# Image reconstruction

## The role of regularization


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# Image Reconstruction

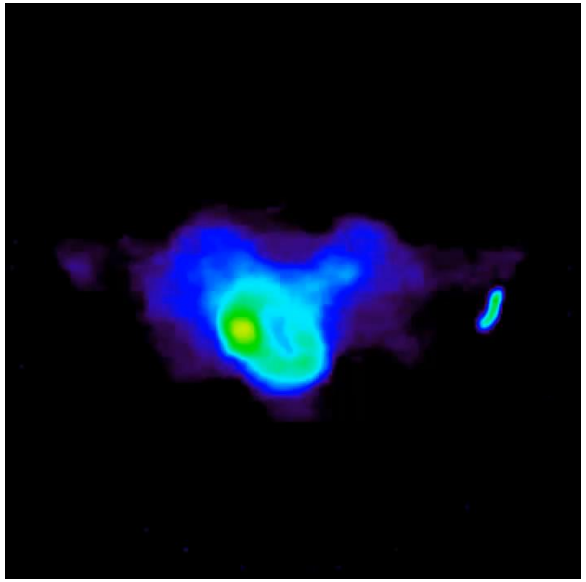
## The underestimated part of imaging

Images (Videos) and their manipulation are part of our daily life

First step of image formation often underestimated, although often the enabling part, cf. **CT = Computed** Tomography

Information / quality **loss in** image formation / **reconstruction can hardly be recovered later**

Strong demand on methods for reconstruction and uncertainty quantification in many application fields, from nano to macro

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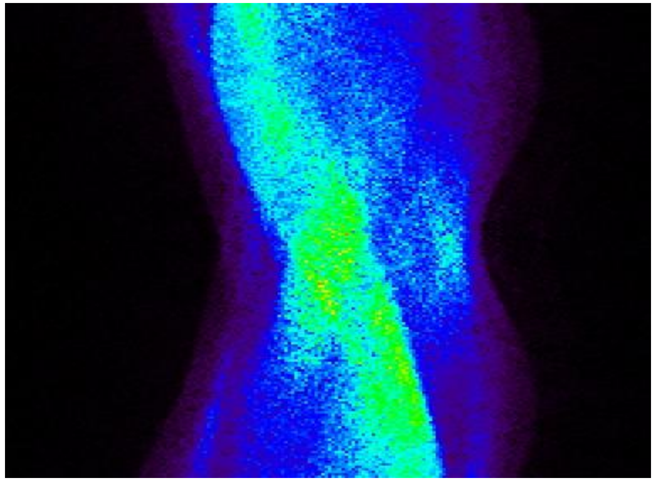
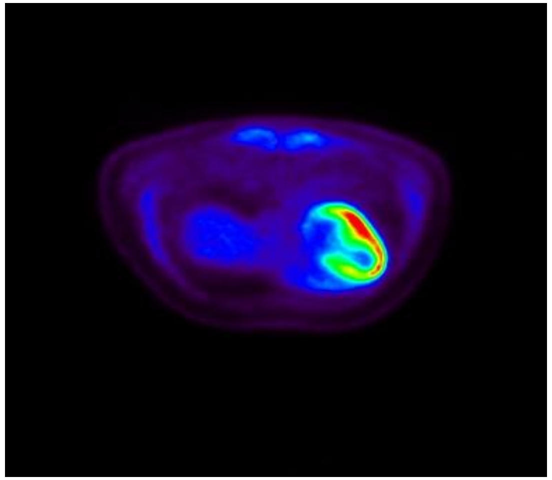
# Emission Tomography

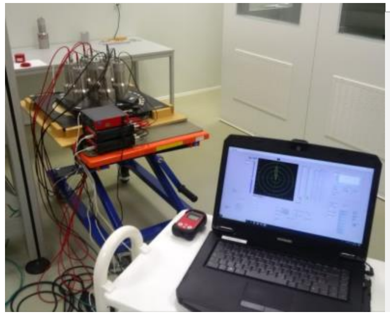
## Active / Passive

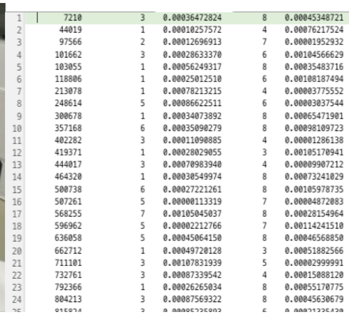
Idea: detect photons emitted e.g. from radioactive decay, with some kind of directional information

- Coincidence based (e.g. PET)
- Collimator based (e.g. SPECT)
- Energy based (Compton effect)
- …









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| 3 | 97566 | 2 | 0.00012696913 | 7 | 0.00001952932 |
| 4 | 101662 | 3 | 0.00028633370 | 6 | 0.00104566629 |
| 5 | 103055 | 1 | 0.00056249317 | 8 | 0.00035483716 |
| 6 | 118806 | 1 | 0.00025012510 | 6 | 0.00108187494 |
| 7 | 213078 | 1 | 0.00078213215 | 4 | 0.00003775552 |
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| 9 | 300678 | 1 | 0.00034073892 | 8 | 0.00065471901 |
| 10 | 357168 | 6 | 0.00035090279 | 8 | 0.00098109723 |
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| 12 | 419371 | 1 | 0.00028029055 | 3 | 0.00105170941 |
| 13 | 444017 | 3 | 0.00070983940 | 4 | 0.00009907212 |
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| 16 | 507261 | 5 | 0.00000113319 | 7 | 0.00004872083 |
| 17 | 568255 | 7 | 0.00105045037 | 8 | 0.00028154964 |
| 18 | 596962 | 5 | 0.00002212766 | 7 | 0.00114241510 |
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| 20 | 662712 | 1 | 0.00049720128 | 3 | 0.00051882566 |
| 21 | 711101 | 3 | 0.00107831939 | 5 | 0.00002999991 |
| 22 | 732761 | 3 | 0.00087339542 | 4 | 0.00015088120 |
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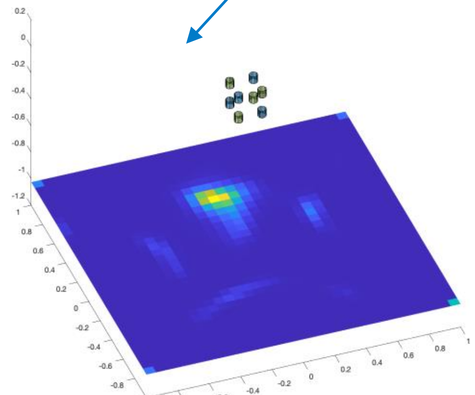
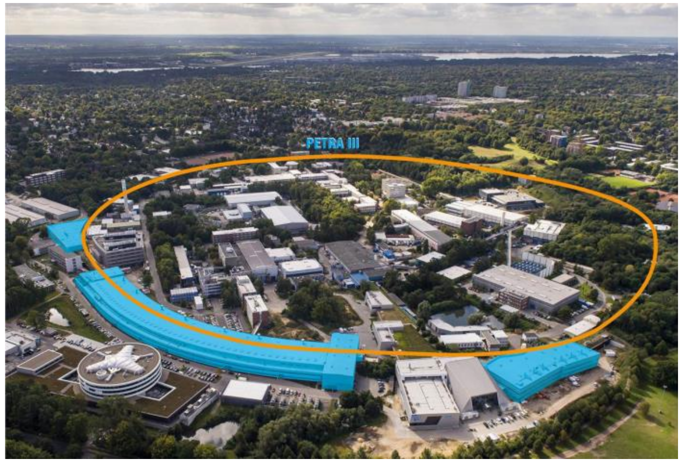
| 24 | 804213 | 3 | 0.00087569322 | 8 | 0.00045630679 |

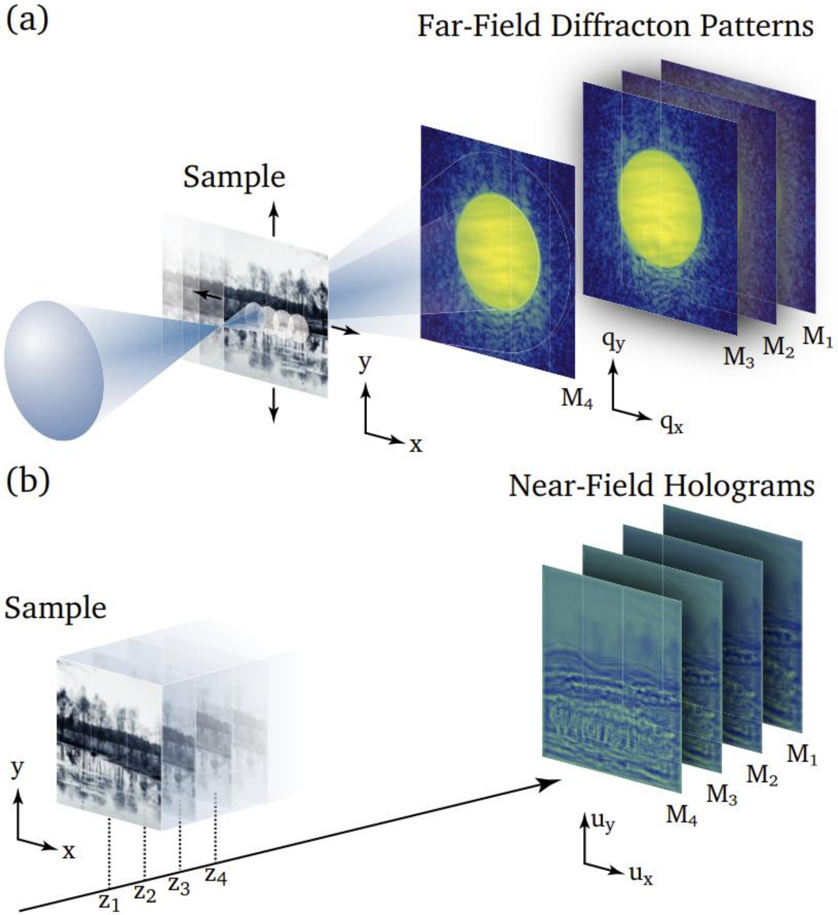


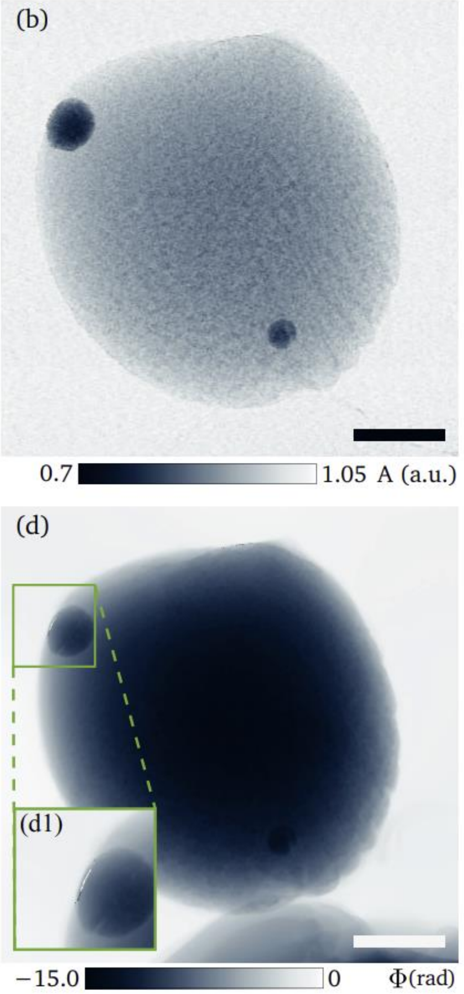
# Image reconstruction from synchrotron x-ray sources

## Ptychographic / Holographic Tomography

### Wittwer et al 2023

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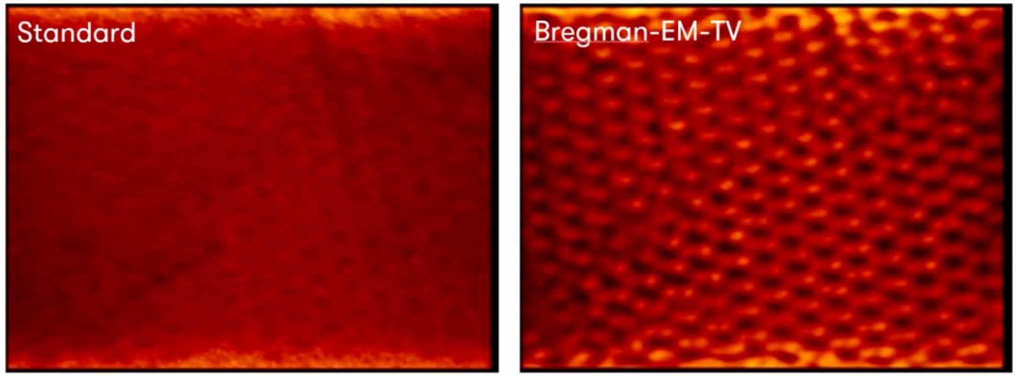
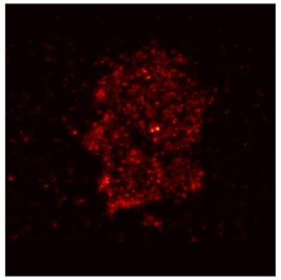
# Image reconstruction across scales and planets

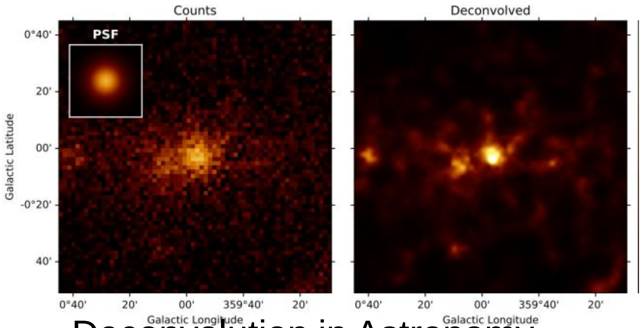
## From nano to macro, from intracellular to outer space



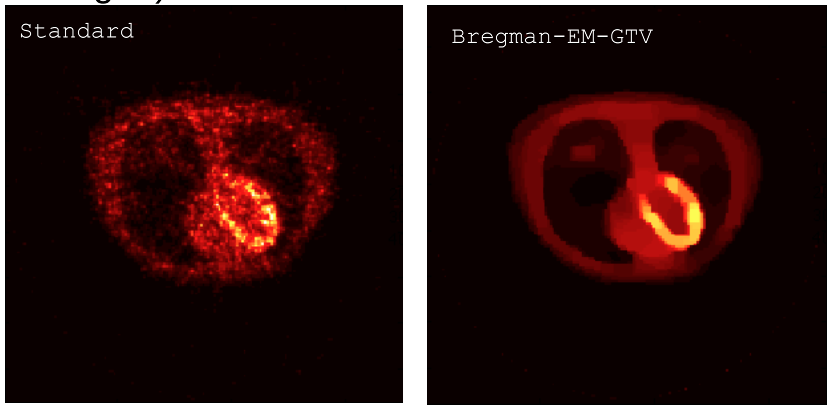

STED Deconvolution of Bead Crystal Structure (with Hell Lab, Göttingen)



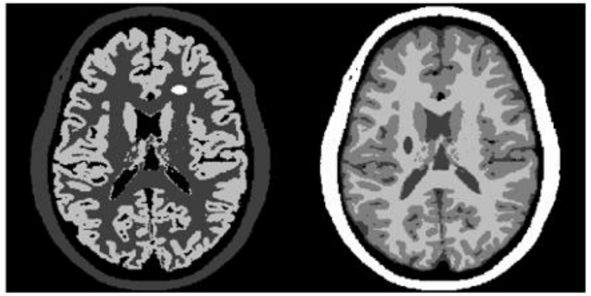
4Pi Deconvolution of Syntaxin PC12 (with Hell Lab, Göttingen)



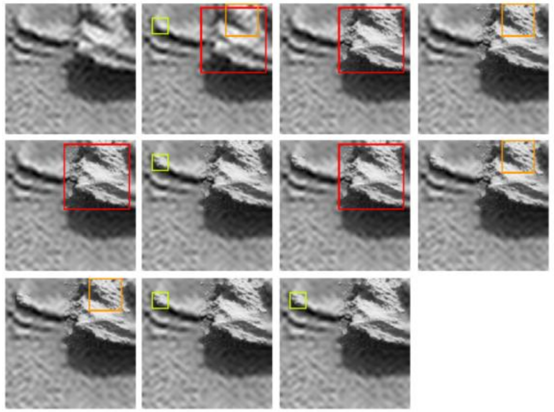

Deconvolution in Astronomy Donath et al 2022




$^{18}$FDG-PET Reconstruction from short time data (with Nuclear Medicine, Münster)



PET-MR, Rasch-Brinkmann-Burger 2017



Energy Efficient THZ Imaging on Mars, with DLR Berlin

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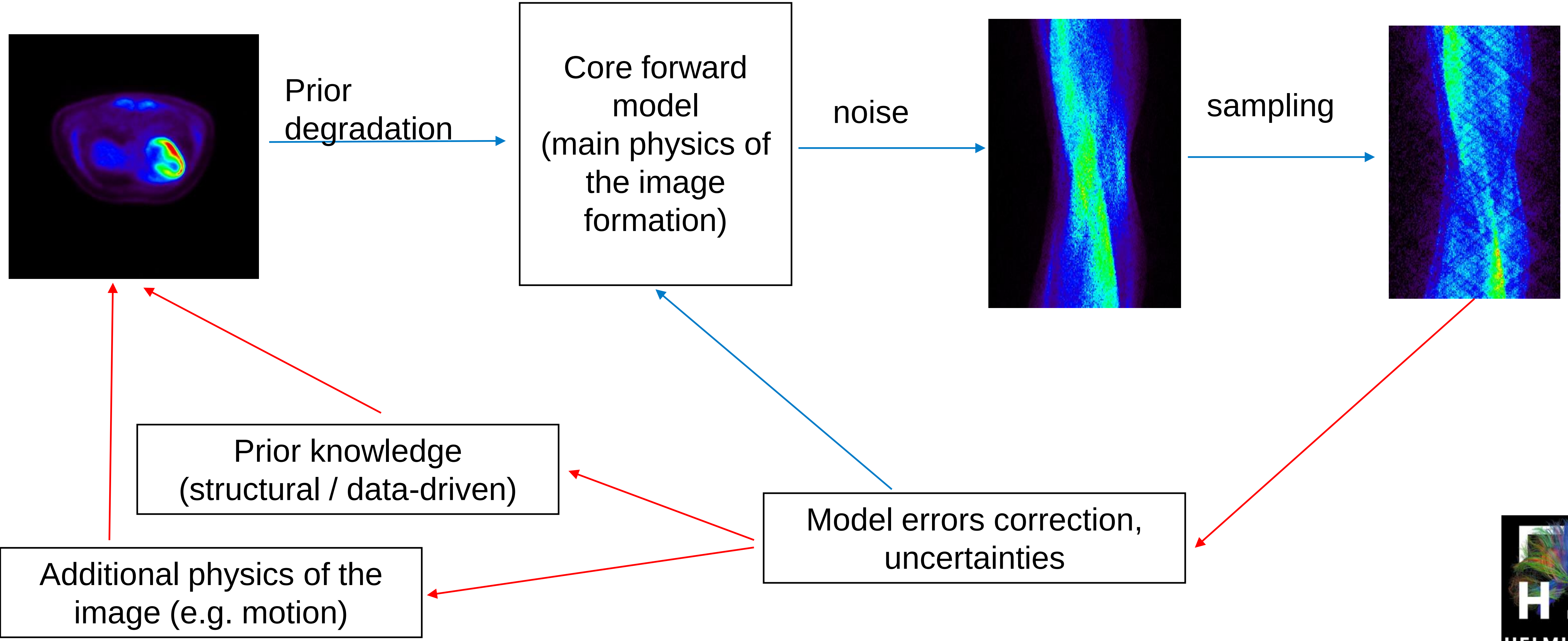
# Modern image reconstruction

## Model based view



# Model based approaches

## The classical way of image reconstruction

**Formulation as an inverse problem**

- Derive physical model of(idealized) forward operator mapping from image to data
- Derive statistical model of noise (e.g. Poisson distribution for photon counts)
- Derive mathematical model of favourable images and structures (e.g. sparsity)
- Possibly add uncertainties

**Condensed in Bayesian posterior model**

$$\pi(u|f) = \frac{1}{\pi_*(f)}\pi(f|u)\pi_0(u)$$

**Likelihood** (from u to f) includes forward and noise model**, prior** includes model of favourable images

# Model based variational methods

## Point estimates

Bayesian **MAP estimate**

$$\hat{u} \in \arg\min_u \left( -\log \pi(f|u) - \log \pi_0(u) \right)$$

Related to **variational regularization method**

$$\hat{u} \in \arg\min_u \Big( F(Ku, f \;) + \alpha J(u) \Big)$$

Simplest case: Gaussian likelihood / prior = quadratic functional = linear equation

Forward operator *K*, data fidelity *F*, regularization functional *J*

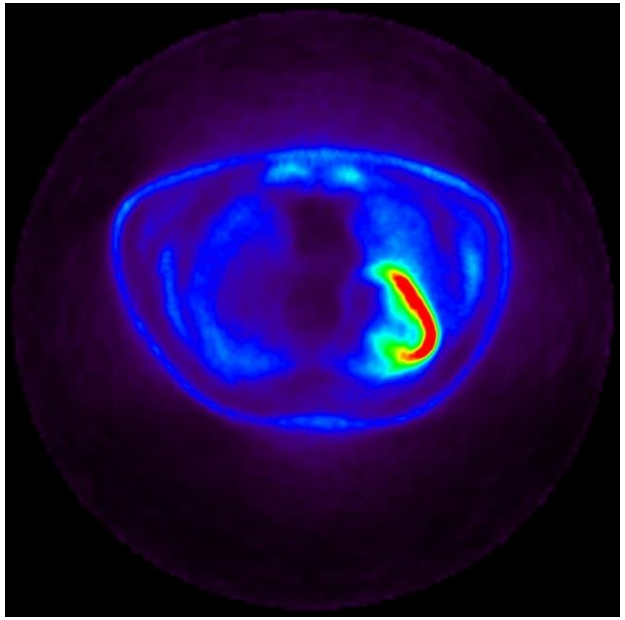
**Forward operator:** physics (examples: convolution, Radon transform, wave propagation, …)
**Data fidelity:** stochastics (examples: additive Gaussian noise, Poisson distribution, …)
**Regularization:** art ?  How to translate structural properties into a functional ?

# Model based variational methods

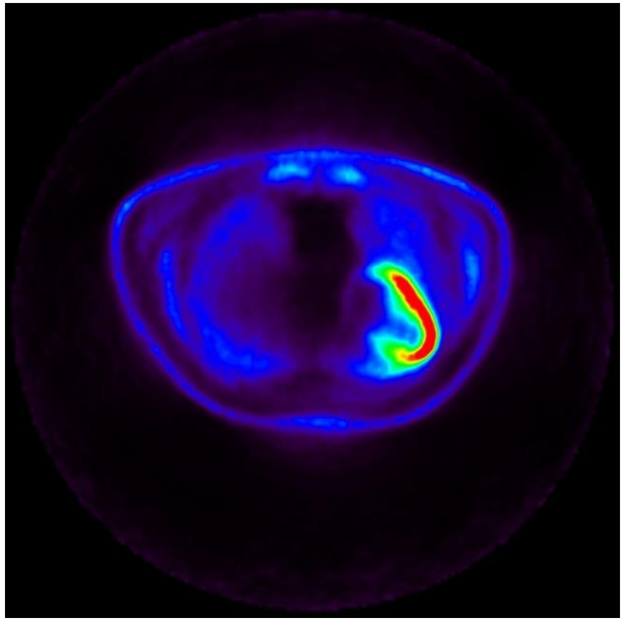
## Improving forward models

Example: PET

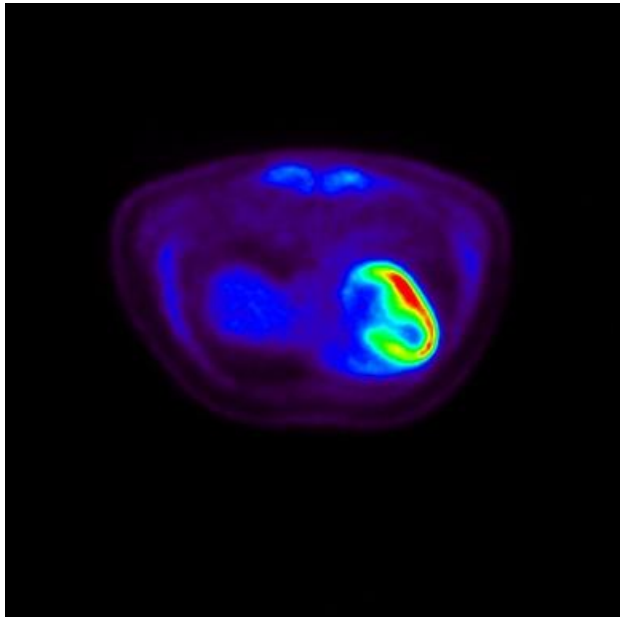
**Radon + photon count noise**



**Radon + photon + scattering**



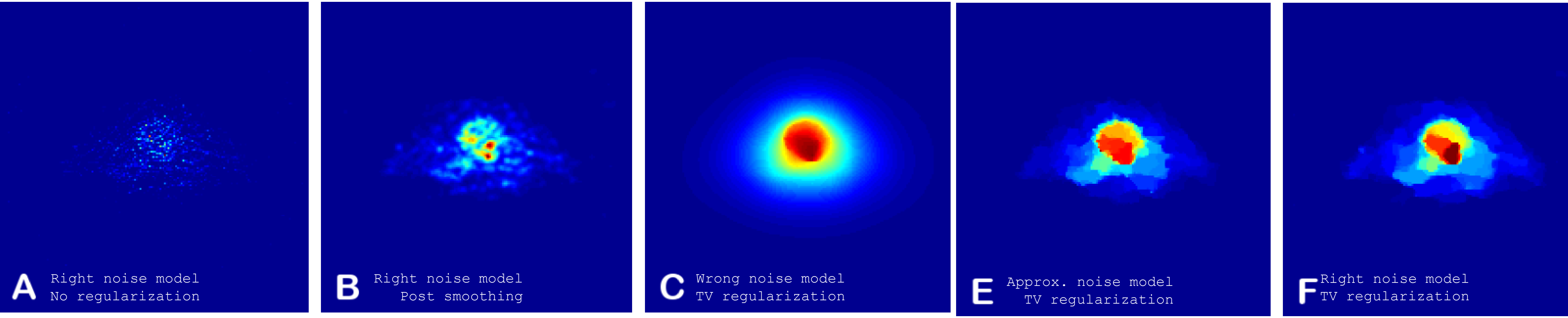
**Radon + photon + scattering + attenuation**



# Model based variational methods

## Improving noise models

Example: PET




Cardiac $^{15}H_2O$ PET: Sawatzky, Brune, Müller, Burger 2009

# Variational Models

## The role of regularization

Recall variational model

$$\hat{u} \in \arg\min_u \left( F(Ku, f) + \alpha J(u) \right)$$

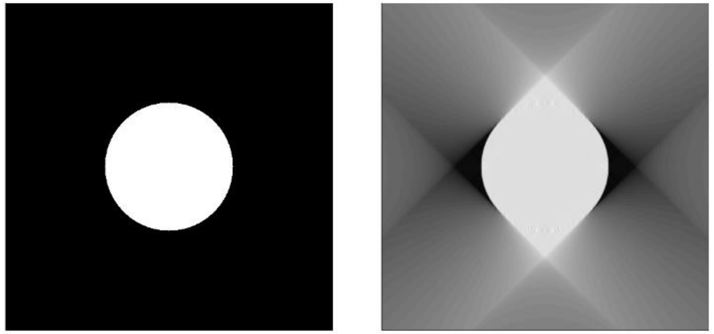
Optimality condition

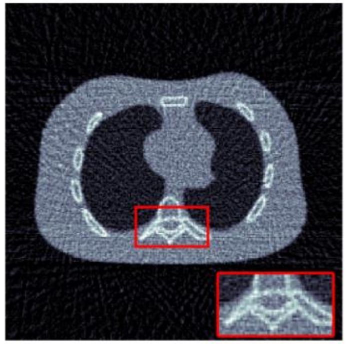
$$K^* \partial_x F(Ku, f) + \alpha p = 0, \qquad p \in \partial J(u).$$

Every solution satisfies the source condition (range condition)

$$p = K^* w \qquad\qquad J(u) = \frac{1}{2}\|u\|^2 \Rightarrow p = u$$

This is an abstract smoothness condition, determines essentially which solutions are preferred / artefacts





# Folklore of Reconstruction

## Use no / minimal prior knowledge

Every reconstruction method uses some prior knowledge, but often it is hidden

Example: fixed point iteration for $Ku = f$ / gradient method for least squares $\|Ku - f\|^2$

$$u^{k+1} = u^k - \tau^k K^*(Ku^k - f)$$

Compute

$$u^1 = u^0 + K^*(\tau^0 f - \tau^0 K u^0) = u^0 + K^* w^0$$

And

$$u^2 = u^1 + K^*(\tau^1 f - \tau^1 K u^1) = u^0 + K^* w^1 + K^*(\tau^1 f - \tau^1 K u^1) = u^0 + K^* w^2$$

Inductively we see

$$u^k = u^0 + K^* w^k$$

# Model based regularizations

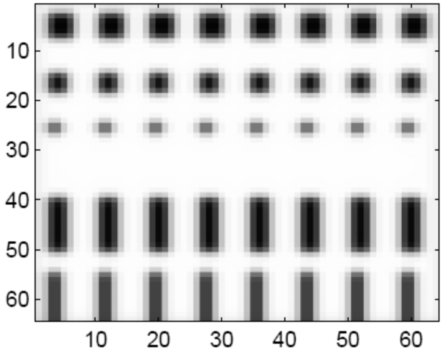
## Images with sharp edges

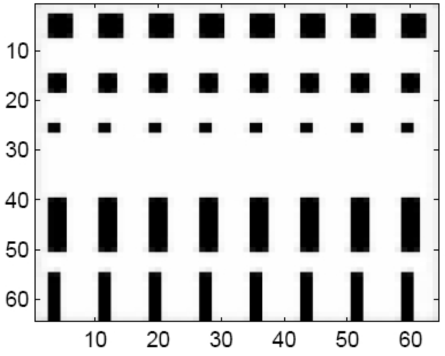
Basic idea from denoising: want to smooth out random noise – local averaging

Simplest idea: Dirichlet energy - quadratic gradient regularization (Gaussian prior)

$$J(u) = \int |\nabla u|^2 \, dx$$

Leads to oversmoothing – no sharp edges




Regularity theory works against us: take $K : L^2 \to Y$

Optimality condition yields $p = -\Delta u = K^* w \in L^2$

Regularity at least $u \in H^2$ does not allow sharp edges

# Model based regularizations

## Images with sharp edges

Alternative idea: p-Laplacian energy

Similar regularity for p > 1

Limit: total variation

$$TV(u) = |u|_{BV} := \sup_{g \in C_0^\infty(\Omega)^d, g \in \mathcal{C}} \int_\Omega u \nabla \cdot g \; dx$$

$$\mathcal{C} = \{g \in L^\infty(\Omega) \mid |g(x)| \leq 1 \text{ a.e. in } \Omega\}$$

Optimality condition

$$K^* \partial_x F(Ku, f) + \alpha \nabla \cdot g = 0$$

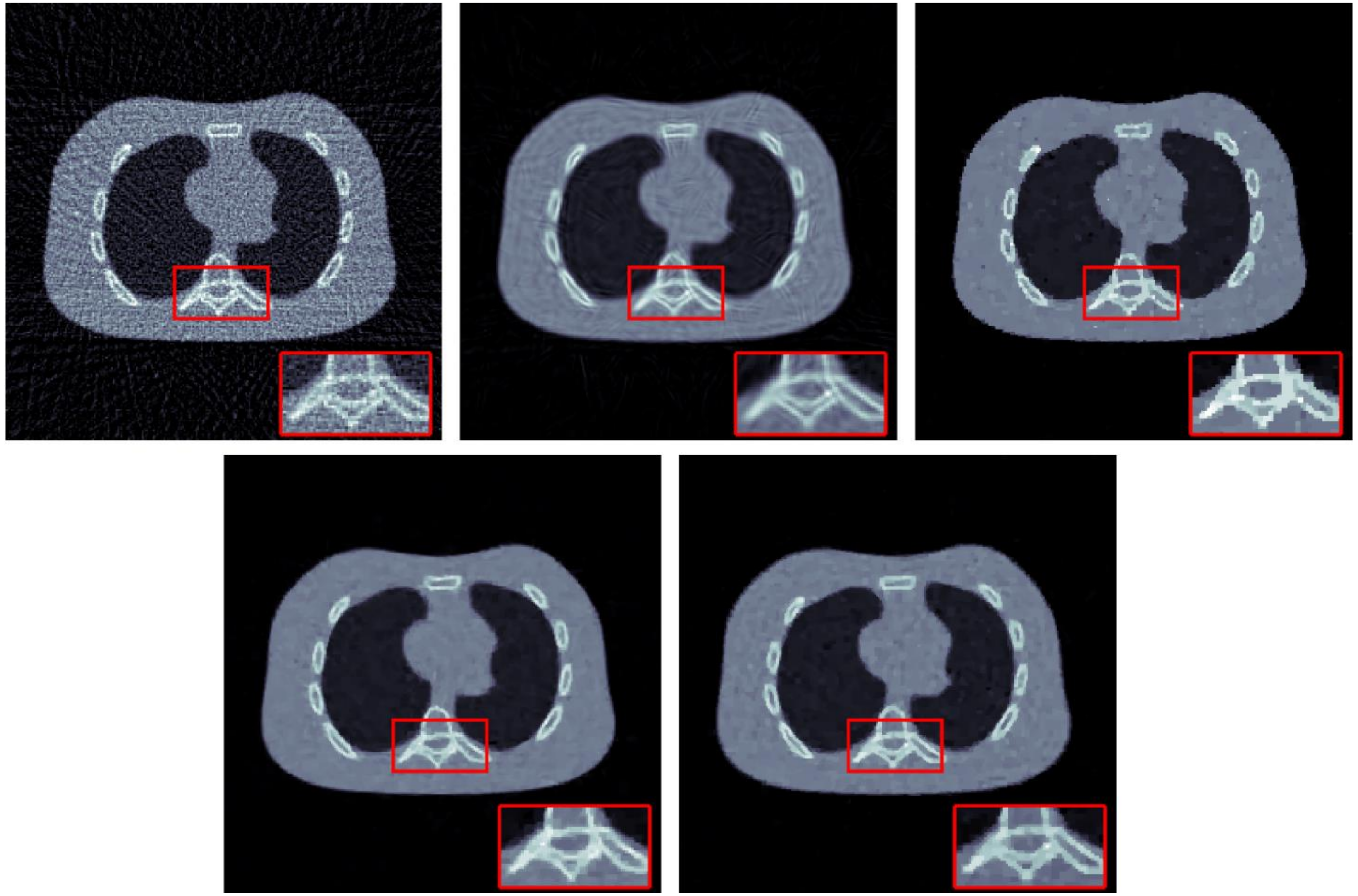
$$g \in \mathcal{C} \qquad \int_\Omega g \cdot dDu = |u|_{BV}$$

Various extensions to cure bias (Bregman iterations) and to avoid staircasing (total generalized variation)

# Model based regularizations

## TV models on sparse angle tomography

Only 50 angles between -90 and 90 degrees measured (Göppel et al 2023)



# Choice of regularization



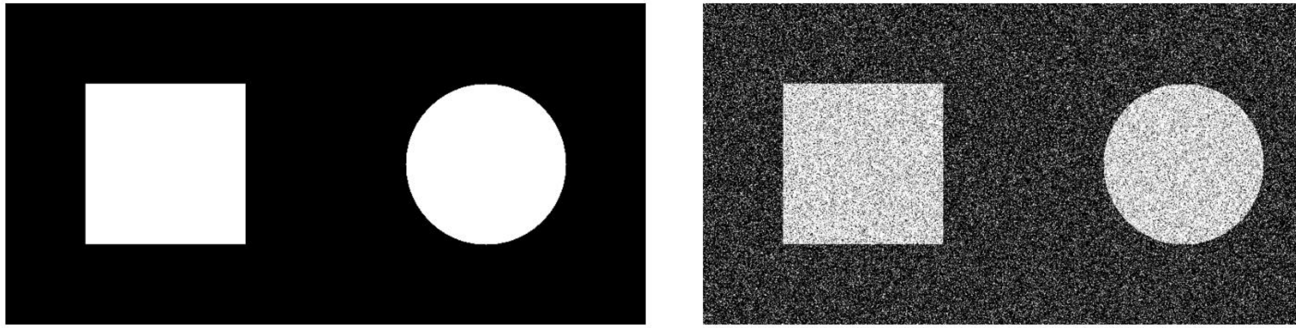
Source condition

$$\nabla \cdot g = K^* w$$

Note that g corresponds to (generalized) normal vector field on level sets (discontinuity sets) of u, its divergence equals mean curvature

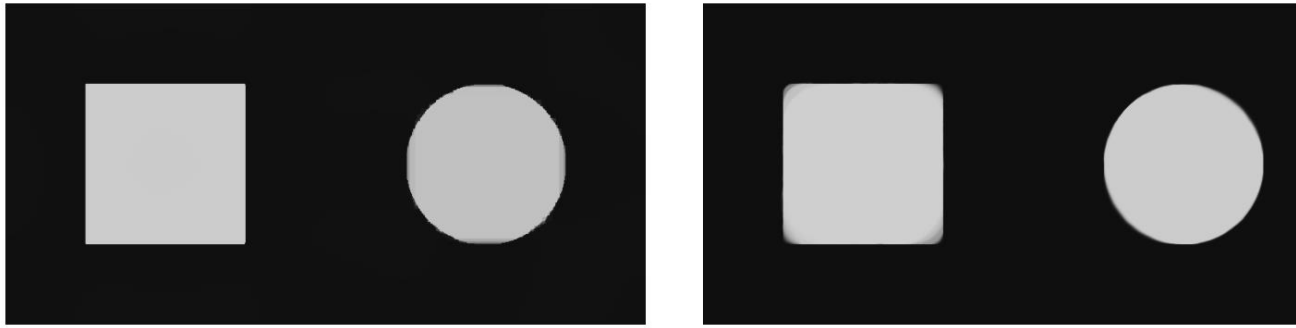
Consequence: solutions of TV regularization can be discontinuous, but have nice discontinuity sets (smooth curvature)

Similar for sparsity and other one-homogeneous regularizations



**(a)** Test image (ground truth)

**(b)** Test image corrupted by additive Gauss noise ($\boldsymbol{\mu} = 0$, $\sigma^2 = 0.25$)



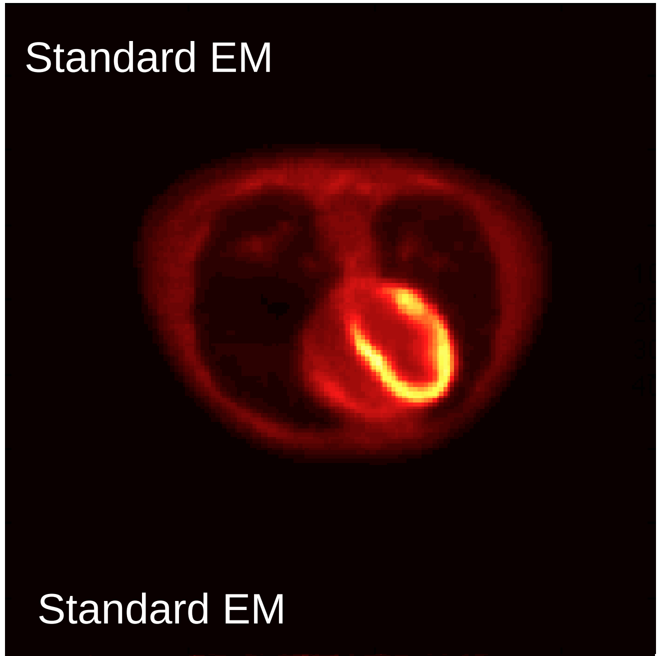
**(c)** Anisotropic TV denoising result ($\alpha = 10$)

**(d)** Isotropic TV denoising result ($\alpha = 1$

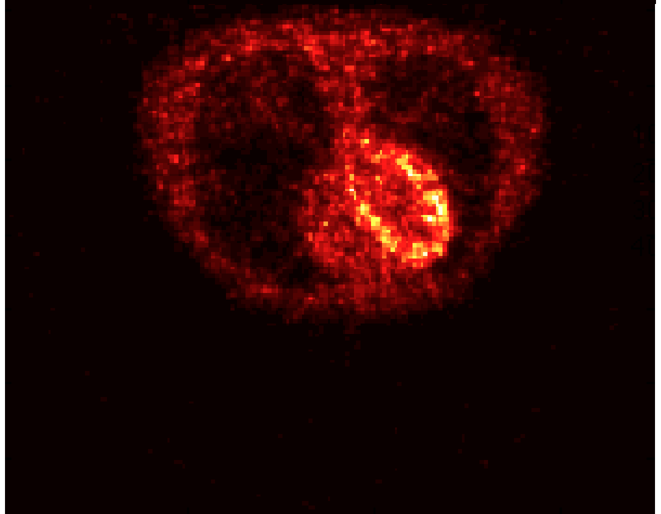
# Total Variation Regularization

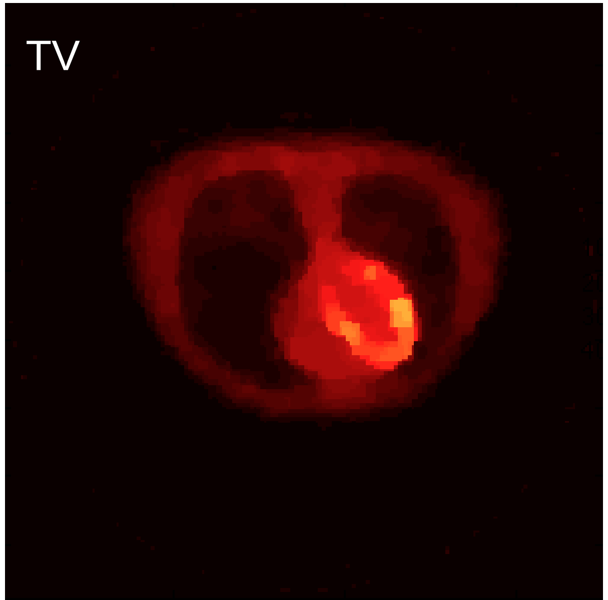
Example: PET reconstruction (inversion of Radon transform with Poisson noise)
[Müller et al 2013]

20min data
(low noise)




5s data
(high noise)



# Variants of total variation



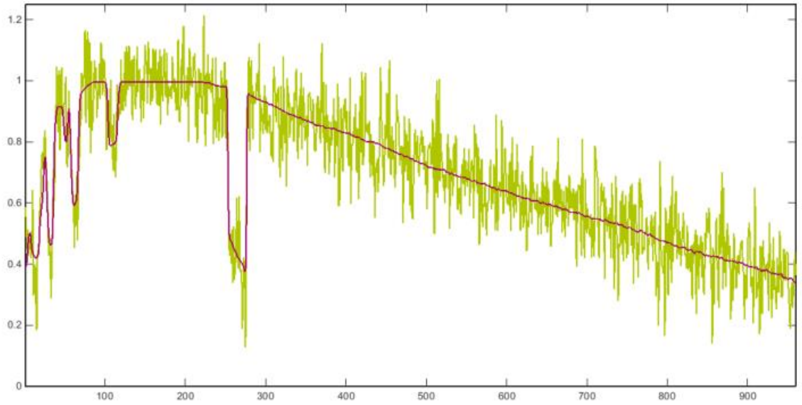
TV regularization suffers from staircasing: piecewise smooth parts often reconstructed by stair-type structure

Example: denoising
K = embedding operatorto $L^2$

[Rudin-Osher-Fatemi 1992]

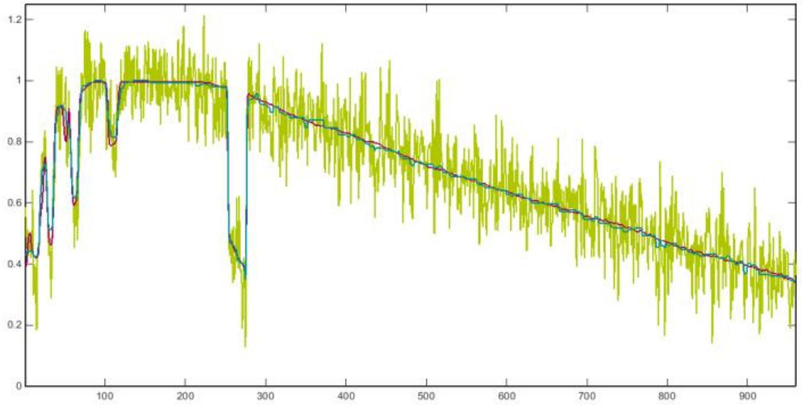
[PhD Brinkmann 2019]

**(g)** Noisy grayscale photo corrupted by additive Gaussian noise ($\mu = 0, \sigma^2 = 0.01$).




**(h)** Line profile of the noisy image and the corresponding row in the noiseless image.

**(i)** TV denoising result with $\alpha = 0.095$.




**(j)** Line plot of the reconstructed image compared against the noisy and the noiseless image.

# Variants of total variation



TV regularization suffers from staircasing: piecewise smooth parts often reconstructed by stair-type structure

Improved versions by infimal convolution [Chambolle-Lions 1997]

$$J(u) = \inf_{u_1+u_2=u} \left( |u_1|_{BV} + |\nabla u_2|_{BV} \right)$$

or total general variation [Bredies-Kunisch-Pock 2010]

$$J(u) = \inf_{Du_1+u_2=Du} \left( |u_1|_{BV} + |u_2|_{BV} \right)$$

Various other generalizations to higher-dimensional (spectral) and time-dependent images

# Model based regularizations

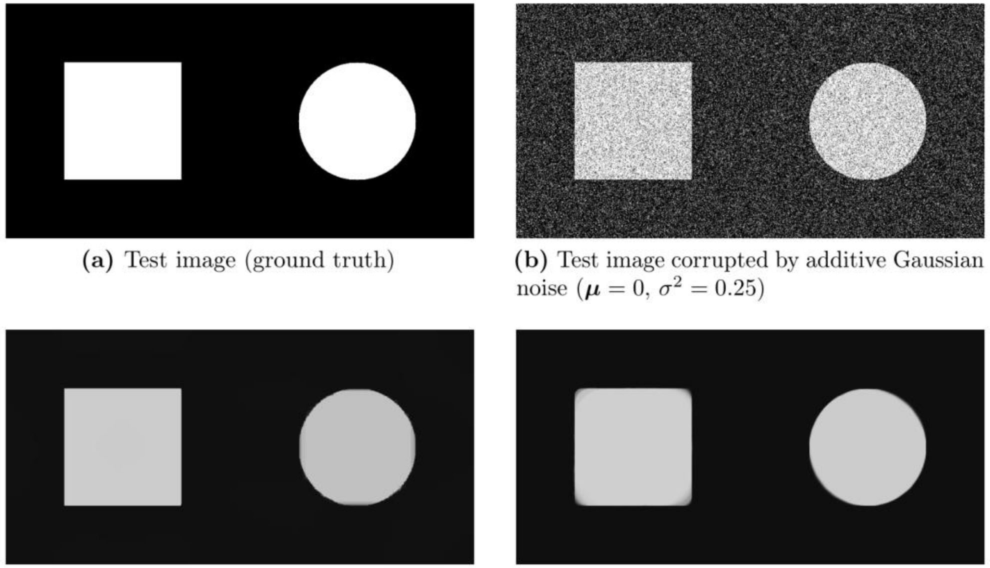
## Total variation and related regularization

Optimality (source condition) $$\nabla \cdot g = K^* w$$

*g* corresponds to (generalized) normal vector field on level lines (surfaces)

Divergence of *g* corresponds to mean curvature

Hence, total variation allows nonsmooth solutions, but smoothes discontinuity sets

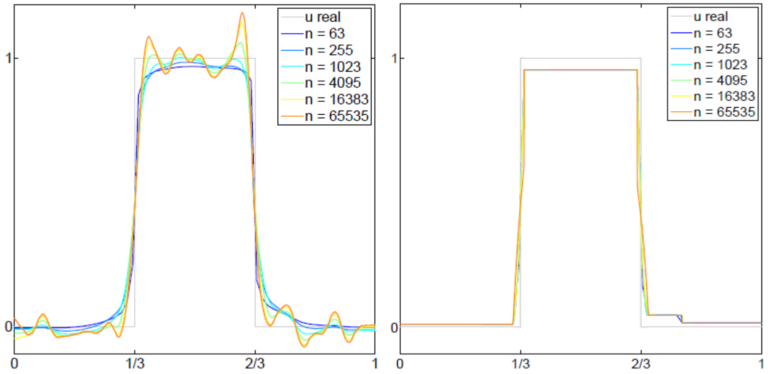


(a) Test image (ground truth)
(b) Test image corrupted by additive Gaussian noise ($\mu = 0$, $\sigma^2 = 0.25$)
(c) Anisotropic TV denoising result ($\alpha = 10$)
(d) Isotropic TV denoising result ($\alpha = 10$)

Problem: modelling very indirect

Prior itself not informative, but only structure of minimizers

Bayesian models for UQ questionable




(a) CM (b) MAP

# Bias

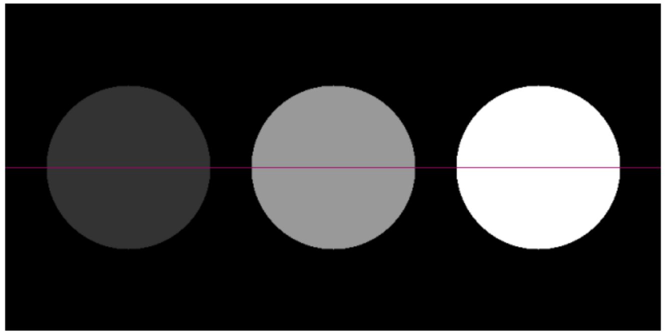


Variational egularization suffers from strong bias

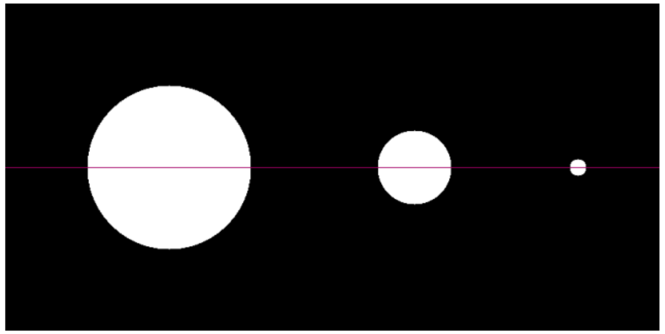
In total variation regularization bias = loss of contrast

[Meyer 2002]

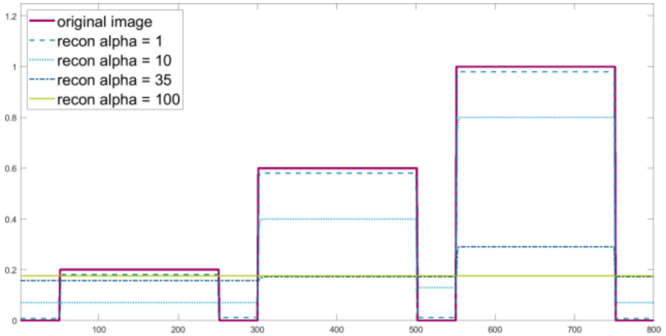
[PhD Brinkmann 2019]



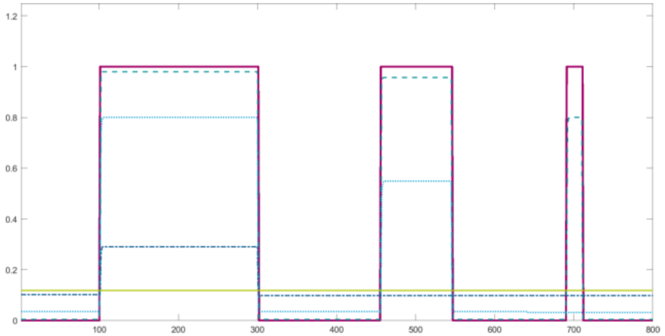
(a) Test image “three circles of equal size”. The pink line corresponds to the line profiles below.



(b) Test image “three circles of equal intensity”. The pink line corresponds to the line profiles below.




(c) Line profiles of ROF reconstructions for the above image for several values of $\alpha$ compared against the original (pink).



(d) Line profiles of ROF reconstructions for the above image for several values of $\alpha$ compared against the original (pink).

# Bias correction



Unfortunately local loss of contrast = missing structures

Example: denoising

clean noisy u f-u

# Bregman iteration



Approximation with penalty

$$\text{minimize } F(Ku,f) + \frac{1}{\tau}\left(J(u) - J(\hat{u}) - \langle \hat{p}, u - \hat{u}\rangle\right)$$

Can be done in multiple steps: Bregman iteration [Bregman 1967] [Hestenes 1969, Powell 1969] [Osher-mb-Goldfarb-Xu-Yin 2005]

$$u^{k+1} \in \arg\min_u F(Ku,f) + \frac{1}{\tau}\Big(J(u) - J(u^k) - \langle p^k, u - u^k\rangle\Big)$$

Optimality condition = dual update

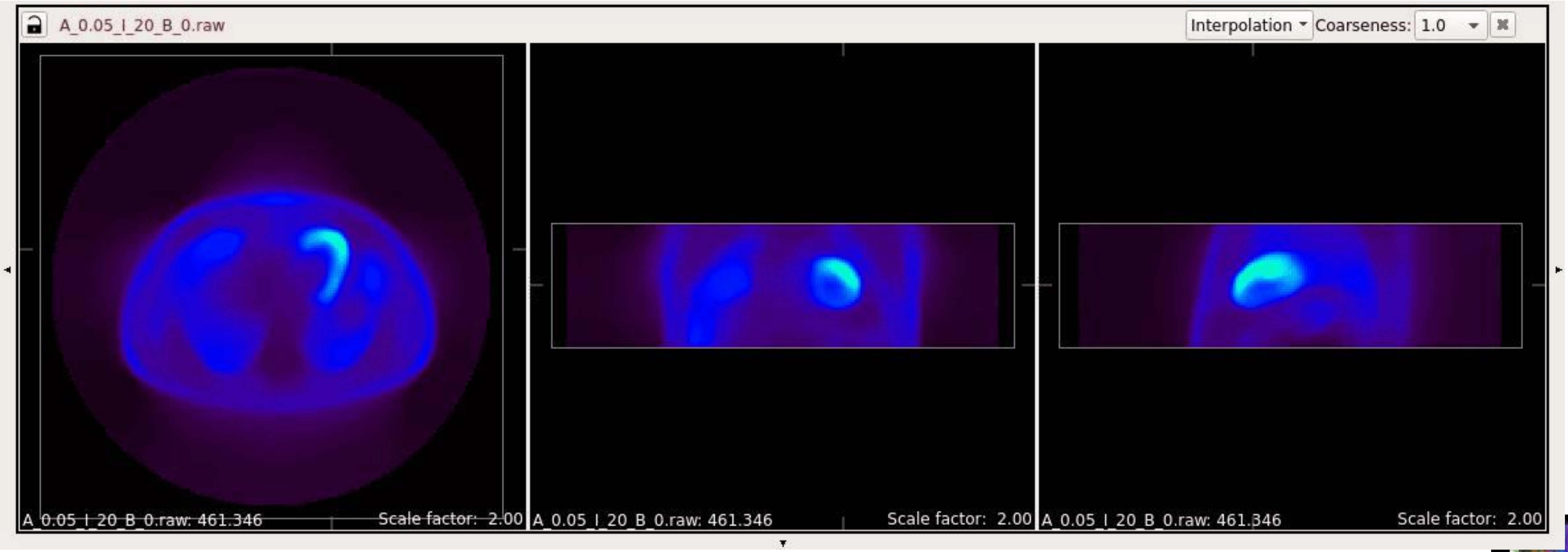
$$p^{k+1} = p^k + \tau K^* \partial F(Ku^{k+1}, f)$$

Bayesian interpretation: recenter prior around last reconstruction (Gauss: shift of mean)

# PET Reconstruction



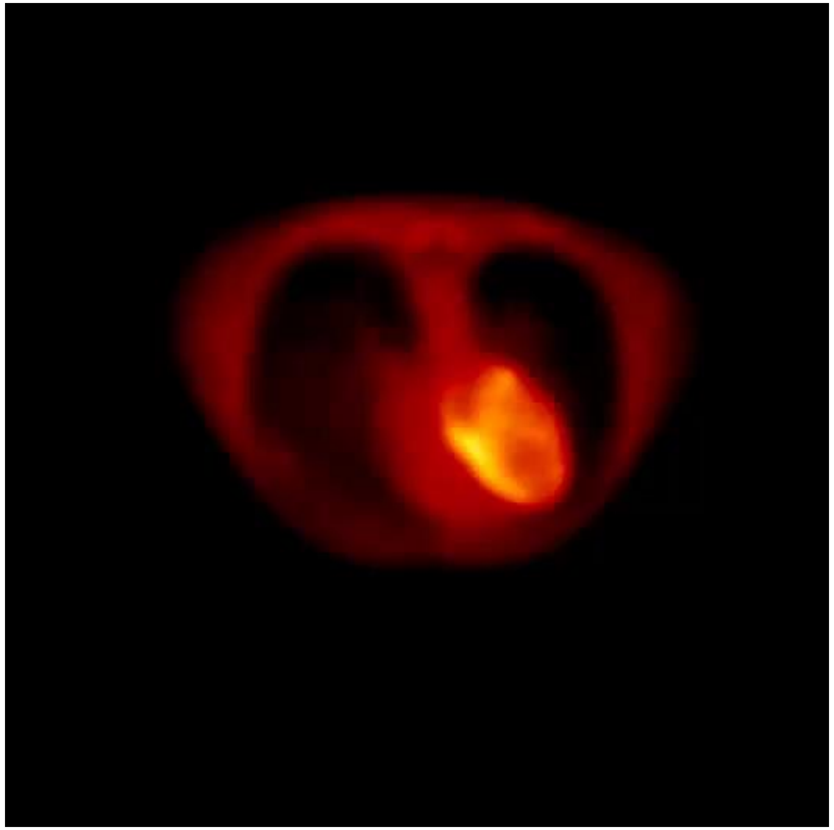
Increasing Bregman iterations



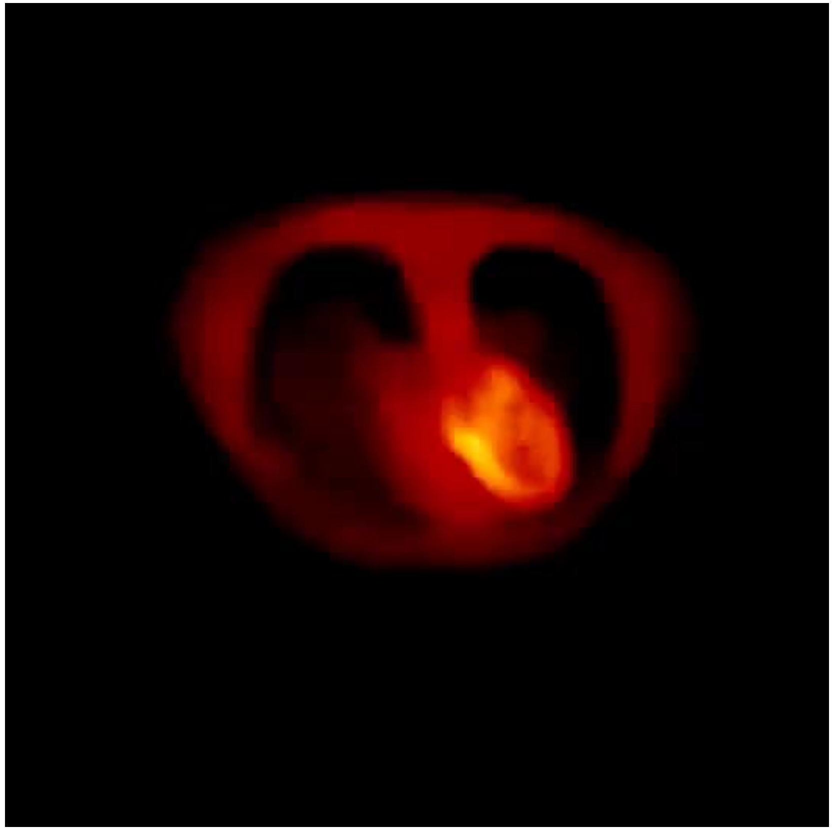
# Model based regularizations

## Cardiac PET Reconstructios

20 min data, EM reconstruction



5s data, Bregman-TGV regularization



# Model based regularizations

## Sparsity regularization

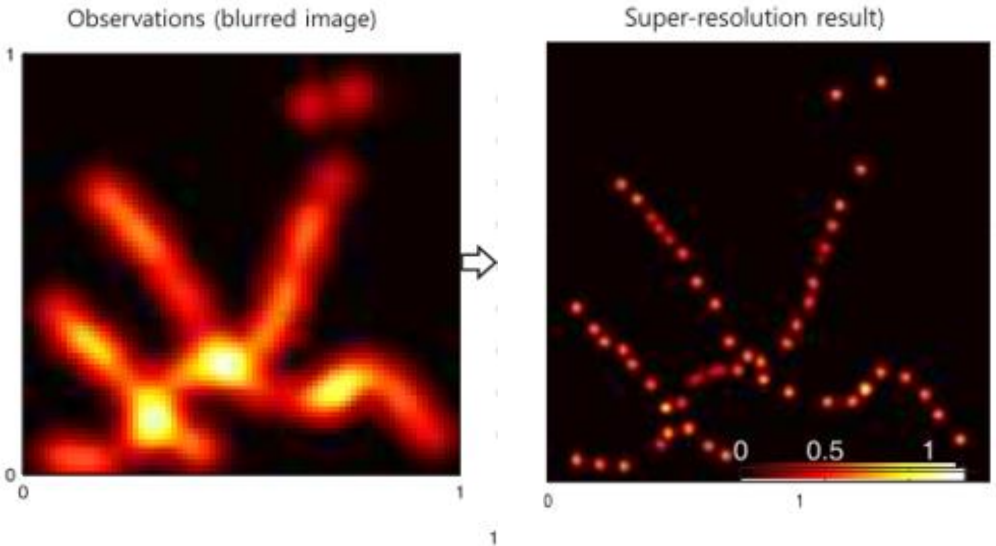
Idea from compressed sensing: choose simple solution (minimal combinations), relax to l1
[Donoho 2006, Candes-Tao 2006]

Analysis formulation: for some frame system choose

$$J(u) = \sum_i |\langle u, \phi_i \rangle|$$

Synthesis formulation (equivalent in case of orthonormal basis)

$$J(u) = \sum_i |c_i| \qquad \text{where } u = \sum_i c_i \phi_i$$



# Bregman iteration, Inverse Scale Space Flow



Bregman Iteration

$$p^{k+1} = p^k + \tau K^* \partial F(Ku^{k+1}, f)$$

Can also be interpreted as implicit Euler discretization with time step *t*

Limit is rather degenerate evolution equation, inverse scale space flow

$$\partial_t p = -K^* \partial F(Ku, f), \quad p \in \partial J(u)$$

[mb-Gilboa-Osher-Xu 2006, mb-Frick-Osher-Scherzer 2007, Brune-Sawatzky-mb 2011,mb-Möller-Benning-Osher 2012]

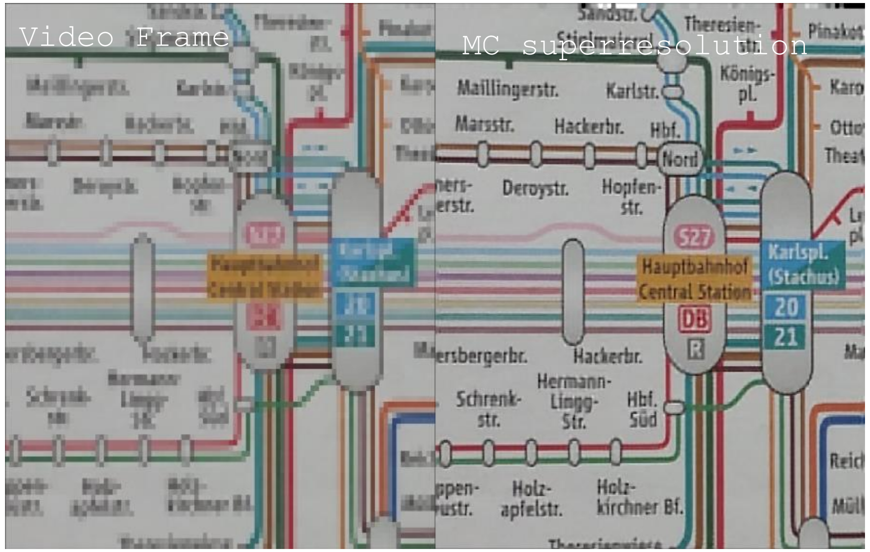
Recent development: stochastic linearized Bregman methods for training sparse deep neural networks [Bungert-Roith-Tenbrinck-mb 2021]

# Dynamic Imaging: motion

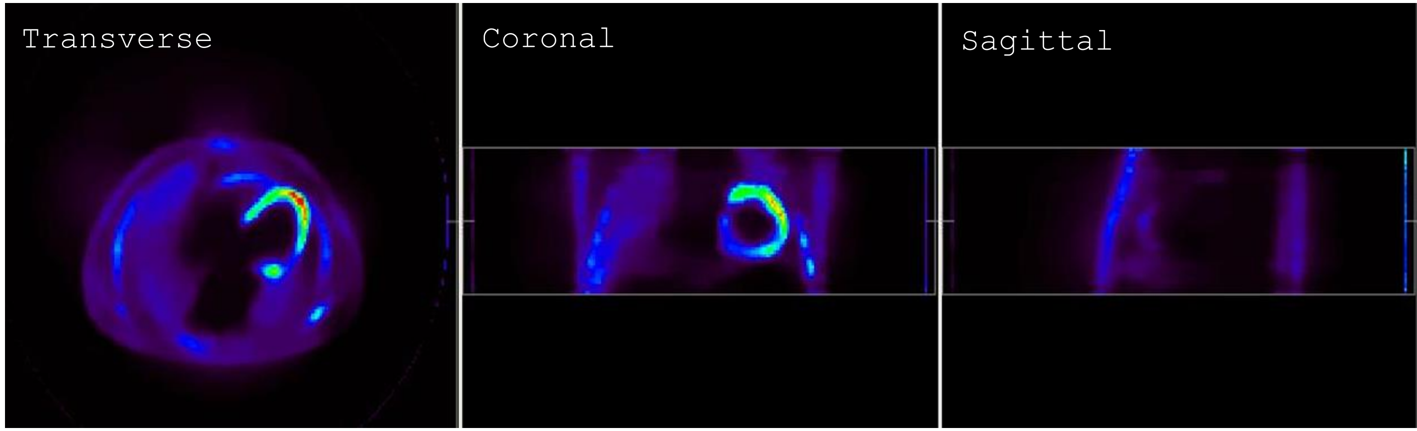
## Additional physics into regularization

Motion-corrected reconstruction,
hyperelastic energy / fluid models for deformation

$F$




Conventional cell phone image from driving subway wagon (H. Dirks)




$^{18}$FDG Cardiac PET, correction of heart and breathing motion

Burger-Dirks-Schönlieb 2018, Burger-Dirks-Frerking-Hauptmann-Helin-Siltanen 2017, Mannweiler Phd 2018

# Regularization and Physics: Motion-Corrected Reconstruction

Measurement of sampled projections at different time steps, motion in between

Simple case: same projection and same noise statistics at each time step (discrete or continuous time)

Lagrangian: transformation operators

$$T(v)u = u(v) \ \det \ \nabla v$$

Eulerian: transformation operators by solving continuity equation

$$\partial_t u + \nabla \cdot (\mathbf{v} u) = 0$$

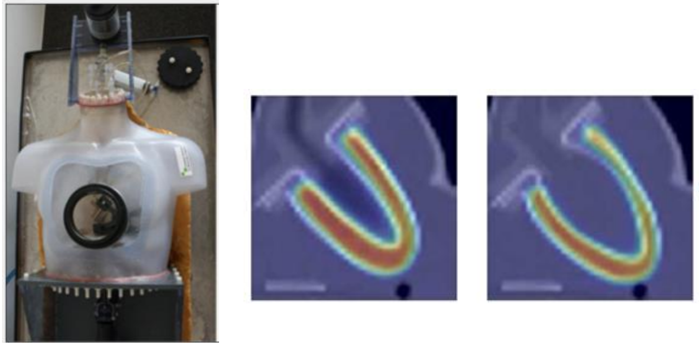
# Regularization and Physics: Motion-Corrected Reconstruction

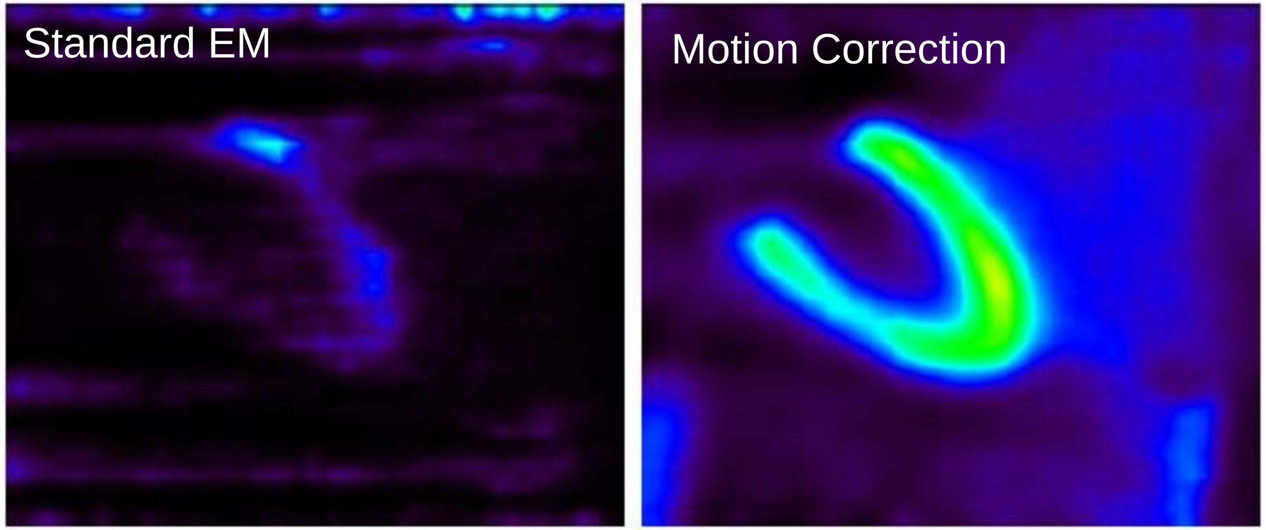
Model in abstract framework: minimize

$$J(u,v) = \sum_{t=0}^{N} D(K_t T(v_t)u, f_t) + \alpha R(u) + \sum_{t=1}^{N} \beta_t S(v_t)$$

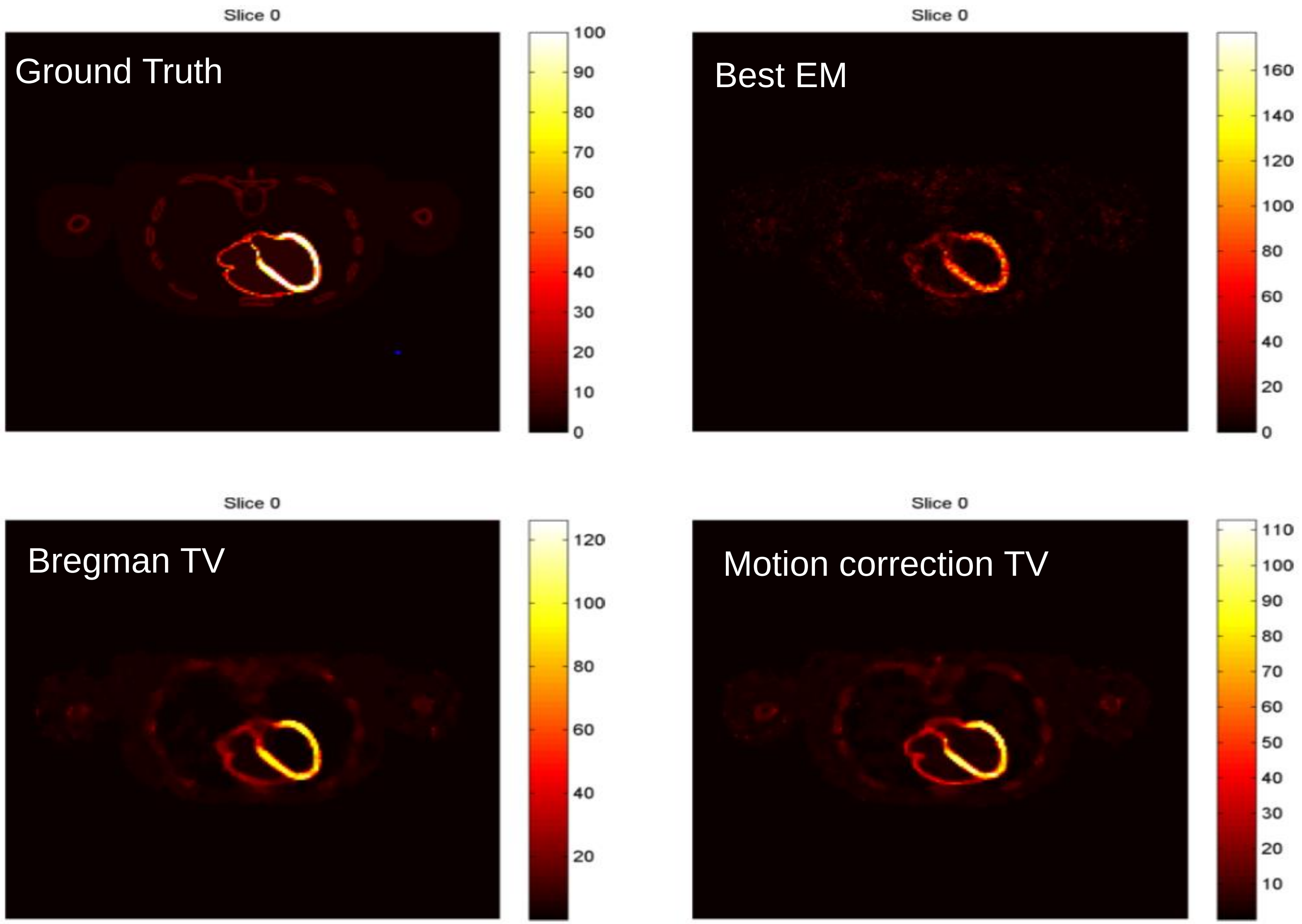
$K_t$ is forward operator, e.g. sampling from Radon transform

PET motion phantom Wilhelm
(University hospital münster)





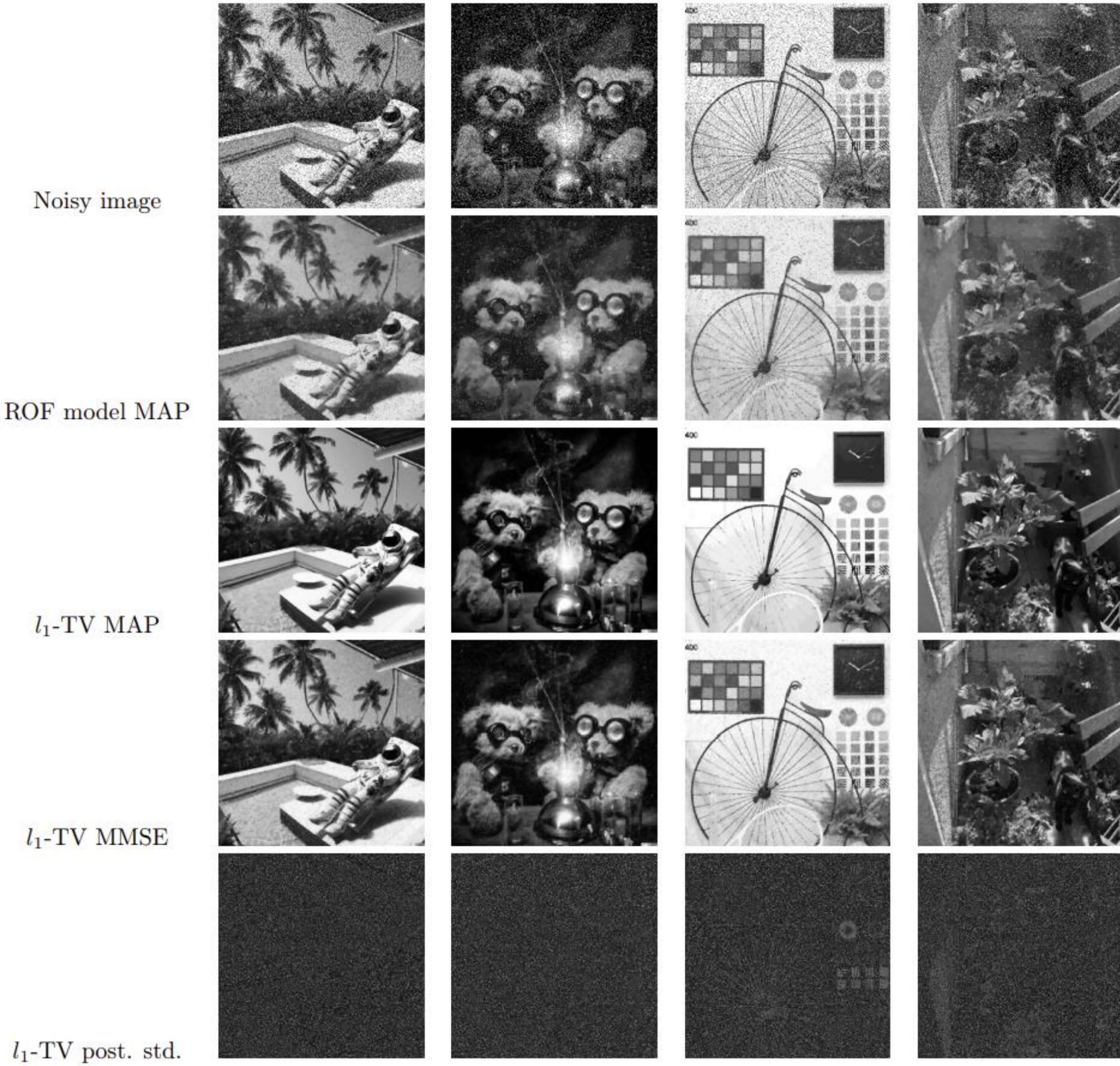
# Regularization and Physics: Motion-Corrected Reconstruction



# Computational Uncertainty Quantification

Modelling, development
of efficient computational
sampling

Here: Primal Dual Sampling,
Lorenz Kuger



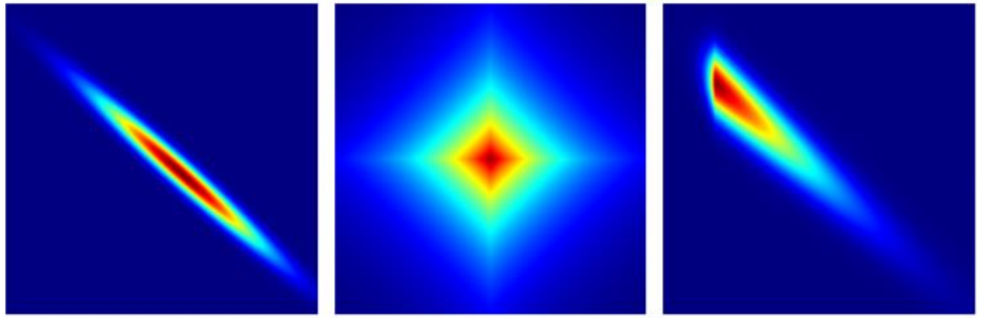
# Uncertainty quantification

## Sampling from the posterior

To evaluate variances, confidence intervals, posterior mean etc sampling schemes are needed

Classical Monte Carlo: Metropolis-Hastings, Gibbs can become inefficient for large-scale problems

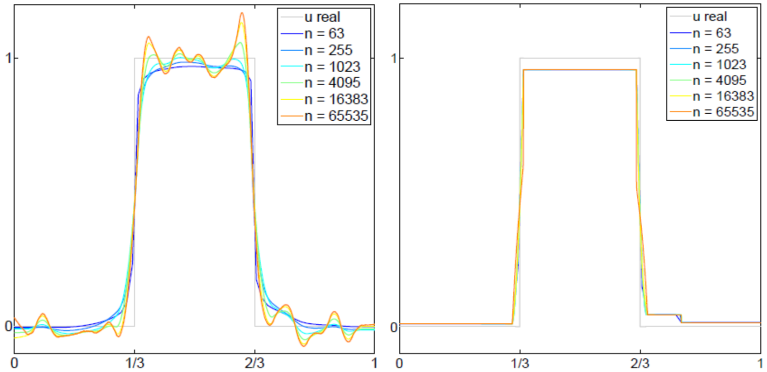
Alternative: Langevin sampling and similar algorithms (modifying optimization by noise)



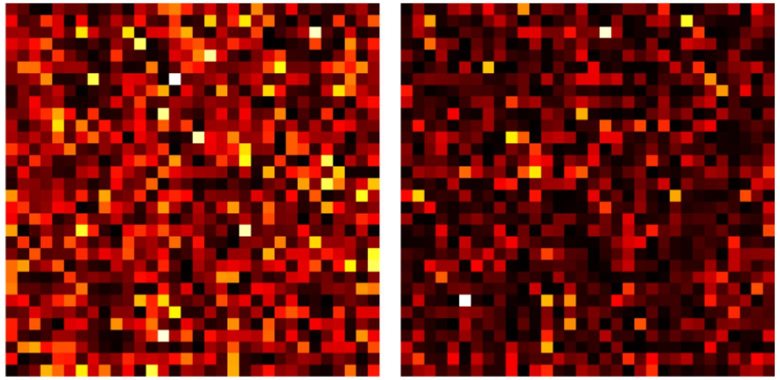
(a) Gaussian likelihood (b) Prior: $p(u) \propto \exp(-\lambda|u|_1)$ (c) Resulting posterior

Remaining difficulty: modelling very indirect. Prior itself eventually not informative, but only structure of minimizers

Bayesian models for UQ questionable. Can better priors be learned ?




(a) CM (b) MAP



(a) $p(u) \propto \exp\left(-\frac{1}{2}\|u\|_2^2\right)$ (b) $p(u) \propto \exp\left(-|u|_1\right)$

# Learning in Inverse Problems

## Supervised learning

### Obvious idea: supervised learning

Use data pairs for input-output related by

$$f^\delta = Ku + \eta$$

Minimize risk with appropriate loss *L* over some neural network architecture

$$\min_\theta \mathbb{E}_{(u,f^\delta)}(L(u,\mathcal{N}_\theta(f^\delta)) = \mathbb{E}_{(u,\nu)}(L(u,\mathcal{N}_\theta(Ku+\nu)))$$

### Issues of supervised learning

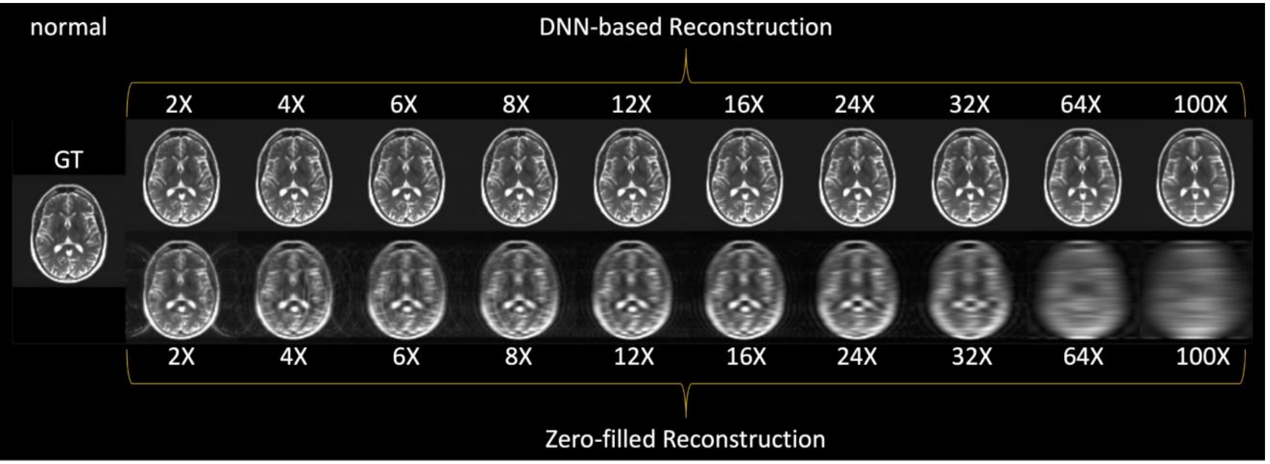
- (Computational) complexity of the inverse problem
- Bad generalization (network for inversion needs huge Lipschitz constant)
- Missing pairs of input-output data

# Learning in Image Reconstruction

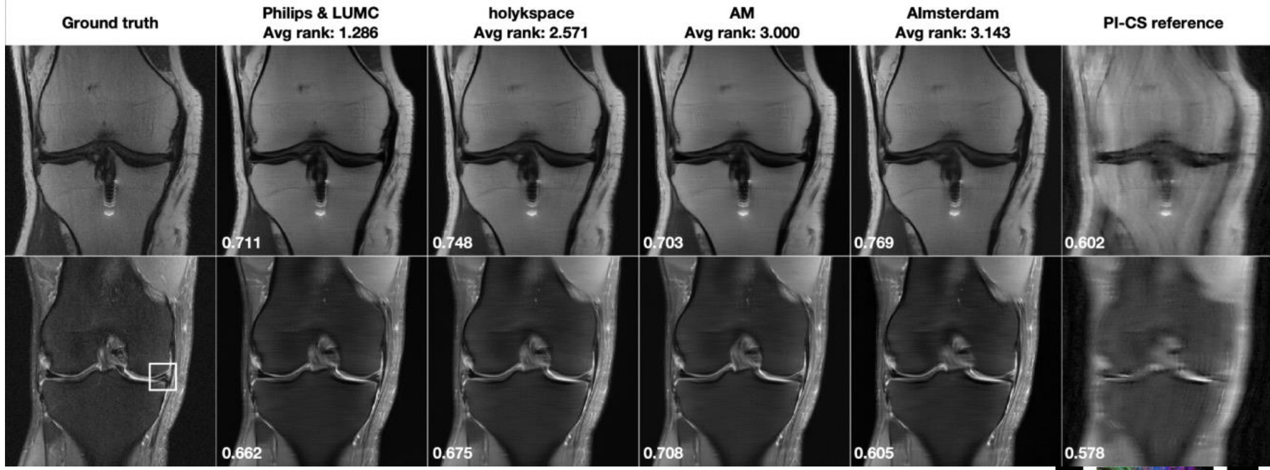
## Undersampled MRI

Undersampling in MRI does not suffer from these issues (partly also in CT):

- Lower complexity, since forward operator just Fourier transform, low noise
- Isometry property of Fourier transform leads to low Lipschitz constant of inverse
- Data pairs from existing fully sampled measurements and reconstructions




Radmanesh Radiology AI 2022




Knoll MRM 2019 / 2020 (fast MRI challenge)

# Learning in Image Reconstruction

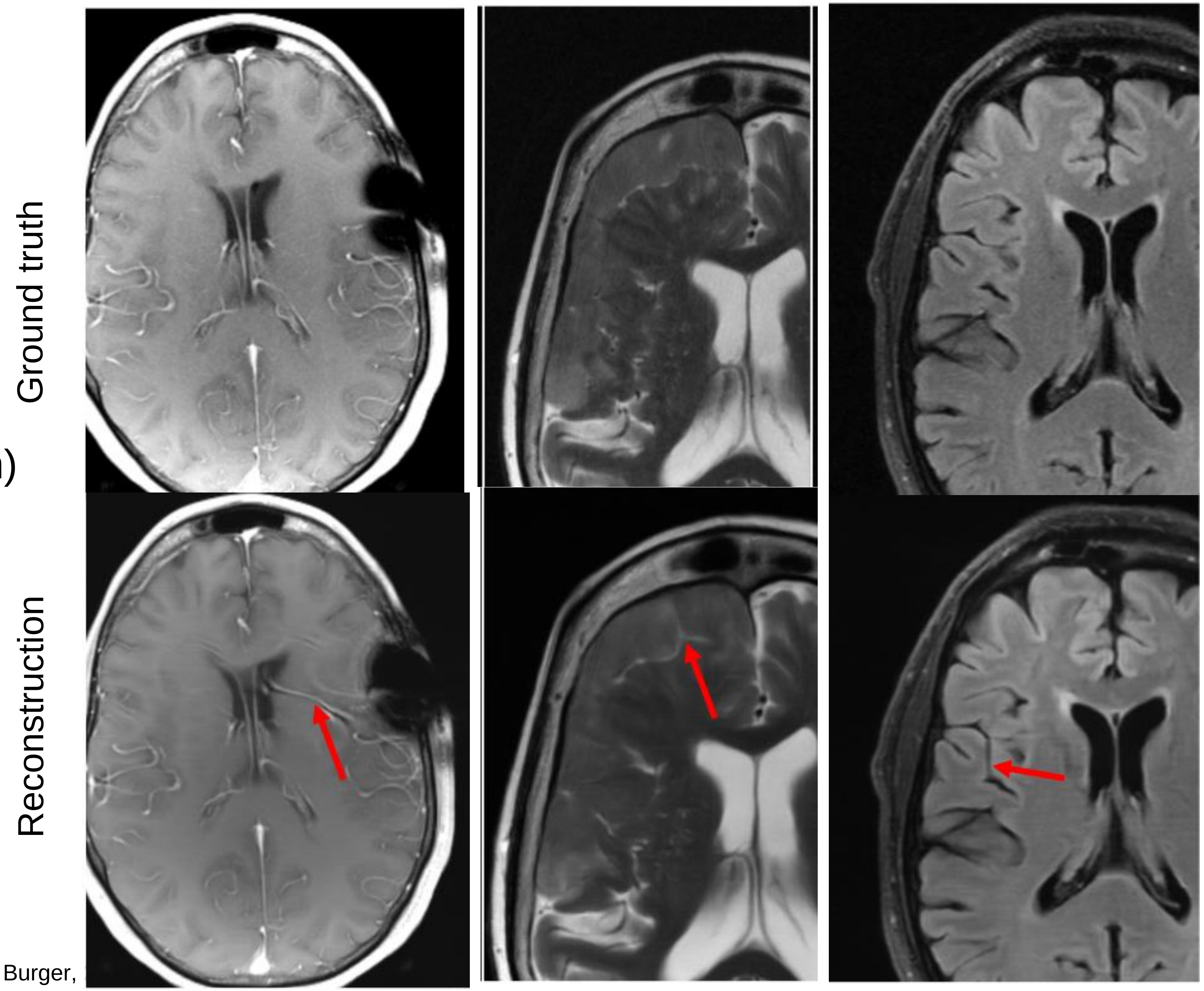
## Undersampled MRI

Majority of results convincing

But possible hallucinations
on few data sets

Not recognizable by
experienced radiologists

(courtesy Florian Knoll, Erlangen)




Muckley TMI 2021

# Further issues in supervised learning

## „Semi-Supervised learning“

**Paradigm: still solve**

$$\hat{u} \in \arg\min_u \left( F(Ku, f^\delta) + \alpha J(u) \right)$$

but with regularizer *J* (and possibly regularization parameter) learned from a database of images (and possibly unrelated noisy data

Bayesian interpretation: directly learn prior, form posterior with forward model

**Examples**

- Adversarial regularizations
- Plug and play priors: trained by denoising on images solely
- Score-based diffusion models: transform prior into Gaussian, construct biased Langevin sampling to go back to approximate sampling of posterior

# Further issues in supervised learning

## Adversarial regularizers

Example: adversarial learning [Lunz-Öktem-Schönlieb 18]

Given favourable images $\{u_i\}_{i=1}^n$ and unfavourable ones $\{v_k\}_{k=1}^m$
minimize (with respect to parameters)

$$\frac{1}{n}\sum_{i=1}^{n} J(u_i) - \frac{1}{m}\sum_{k=1}^{m} J(v_k) + \lambda \mathbb{E}[(\|\nabla J\| - 1)_+^2]$$

Learned regularization method is itself a random variable in terms of training data.
As n and m tend to infinity and under assumption of i.i.d. sampling from appropriate distributions expect convergence to minimizer of deterministic population risk

$$\mathbb{E}_u(J) - \mathbb{E}_v(J) + \lambda \mathbb{E}[(\|\nabla J\| - 1)_+^2]$$

Detailed properties of regularizer and subsequent solutions of inverse problem remain unclear

So far, functionals learned based on data sets, but independent of inverse problem (forward operator *K*).

**Unclear if training data could even be solution of inverse problem**

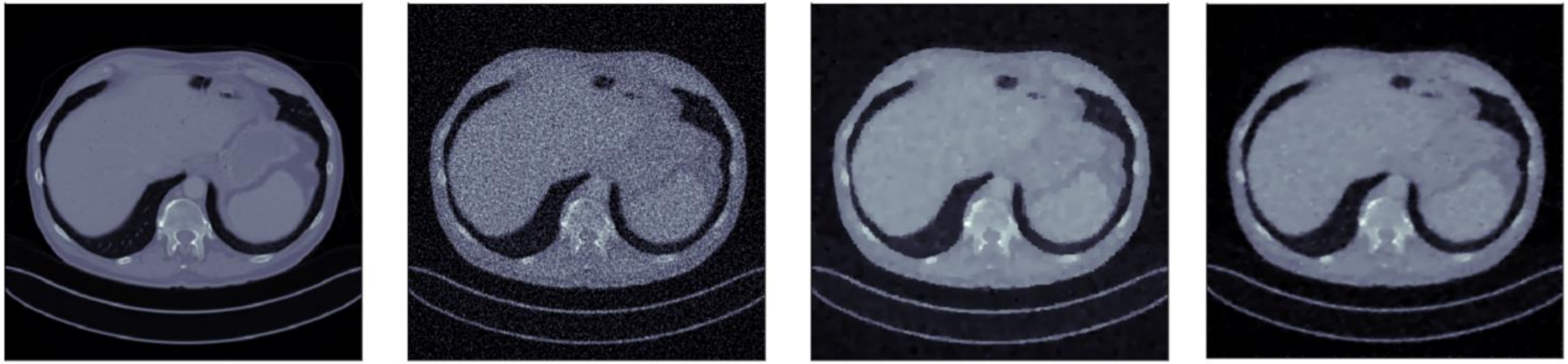
# Learned Regularizers

## Adversarial regularization with source condition

Augment with penalty that ensures training data satisfy source condition [mb-Mukherjee-Schönlieb, NeurIPS Workshop 2021]

$$\frac{1}{n}\sum_{i=1}^{n}\|(K^*)^{-1}\partial_u J(u_i)\|^2 \qquad\qquad \mathbb{E}_u(\|(K^*)^{-1}\partial J(u)\|^2)$$

Undersampled and noisy CT reconstruction  (Mayo Clinic Low Dose dataset)



(a) ground-truth (b) FBP: 21.19, 0.22 (c) TV: 29.85, 0.79 (h) ACR-SC: 30.93, 0.85

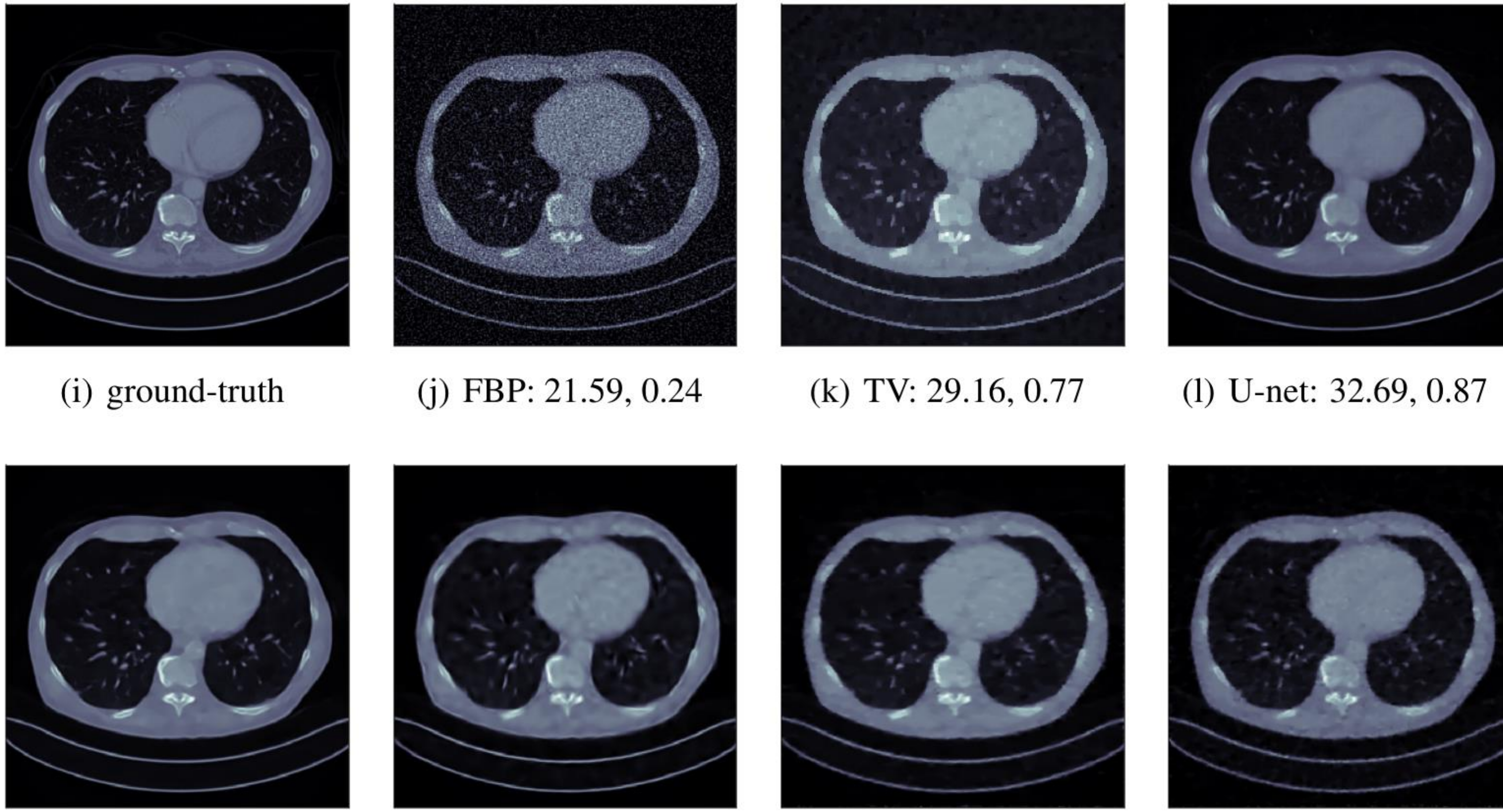
# Learned Regularizers

## Adversarial regularization with source condition

Best case comparison:
Supervised learning and methods with less constraints superior

However, more interpretability and robustness with source condition constraint

| method | PSNR (dB) | SSIM | # param. | reconstruction time (ms) |
|---|---|---|---|---|
| FBP | $21.28 \pm 0.13$ | $0.20 \pm 0.02$ | 1 | $37.0 \pm 4.6$ |
| TV | $30.31 \pm 0.52$ | $0.78 \pm 0.01$ | 1 | $28371.4 \pm 1281.5$ |
| *Supervised methods* | | | | |
| U-Net | $34.50 \pm 0.65$ | $0.90 \pm 0.01$ | 7215233 | $44.4 \pm 12.5$ |
| LPD | $35.69 \pm 0.60$ | $0.91 \pm 0.01$ | 1138720 | $279.8 \pm 12.8$ |
| *Unsupervised methods* | | | | |
| AR | $33.84 \pm 0.63$ | $0.86 \pm 0.01$ | 19338465 | $22567.1 \pm 309.7$ |
| ACR | $31.55 \pm 0.54$ | $0.85 \pm 0.01$ | 606610 | $109952.4 \pm 497.8$ |
| ACR-SC | $31.28 \pm 0.50$ | $0.84 \pm 0.01$ | 590928 | $105232.1 \pm 378.5$ |



(i) ground-truth (j) FBP: 21.59, 0.24 (k) TV: 29.16, 0.77 (l) U-net: 32.69, 0.87
(m) LPD: 34.05, 0.89 (n) AR: 32.14, 0.84 (o) ACR: 30.14, 0.83 (p) ACR-SC: 29.88, 0.8

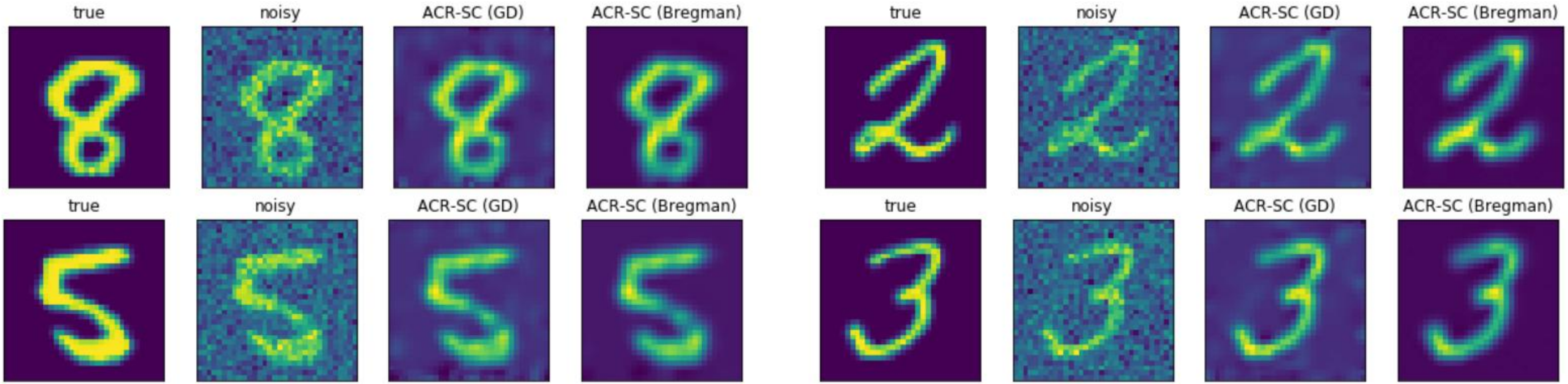
# Learned Regularizers

## Adversarial regularization with source condition

Additional advantage: interpretable method allows to use superior approaches developed for variational models

Example: Bregman Iteration for Bias Correction, iterative recentering of prior. **Mean SSIM improvement > 10 %**
[Bregman 1967] [Hestenes 1969, Powell 1969] [Osher-mb-Goldfarb-Xu-Yin 2005]

$$u^{k+1} \in \arg\min_u F(Ku, f) + \frac{1}{\tau}\Big(J(u) - J(u^k) - \langle p^k, u - u^k\rangle\Big)$$
$$p^{k+1} = p^k + \tau K^* \partial F(Ku^{k+1}, f)$$



# Learning in Image Reconstruction

## State of the Art

**Common approach:**

- Use appropriate neural network for data at fixed resolution
- Use appropriate, often synthetic data set to train
- Display results and compare with reconstruction method that do not use any training data
- Find out that learning surpringly leads results that look better

Few approaches to provide theoretical insights, often in finite dimension or with assumptions that make the original image reconstruction problem well-posed

2019 | OriginalPaper | Buchkapitel

**Deep Learning for Trivial Inverse Problems**

verfasst von : Peter Maass

Erschienen in: Compressed Sensing and Its Applications

Verlag: Springer International Publishing

Deep neural networks can stably solve high-dimensional, noisy, non-linear inverse problems

Andrés Felipe Lerma Pineda* Philipp Christian Petersen†

IEEE SP MAGAZINE SPECIAL ISSUE ON PHYSICS-DRIVEN MACHINE LEARNING FOR COMPUTATIONAL IMAGING

Learned reconstruction methods with convergence guarantees

Subhadip Mukherjee*[1], Andreas Hauptmann*[2,3], Ozan Öktem[4], Marcelo Pereyra[5], and Carola-Bibiane Schönlieb[1]

Learning the optimal Tikhonov regularizer for inverse problems

Giovanni S. Alberti[1], Ernesto De Vito[1], Matti Lassas[2], Luca Ratti[1], Matteo Santacesaria[1]

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Inverse Problems 38 (2022) 115005 (21pp) https://doi.org/10.1088/1361-6420/ac9011

**Regularization theory of the analytic deep prior approach**

Clemens Arndt*

# Learning in Image Reconstruction

## Open issues

- How do learned methods behave in the infinite-dimensional limit ?
- Do learned methods provide regularization with respect to data noise ? (Guarantees in certain metrics)
- How do typical solutions of a learned regularization method look like ? (Smoothness, bias, ..)
- What is the impact of the specific training approach
- Generalization aspect: do we obtain a convergent regularization method with high probability when trained on finite data ?
- ….

# Credits

**Samira Kabri, Tim Roith, Lorenz Kuger, DESY**

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**Michael Möller, Alexander Auras, Siegen**

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