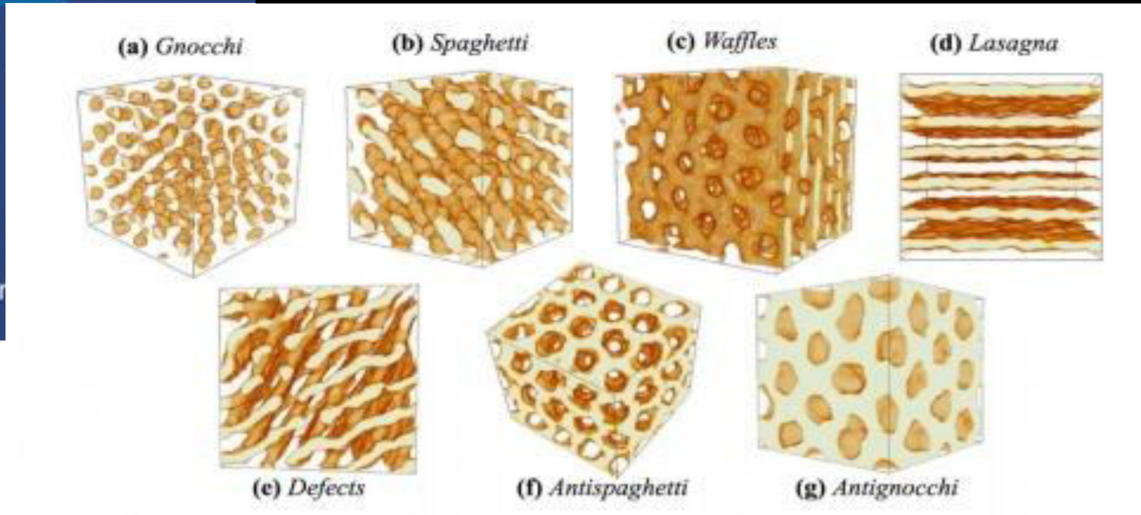
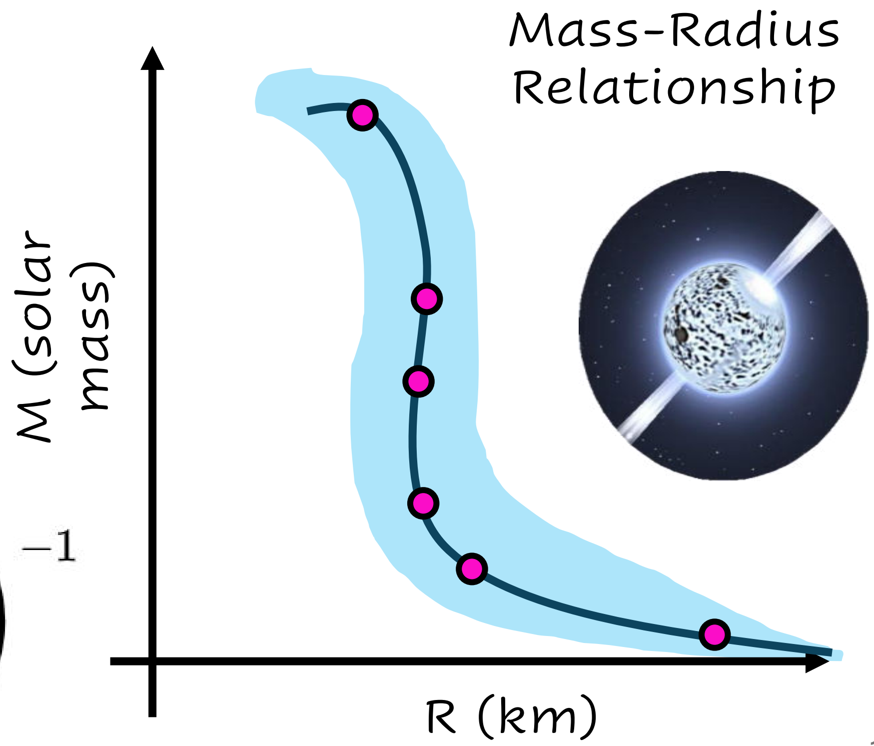




# Math

- Structure is governed by GR
  - Tolman-Oppenheimer-Volkoff equations
- Provide an equation of state → solve on a grid




$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

$$\frac{dM}{dr} = 4\pi\rho r^2$$

- Define some step size and central pressure (density)
- Integrate outward until P=0

# More Math

- Often also want to solve for Tidal Deformability
  - Constrained from GWs
  - Describes quadrupole gravitational radiation
- Important for nuclear structure due t
  large R dependence

$$\Lambda = \frac{2}{3} k_2 \left(\frac{Rc^2}{GM}\right)^5$$

$$\begin{aligned}
k_2 = &\frac{1}{20}\xi^5(1-\xi)^2\Big[(2-y_R)+(y_R-1)\xi\Big] \\
&\times\Big\{\Big[(6-3y_R)+\frac{3}{2}(5y_R-8)\xi\Big]\xi \\
&+\frac{1}{2}\Big[(13-11y_R)+\frac{1}{2}(3y_R-2)\xi+\frac{1}{2}(1+y_R)\xi^2\Big]\xi^2 \\
&+3\Big[(2-y_R)+(y_R-1)\xi\Big](1-\xi)^2\ln(1-\xi)\Big\}^{-1}
\end{aligned}$$

$$r\frac{dy}{dr} + y^2 + F(r)y + r^2 Q(r) = 0$$

$$F(r) = \frac{r - 4\pi G r^3(\rho(r) - p(r))}{r - 2GM(r)}$$

$$\begin{aligned}
Q(r) = &\frac{4\pi r}{r-2GM(r)}\left[G\left(5\rho(r)+9p(r)+\frac{\rho(r)+p(r)}{c_s^2(r)}\right)\right. \\
&\left.-\frac{6}{4\pi r^2}\right] - 4\left[\frac{G(M(r)+4\pi r^3 p(r))}{r(r-2GM(r))}\right]^2
\end{aligned}$$

# How it works

*Reducing your model in two easy steps:*

1) Find good reduced coordinates

2) Find equations for them

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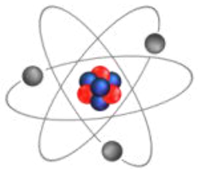
 $\sim 10^{24}$ 

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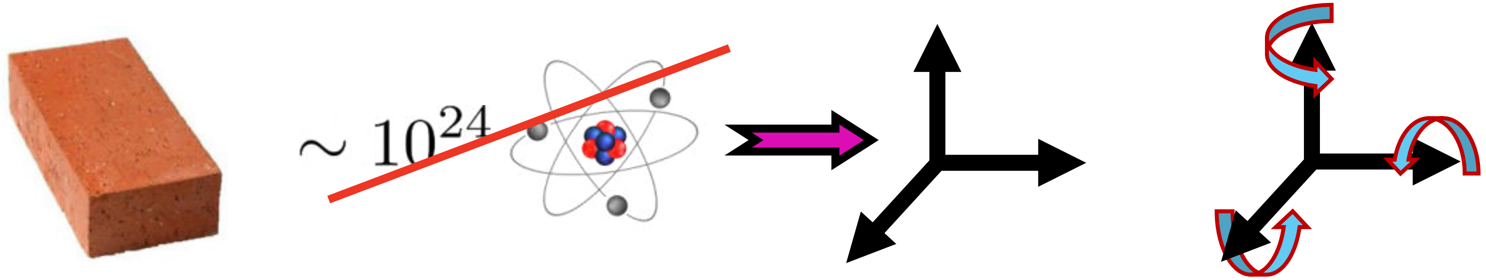


3 translations + 3 rotations

2) Find equations for them

# How it works

*Reducing your model in two easy steps:*

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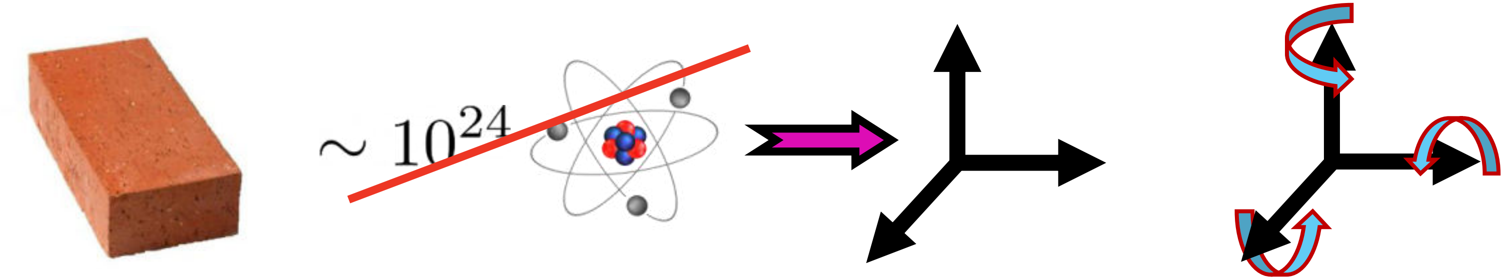



3 translations + 3 rotations

2) Find equations for them

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot{q}_k}\right) - \frac{d\mathcal{L}}{dq_k} = 0$$

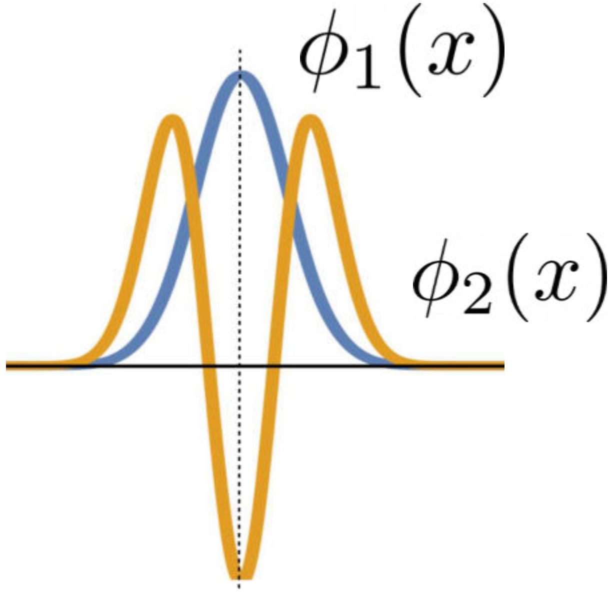
# How it works

*Reducing your model in two easy steps:*

1) Find good reduced coordinates

$$\hat{\phi}(x) = \phi_0 + \sum_{k}^{n} a_k \phi_k(x)$$

Principal Component Analysis




2) Find equations for them

- Neural Network mapping
- Gaussian Processes
- Galerkin Projection
- Black Box Techniques (ask me about this)

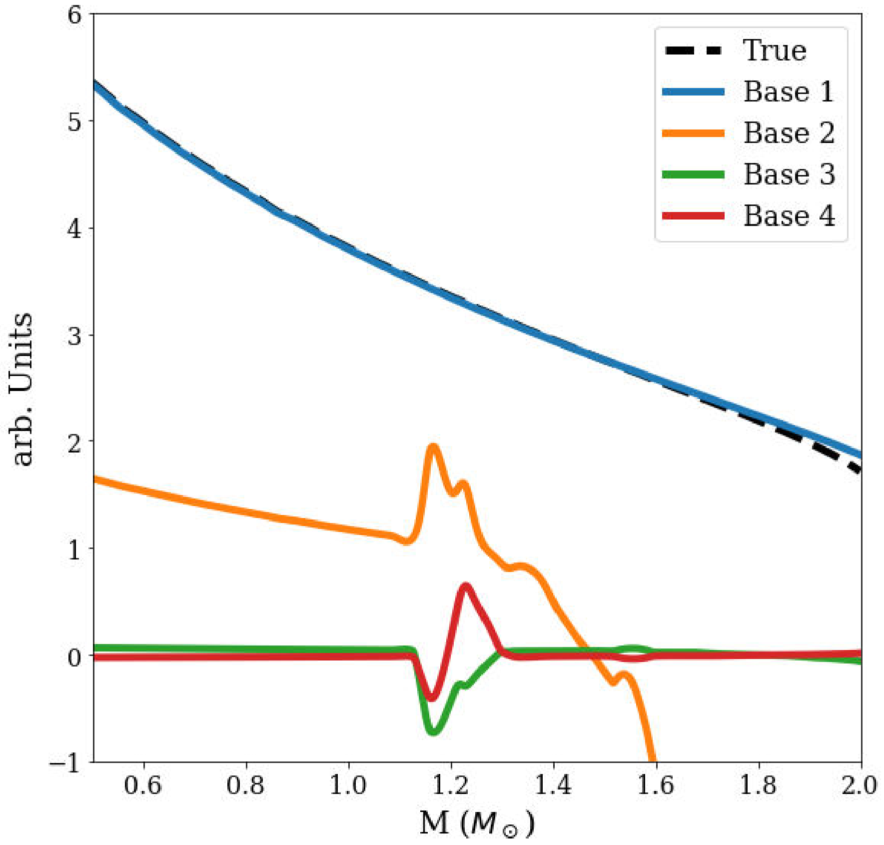
# How it works

Reducing your model in two easy steps:

1) Find good reduced coordinates

$$\hat{\phi}(x) = \phi_0 + \sum_k^n a_k \phi_k(x)$$

Principal Component Analysis



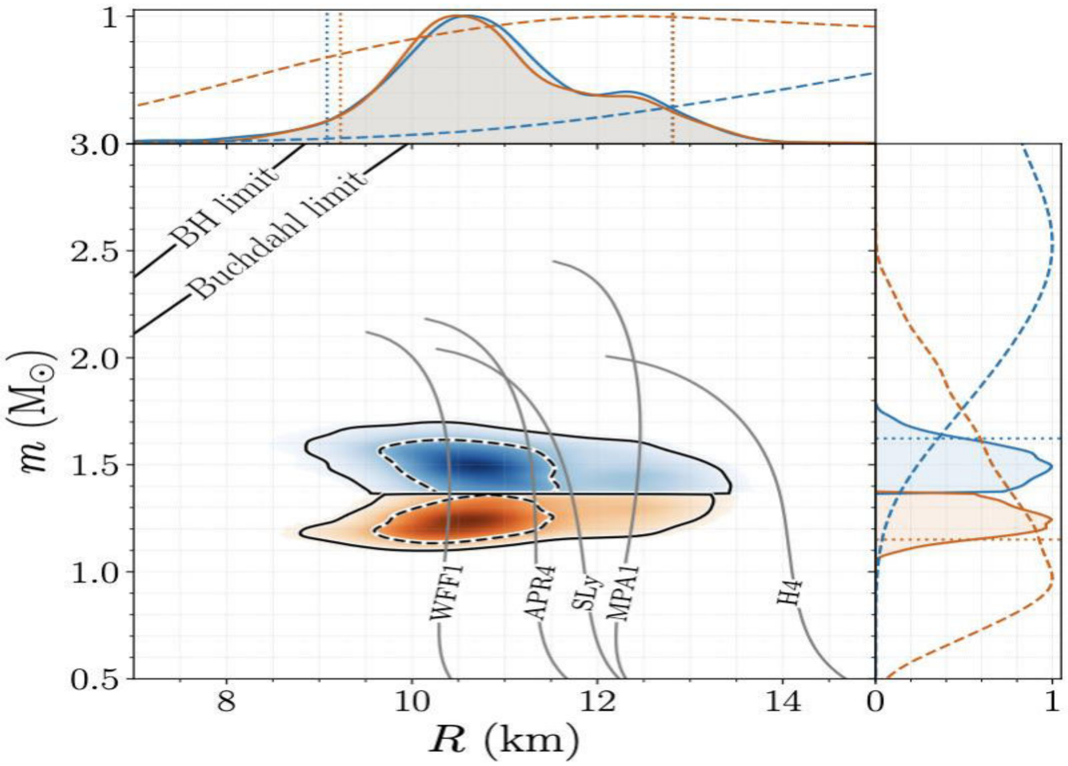

2) Find equations for them

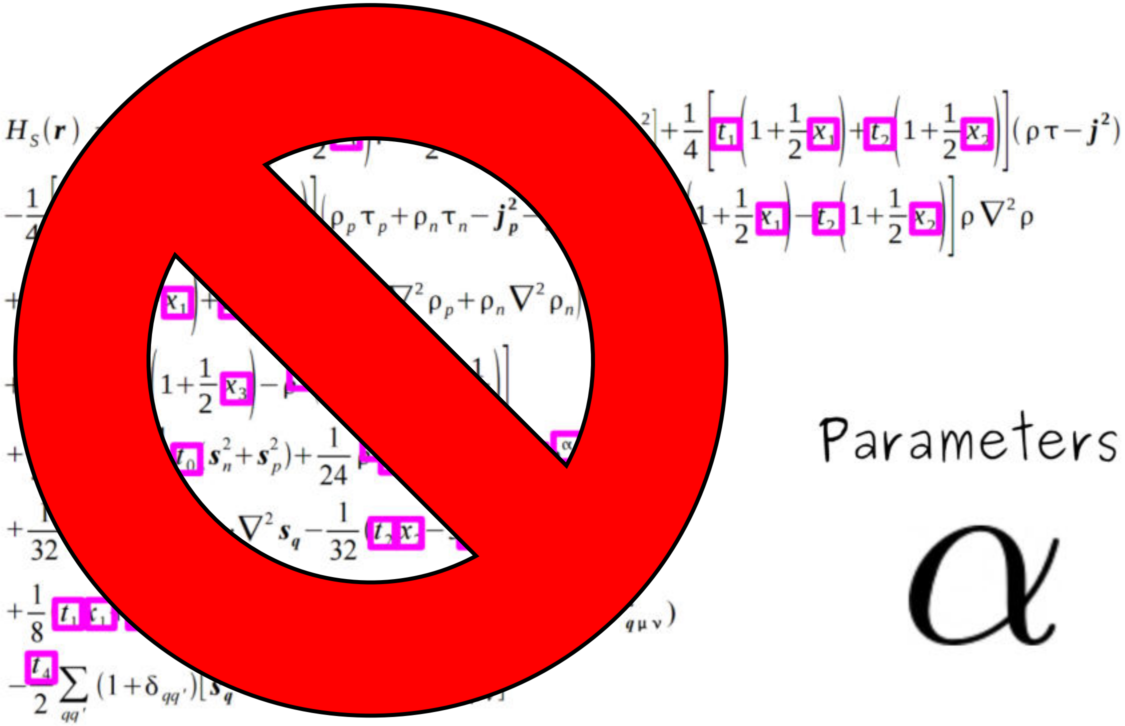
$$\hat{H}(\alpha)\,\hat{\phi}(x) = \lambda\hat{\phi}(x)$$

$$\vec{a} = \vec{\lambda}(\alpha)\lambda$$

# Finding Alphas

- Consider your data
  - What are the inputs?
  - Is there any preprocessing that can be done?
- How complex is the system?
  - Are the models stochastic or smooth?



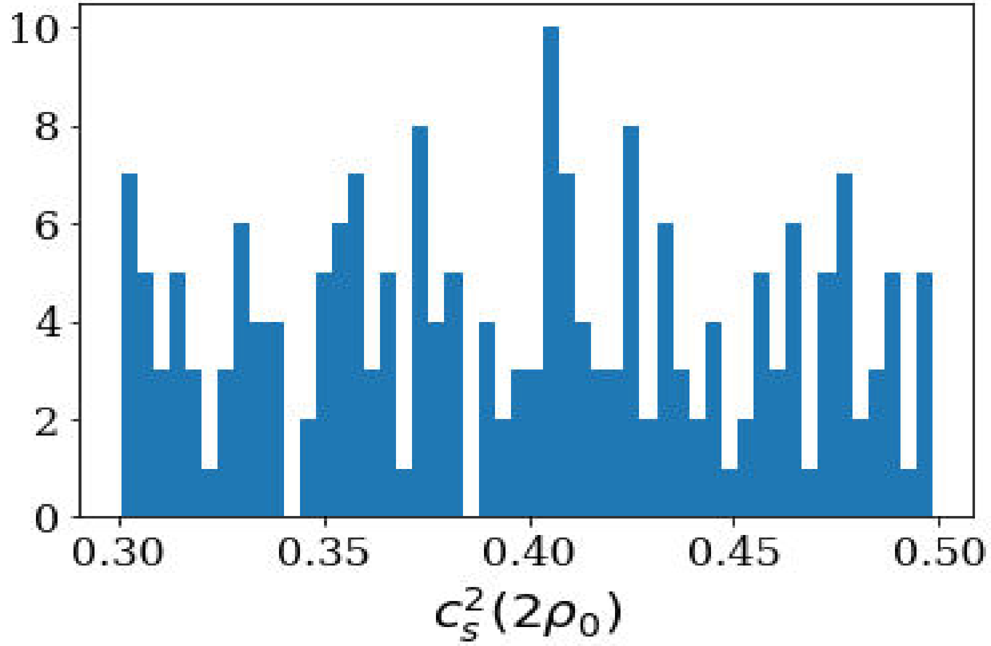



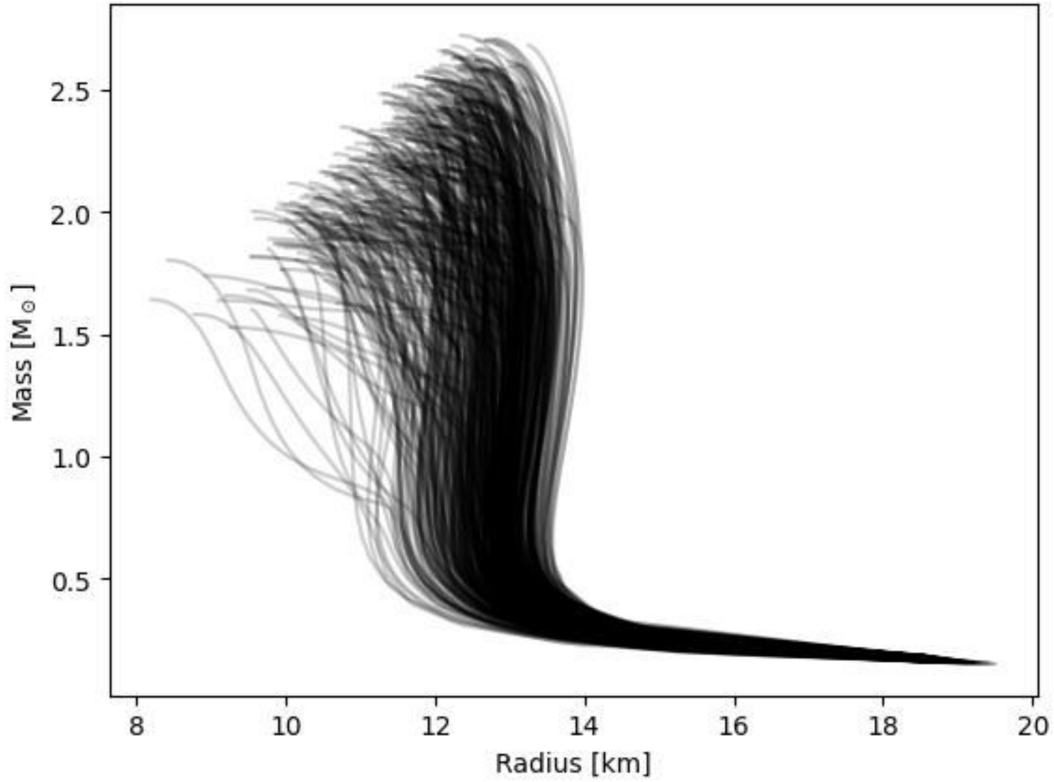
Parameters

$\alpha$

# Our Parameters

- Nuclear EOS
  - $\rho_0, \frac{E}{A}, K_{sat}, J, L, K_{sym}, c_s^2(2\rho_0), c_s^2(3\rho_0)$ …
- Use a large sample (~100,000)
- Keep some for validation (~10%)



# Building the Emulator

$$\hat{R}(x) = R_0 + \sum_{k=1} a_k^R(\alpha) R_k(x)$$

$$\hat{\Lambda}(x) = \Lambda_0 + \sum_{k=1} a_k^\Lambda(\alpha) \Lambda_k(x)$$

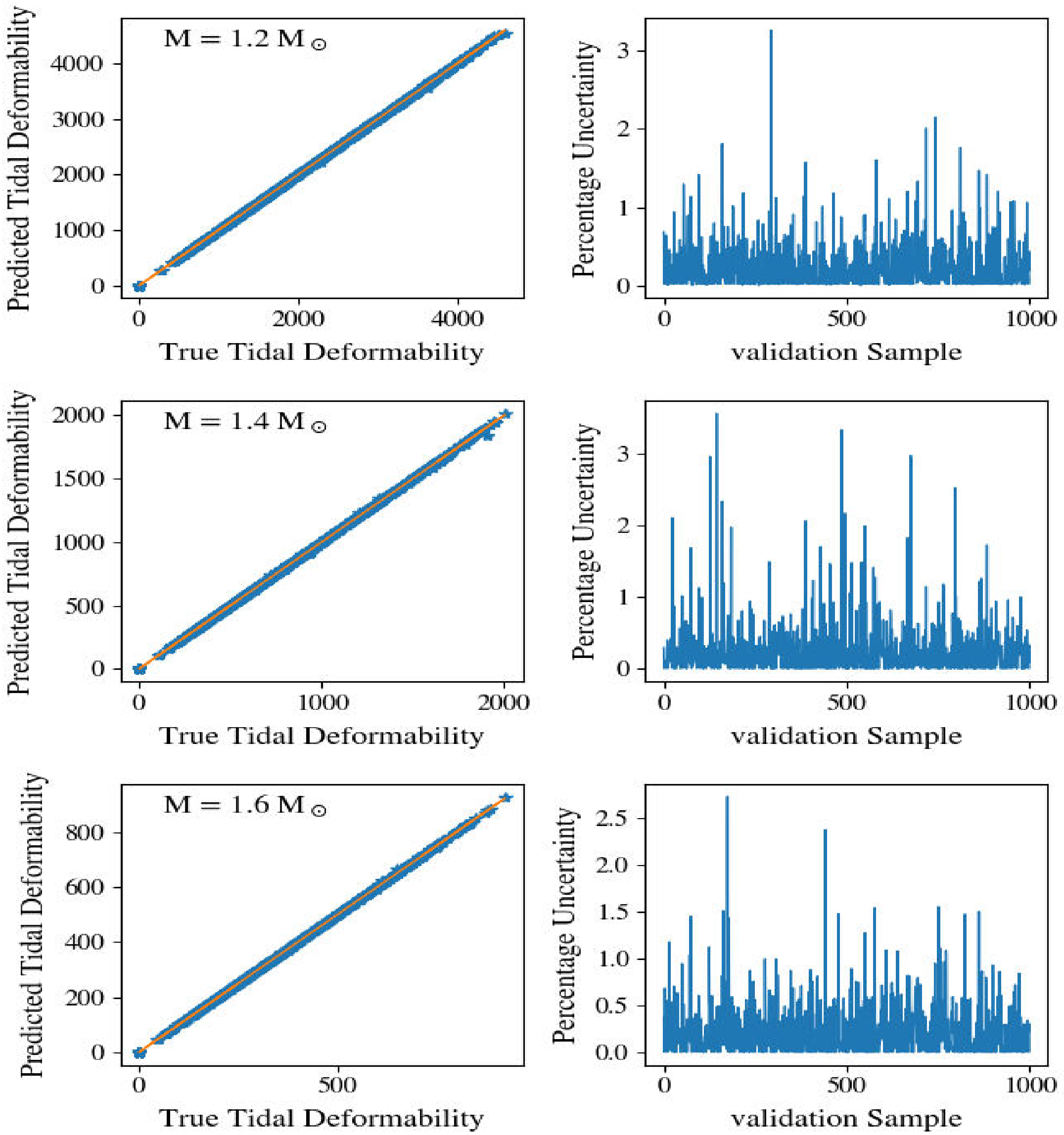
- Functions of model parameters
- Must be determined from data

- Functions determined from SVD
- Stay fixed regardless of model parameters

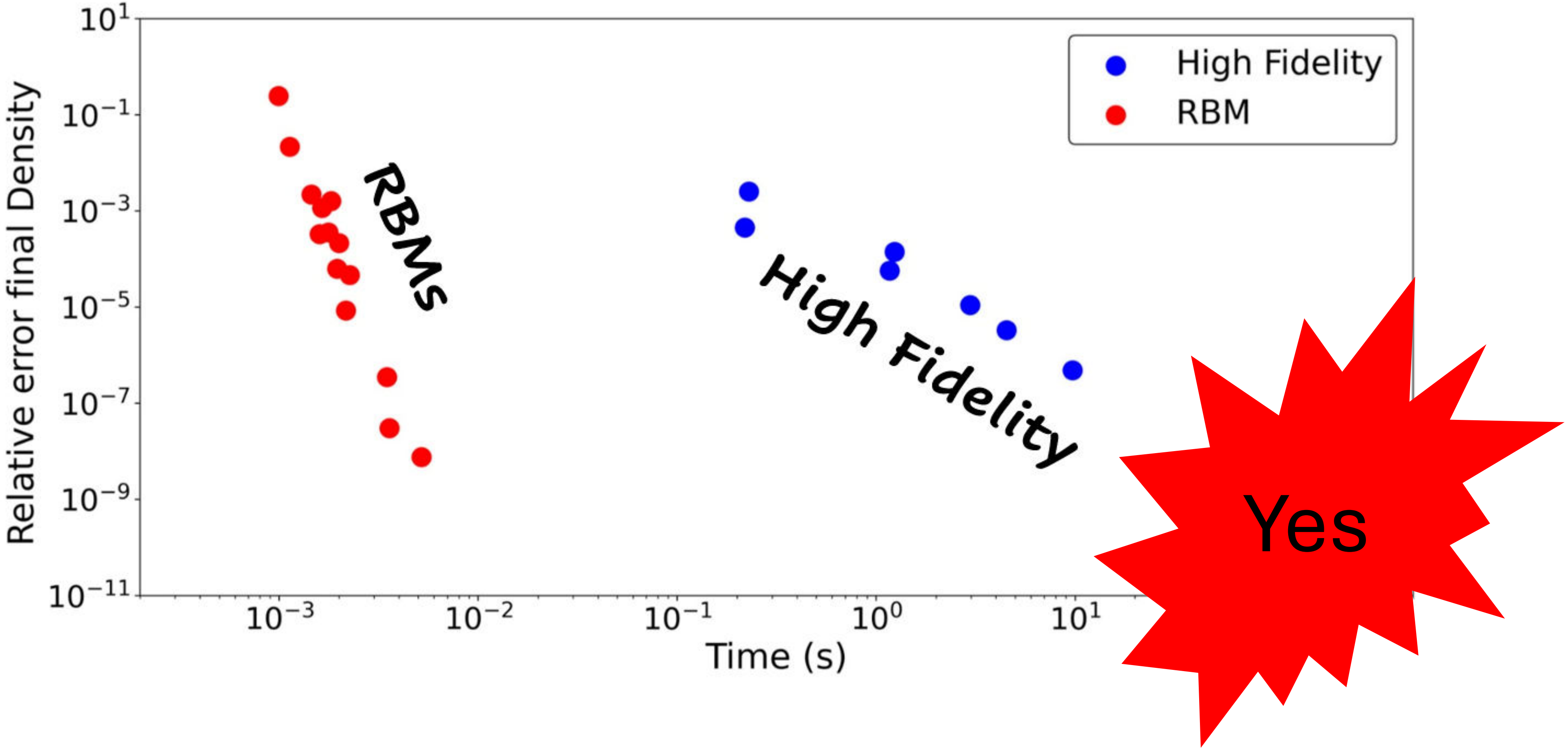
# How To Determine Coefficients

- Neural Networks
- Gaussian Processes
- Galerkin Projection
- Black Box Techniques (ask me about this)

$$\alpha \to a_k$$



# Does this Improve Things?



# Thank-yous

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NSF

BAND Bayesian Analysis of Nuclear Dynamics

## ASCSN Team

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Sudhanva Lalit
Magda Kuich
Ingo Twes
Amy Lovell
Ana Posada
Witek Nazarewicz
Daniel Phillips
Dirac ...

And thank you, for listening

# Further resources

Watch on YouTube

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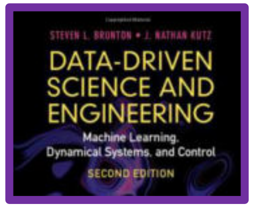
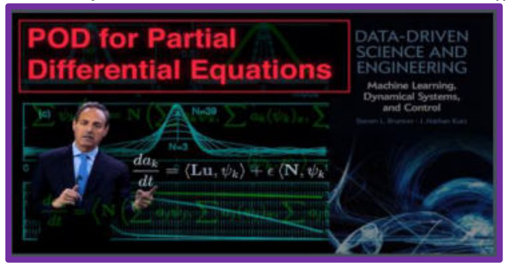



http://databookuw.com/


https://www.youtube.com/watch?v=n9Kgb72Ozgk

Dimensionality Reduction in Nuclear Physics
Presented by ASCSN

https://dr.ascsn.net




FRIB-TA Summer School: Practical Uncertainty Quantification and Emulator Development in Nuclear Physics

https://forum.ascsn.net/t/about-the-2023-frib-ta-summer-school/42

Link to slides