“Frozen and β-equilibrated f and p modes of cold neutron stars: metamodel predictions”

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1 Introduction

Neutron stars exhibit a variety of oscillation modes due to their stratified internal structure and composition. These oscillations include fundamental (f), pressure (p), and gravitational (g) modes, among others. The analysis of these modes offers insights into the equation of state of dense nuclear matter, and, possibly, their internal composition. Most studies makes use of a barotropic EoS. Here we explore possible effect of frozen composition on the f and p mode by performing a Bayesian inference. From it we can also provide the distribution of the frequencies for 10^5 models in accordance with latest astrophysical and nuclear data.

2 Metamodel representention of EoS and composition

The Meta-Model is a flexible, phenomenological, representation of the energy per baryon $e(n_b, \delta(n_b))$ for a given nuclear asymmetry delta, see Margueron et al. (2018) for further details [1]. This allows to explore the energy of nuclear matter outside the β-equilibrium. First we find the beta equilibrated composition, which allows to obtain the barotropic equation of state for the NS core. The EoS is then extended with a consistent crust in order to have a unified EoS model. For what concerns our Bayesian analysis, the meta-model is a procedure M

$$M : \mathbf{X} \to \{EoS, \delta(n_b), v_s^{\beta}, v_s^{FR} \dots \}$$

that takes in input the values for some nuclear matter parameters X and output a β-equilibrated EoS, the barotropic and frozen sound speed and the nuclear asymmetry $\delta(n_b)$.

3 Bayesian inference

In order to constrain the parameters X and assign a likelihood to each matter model $M(\mathrm{X})$ we perform a Bayesian inference. We used information coming from both “nuclear” and “astro” physics.

“Nuclear” constraints

- The band of pure neutron matter energy from χ-EFT [2]
- Ame2020 masses [3]

“Astro” constraints

- Maximum observed NS mass from radio-timing of PSRJ0348 [4]
- Tidal deformability constraints from GW170817 by LIGO/Virgo [5]
- NICER+XMN M-R measurements of PSRJ0030 [6], PSRJ0347 [7] and PSRJ0740 [8]

4 Perturbation equations and speed of sound

We work within the Cowling approximation, i.e. the metric is fixed. The equations to be solved are

$$\frac{dW}{dr} = \boxed{\frac{d\epsilon}{dP}}\left[\omega^2 r^2 e^{\Lambda - 2\Phi} V + \frac{d\Phi}{dr} W\right] - l(l+1)e^{\Lambda} V \qquad \frac{dV}{dr} = 2\frac{d\Phi}{dr}V - \frac{1}{r^2}e^{\Lambda}W$$

With the following boundary condition at the surface, $r = R$, it becomes an eigenvalue problem for the real frequency ω

$$\omega^2 R^2 e^{\Lambda - 2\Phi} V + \frac{d\Phi}{dr} W = 0$$

Barotropic mode:

$$v_{\beta}^2 = \boxed{\frac{dP}{d\epsilon}} = \frac{dP(n, \delta(n))/dn}{d\epsilon(n, \delta(n))/dn}$$

Frozen composition mode:

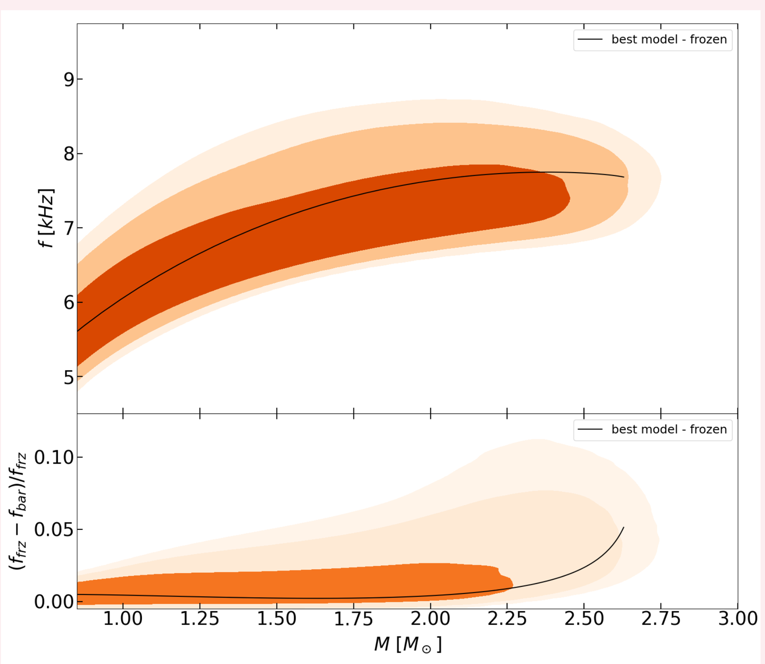
$$v_{FR}^2 = \boxed{\frac{\partial P}{\partial \epsilon}}\Big|_{\delta} = \frac{\partial P(n, \delta(n))/\partial n}{\partial \epsilon(n, \delta(n))/\partial n}$$

5.1 Frozen vs β-equilibrated frequencies

F-mode: differences < 0.5%.

P-mode: difference <3% except for $M \to M_{max}$ where they reach up to 10%

Since the differences are small, if v_{FR} is not available, it is possible to use the barotropic frequencies with very little errors.



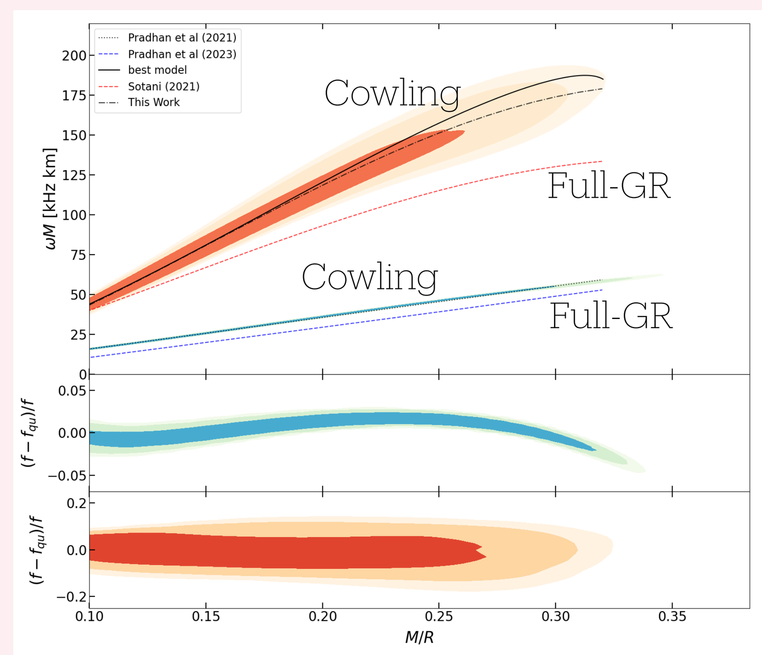
5.2 Quasi-universal Test

The quasi-universal (QU) relation for the f and p-mode is [9] [10]

$$\omega M = a_3\left(\frac{M}{R}\right)^3 + a_2\left(\frac{M}{R}\right)^2 + a_1\frac{M}{R} + a_0$$

It is accurate within 5% for the f-mode and 10% for the p-mode

This QU relation can be used to extract the stars feature or to obtain the frequencies by evaluating only the M-R relation

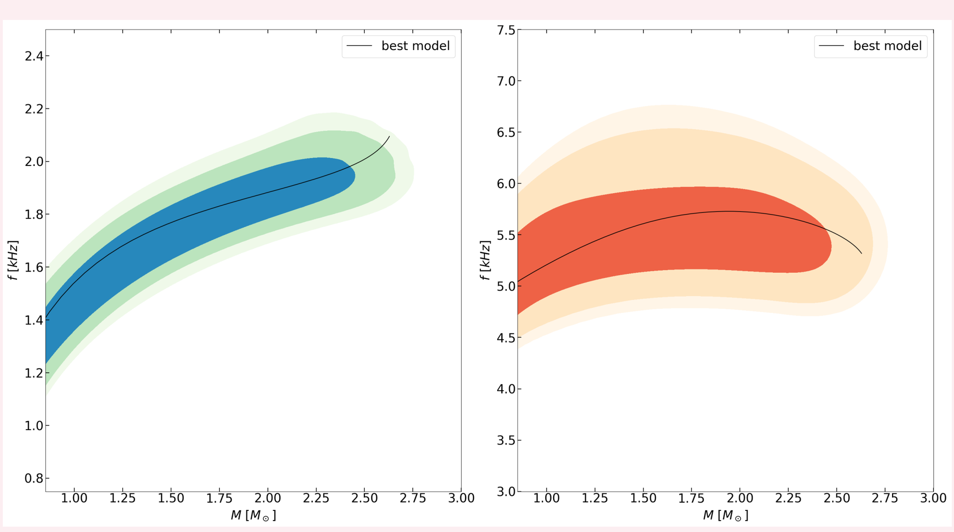


5.3 Synthetic full-GR frequencies

Inserting the M-R inside the QU can underestimate the frequencies dispersion. To obtain a synthetic Full-GR prediction we use the prescription

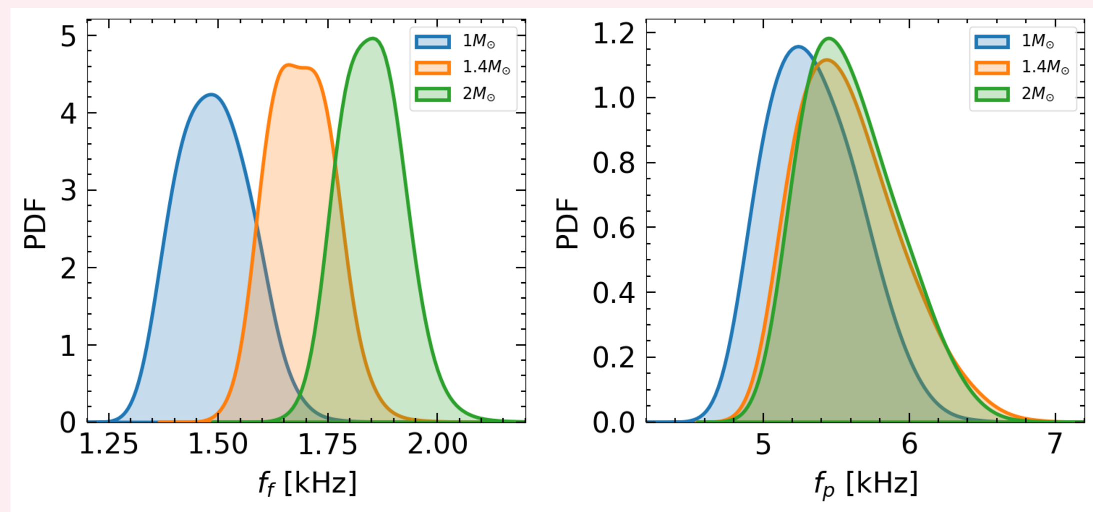
$$\omega_{GR} = (1 + \Delta)QU_{GR} \qquad \Delta = \frac{\omega_C - QU_C}{QU_C}$$

where $QU_{GR,C}$ is the quasi-universal obtained in full-GR or in cowling



The distribution of the f-mode are less overlapped than the p ones.

A constraint from an f-mode observation is easier to obtain and more precise



6 Conclusion

Both f and p modes do not depend much on the "barotropic" or "frozen" character of the speed of sound. As a result, studies that assume phenomenological models for the barotropic EOS are accurate within a few percent. This behavior is reflected in the goodness of the QU relation, which can therefore be used to estimate the mode frequencies without solving the perturbation equations. This is crucial in Bayesian studies, where millions of EOS are used. Finally, we obtained a synthetic full-GR prediction of f and p-mode frequencies as a function of the NS mass: while an f-mode detection could constrain the NS mass, this information is lost for p-modes.

7 References

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