Influence of Density Dependence of Symmetry Energy on Astrophysical S-factor in Heavy-ion Fusion Reactions

G. Saxena

Department of Physics (H&S), Govt. Women Engineering College, Ajmer-305002, India

Nuclear Physics in Astrophysics XI (NPA-XI) conference
Friday 13th September, 2024

Introduction

- The nuclear symmetry energy and its density dependence are instrumental in determining the properties of systems ranging from asymmetric nuclei to neutron stars and various other astrophysical phenomena.
- The neutron skin thickness in asymmetric nuclei is sensitive to the slope of symmetry energy (L_0) at the saturation density ($\rho_0 \sim 0.16$ fm^{-3}).
- PREX-II $\rightarrow$ (Δr_{np}) $\rightarrow$ 0.283 $\pm$ 0.071 fm for ^{208}Pb
  CREX $\rightarrow$ (Δr_{np})$\rightarrow$ 0.121 $\pm$ 0.026(exp) fm for ^{48}Ca
- The values of L_0= 76$\sim$165 MeV from Δr_{np} (^{208}Pb) and L_0= 0$\sim$51 MeV from Δr_{np} (^{48}Ca) have been deduced by using 207 non-relativistic and relativistic mean-field models.

- This discrepancy poses a challenge for nuclear theory, as current models struggle to fit the results from PREX-II and CREX simultaneously. It suggests that there might be more complexity in the nuclear symmetry energy and its density dependence than previously thought.
- One of the several alternative ways to investigate the density dependence of the symmetry energy or the neutron skin thickness is "Nuclear Reactions (Heavy-Ion Collisions)":
  1. Description: Heavy-ion collisions between neutron-rich nuclei at intermediate energies produce nuclear matter at densities and temperatures similar to those in astrophysical environments.
  2. Experiments:
     Radioactive ion beam facilities (e.g., RIKEN, FRIB) can provide collisions of exotic neutron-rich isotopes.
     Heavy-ion collisions (e.g., at GANIL, GSI) can be used to extract information on the symmetry energy at supra-saturation densities.

Present Work in Nutshell

We study the sensitivity of density dependence of the symmetry energy to the sub-barrier fusion cross-sections and the resulting astrophysical S-factor for a few asymmetric nuclei.

↓

^{24}O+^{24}O, ^{54}Ca+^{54}Ca, ^{60}Ca+^{60}Ca, ^{78}Ni+^{78}Ni, ^{124}Sn+^{124}Sn, and ^{132}Sn+^{132}Sn.

↓

Density profiles are obtained by Skyrme-Hartree-Fock-Bogoliubov and RMF models corresponding to different values of L_0.

↓

DFM potentials are calculated using the DFMSPH22 code applying M3Y-Paris parameterization without density dependence (PDD0) and with density dependence (PDD1) (density dependence of the exchange part of the NN-forces, V_{NN})

↓

These DFM potentials are used in single-channel CCFULL code to calculate fusion cross-section (σ(E)).

Methodology

From the partial wave analysis of formal nuclear reaction theory results the following formula of the cross-section for two nuclei undergoing nuclear reaction

$$\sigma(E) = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l(E) \qquad (1)$$

where, $k = \frac{\sqrt{2\mu E}}{\hbar}$, μ being the reduced mass of the interacting nuclei and the transmission coefficient $T_l(E)$ for l^{th} partial wave is given by

$$T_l(E) = exp(-\frac{2}{\hbar}\int_{r_1}^{r_2}\sqrt{2\mu[V_{eff}(r) - E]}dr) \qquad (2)$$

within WKB approximation, where r_1 and r_2 are classical turning points, E is the energy in the centre of mass frame and $V_{eff}(r)$ is the effective barrier potential expressed as

$$V_{eff}(r) = V_n(r) + V_c(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} \qquad (3)$$

In our calculations, the double-folding model (DFM) is used to generate the nucleus–nucleus potential which reads as,

$$V_n(\mathbf{R}) = \int\int V_{NN}(|\mathbf{R} - \mathbf{r}_t + \mathbf{r}_p|)\rho_p(\mathbf{r}_p)\rho_t(\mathbf{r}_t)d\mathbf{r}_p d\mathbf{r}_t \quad (4)$$

- $\mathbf{r}_p$ and $\mathbf{r}_t$ are the radius vectors of two interacting points
- $\mathbf{R}$ denotes the vector joining their centers of mass
- $\rho_p(\mathbf{r}_p)$ and $\rho_t(\mathbf{r}_t)$ stand for the target and projectile nuclear matter densities

The expression of the cross-section (Eq. 1) can be decomposed into the term that depends strongly on energy and another one that varies weakly with energy as,

$$\sigma(E) = E^{-1}exp(-2\pi\eta)S(E) \quad (5)$$

- E is the center of mass energy of the reactants
- $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$ is the Sommmerfeld parameter
- $v = \sqrt{\frac{2E}{\mu}}$ denotes the relative velocity

Results from Skyrme parameters SLy4 and SkO

We first present our results in detail for SLy4 and SkO Skyrme effective interactions [56,57] which mainly differ in the behavior of the symmetry energy.

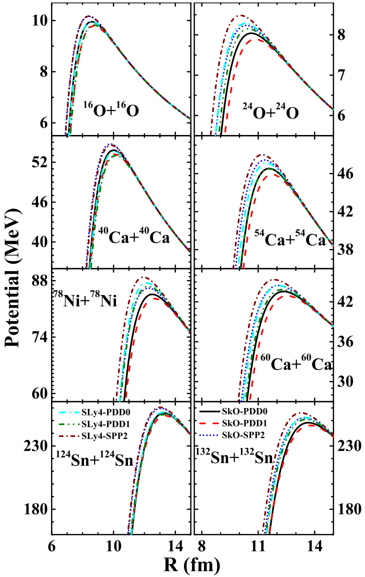
Table: Nuclear matter properties at the saturation density ρ_0 using the SLy4 and SkO effective interactions.

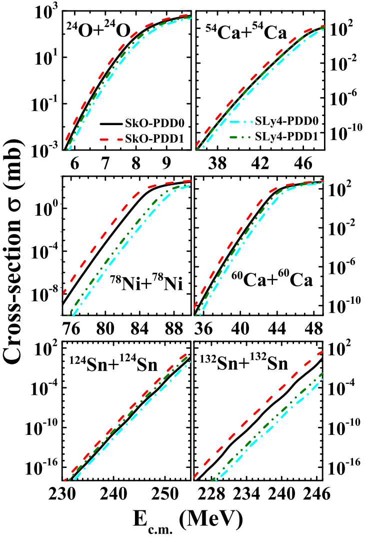
Model	ρ_0	ϵ_0	K_0	m_0^*	J_0	L_0	$K_{sym,0}$
SLy4	0.16	-15.97	229.9	0.69	32.0	45.94	-119.73
SkO	0.16	-15.84	223.3	0.90	32.0	79.14	-43.17

Table: Ground state Properties of some nuclei

Nucleus	Expt.		SLy4			SkO			Δr_{np}^{diff}
	BE/A	r_p	BE/A	r_p	Δr_{np}	BE/A	r_p	Δr_{np}	
^{16}O	7.976	2.578	8.029	2.702	-0.023	7.406	2.694	-0.021	0.002
^{24}O	7.040	-	7.208	2.762	0.479	6.950	2.760	0.628	0.149
^{40}Ca	8.551	3.384	8.606	3.419	-0.047	8.397	3.401	-0.041	0.006
^{54}Ca	8.247	-	8.311	3.534	0.350	8.267	3.509	0.466	0.116
^{60}Ca	7.627	-	7.750	3.615	0.483	7.818	3.584	0.632	0.149
^{78}Ni	8.238	-	8.253	3.925	0.310	8.254	3.929	0.451	0.141
^{124}Sn	8.467	4.605	8.472	4.626	0.180	8.471	4.614	0.226	0.046
^{132}Sn	8.354	4.641	8.358	4.670	0.222	8.320	4.670	0.320	0.098

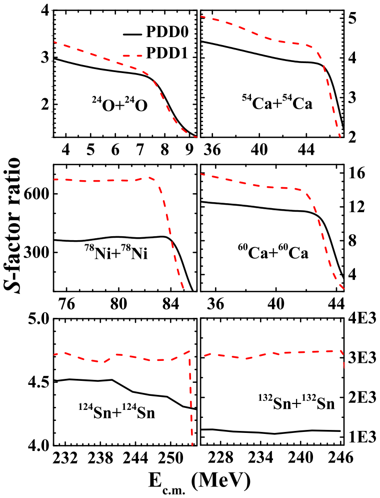
Effect on potential and cross-section





- The SkO interaction associated with larger skin thickness yields smaller barrier heights and widths.
- The area of the barrier is determined by its height and width that governs the values of cross-section exponentially. Thus, small changes in the barrier parameters could significantly affect the cross-section and astrophysical S-factor.
- Due to the reduction in barrier height and width, the SkO interaction leads to a larger cross-section.
- The cross-section for $^{24}O+^{24}O$ is enhanced by three to four times due to increase in skin thickness by 0.15 fm for SkO in comparison with that of SLy4 interaction.
- For $^{132}Sn+^{132}Sn$ case, the cross-section enhances by two to three orders of magnitude due to an increase in neutron skin thickness by 0.10 fm.

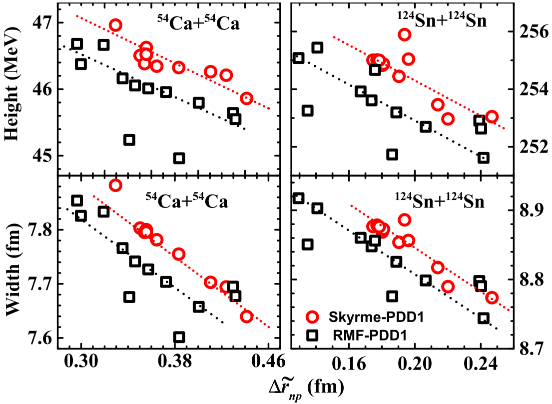
Impact on Astrophysical S-factor



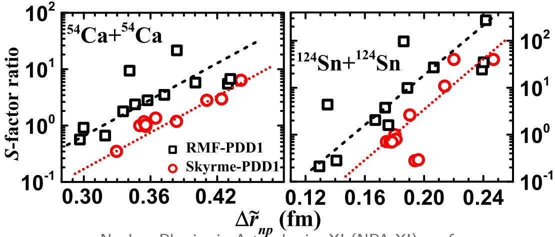
- We plot the ratio of the S-factor obtained for the SkO interaction to those for the SLy4 interaction.
- The rapid increase in the values of the *S*-factor in these nuclei for SkO force clearly suggests its sensitivity to the neutron skin thickness, which grows stronger with the increase in proton number.
- The *S*-factor increases by one order of magnitude for $^{54}Ca+^{54}Ca$ with the increase in skin thickness by 0.1 fm and a similar increase is observed for $^{124}Sn+^{124}Sn$ with the increase in skin thickness only by 0.05 fm.

S-factor by varying the neutron skin thickness

- We also calculate the cross-section and *S*-factor by varying the neutron skin thickness for ^{54}Ca+^{54}Ca and ^{124}Sn+^{124}Sn, which might be within the experimental reach.
- The different Skyrme models used are SLy4, SLy5, SLy6, SLy7, SkO, SIII, SkM, SkP, S255, SkI3, HFB9.
- RMF models used are NL3, NL3*, FSU-Gold, FSU-Garnet, DD-PC1, DD-PCX, DD-ME2, DD-MEa to DD-MEe.
- The overall decrease in the height, as well as the width of the barrier on increasing neutron skin thickness, is evident from the figure for both ^{54}Ca+^{54}Ca and ^{124}Sn+^{124}Sn.
- The *S*-factor readily changes by an order of magnitude for ^{54}Ca and about two orders of magnitude for ^{124}Sn with an increase in neutron skin thickness.



^{54}Ca+^{54}Ca
^{124}Sn+^{124}Sn
Height (MeV)
Width (fm)
Skyrme-PDD1
RMF-PDD1
$\Delta\tilde{r}_{np}$ (fm)



^{54}Ca+^{54}Ca
^{124}Sn+^{124}Sn
S-factor ratio
RMF-PDD1
Skyrme-PDD1
$\Delta\tilde{r}_{np}$ (fm)

Conclusion

- We have performed the calculations of the cross-section for the sub-barrier fusion and astrophysical S-factor for several asymmetric nuclei.
- The double-folding model potentials required to compute cross-sections are obtained by folding nucleon-nucleon interactions with the density profiles for nucleons inside the nucleus obtained from mean-field models.
- The mean-field models employed are based on the non-relativistic Skyrme-type effective interactions and different variants of relativistic effective Lagrangian associated with a wide range of the symmetry energy slope parameter or the neutron skin thickness.
- The barrier parameters such as its height and width decrease with an increase in neutron skin thickness which leads to the enhancement of cross-section and astrophysical S-factor up to one or two orders of magnitude.
- The sensitivity of the cross-section or the S-factor to the neutron skin thickness grows stronger with the increase in the proton number.
- The precise measurement of sub-barrier fusion cross-section or astrophysical S-factor in asymmetric nuclei may provide an alternative probe to determine the neutron skin thickness or the slope parameter of the symmetry energy.

Acknowledgements

- I would like to express my gratitude to the following research group members:
  - Prof. B. K. Agrawal, SINP, India
  - Prof. Ushasi Datta, SINP, India
  - Dr. Sangeeta, CU, India
  - Dr. T. Ghosh, SINP, India
- I am deeply grateful to Prof. Nils Paar for his kind hospitality during my stay at the University of Zagreb, Croatia.
- Financial support from SERB (DST), Government of India, under ITS/2024/003332, and from the NPA-XI conference organizers, is gratefully acknowledged.

Thank you.
Questions are welcome.

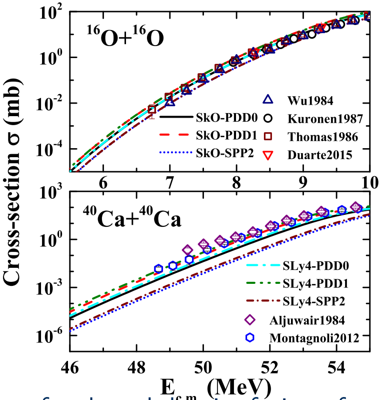


Figure: Cross-sections for the sub-barrier fusion of symmetric systems $^{16}O+^{16}O$ and $^{40}Ca+^{40}Ca$ as a function of energy in the center of mass frame (E_{cm}) calculated for different DFM potentials and density profiles.