# Incorporating thermal effects into alpha decay half-life calculations for nucleosynthesis investigations

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## Abstract

$\alpha$ decay is one of the prominent decay modes in the nucleosynthesis of heavy and super-heavy elements synthesized at temperatures of the order of Giga Kelvin. To facilitate the investigation of the role played by the $\alpha$ decay half-lives of thermally excited nuclei in nucleosynthesis calculations, an empirical formula based on a model for the $\alpha$ decay of nuclei in their ground and excited states to daughter nuclei in their ground or excited states is presented. Under the assumption that thermal equilibrium has been reached between nuclear states, temperature ($T$)-dependent half-lives, $t_{1/2}(T)$, for several of the experimentally studied $\alpha$ emitters are presented using available data on the half-lives of excited nuclei. Though the general trend is a decrease in $t_{1/2}(T)$ at elevated temperatures, exceptional cases with increased half-lives are found in the case of some isomeric states [1].

## Model for $\alpha$ decay of excited nuclei

To extend the calculation of $\alpha$ decay half-lives to excited states, i.e., the $\alpha$ decay of nuclei in either ground or excited states, decaying into daughter nuclei that can also exist in ground or excited states, we begin by defining an effective $Q$ value. By applying energy conservation and neglecting the recoil energy of the heavy daughter nucleus, we define the effective $Q$ value

$$Q_{eff} \equiv Q_\alpha + E_p^* - E_d^* = Q_\alpha + \Delta E^* \,, \tag{1}$$

as the energy available for the tunneling $\alpha$ particle. The data in Fig. 1 shows the experimental $\alpha$ decay half-lives, $t_{1/2}^{\mathrm{exp}}$, of excited parent nuclei to daughters in the excited or ground state, as a function of $Q_{eff}$.

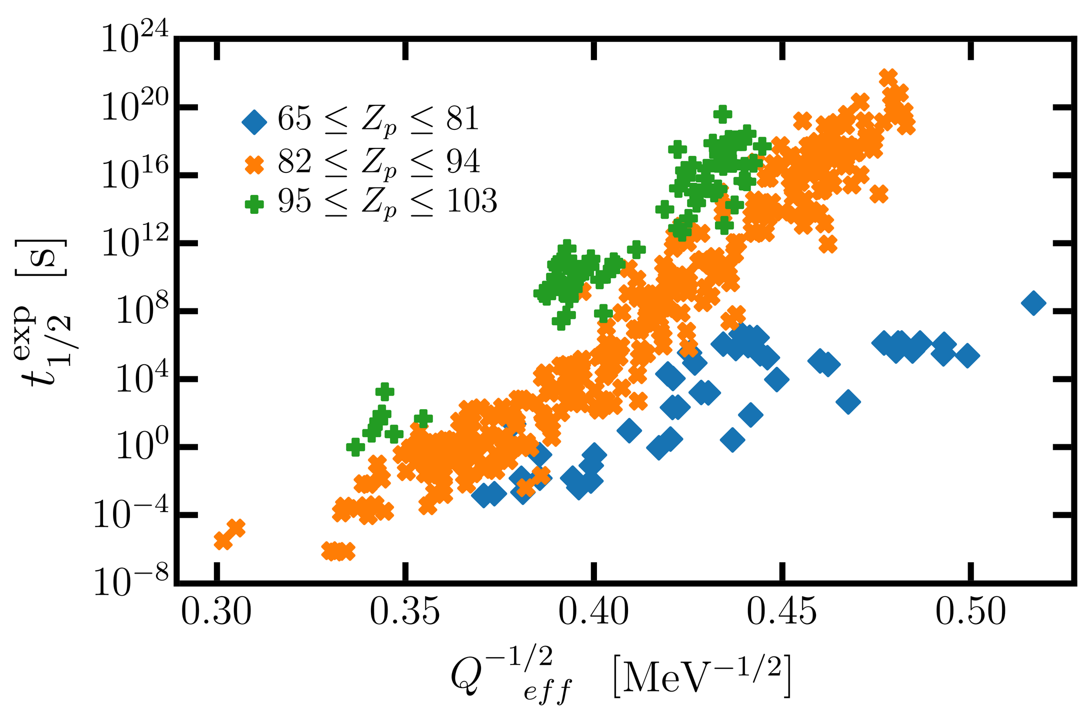


**Figure 1:** Measured $\alpha$ decay half-lives [2] for 592 decays, where the parent nucleus, daughter nucleus, or both can have multiple excited states associated with the $\alpha$ decay process.

This behavior is studied using a model that, in addition to standard quantities like the $Q$-value and the number of nucleons, includes terms dependent on the excitation energies of the nuclei involved. By fitting model parameters to available data on $\alpha$ decays for nuclei with parent atomic numbers $82 \le Z_p \le 94$, where either the parent, the daughter, or both are in excited states, we derive a semi-empirical formula. The excitation-energy-dependent decay law can be expressed as

$$\log_{10} t_{1/2}^{\mathrm{exc}} = \beta_1\,\chi' + \beta_2\,\chi'\,\frac{\Delta E^*}{Q_\alpha} + \beta_3\,\rho' + \beta_4\,\frac{\ell(\ell+1)}{\rho'} + \beta_5\,\delta_{oe}\,, \tag{2}$$

where $\rho' = \sqrt{Z_\alpha Z_d A_\alpha A_d \left(A_\alpha^{1/3} + A_d^{1/3}\right)/A_p}$ and $\chi' = Z_\alpha Z_d \sqrt{A_\alpha A_d / A_p Q_\alpha}$. The constants $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, and $\beta_5$ are treated as adjustable parameters, fitted to experimental $\alpha$ decay half-life data to account for approximations made in deriving the decay law that can be used to predict $\alpha$ decay half-lives at elevated temperatures. This model is particularly useful in astrophysical environments where heavy elements in this mass range are synthesized. Evaluation of the $r$-squared and MSE metrics yielded values of 0.976 and 1.08, respectively.

## $\alpha$ decay half-lives at elevated temperatures

It is natural to expect that at elevated temperatures, the excited states of $\alpha$-emitting nuclei would be populated, thus influencing the total half-lives. Understanding the contribution of these excited states is essential for predicting nucleosynthetic outcomes in environments where high temperatures play a significant role.

To examine the change of $\alpha$ particle emission rates from excited states of heavy nuclei under conditions of finite temperature, we fix the temperature and calculate the half-lives using available experimental data. The stellar $\alpha$ decay half-life is then given by

$$t_{1/2}(T) = \left[\frac{1}{\mathcal{G}} \sum_i \frac{g_{p_i} \exp\left(-E_{p_i}^*/k_B T\right)}{t_{1/2}\left(E_{p_i}^*\right)}\right]^{-1}, \tag{3}$$

where $T$ is the ambient temperature, $k_B$ is the Boltzmann constant, and the sum runs over the excited levels of the parent ($p_i$). $\mathcal{G} = \sum_i g_{p_i} \exp\left(-E_{p_i}^*/k_B T\right)$ is the partition function, and $g_{p_i} = (2J_{p_i}+1)$ is the statistical weight of the parent's excited state with spin $J_{p_i}$. The term $t_{1/2}(E_{p_i}^*)$ represents the $\alpha$ decay half-life of the parent's excited state $i$.

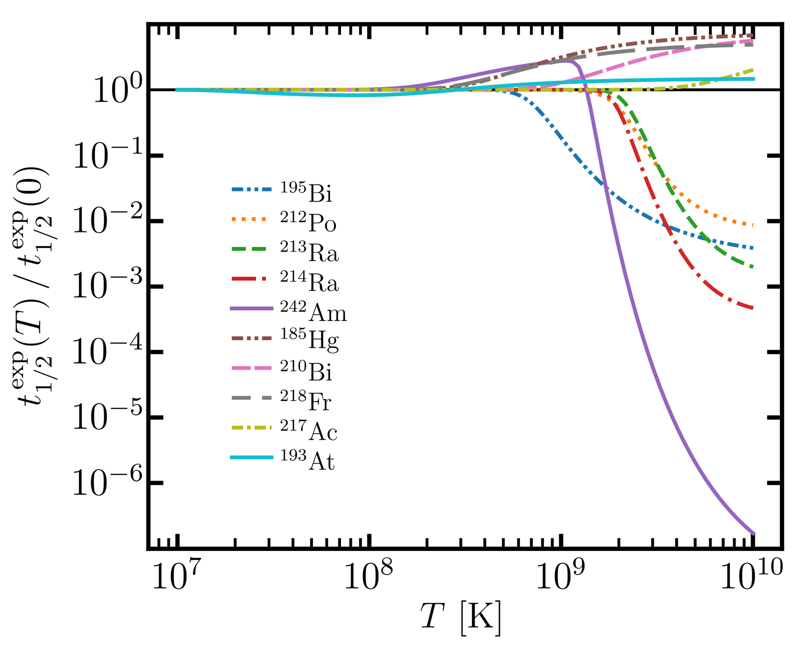


**Figure 2:** Temperature dependence of the $\alpha$ decay half-lives evaluated as in Eq. (3).

Using the data from [2], we calculate the half-lives as a function of temperature, denoted $t_{1/2}(T)$, for all experimentally studied $\alpha$ emitters with $65 \le Z_p \le 94$. We label this result as $t_{1/2}^{\mathrm{exp}}(T)$ to indicate that only those levels with known $\alpha$ decay branches were used in the stellar rate (Eq. 3). Fig. 2 displays the results, normalized to the zero-temperature values (i.e., the ground-state half-lives) for some of the nuclei under consideration. The semi-empirical excitation energy-dependent decay law can be rewritten as [3]

$$\log_{10}[t_{1/2}(E_p^*, E_d^*)] = \log_{10}[t_{1/2}(g.s. \to g.s.)] - \beta_2 \chi' \frac{\Delta E^*}{Q} \tag{4}$$

where $t_{1/2}(g.s. \to g.s.)$ represents the half-life when the decay occurs from the ground-state parent to the ground-state daughter.

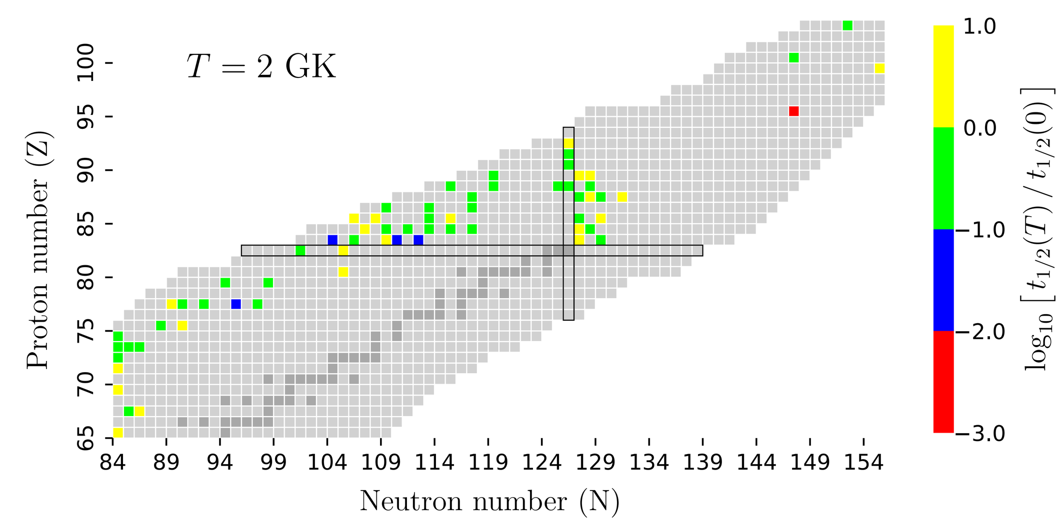


**Figure 3:** Temperature-dependent half-lives $t_{1/2}^{exp}(T)$ normalized by those at $T = 0$.

Trends in half-lives are shown in Fig. 4, where we plot the ratio of stellar half-lives to ground-state values across the nuclear landscape. The temperature is fixed at $T = 2$ GK. The colored squares correspond to nuclei with excited states that undergo $\alpha$ decay. At moderate temperatures (a few GK), a substantial fraction of nuclei exhibit an increase in half-life by a factor of 10. The isomeric states are responsible for the increase and the asymptotic behavior of $\alpha$ decay half-lives at higher temperatures. Conversely, a larger group of nuclei show decreasing half-lives, with changes of up to an order of magnitude. If these nuclei have short-lived, $\alpha$-emitting excited states, they will decay faster. For nuclei with decreasing half-lives at 2 GK, we expect a further reduction in half-lives as increased temperatures populate more excited $\alpha$-emitting states.

We now apply our model, defined in Eq. (2) to several $\alpha$ emitters and validate our approximation by comparing two approaches. Specifically, we use Eq. (2) to estimate the half-lives of the excited states (with known $\alpha$ decay branches) that enter Eq. (3) and denote this rate by $t_{1/2}^{\mathrm{cal}}(T)$.

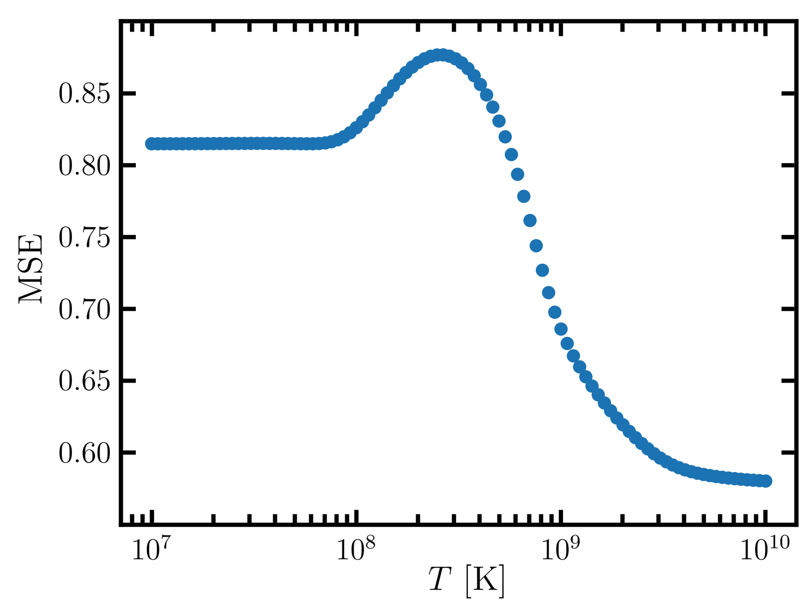


**Figure 4:** MSE of predicted $T$-dependent half-lives to those based on experimentally data.

## Summary and Outlook

Given that $\alpha$ decay can occur in astrophysical environments at temperatures around Giga Kelvin, it is essential to account for the temperature dependence of nuclear half-lives, $t_{1/2}(T)$. The simplest approach sums the half-lives of all excited states of a nucleus, weighted by their population probabilities at a given temperature. The stellar decay rates calculated here assume thermal equilibrium between ground and excited states, including isomers. The validity of the proposed decay law was tested by comparing $t_{1/2}(T)$ derived from the empirical formula with experimental data, $t_{1/2}^{\mathrm{exp}}(T)$. With consistently low MSE across a wide temperature range, this empirical model is a reliable tool when data is unavailable.

## References

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