Stellar modeling and evolution: an introduction

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INAF ISTITUTO NAZIONALE DI ASTROFISICA OSSERVATORIO ASTRONOMICO D'ABRUZZO

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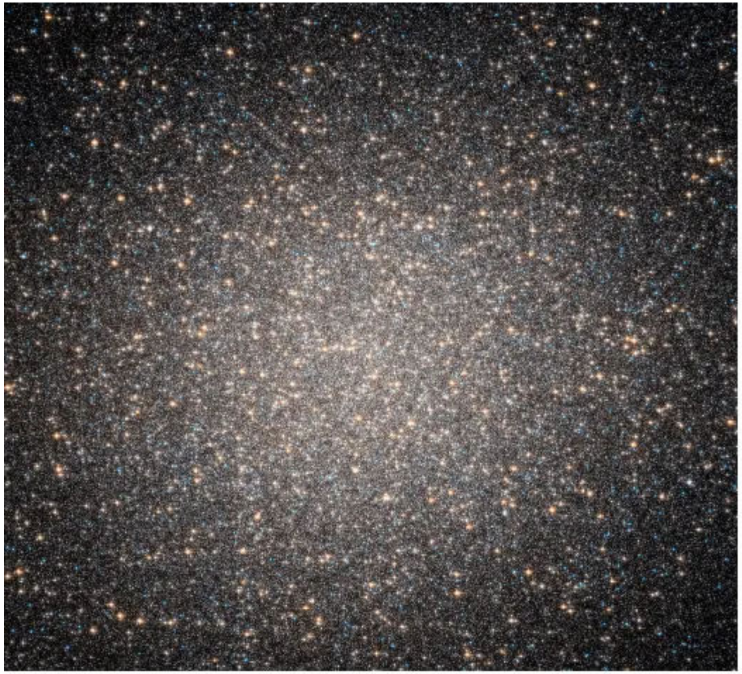
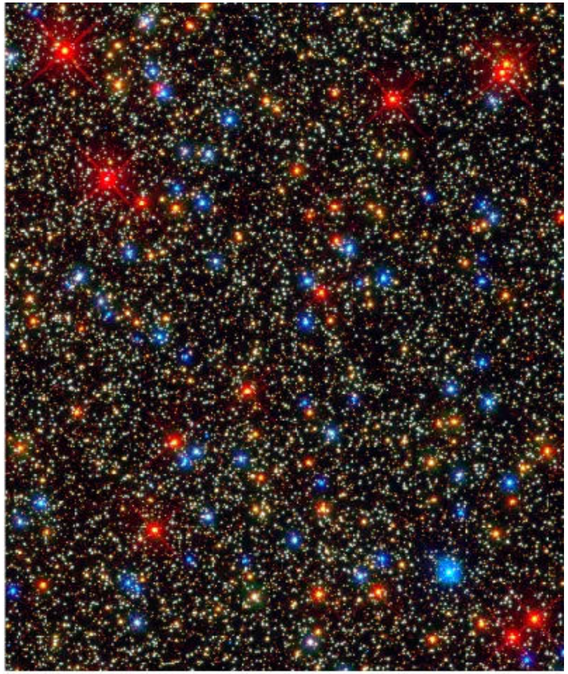
What is a star?

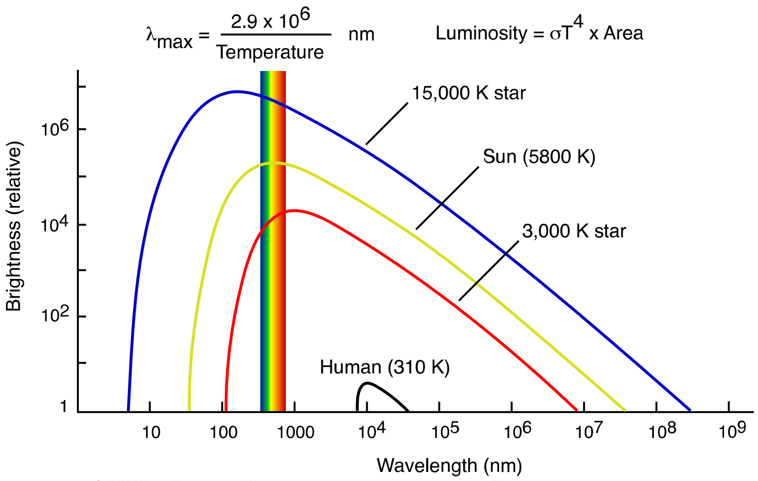



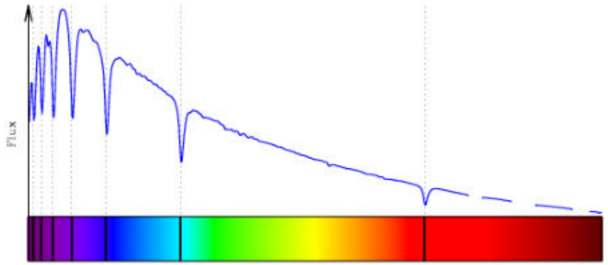
Omega Centauri

- Stars have a range of colors
- Some stars are intrinsically brighter than others

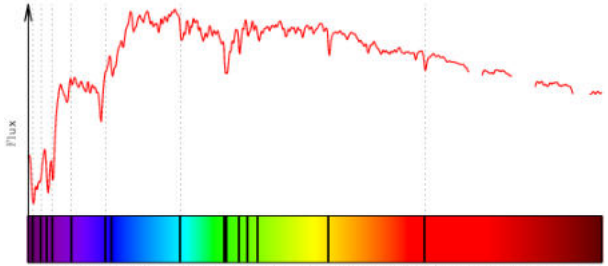
(Some) properties of stars

- **Surface temperature**
  - Wien's law (color-temperature relation) → $\lambda_{max} = 2.9 \text{ mm}/T$
  - Measure the spectrum and get the spectral type
- **Luminosity**
  - Measure star's apparent brightness and compensate for distance
- **Chemical composition**
  - Spectral lines observed in a star
- **Radius**
  - Stefan-Boltzmann law → $L = 4\pi R^2 \sigma T^4$
- **Mass**
  - Modified form of Kepler's third law applied to binary stars





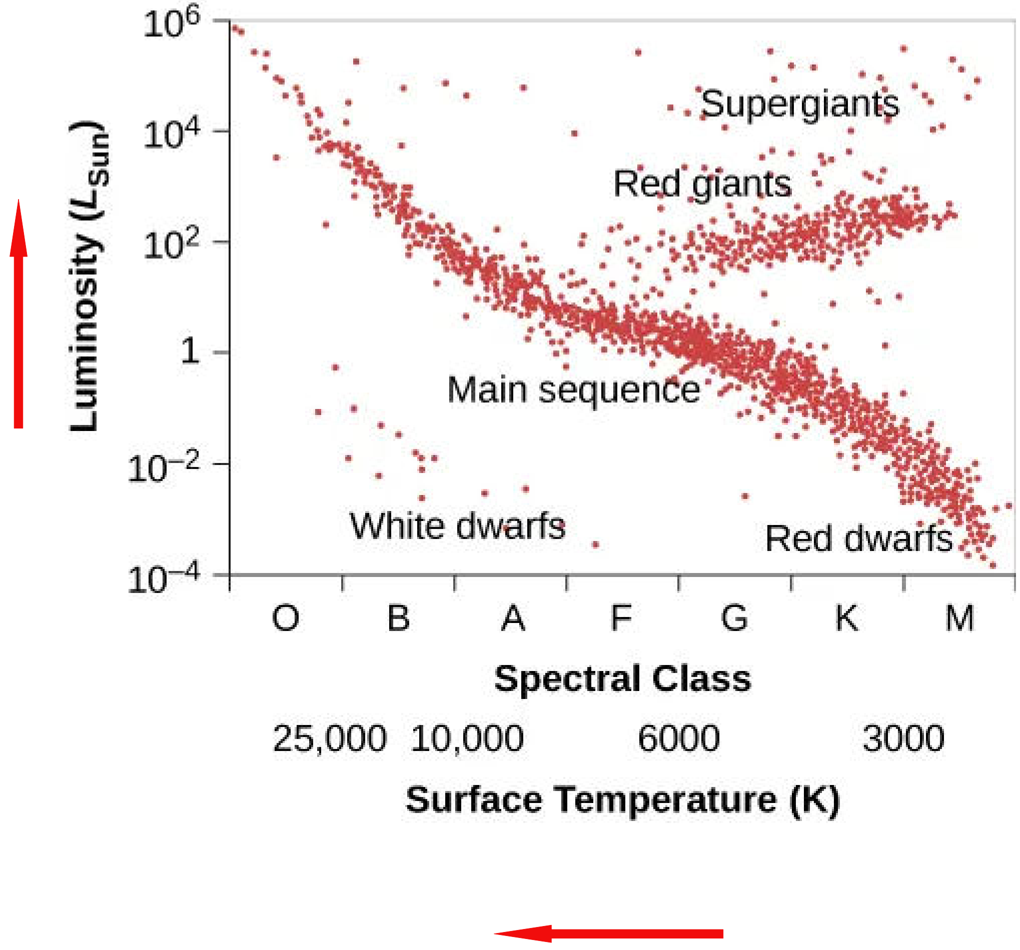
A-star spectra show strong hydrogen absorption features (marked with dotted lines).



G-star spectra have weak hydrogen features, and also lines from many other elements.

The Hertzsprung-Russell Diagram

- The H-R diagram is a plot of **stellar temperature vs. luminosity**
- Most of the stars on the H-R diagram lie along a smooth <u>diagonal</u> running from **hot, blue, luminous** stars to **cool, red, dim** ones
- The diagonally running group of stars on the H-R diagram is referred to as the ***main sequence***
- Generally, <u>90%</u> of a group of stars will be on the main sequence
- Few stars will be cool but very luminous while others will be hot and dim



The Hertzsprung-Russell Diagram

- $L = 4\pi R^2 \sigma T^4$

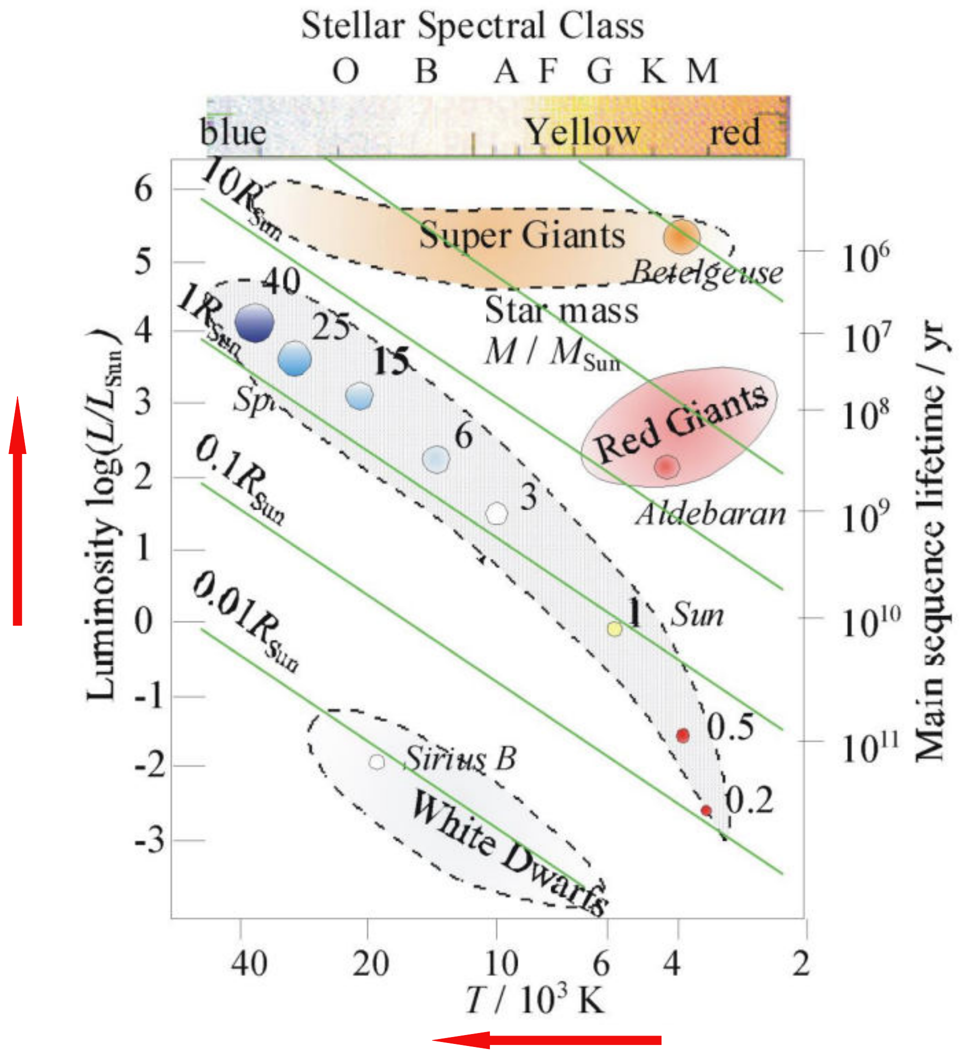
- Stars in the upper right are called ***red giants***

- Stars in the lower left are ***white dwarfs***

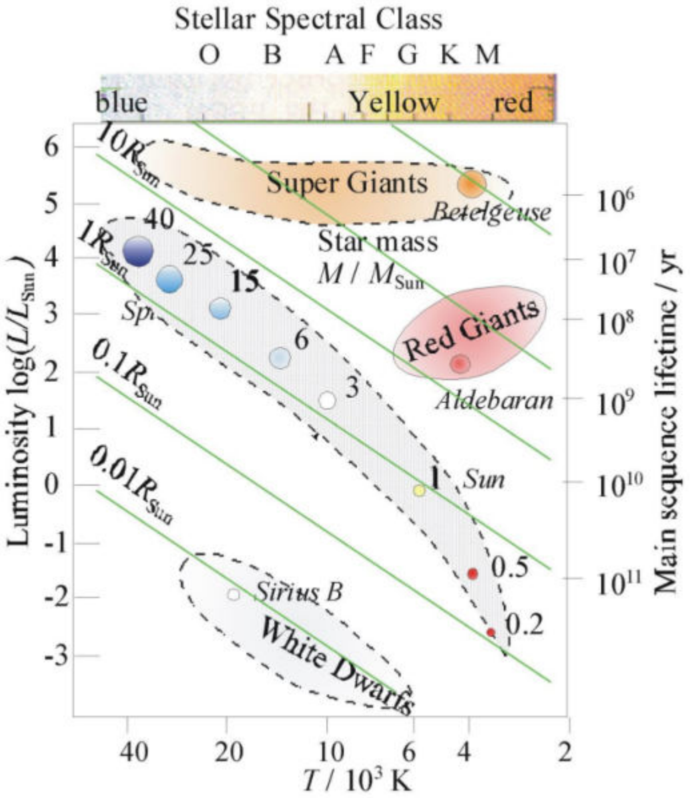
- <u>Three main stellar types</u>: main sequence, red giants, and white dwarfs

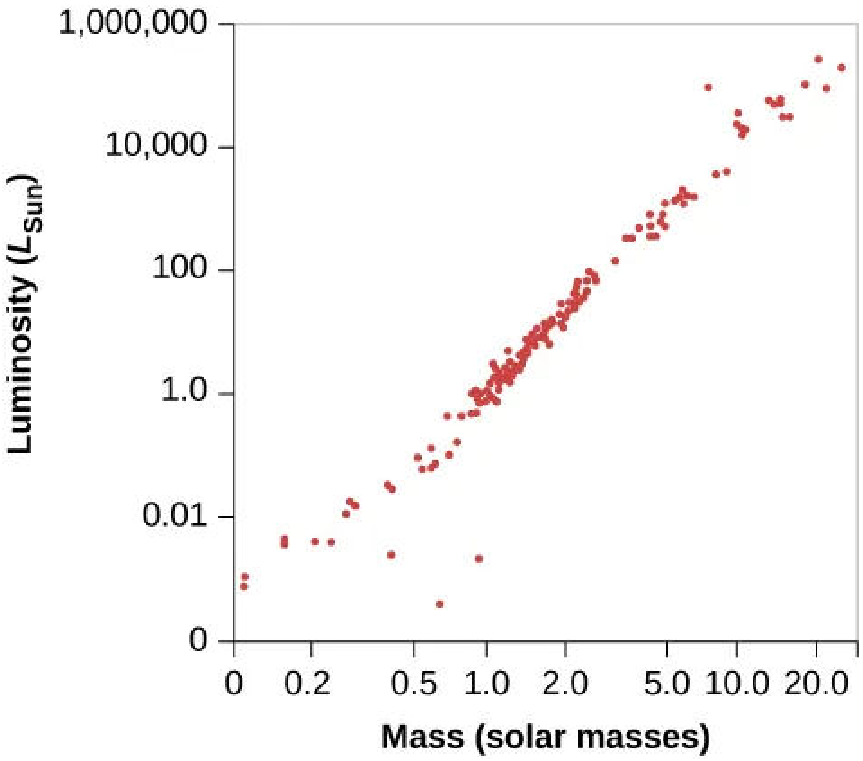
- Giants, white dwarfs, and main sequence stars also <u>differ in average density</u>, not just diameter

- Typical density of main-sequence star is 1 g/cm^3, while for a giant it is 10^{-6} g/cm^3



The Mass-Luminosity Relation



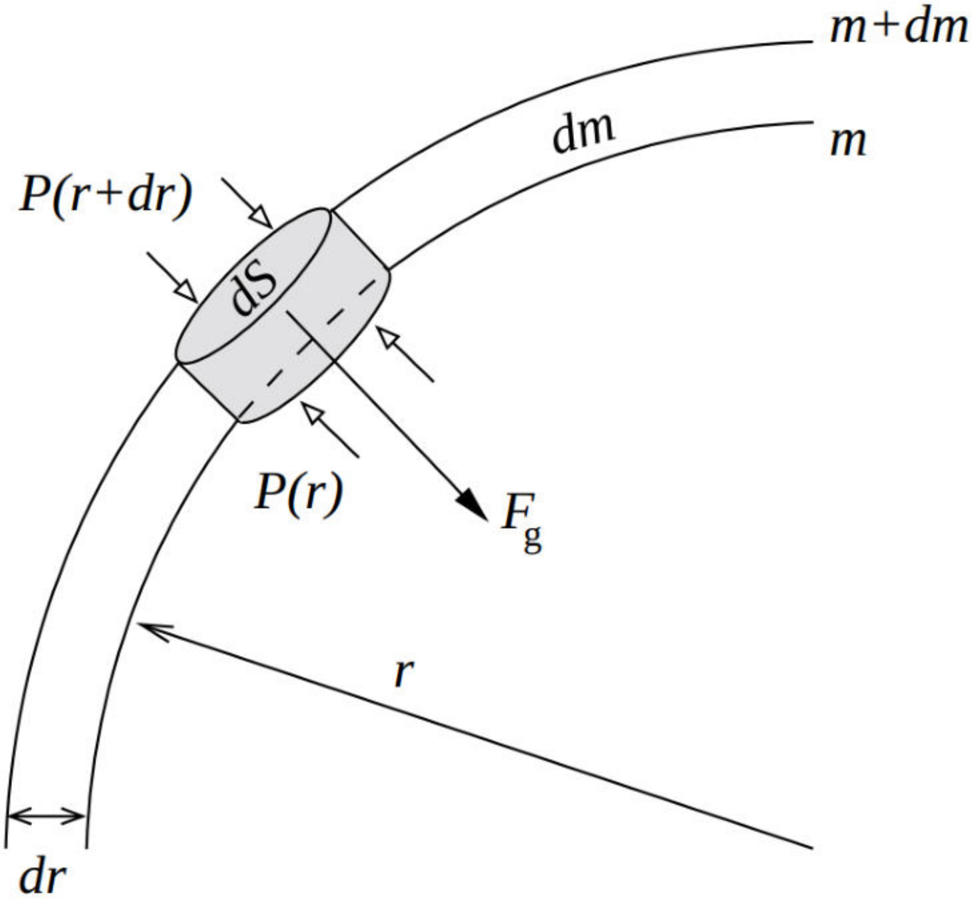


- Main-sequence stars obey a mass-luminosity relation, approximately given by: $L \propto M^{3.5}$
- <u>Stars at top of main-sequence are more massive than stars lower down</u>

Stellar models

- Stars require millions to billions of years to evolve
- A star's evolution can be studied two ways:
  - **Observations** – different stars represent different snapshots in the life of a star
  - **Stellar models** via computer calculations that take into account the relevant physics
- This involve formulating a comprehensive set of differential equations for the stellar structure
- The solution provides internal profiles of various physical and chemical properties
- This allows inferring the temporal evolution of observable quantities
- The computed stellar models evidently depend on the assumptions about stellar structure and physical properties of matter that went into the calculations
- By **comparing** the results of the calculations with the different kinds of observations we are effectively **testing** the underlying physics, often under conditions where it is impossible to carry out tests in the laboratory

Hydrostatic equilibrium



Mass conservation

$$dm = 4\pi r^2 \rho \, dr$$

$$\boxed{\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}}$$

Hydrostatic equilibrium

$$\ddot{r}\, dm = -g\, dm + P(r)\, dS - P(r+dr)\, dS$$

$$\ddot{r} = -\frac{Gm}{r^2} - \frac{1}{\rho}\frac{dP}{dr}$$

$$\boxed{\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}}$$

Energy conservation

Energy leaks

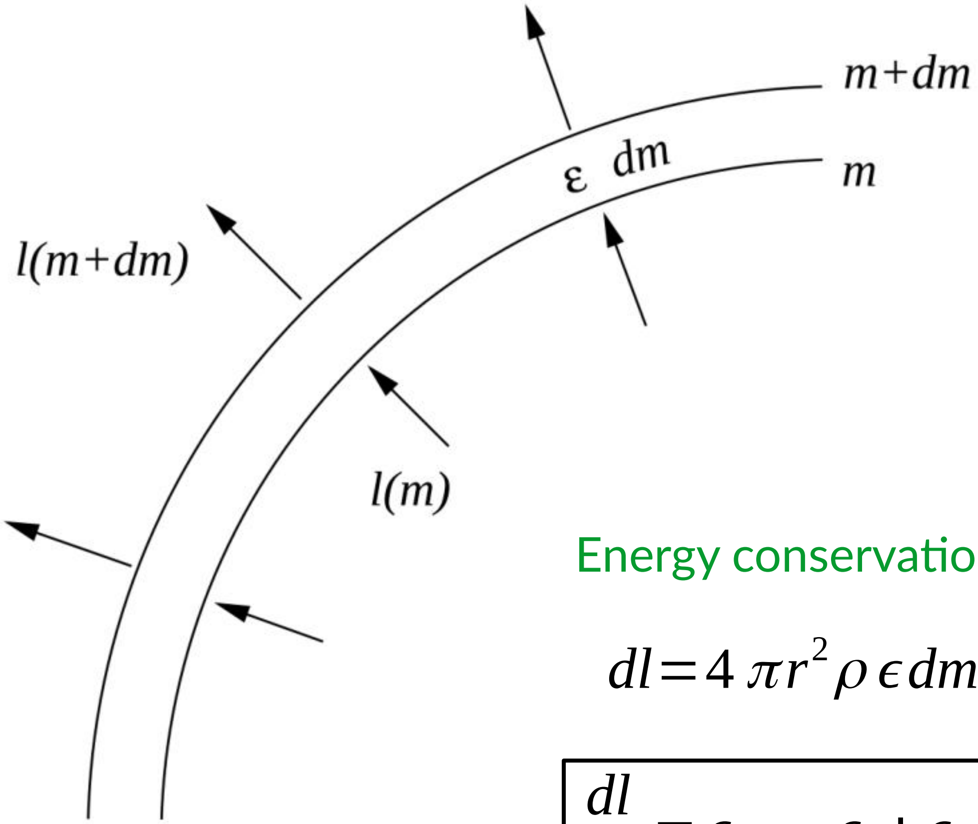
$$\epsilon_\gamma = \text{radiation} = \frac{dl}{dm}$$

$\epsilon_\nu =$ nuclear reactions + spontaneous emission

Energy sources

$$\epsilon_{\text{grav}} = -\frac{dU}{dt} + \frac{P}{\rho^2}\frac{d\rho}{dt}$$

$$\epsilon_{\text{nuc}} = \sum_k Y_i Y_j \rho N_A \langle \sigma v \rangle_k Q_k$$



Energy conservation

$$dl = 4\pi r^2 \rho \epsilon dm$$

$$\boxed{\frac{dl}{dm} = \epsilon_{\text{nuc}} - \epsilon_\nu + \epsilon_{\text{grav}}}$$

Energy transport: heat diffusion

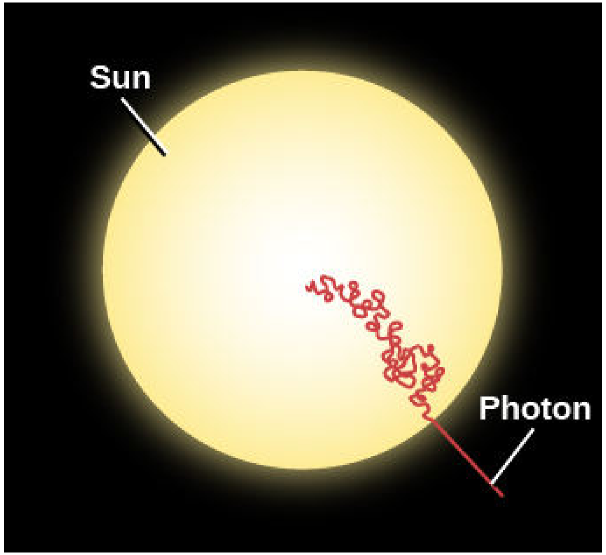
$$F = -K \nabla T \quad \text{with} \quad K = \frac{1}{3} \bar{v} \, l \, C_V$$

- Heat diffusion proceeds through the random thermal motion of particles across gradients in temperature
- Photons (**radiative diffusion**) or gas particles (**conduction**) → $K = K_{rad} + K_{cond}$

$$F = -\frac{4acT^3}{3\kappa\rho} \nabla T \quad \text{with} \quad \frac{1}{\kappa} = \frac{1}{\kappa_{\text{rad}}} + \frac{1}{\kappa_{\text{cond}}}$$

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4acT^3} \frac{l}{4\pi r^2}$$

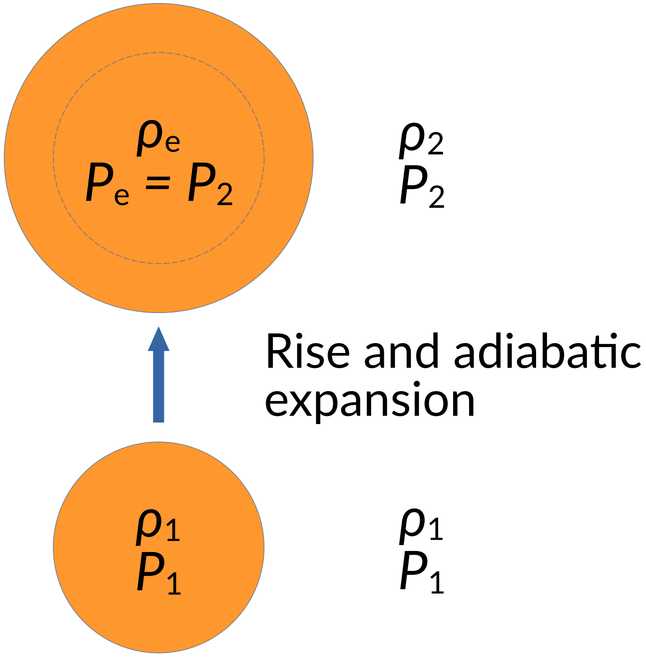
$$\boxed{\frac{dT}{dm} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{4\pi r^2 T^3}}$$



Energy transport: convection

- Radiative diffusion can transport energy outwards, however the higher the luminosity, the higher the temperature gradient required

- There is a limit for such a gradient above which an instability in the stellar plasma sets is. This instability is called **convection**



Unstable if $\rho_e < \rho_2$ $\longrightarrow$ $$\frac{d \log \rho}{d \log P} < \frac{1}{\gamma_{ad}}$$

Stable if $\rho_e > \rho_2$

with $$\gamma_{ad} \equiv \left(\frac{\partial \log P}{\partial \log \rho} \right)_s$$

The hot/cool convective cells move up/down and lose their identity after a typical **mixing length** distance in their new surroundings, depositing/absorbing energy there

➔ **Net upward heat flux**

➔ No net mass flux

Occurrence of convection

Note:

$$\gamma = \frac{c_P}{c_V}$$

$$\chi_T = \left(\frac{d\log P}{d\log T}\right)_{\rho,\mu}$$

$$\chi_\mu = \left(\frac{d\log P}{d\log \mu}\right)_{\rho,T}$$

$$\nabla_{\mathrm{rad}} = \left(\frac{d\log T}{d\log P}\right)_{\mathrm{rad}} = \frac{3}{16\pi a c}\frac{\kappa l P}{m T^4}$$

$$\nabla_{\mathrm{ad}} = \left(\frac{d\log T}{d\log P}\right)_{\mathrm{ad}} = \frac{\gamma-1}{\gamma\chi_T}$$

$$\boxed{\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}}}$$

Schwarzchild criterion

$$\boxed{\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} - \frac{\chi_\mu}{\chi_T}\nabla_\mu}$$

Ledoux criterion

Convection occurs with:

- A large value of the opacity κ. Convection occurs in opaque (and/or cool) regions of a star
- A large value of l/m, i.e. regions with a large energy flux
- A small value of ∇_{ad}, e.g. in partial ionization zones at relatively low temperatures

Energy transport: general formalism

- Under hydrostatic equilibrium, the transport equation can be written as:

$$\frac{dT}{dm} = \frac{dP}{dm}\frac{dT}{dP} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\frac{d\log T}{d\log P} \longrightarrow \boxed{\frac{dT}{dm} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla}$$

- **$\nabla = \nabla_{rad}$** in radiative zones where $\nabla_{rad} < \nabla_{ad}$
- The temperature gradient in a convective region is $\nabla_{ad} \leq \nabla \leq \nabla_{rad}$. Convection may or may not be active, while radiative transport in the presence of a temperature gradient is always active.
- **$\nabla = \nabla_{ad}$** in deep stellar interiors. There, the huge heat capacity of the stellar matter makes the convective energy transport very efficient compared to the radiative one ($F_{conv} \gg F_{rad}$)
- **∇ as given by the solution of the mixing-length theory** for envelope convection. The marked decrease in heat capacity, resulting from the decreased density of matter, makes F_{conv} much smaller and the superadiabaticity becomes substantial (**$\nabla > \nabla_{ad}$**)
- As the surface is approached, convection becomes very inefficient at transporting energy. Then $F_{conv} \ll F_{rad}$ so that radiation effectively transports all the energy, and **$\nabla \approx \nabla_{rad}$** despite convection taking place

The equations of stellar evolution

$$\frac{dP}{dm} = -\frac{G\,m}{4\,\pi r^4}$$

Hydrostatic equilibrium

$$\frac{dr}{dm} = \frac{1}{4\,\pi r^2 \rho}$$

Mass conservation

$$\frac{dT}{dm} = -\frac{G\,m\,T}{4\,\pi r^4 P}\nabla$$

Energy transport

$$\frac{dl}{dm} = \epsilon_{\mathrm{nuc}} - \epsilon_{\nu} + \epsilon_{\mathrm{grav}}$$

Energy conservation

+

$$\rho = \rho(P, T, Y_i)$$

Equation of state

$$\kappa = \kappa(\rho, T, Y_i)$$

Opacity

$$\epsilon_j = \epsilon_j(\rho, T, Y_i)$$

Energy generation rates

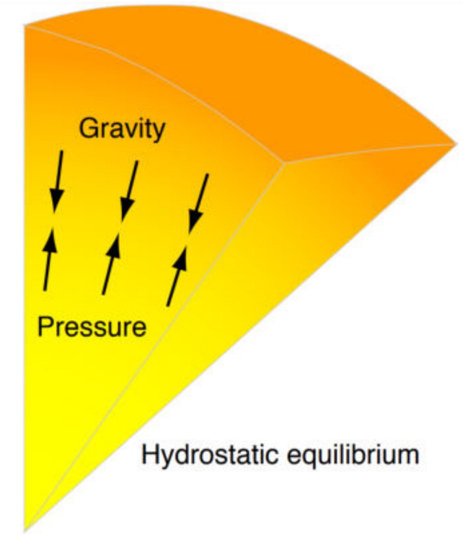
$$\frac{dY_i}{dt} = \left(\frac{dY_i}{dt}\right)_{nucl} + \left(\frac{dY_i}{dt}\right)_{mix}$$

Composition changes

- Can be numerically solved, once fixed the mass and the stellar original chemical composition

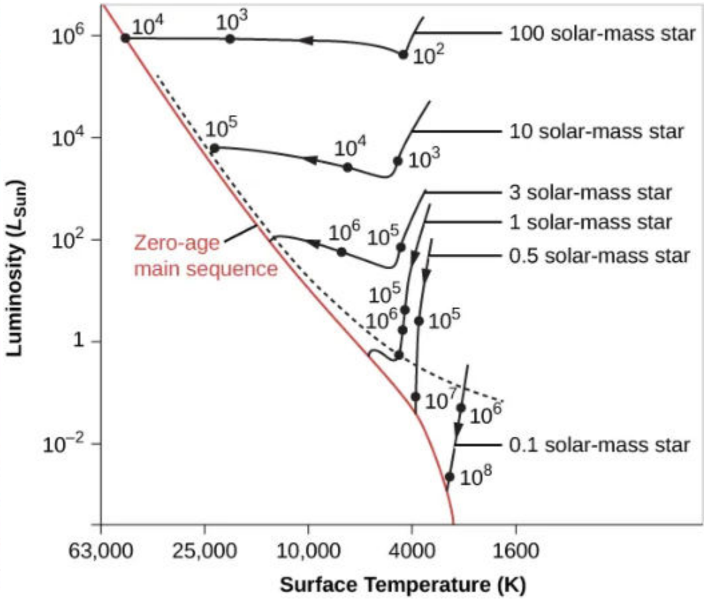
Principles of stellar structure and evolution

- *Gravity* holds a star together while the *pressure* of its gases supports it against gravity's pull
- Gravity and pressure forces must balance (***hydrostatic equilibrium***)
- Pressure is created by high temperature
- High temperature causes *heat* to flow from core to surface, where it escapes into space as the star's *luminosity* (starlight)
- Escaping heat is replenished by *nuclear fusion* in the core
- The nuclear fuel cannot last forever → the star must evolve (age)
- Gravity's force is no longer counterbalanced → the star resumes its contraction



- Heats up until it reaches temperature and density high enough to fusion reactions to occur
- The evolution of a star proceeds through a **sequence of phases**, in which energy is mainly generated by nuclear reactions or by contraction

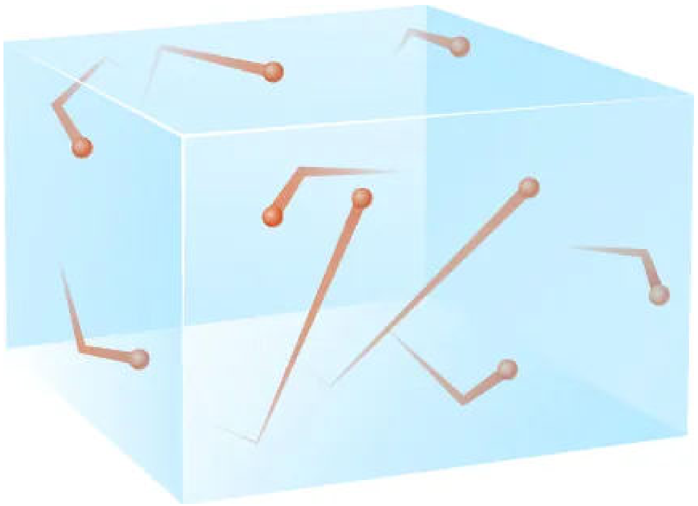
The Birth of a Star



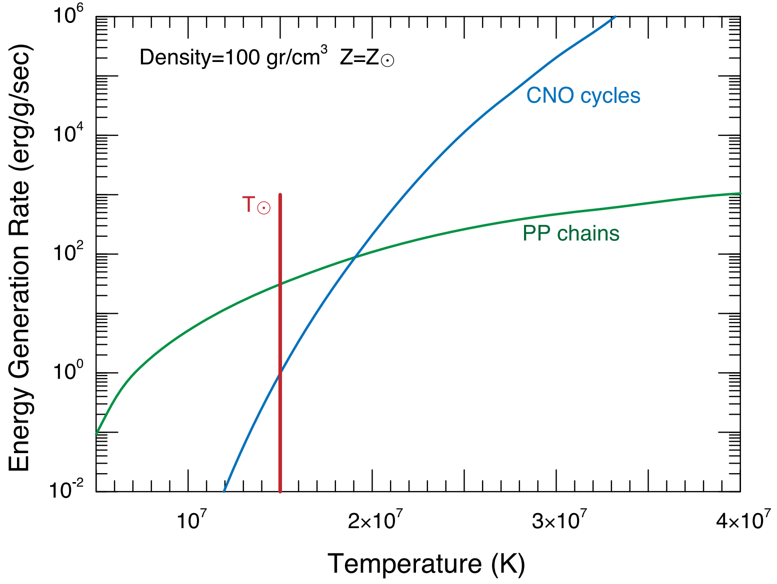
- A cold molecular gas cloud is disturbed from equilibrium
- Gravity takes over as the cloud collapses, converting gravitational potential energy into kinetic energy (heat)
- Temperature and density increase → **Hydrogen fusion** begins
- Thermonuclear reactions then provide non-gravitational energy, countering further collapse and and the star settles onto the ***main sequence***

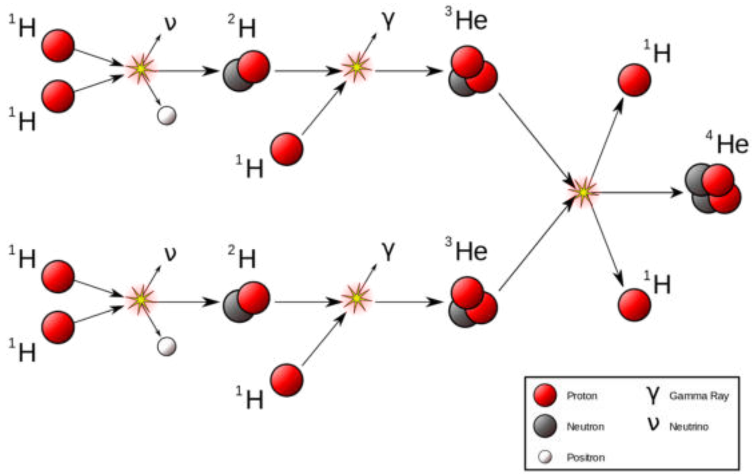
Mass - Core Temperature

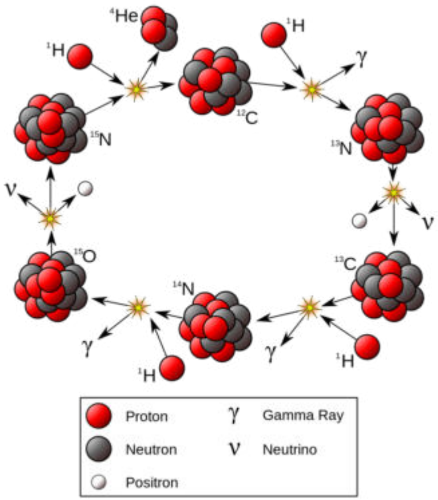
- The properties of a main sequence star depend greatly on its **mass**
- A more massive star has a higher gravitational attraction than a less massive star
- Hydrostatic equilibrium then requires a **higher gas pressure** for the **larger gravity** of a massive star
- The higher pressure can be achieved by a higher temperature



H-burning

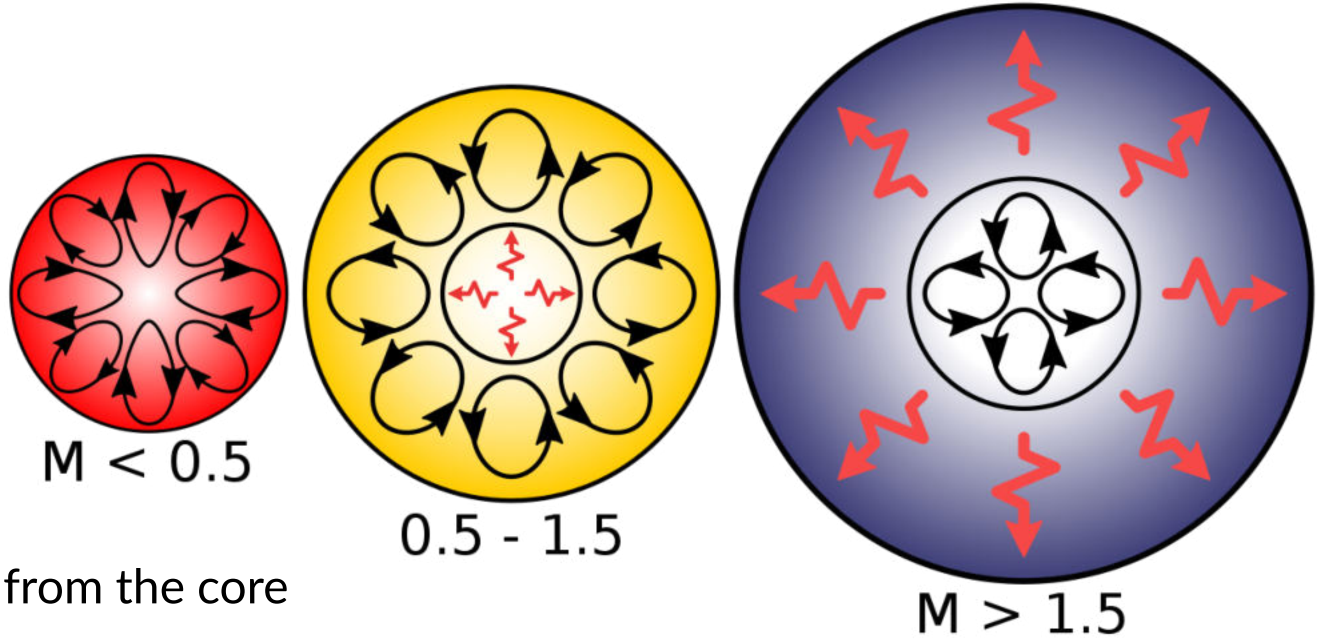






- Fusion in the core
  - <u>Low-mass</u> stars: ***proton-proton chain***
  - <u>High-mass</u> stars: ***CNO cycle*** – carbon, nitrogen, and oxygen act as catalysts for H fusion at higher core temperatures

Structure of Stars



- Energy transport from the core
  - <u>Low-mass</u> stars: Inner radiative zone, outer convection layer
  - <u>High-mass</u> stars: Inner convection zone, outer radiative layer
  - <u>All</u> stars: Outer layers of hydrogen gas are unavailable for fusion reactions in the core

Stellar Lifetimes

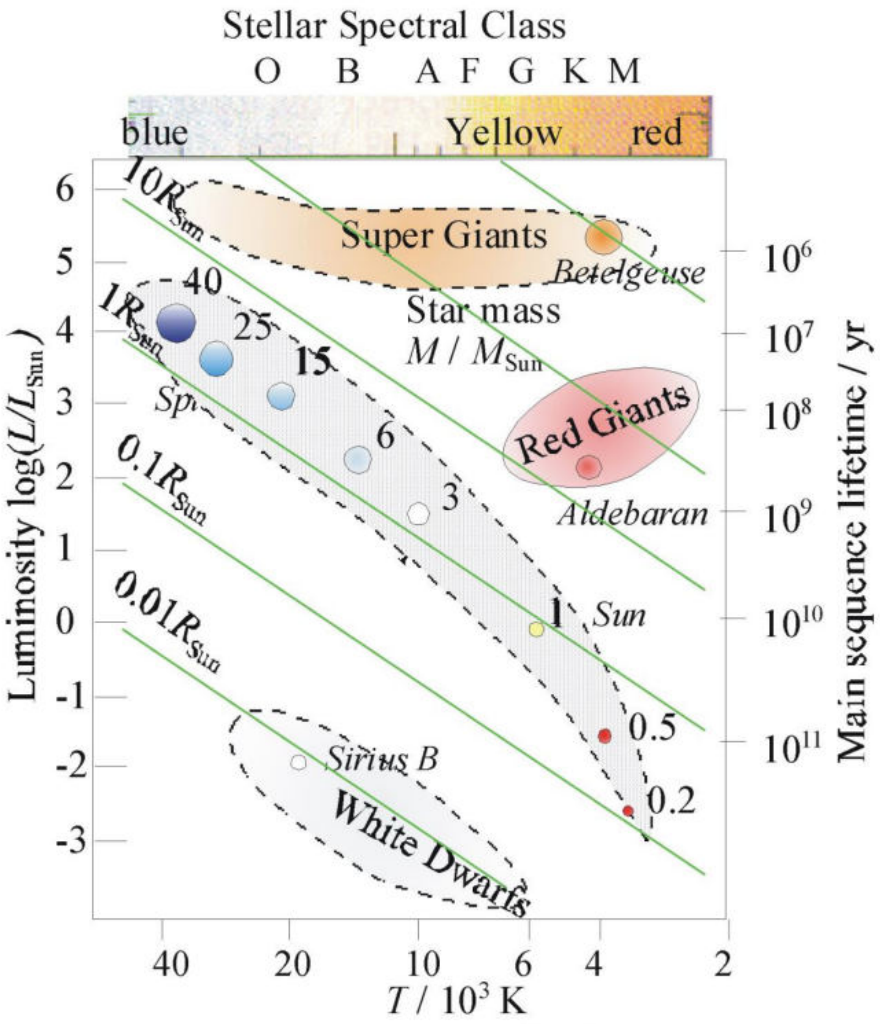
- The time a star stays on the main sequence is called the ***main-sequence lifetime***

- The amount of time t_{ms} a star will spend on the main sequence depends on its available fuel (mass *M*) and how fast it consumes it (luminosity *L*)

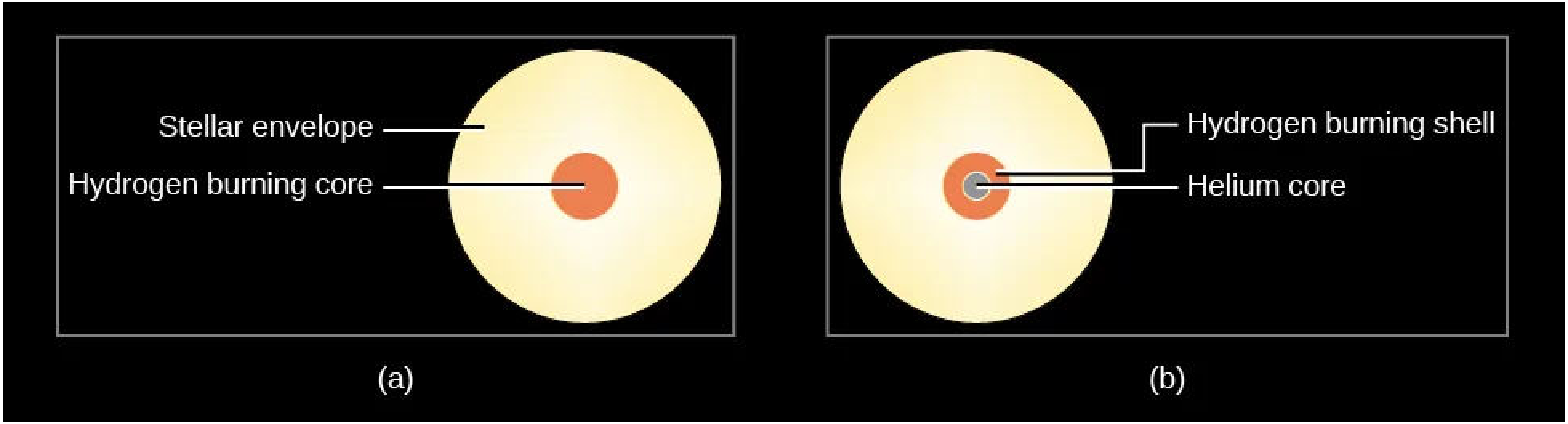
- $t_{ms} \propto M/L \propto M/M^{3.5} \propto M^{-2.5} \approx 10^{10}\,(M/L)$ years (M and L are in solar mass units)

  - 1 $M_\odot$ star with 1 $L_\odot$: 10 billion years
  - 2 $M_\odot$ star with 20 $L_\odot$: 1 billion years
  - 30 $M_\odot$ star with 10^5 $L_\odot$: 3 million years

- Short lifetime of massive main-sequence stars implies blue stars have formed recently



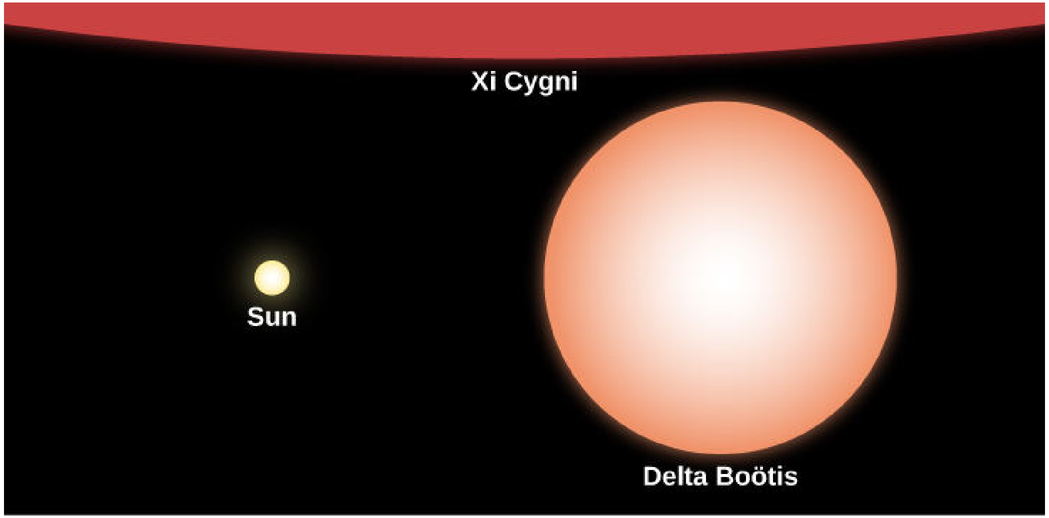
Leaving the Main Sequence

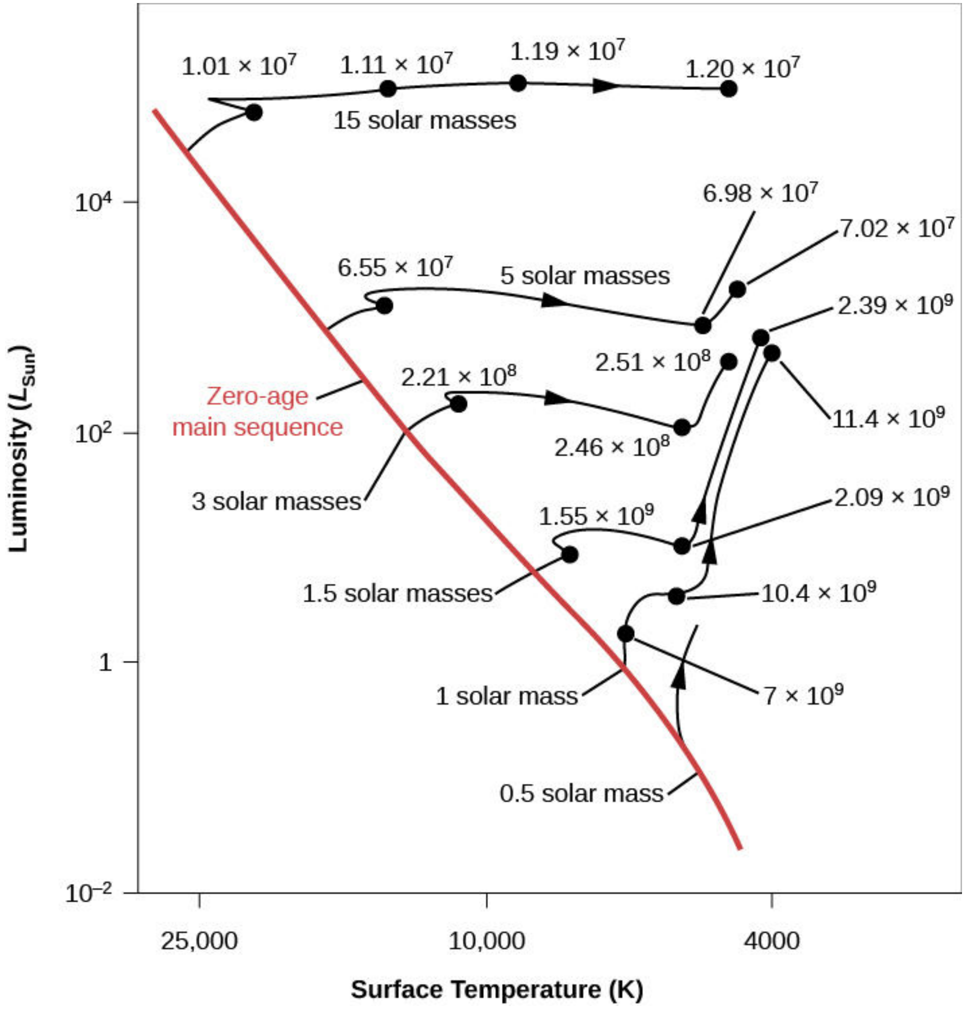


- When a main-sequence star exhausts its fuel, the core drops its pressure, is compressed by gravity, and heats up
- The increasing temperature of the core eventually ignites hydrogen gas just outside the core in a region called the ***shell source***

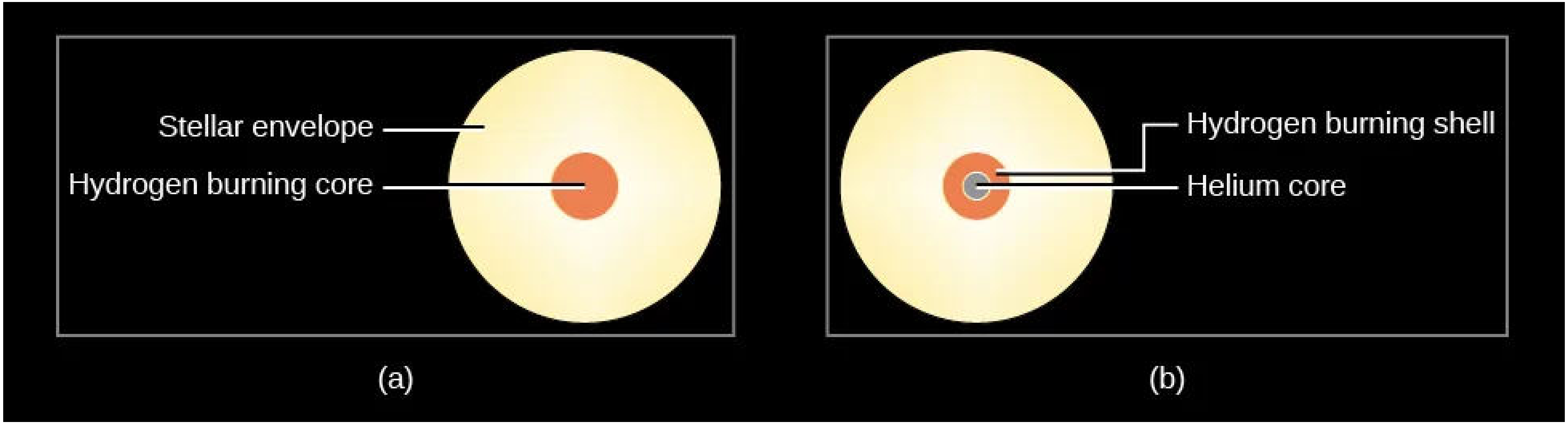
Becoming a Red Giant

- The shell source increases the pressure around the core and pushes surrounding gases outward
- The star expands into a red giant as the radius increases and the surface cools
- Size of red giant depends on initial mass of star





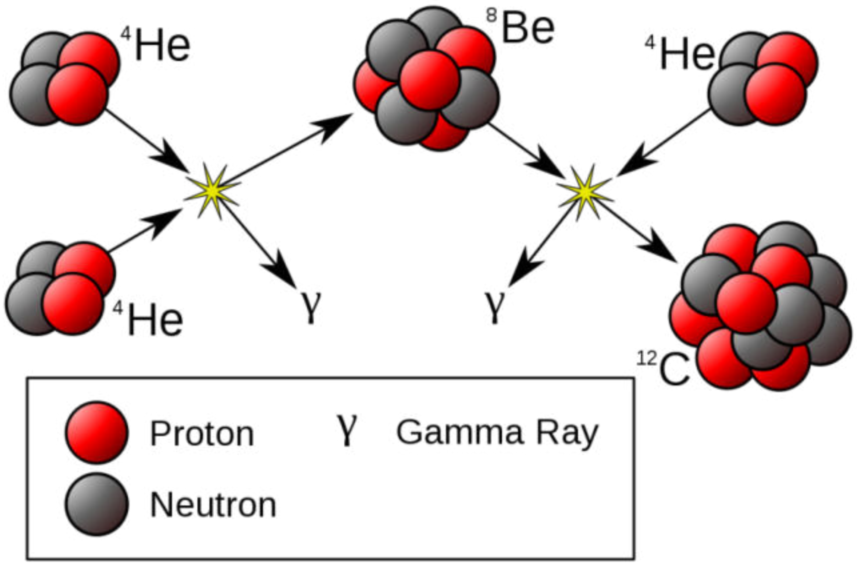
Structure of a Red Giant



- Most of a giant star's volume is in its huge outer envelope, while most of its mass is in its Earth-sized core
- Convection carries energy through the outer opaque envelope to the surface

Triple-Alpha Process and Giant Stars

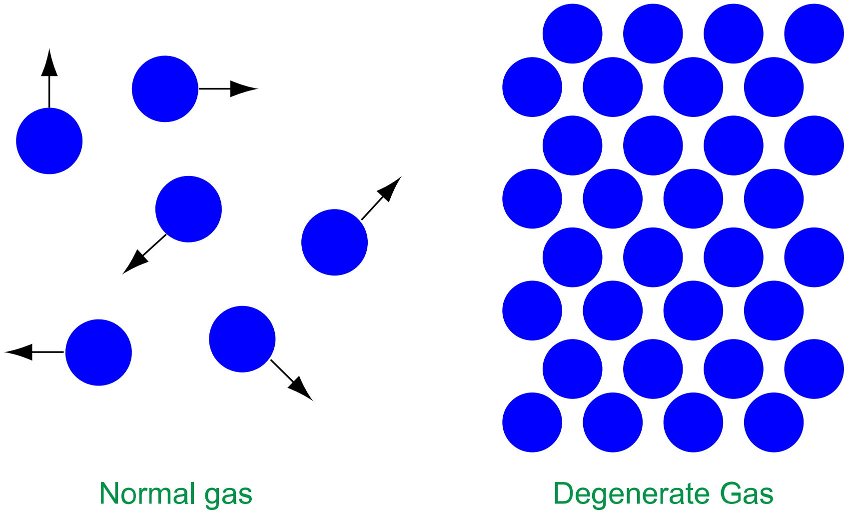
- Nuclear fuels heavier than hydrogen
  - To fuse nuclei containing larger numbers of protons requires higher impact velocities (higher temperatures) to overcome the bigger electrostatic repulsion
- As a giant star compresses its core, higher temperatures are achieved and helium fusion occurs at about 100 million K
- This fusion is referred to as the ***triple alpha process***



- Fusion of helium proceeds smoothly for a high-mass star since its core's pressure and temperature are high to begin with
- A low-mass star must compress its core to such an extent that it first becomes ***degenerate*** before fusing

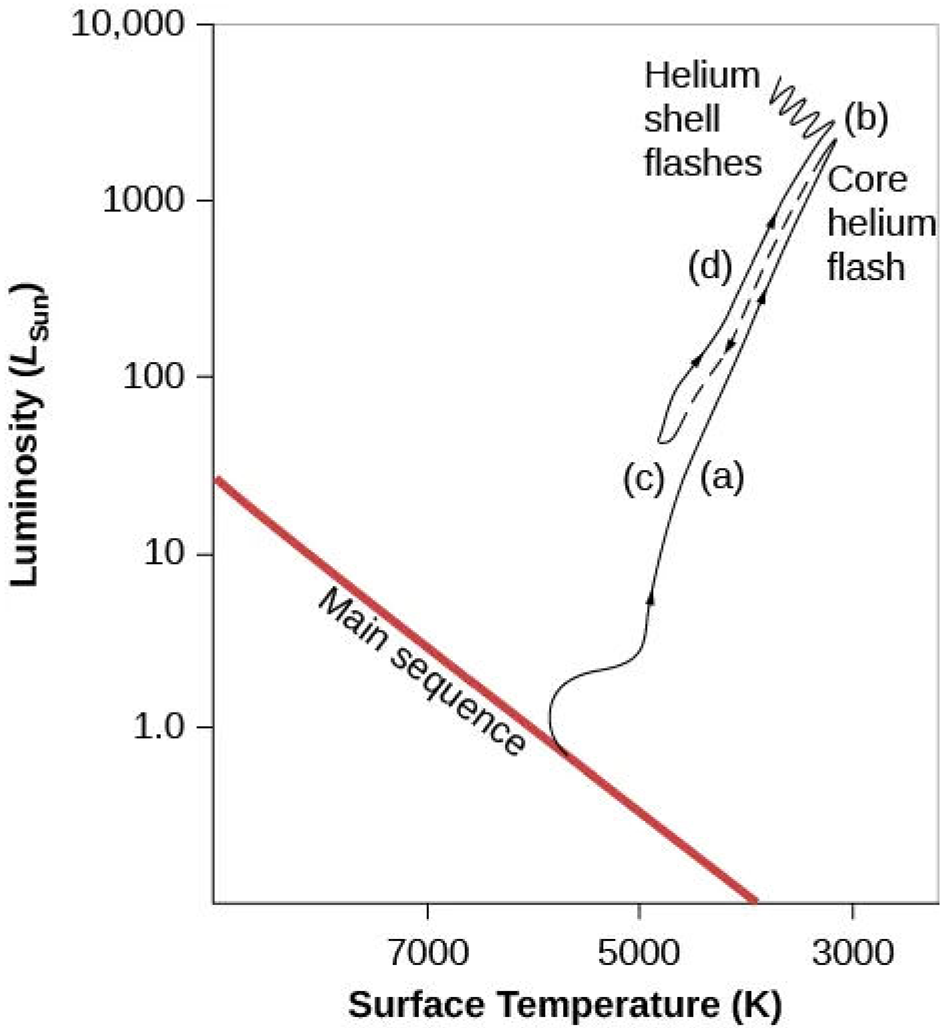
Degeneracy in Low-Mass Giant Stars

- Degenerate gas is so tightly packed that the electrons interact not as ordinary charged particles but according to laws of quantum mechanics [The Pauli Exclusion Principle]
  - Two electrons of the same energy cannot occupy the same volume
  - The degenerate gas behaves more like a solid – *it does not expand as its temperature rises*
- When a degenerate, low-mass star begins to fuse helium, it will not expand
  - The core temperature increases exponentially
  - Helium fusion proceeds explosively in what is called a ***helium flash***
  - No helium flash if $M \gtrsim 2.3\ M_\odot$

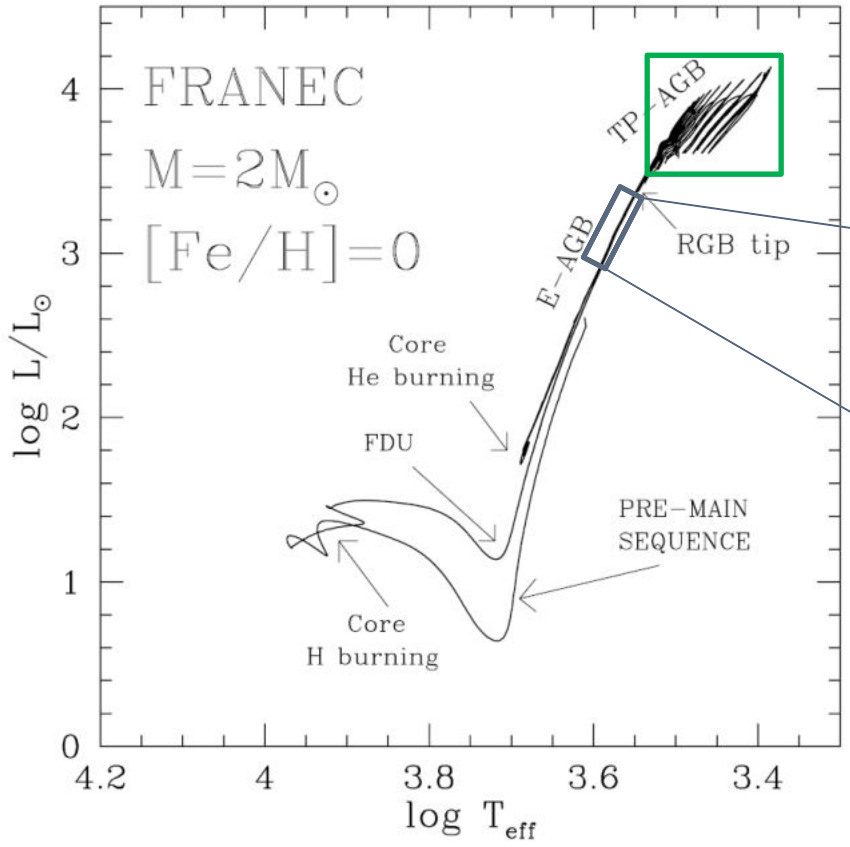


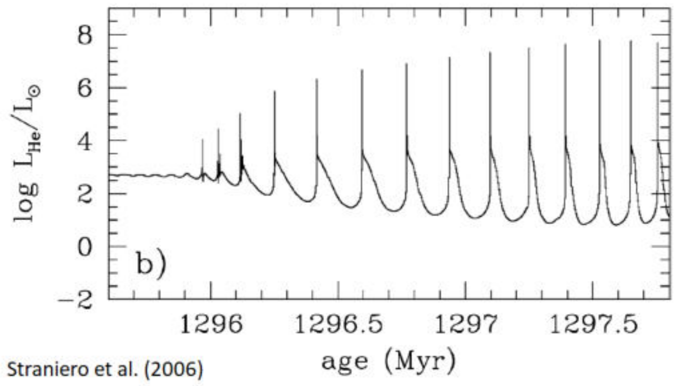
Becoming a Giant Again

- The star then continues to **fuse the helium in its core** for a while
- At a temperature of 100 million K, the inner core is converting its helium fuel to carbon (and a bit of oxygen) at a rapid rate
- Helium exhaustion → gravity takes over → the core shrinks again
- Heat released by the shrinking of the CO core flows into a shell of helium just above
- Farther out there is also a H-burning shell
- The star moves back to the red-giant domain on the H–R diagram for a short time → Asymptotic Giant Branch Star

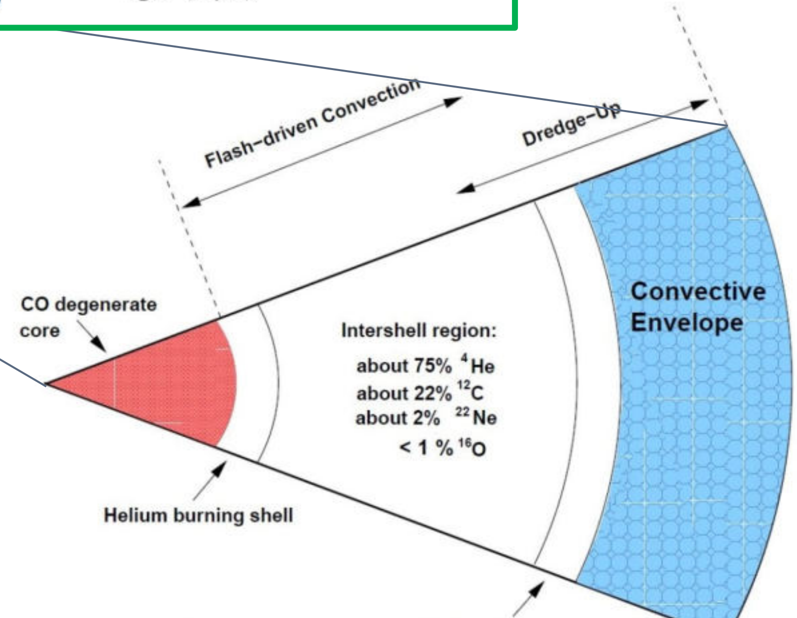


Asymptotic Giant Branch Stars



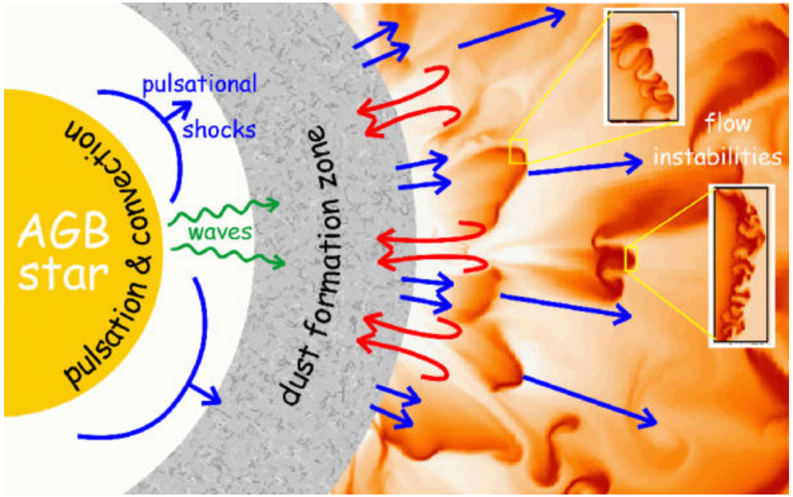


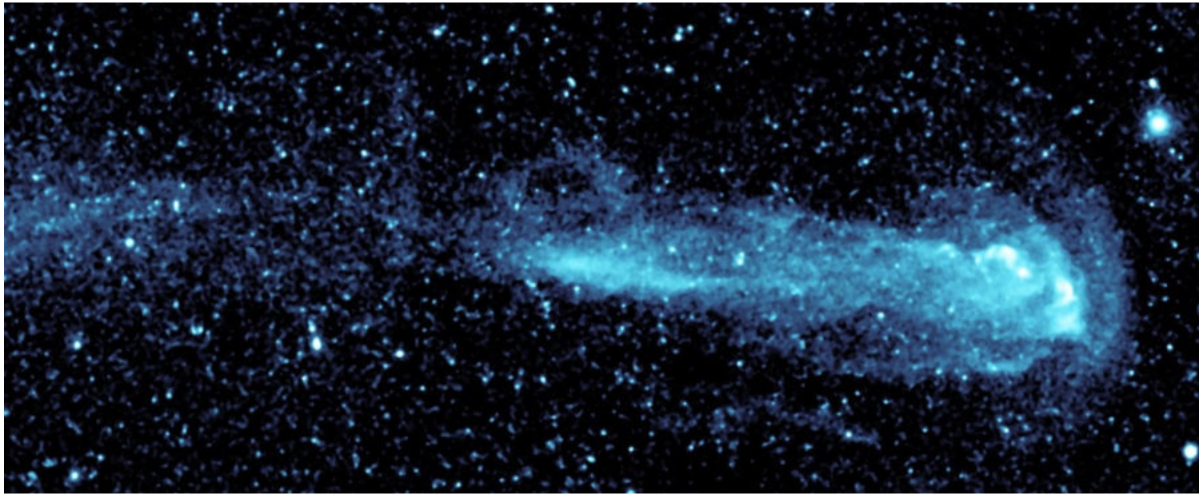
Straniero et al. (2006)



Karakas et al. (2002)

Ejection of a Low-Mass Star’s Outer Layers





- As helium burns in the star’s core, its radius shrinks, but never enough to heat it to carbon-fusing temperatures
- Luminosity increases, the outer surface expands and temperature decreases down to ~2500 K
- Carbon and silicon grains form in this cool environment and are driven out by radiation pressure

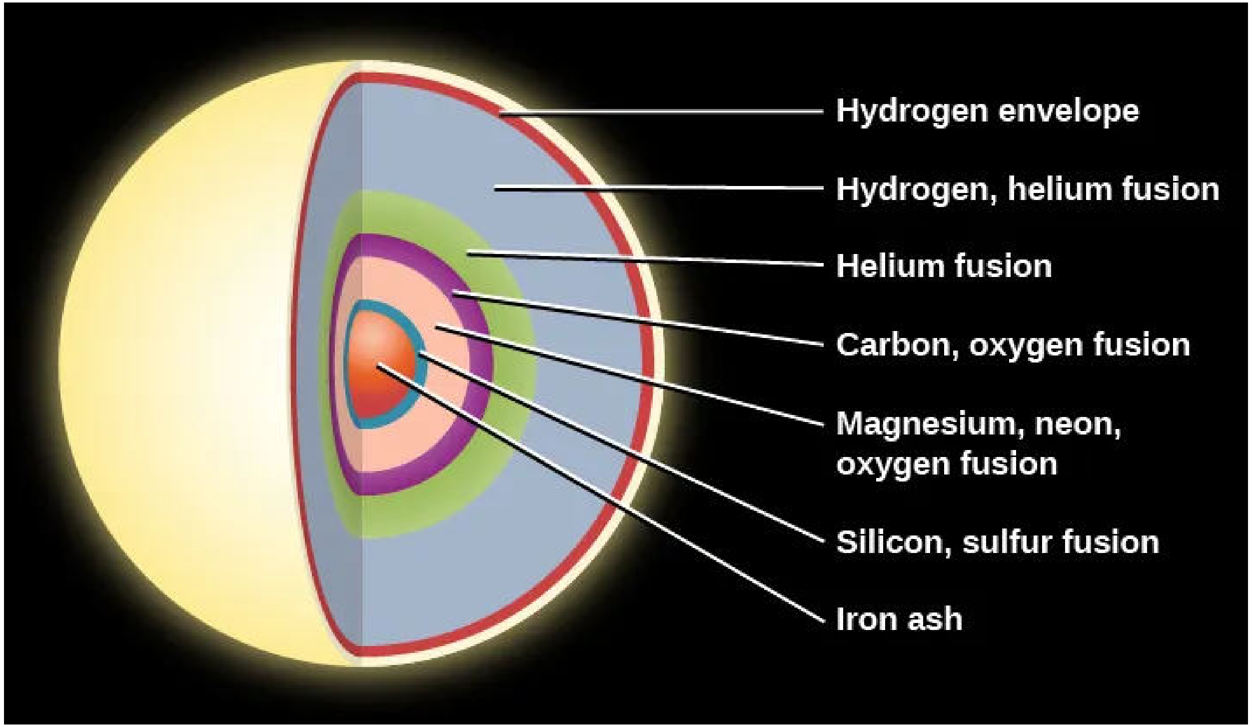
The Planetary Nebula Stage

- The grains carry the gas into space – a ***planetary nebula*** is formed – and the inner core becomes visible

- Planetary nebula (no relation to planets) glows from UV radiation from bare core

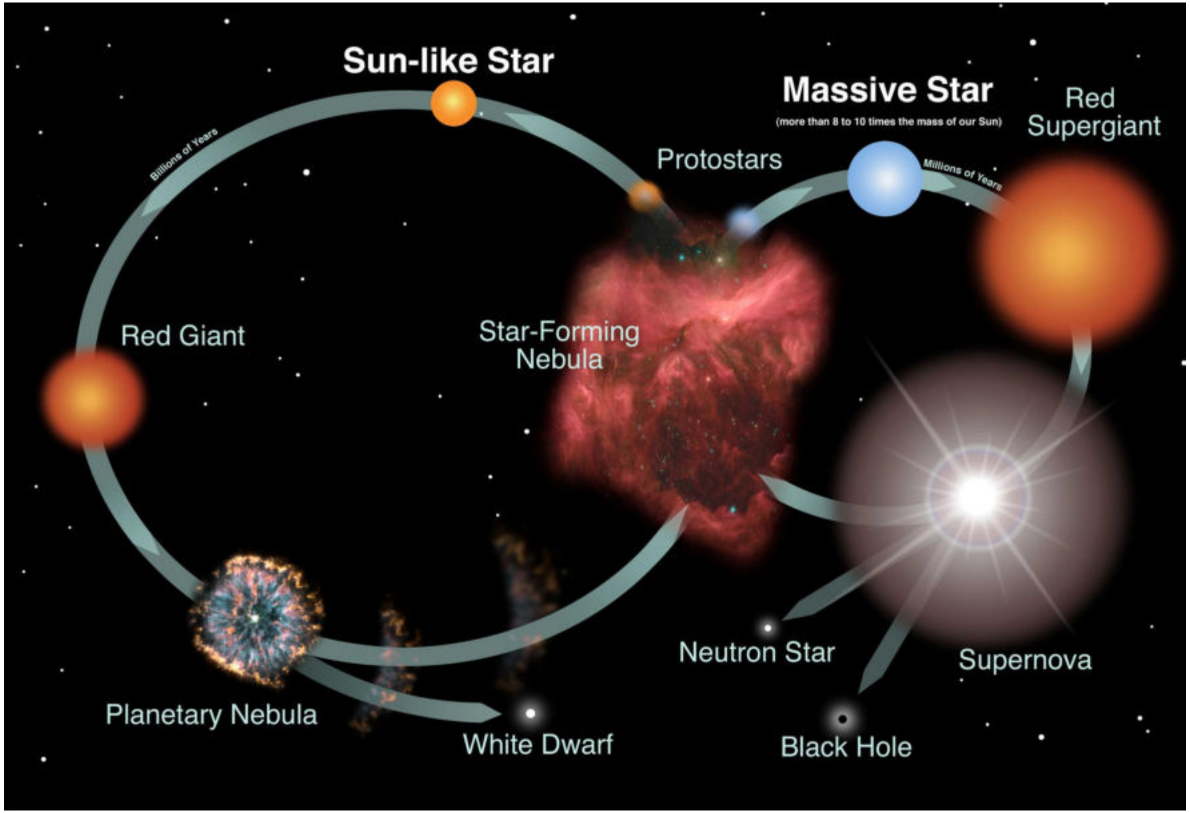
Massive Stars

- Massive stars do not stop with helium fusion – a variety of nuclear reactions creates heavier elements
- Elements in the universe heavier than helium and up to the Fe-peak were created by massive stars
- Once iron is reached, the core is out of fuel and it collapses
- New elements are blown into space along with its outer layers



Cosmic Recycling

- The loss of mass by dying stars is a key step in the cosmic recycling scheme
- Stars form from vast clouds of gas and dust
- As they end their lives, stars return part of their gas to the galactic reservoirs of raw material
- Eventually, some of the expelled material from aging stars will participate in the formation of new star systems



- Matter expelled from such stars include atoms that were “freshly synthesized” inside stars
- The raw material of the Galaxy is not only resupplied but also receives infusions of new elements

Solar System Abundances