

# Probing the equation of state of dense matter with neutron stars

Anthea F. Fantina ([anthea.fantina\[AT\]ganil.fr](mailto:anthea.fantina@ganil.fr))



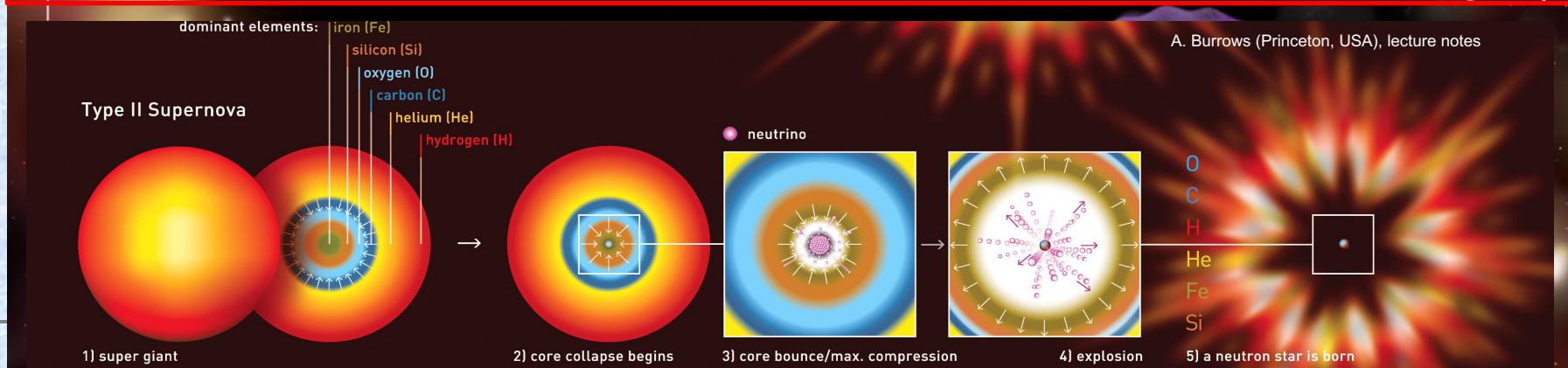
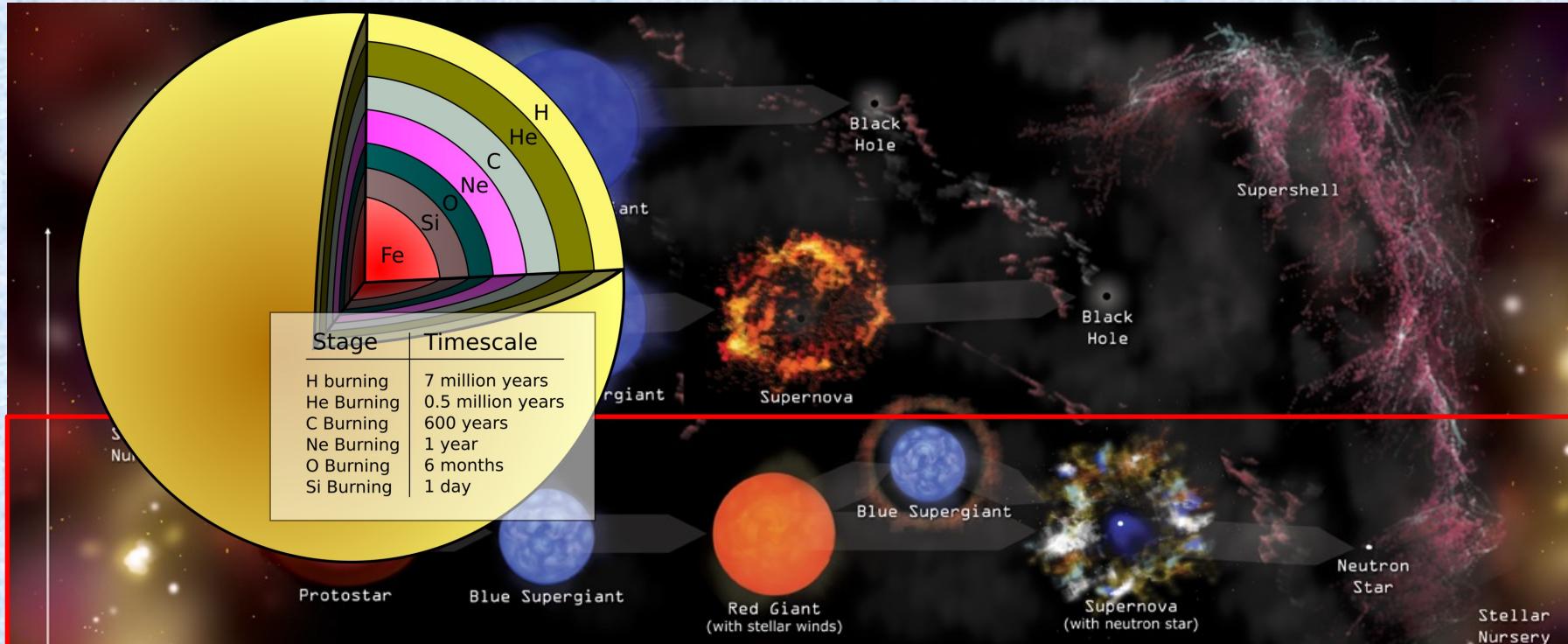
# Outline

- ❖ Introduction
  - Neutron-star (NS) properties
- ❖ Equation-of-state (EoS) modelling
  - NS EoS
    - Outer crust
    - Inner crust
    - Homogeneous matter see also A. Raduta's talk
  - EoS and neutron-star (NS) properties
    - How to build a global model of NS ?
    - How can we get constraints ?
    - Bayesian analysis see also M. Beznogov's talk
- ❖ Conclusions and perspectives

N.B.: In this talk,  $T = 0$  and beta-equilibrium matter



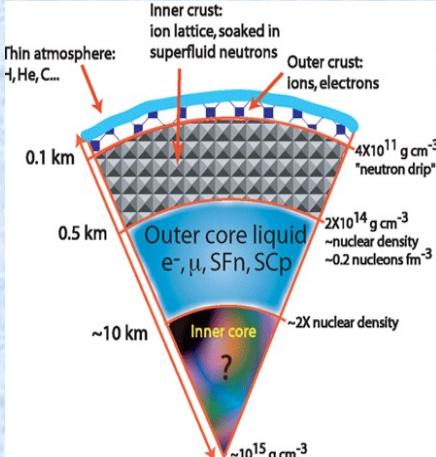
# What is a NS ?





# Scenarios for “ $T = 0$ ” NS EoS

## Mature neutron stars – cold (NSs)

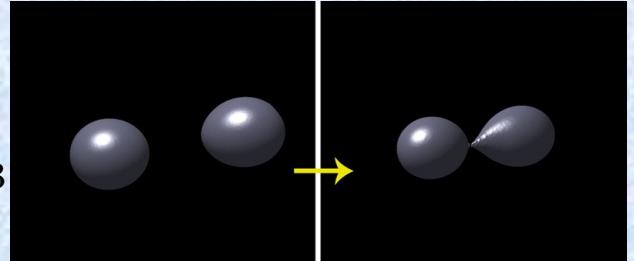


<http://www.physics.montana.edu>

$$\begin{aligned} M &\approx 1 - 2M_{\odot} \\ R &\approx 10 \text{ km} \\ \bar{\rho} &\approx 10^{14} - 10^{15} \text{ g cm}^{-3} \\ T &< 10^8 \text{ K} \end{aligned}$$

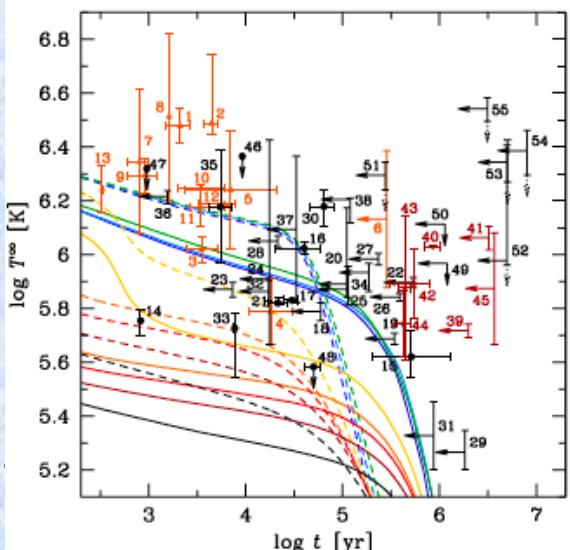
NB:  $T = 0 \Leftrightarrow T \ll T_F$

## Binary NS mergers (“cold” in inspiral phase)



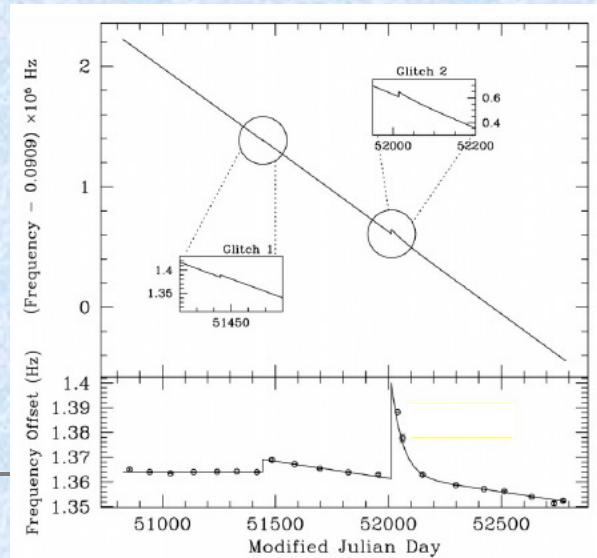
Simulation MPA Garching  
(Goriely, Bauswein, Janka, ApJ 738, 2011)

## Cooling



Potekhin et al., MNRAS 496, 5052 (2020)

## Gliches



from M. Antonelli's talk (2022)



# Fortuitous discovery of a pulsar

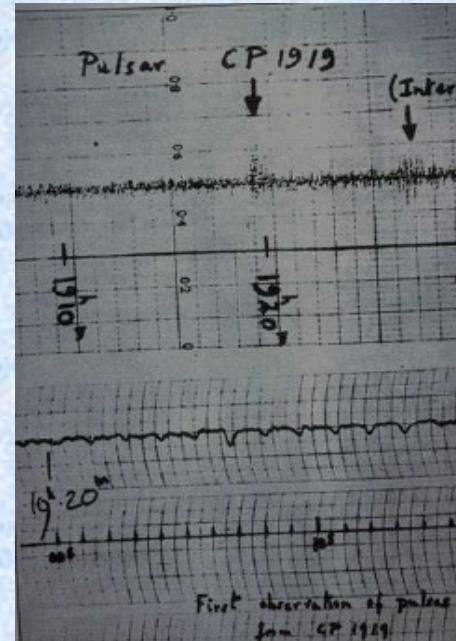


Jocelyn Bell in 1966

1967: J. Bell, PhD thesis at Cavendish Laboratory, Cambridge on radio sources. With a 3.7m diameter telescope, she discovered a very regular source, with a period of 1.3373012 s.

This source was called LGM (“Little Green Man”); a journalist from Daily Telegraph called this source “*Pulsar*”. It is now known as PSR B1919+21.

In 1974, A. Hewish (Bell’s PhD supervisor) received the Nobel prize...





# Fortuitous discovery of a pulsar



Jocelyn Bell in 1966

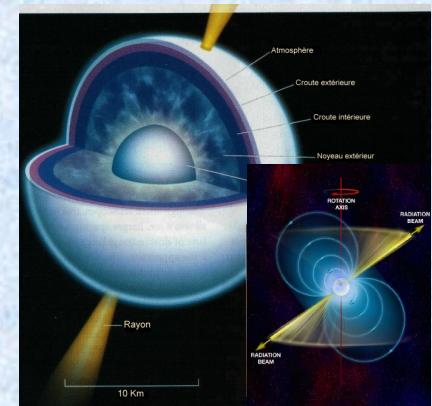
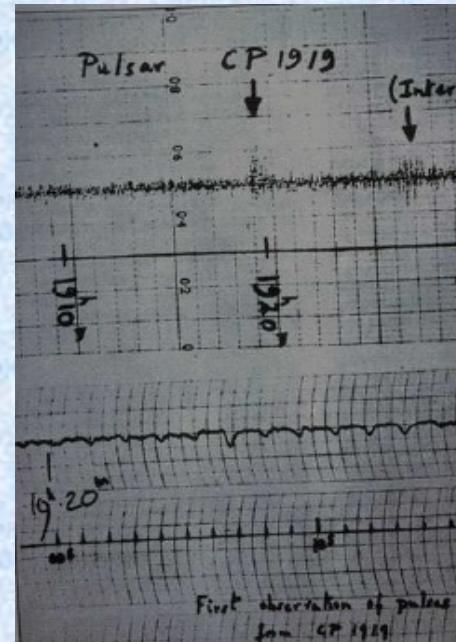
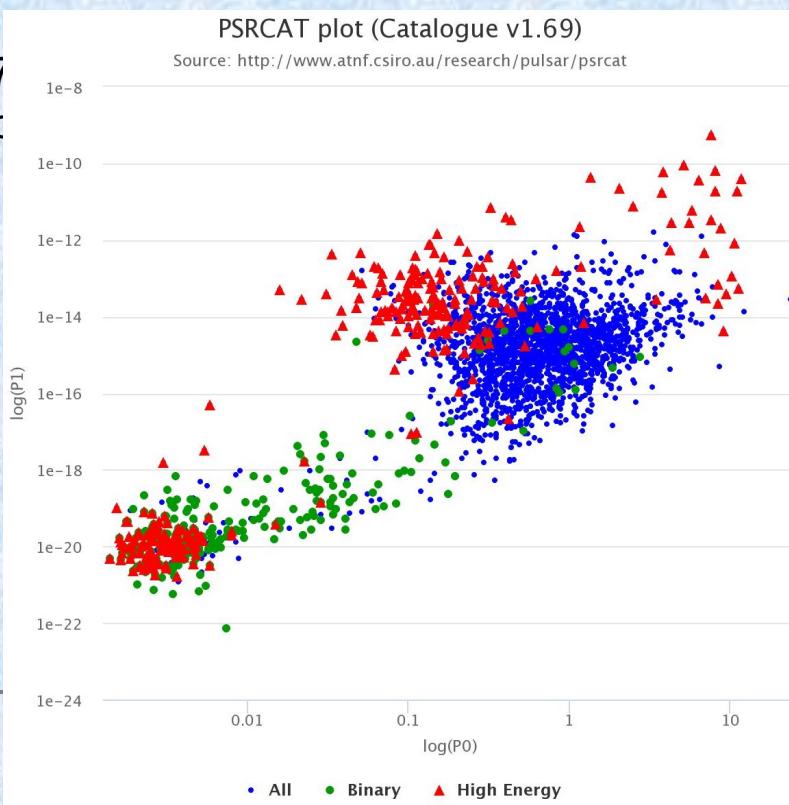
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In 1970  
the No

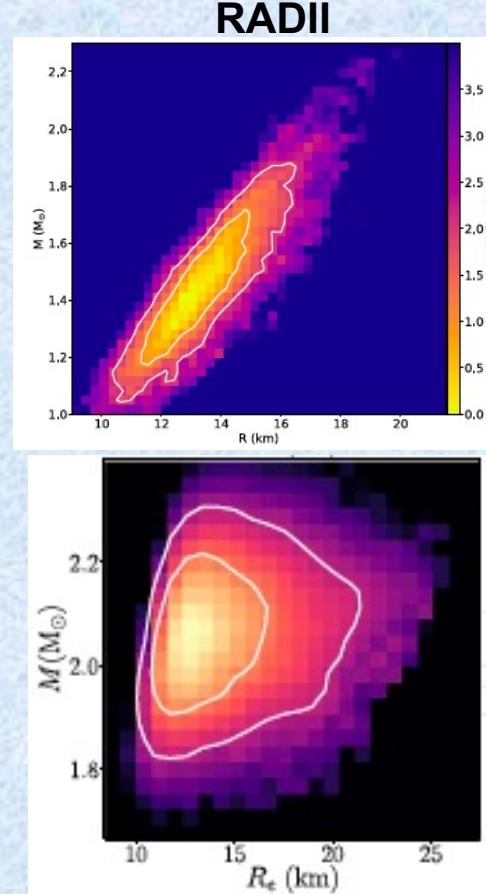
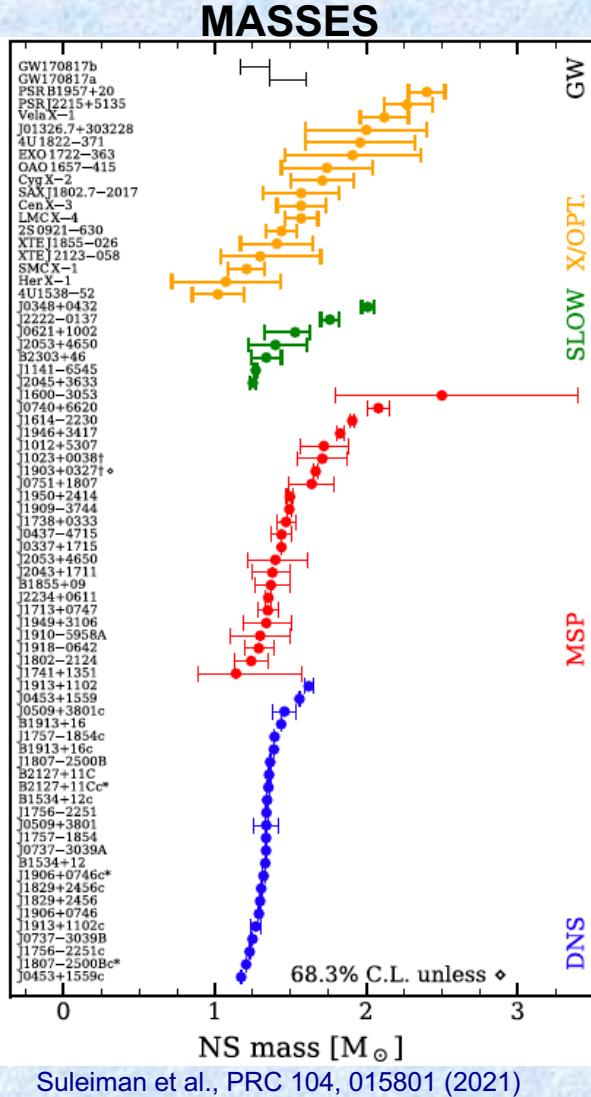
Since 1967, more than 2000 pulsars have been discovered!

Some of them have strong magnetic field



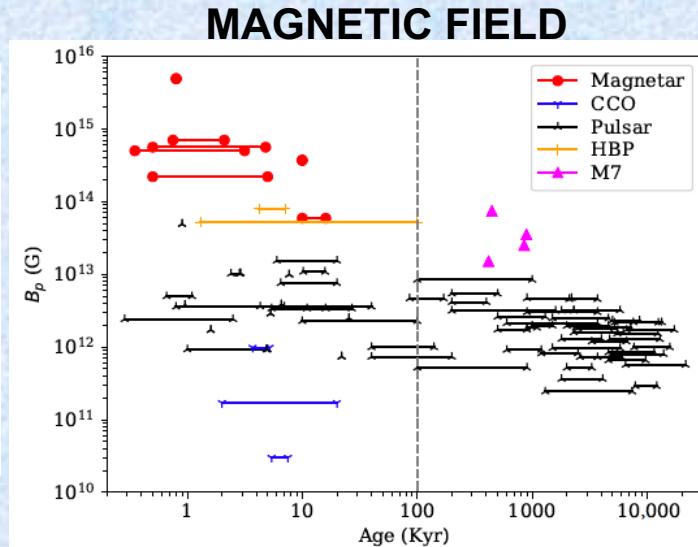


# What other properties do we “observe”?

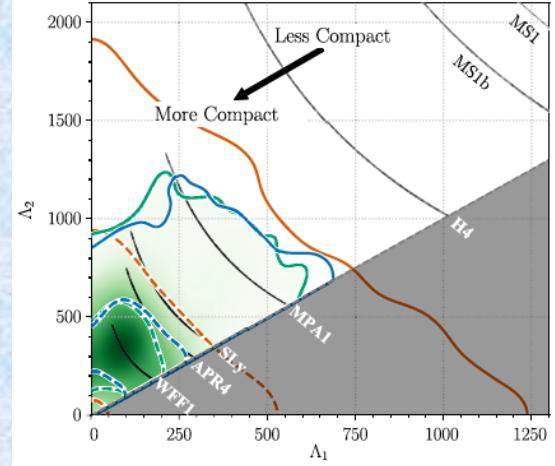


Miller et al., ApJL 2019, 2021;  
also Riley et al., ApJL 2019, 2021

see A. Raduta’s talk



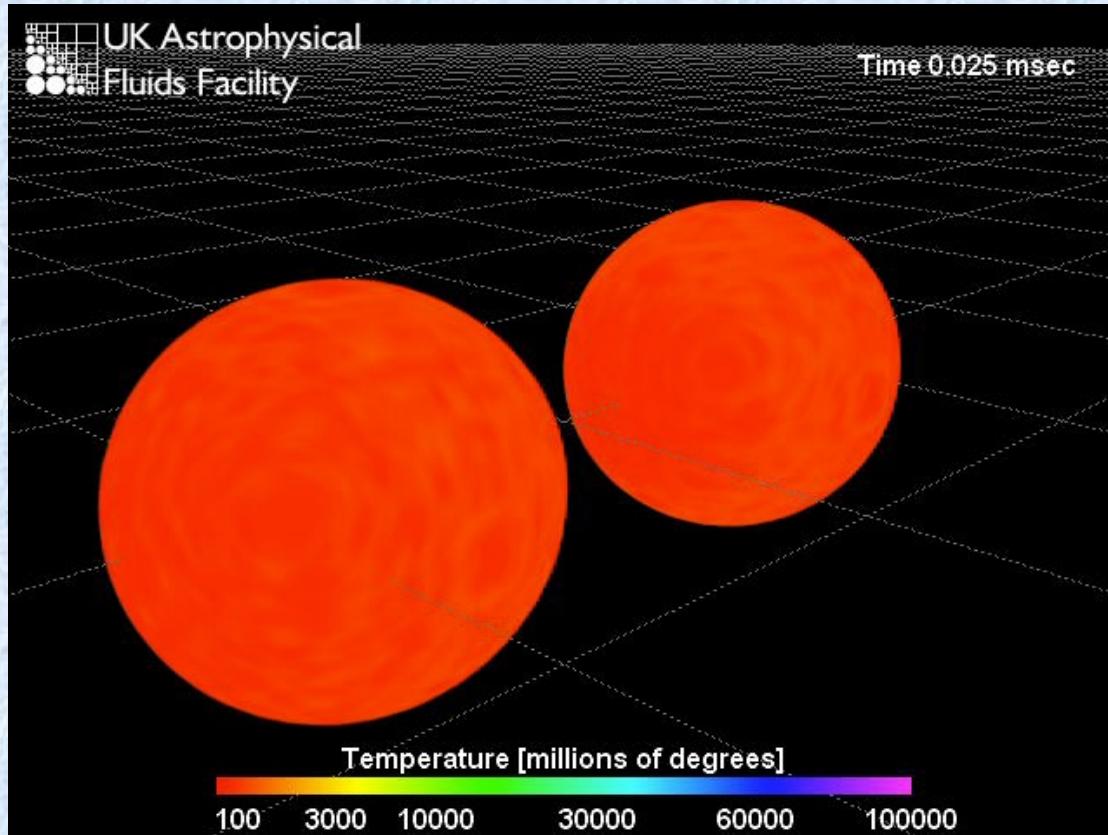
## TIDAL DEFORMABILITY



Abbott et al., PRL 121, 161101 (2018)



# Binary NS merger: simulations



Simulation by S. Rosswog, visualisation R. West  
<http://www.ukaff.ac.uk/movies/nsmerger/>



# Gravitational waves (GW): detection

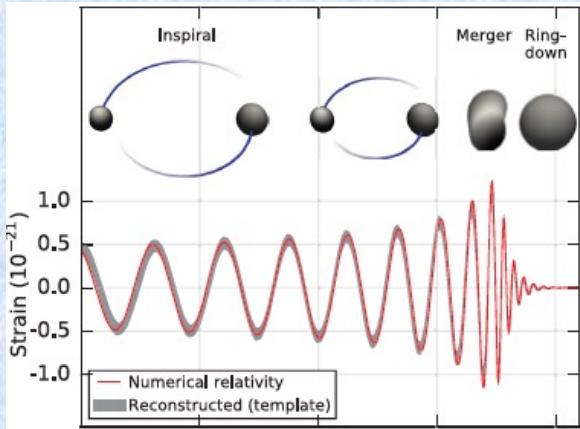
- Predicted by Einstein, observed in 2015 from **black-hole merger**

→ Nobel Prize in physics 2017

2017 NOBEL PRIZE IN PHYSICS



2017 Nobel Prize Physics - gravitational waves (Weiss, Barish, Thorne)  
Credits: The Royal Swedish Academy of Sciences. Ill. N. Elmehed



Abbott et al., PRL 116, 061102 (2016)

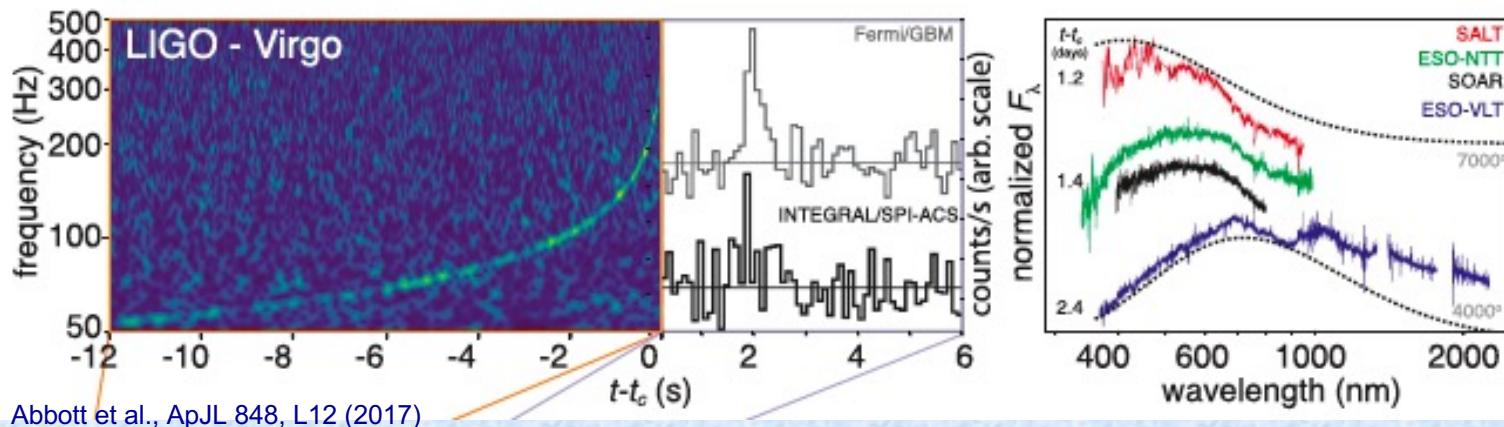


LIGO Hanford Observatory, Washington  
(Credits: C. Gray)



LIGO Livingston Observatory, Louisiana  
(Credits: J. Glairme)

- Detection of GW from **binary NS merger** in 2017 (GW170817)  
→ *multi-messenger astronomy*



A. F. Fantina

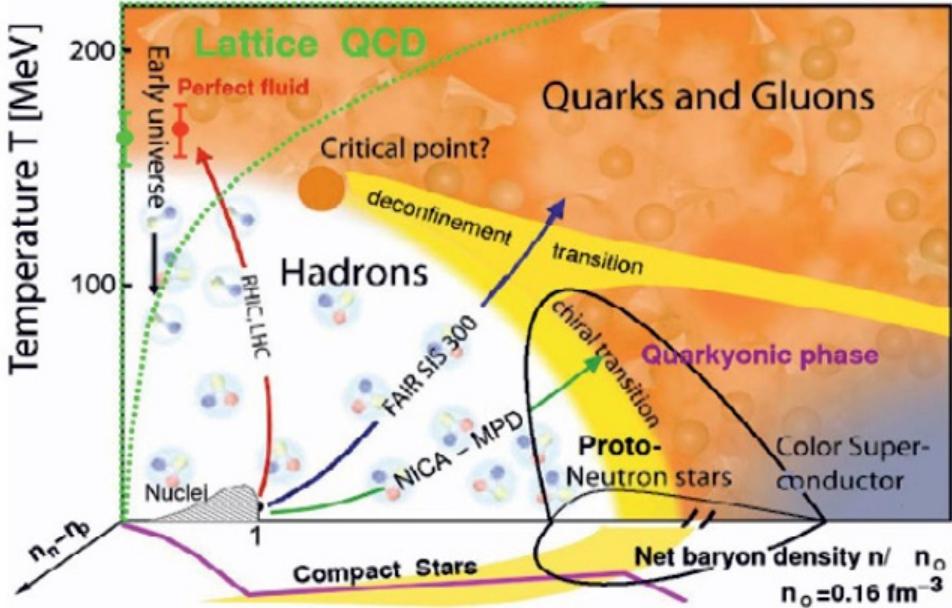
new window on the Univers



Virgo detector, Italy  
(Credits: Virgo Collaboration)



# Probing extreme conditions in NSs



Kekelidze et al., EPJ Web of Conf. 70, 00084 (2014)

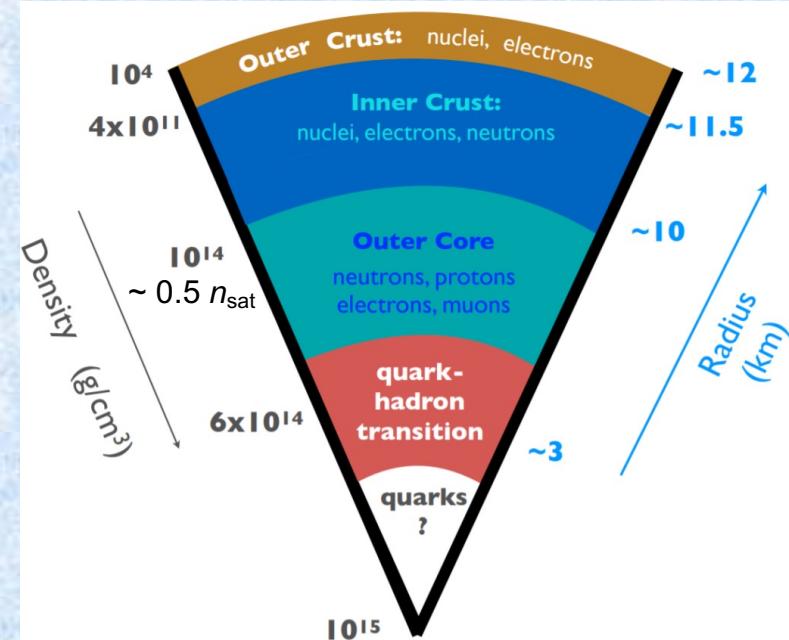


Image Credit: 3G Science White Paper

different states of matter spanned in NSs !

→ inhomogeneous, homogeneous, “exotic” particles (?) → see A. Raduta’s talk )  
+ superfluidity, magnetic field, etc.

**→ not all conditions can be probed in terrestrial labs → theoretical models !**



# Micro to macro through modelling

**Microphysics (inputs)**  
(e.g. EoS, nuclear processes)

**Astrophysical (macrophysics)  
hydrodynamic/static models**  
(simulations)

**Nuclear theory** (with model parameters)

**Nuclear physics Experiments**  
e.g. nuclear masses, resonances, decay rates, ...

**Astrophysical observations**  
(e.g. GW, NS masses, light curves,...)

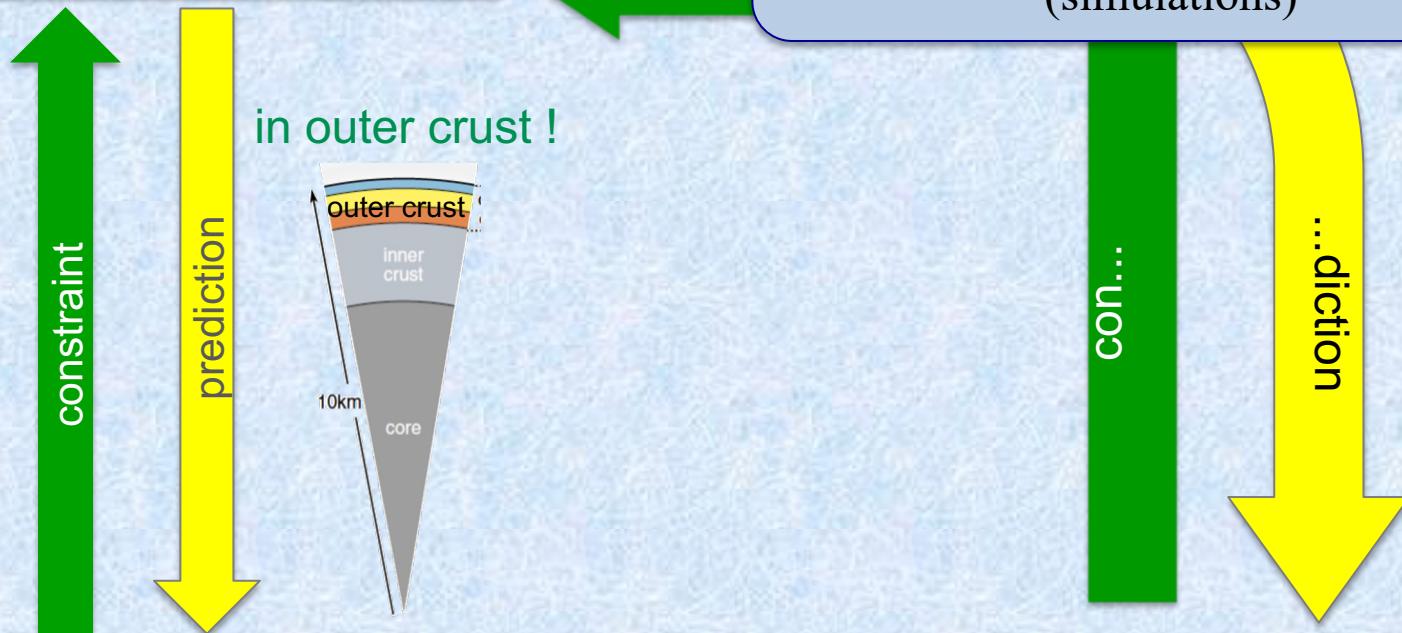




# Micro to macro through modelling

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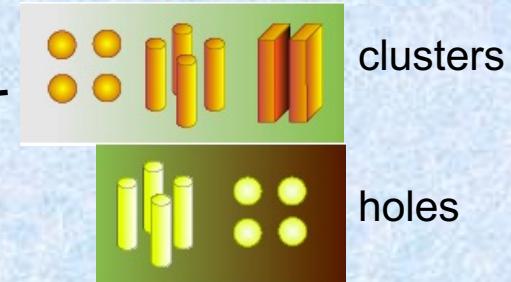
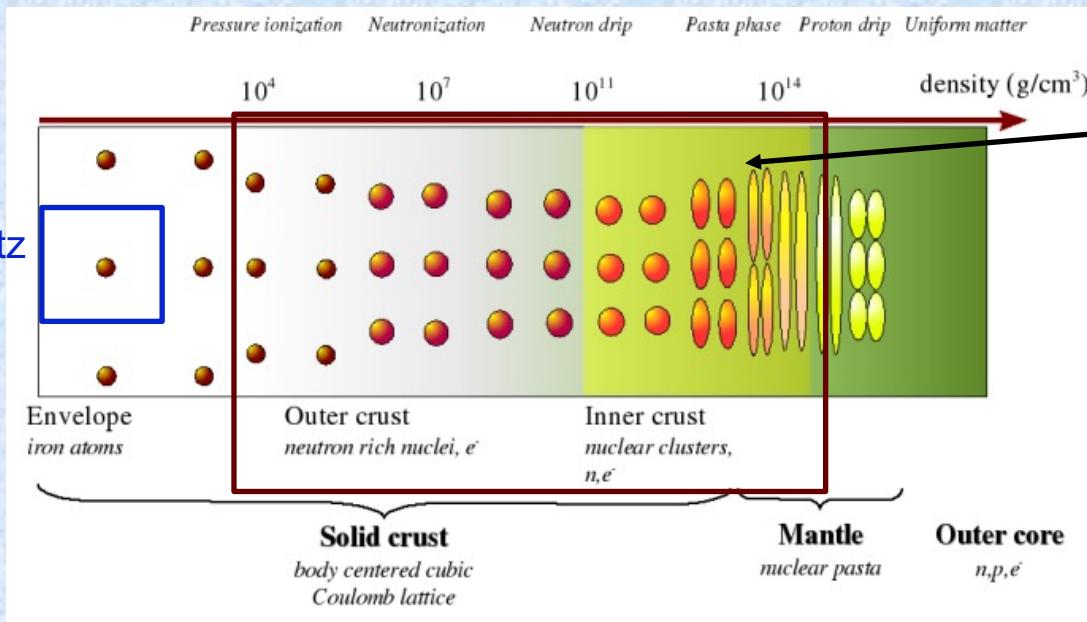
**Astrophysical observations**  
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# NS crust ( $T = 0$ )

“Cold” (catalysed) NS :  $T = 0$  approx., full (thermodynamic + beta) equilibrium

Wigner-Seitz  
(WS) cell



→ possible existence of  
“pasta” layer at the  
bottom of the crust

Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

➤ To obtain the EoS  $\rightarrow$  minimisation  $\varepsilon(n_B) = E_{\text{ws}}/V_{\text{ws}} = \min$

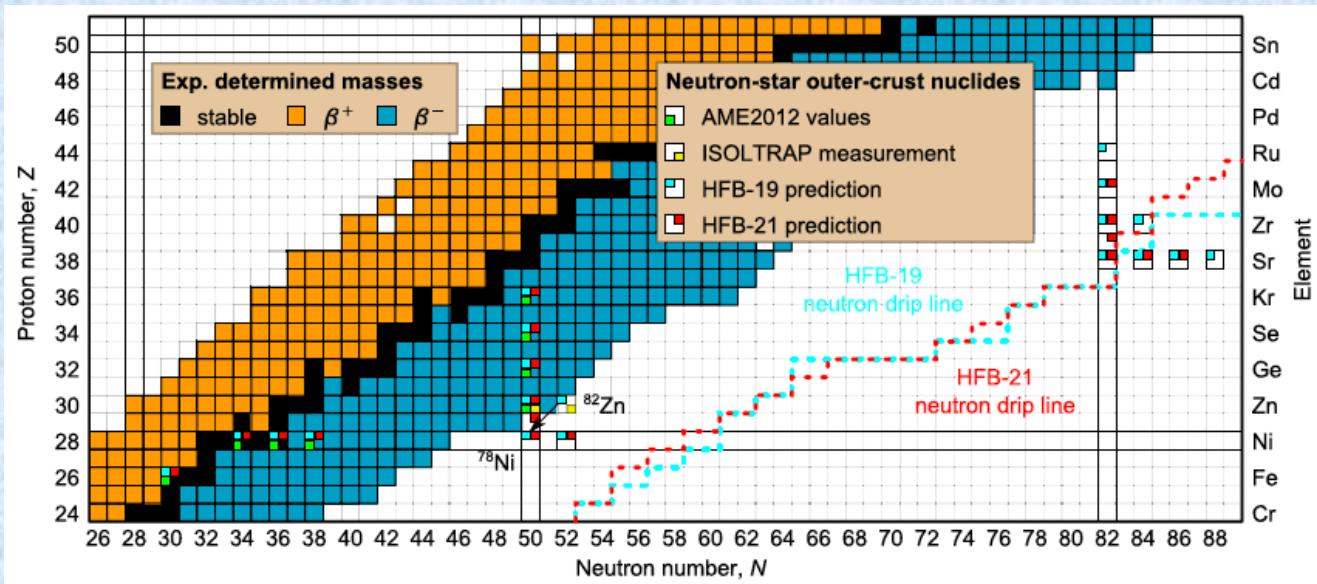
N.B.:  $n_B$  = baryon number density,  
 $\rho c^2 = \varepsilon$  mass-energy density !



# NS outer crust: up to neutron drip

- Nuclei in bcc lattice + electrons:  $\epsilon_{\text{WS}}(n_B) = \frac{M(A, Z)c^2}{V_{\text{WS}}} + \epsilon_e(n_e) + \epsilon_{\text{Coul}}$   
e-e and e-i int.

Only microscopic inputs are nuclear masses → Experimental or mass models



Wolf et al., PRL 110, 041101 (2013)

→ very neutron-rich nuclei  
→ imprint of shell structure

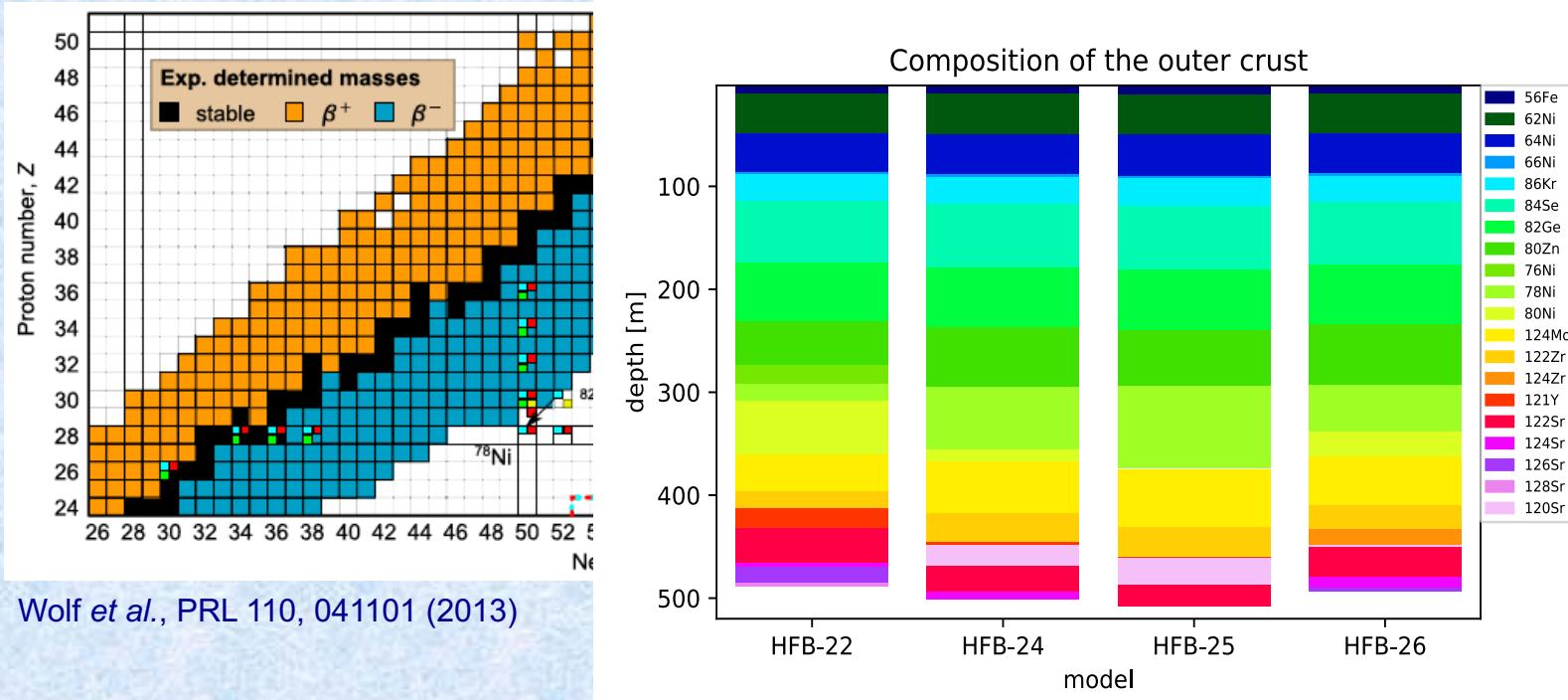
see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018), Blaschke & Chamel, ASSL 457, 337 (Springer, 2018) for a review and refs. therein  
for HFB models shown see also Goriely et al. 2010, 2016, Pearson et al., MNRAS 481, 2994 (2018)



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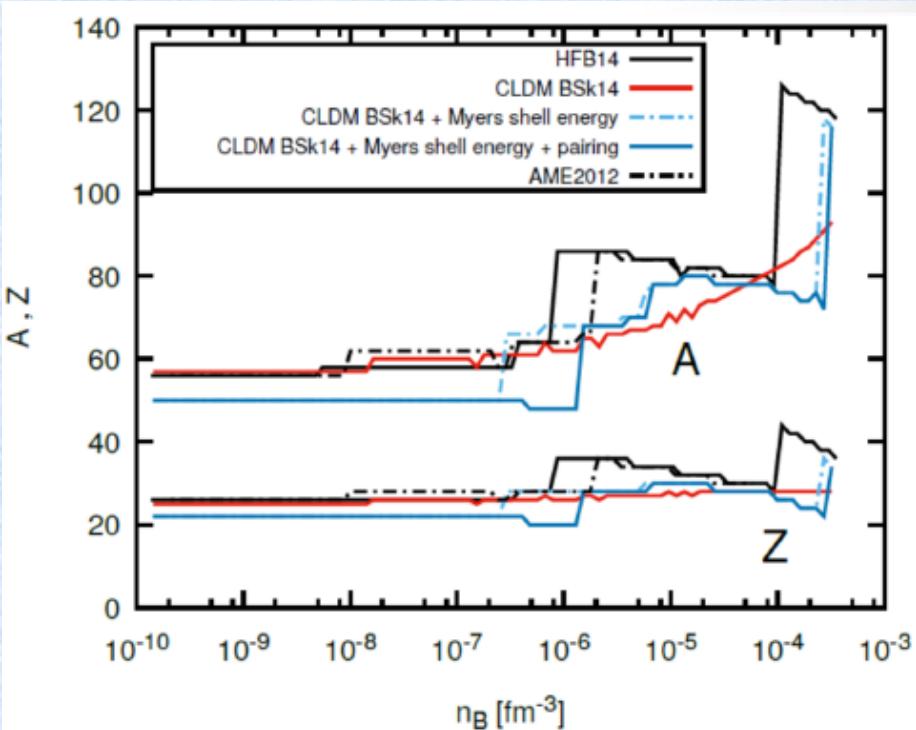
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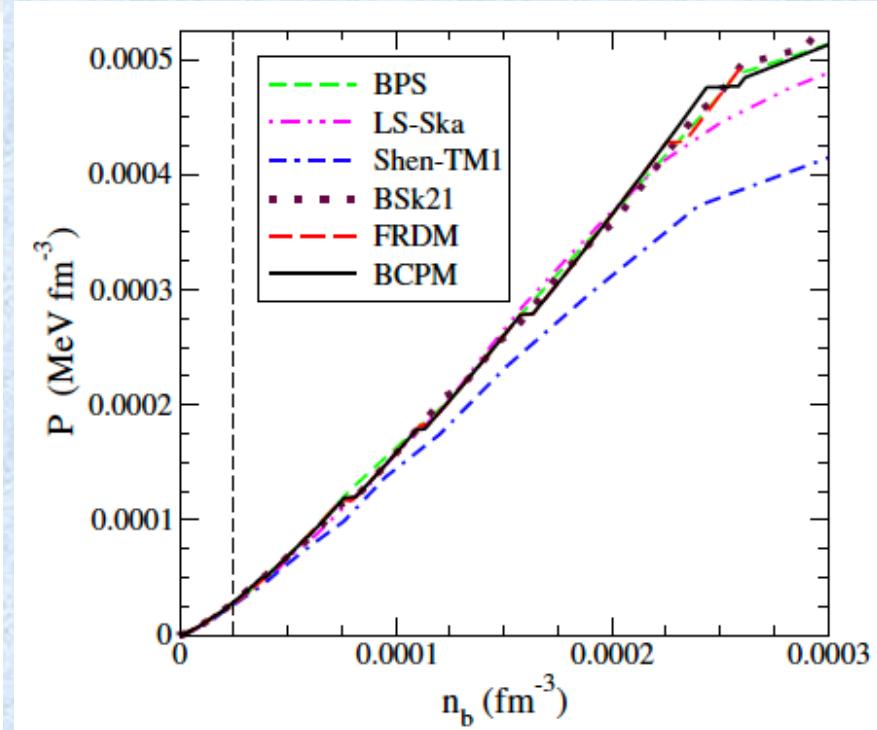
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# NS outer crust: composition and EoS



F. Gulminelli's talk @GMR workshop (2020)



Sharma et al., A&A 584, A103 (2015)



- dependence on the many-body method
- EoS relatively well constrained



# NS inner crust: until cc transition (1)

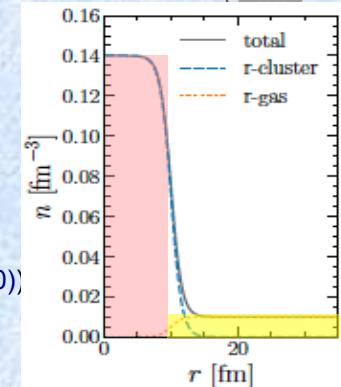
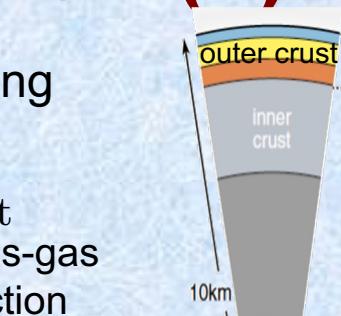
- Nuclei in bcc lattice + electrons + "free" neutrons → nuclear modelling

$$\epsilon_{\text{WS}}(n_B) = n_n m_n c^2 + n_p m_p c^2 + \epsilon_N + \epsilon_e + \epsilon_{\text{Coul}}$$

→ different methods:

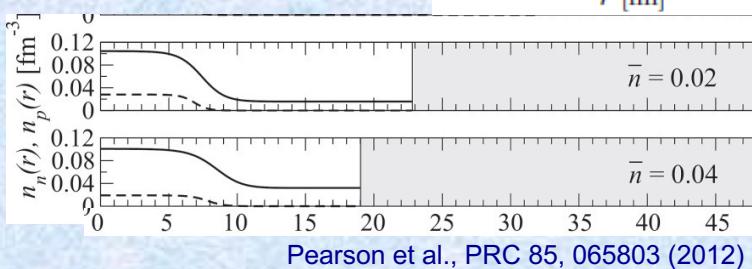
## 1. Compressible liquid-drop (CLD) model

- separation nucleons “inside”/“outside” nuclei (fig. adapted from Carreau, PhD thesis (2020))
  - nuclear energy given by sum of contribution (bulk, surface, Coulomb)



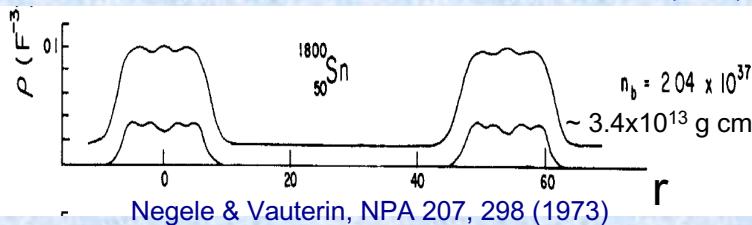
## 2. (Extended) Thomas-Fermi ((E)TF)

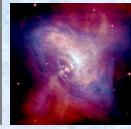
- smooth density profiles
  - nuclear energy functional of density and gradients
  - consistent treatment nucleons “inside”/“outside”



### 3. Hartree-Fock / Hartree-Fock Bogoliubov

- quantum calculations → independent particles/qp





# NS inner crust: until cc transition (2)

- Nuclei in bcc lattice + electrons + "free" neutrons → nuclear modelling

$$\epsilon_{\text{WS}}(n_B) = \frac{M(A, Z)c^2}{V_{\text{WS}}} + \epsilon_e(n_e) + \epsilon_{\text{Coul}} + \epsilon(n_{gn}) + \epsilon_{\text{int}}$$

homog.  
matter

→ model  
dependences

→ different methods : CLD, (E)TF,  
self-consistent mean-field

→ variational problem (beta equilibrium)

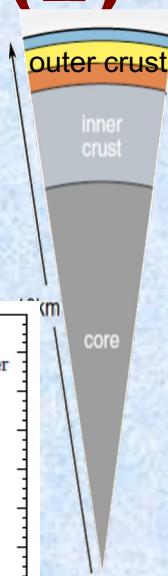
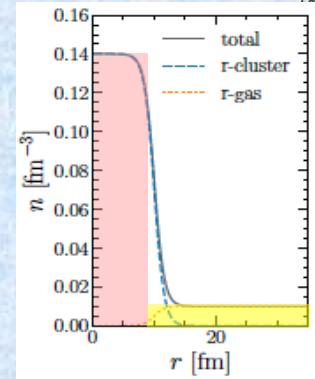
$$d \left[ \epsilon_{\text{WS}}(n_B) - \lambda_1 \left( \frac{A_{\text{WS}}}{V_{\text{WS}}} - n_B \right) - \lambda_2 \left( \frac{Z_{\text{WS}}}{V_{\text{WS}}} - n_p \right) \right] = 0$$

$\lambda_1$  and  $\lambda_2$  related to  $\mu_n$ ,  $\mu_p$

$Z_{\text{WS}} = Z$  if no free p ( $T=0$ );  $n_p = n_e$

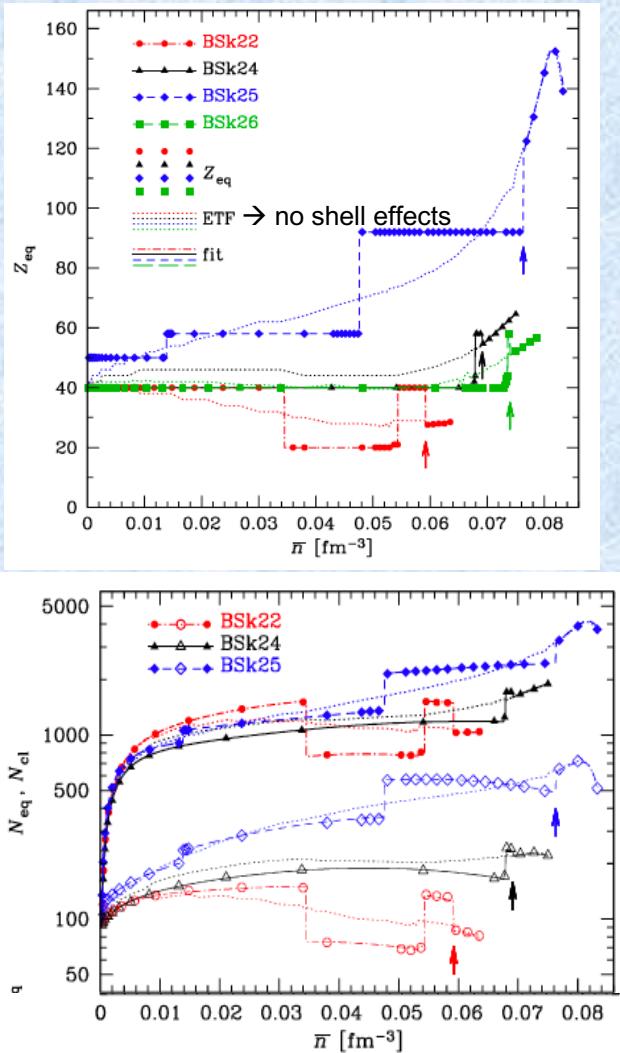
→ at each  $n_B$ , solve a system of coupled eqs.

→ obtain variational variables ( $A, Z, V_{\text{WS}}, n_{gn}$ ) + shape of WS cell if pasta included  
(NB: in HF, minimisation wrt wave functions)

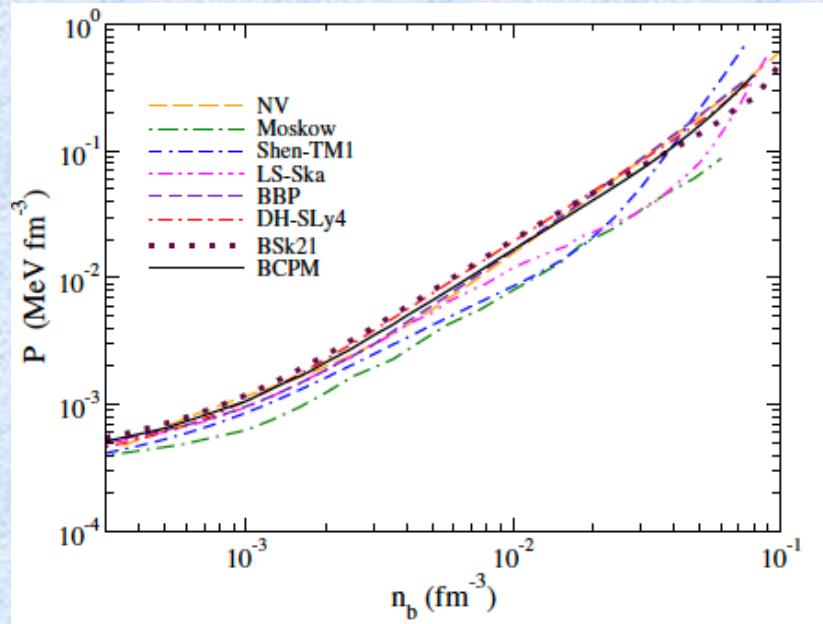




# NS inner crust: composition and EoS



Pearson et al., MNRAS 481, 2994 (2018)



Sharma et al., A&A 584, A103 (2015)

- model-dependence in many-body method
- model-dependence in functional



# Homogeneous matter: various approaches

- **Ab-initio (“microscopic”) approaches**

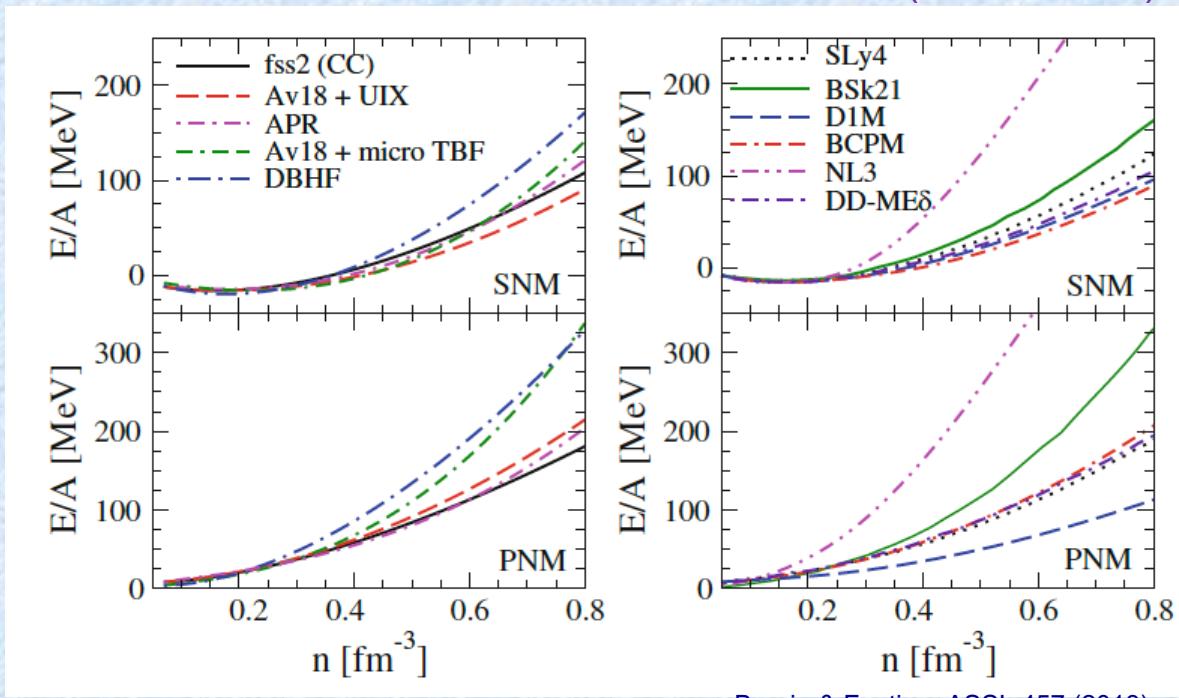
based on quantum many-body theories from realistic nuclear interactions (variational methods, (D)BHF, chiral EFT, Monte-Carlo, Green's func., ...)

→ usually restricted to homogeneous matter (core)

- **Phenomenological approaches**

based on effective interactions with parameters adjusted to reproduced nuclear properties (EDF e.g. Skyrme / Gogny, meta-models, ...)

→ also applicable for inhomogeneous matter (crust + core)



Burgio & Fantina, ASSL 457 (2018)



# Homogeneous matter: mean-field approx.

- Star is neutral  $\rightarrow$  baryons + leptons ( $e^-$  and possibly  $\mu$  in core); thermodynamic limit

$$\epsilon_{\text{tot}} = \epsilon_B + \epsilon_l \quad T = 0$$

- If only nucleons: (see A. Raduta's talk for "exotica")

$$e_{n,p} = \sqrt{(m_{n,p}^* c^2)^2 + (\hbar c k)^2} + V_{n,p}(n_n, n_p)$$

$$\epsilon_B = \epsilon_n(n_n) + \epsilon_p(n_p) = \epsilon_{\text{FG},n+p} + \epsilon(n_n, n_p)$$

Energy-density functional  
→ to be determined

→ *non-relativistic functionals*: Skyrme (zero-range effective interaction), Gogny (finite-range effective interaction) with parameters constrained by experiments but ad-hoc density dependence to mimic many-body effects

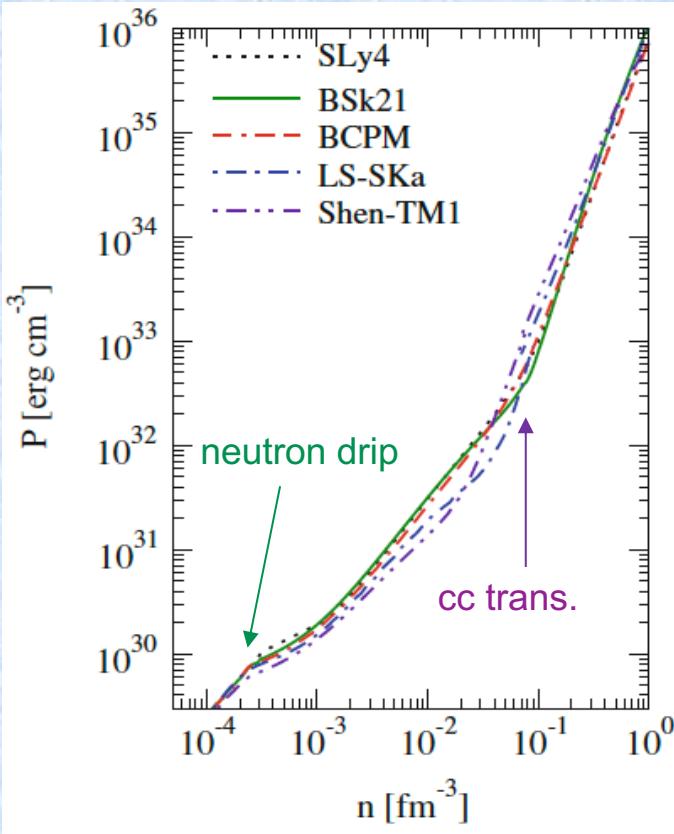
→ *relativistic mean field (RMF)* based on effective Lagrangian, but ad-hoc density dependent or non-linear couplings to mimic many-body effects

→ a functional form has to be chosen ! Extrapolations ?

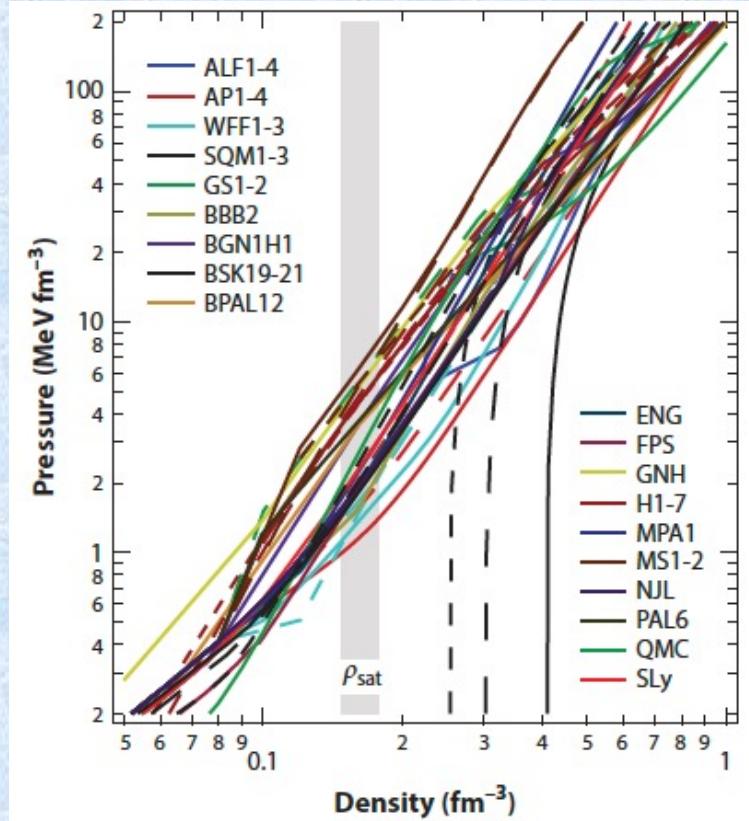
- Plus beyond mean-field: e.g., pairing



# NS EoS: crust and core



Burgio & Fantina, ASSL 457 (2018)



Ozel & Freire, ARAA 54, 401 (2016)

- higher uncertainties in the core → uncertainties in the functional
- ➡ how these uncertainties propagate in NS observables ?



# EoS $\longleftrightarrow$ NS (static) observables (1)

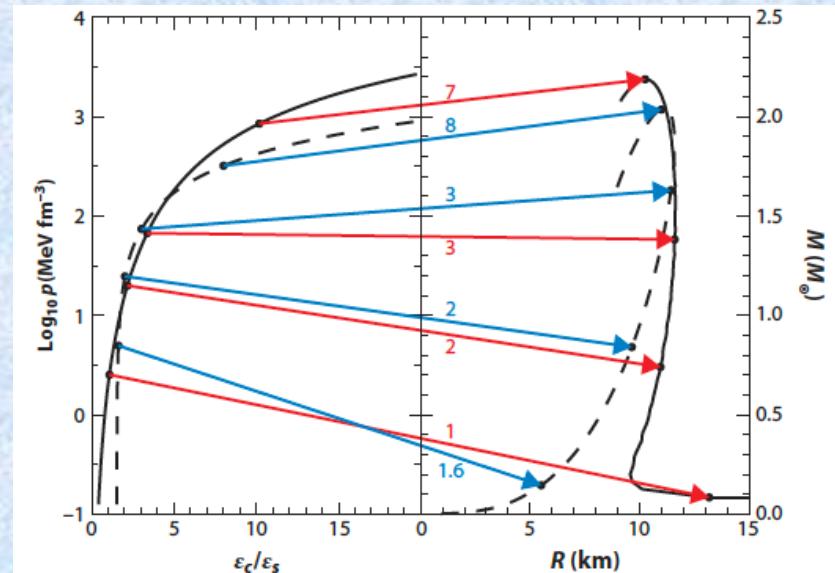
- TOV  $\rightarrow M(R)$  (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel, Potekhin, Yakovlev, Springer 2007)

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} \left[ 1 + \frac{P(r)}{c^2\rho(r)} \right] \left[ 1 + \frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)} \right] \left[ 1 - \frac{2G\mathcal{M}(r)}{c^2r} \right]^{-1}$$

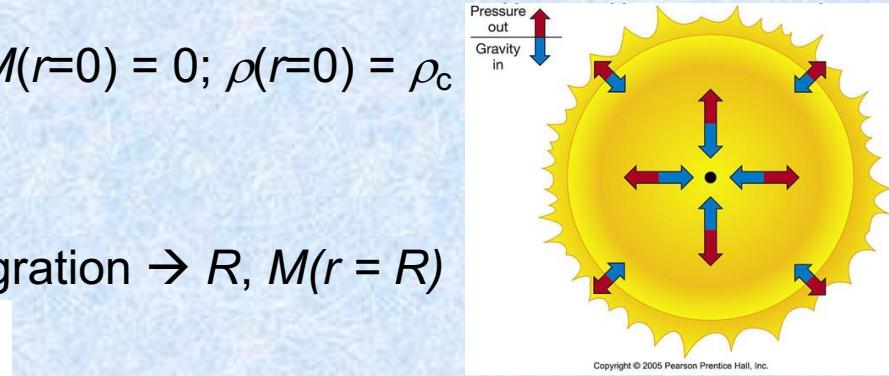
$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r')r'^2 dr' \quad \text{with b.c. } M(r=0) = 0; \rho(r=0) = \rho_c$$

→ only EoS  $P(\rho)$  is needed !

→ for each  $\rho_c$  (or equivalently  $P_c$ )  $\rightarrow$  integration  $\rightarrow R, M(r=R)$



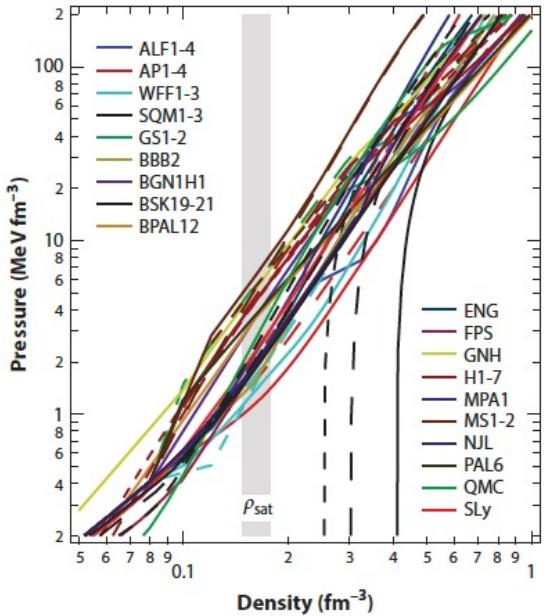
Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)



GR  $\rightarrow$  one-to-one correspondence  
EoS  $\longleftrightarrow$  NS static properties  
(non-rotating mature NS)



# EoS $\longleftrightarrow$ NS (static) observables (2)

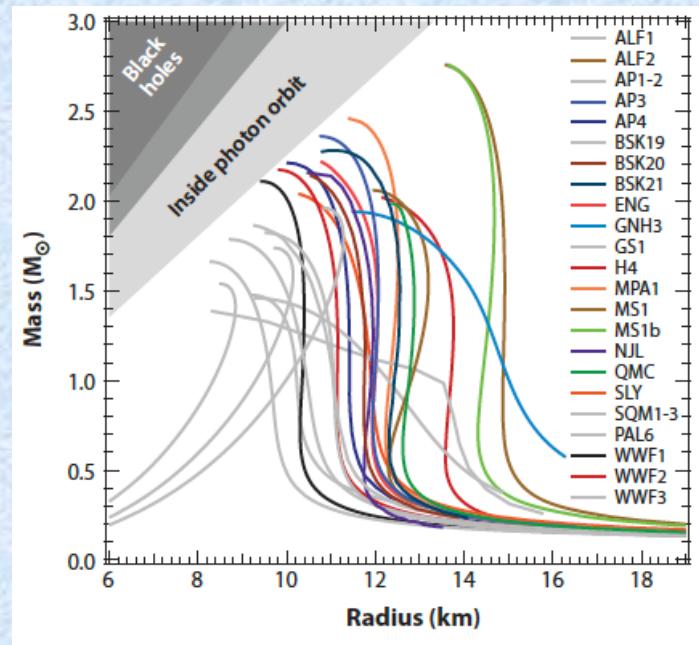


Ozel & Freire, ARAA 54, 401 (2016)

but:

- X EoS model dependent !
- X no ab-initio dense-matter calculations in all regimes  
→ phenomenological models
- X composition  $\longleftrightarrow$  EoS  $\rightarrow M(R)$  ?
- X role of additional d.o.f. ? → see A. Raduta's talk

- ✓ GR  $\rightarrow$  one-to-one correspondence  
EoS  $\longleftrightarrow$  NS static properties  $M(R)$ ,  $\Lambda(M)$ ...  
(non-rotating mature NS)
- ✓ Different EoSs  $\longleftrightarrow$  different NS properties  
 $\longleftrightarrow$  different GW signals  
?  $\rightarrow$  trace back to EoS and composition ?



Ozel & Freire, ARAA 54, 401 (2016);  
see also Burgio & Fantina, ASSL 457, 255 (2018)



# EoS $\longleftrightarrow$ NS (static) observables (3)

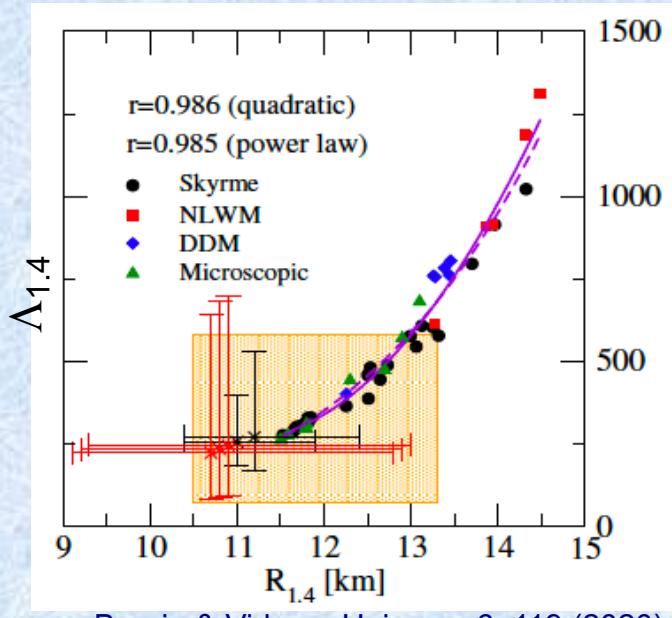
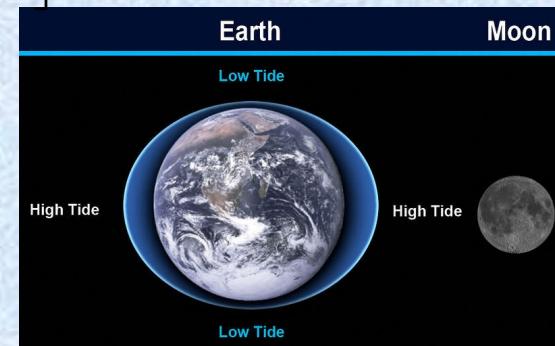
- Influence of second body  $\rightarrow$  eq. for  $H(r) \rightarrow \Lambda(M), \Lambda(R)$  (Thorne & Campolattaro 1967)

$$H''(r) + H' \left( 1 - \frac{2G\mathcal{M}(r)}{c^2 r} \right)^{-1} \left[ \frac{2}{r} - \frac{2G\mathcal{M}(r)}{c^4} r(\rho(r) - P(r)) \right] + H(r)Q(r, P(r), \rho(r))$$

with b.c.  $M(r=0) = 0$ ;  $\rho(r=0) = \rho_c$ ;  $H(r=0) = 0$ ,  $H'(r=0) = 0$

→ only EoS  $P(\rho)$  is needed !

→ for each  $\rho_c$  → integration →  $k_2$  →  $\lambda$  tidal response



Burgio & Vidana, Universe 6, 119 (2020)

→ tidal polarizability  $\Lambda = \lambda(r = R) = \frac{2}{3}k_2 \left( \frac{Rc^2}{GM} \right)^5$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

→ can be extracted from GW signal

N.B.: GR in slow rotation limit w/o magnetic field !



# Micro to macro through modelling

**Microphysics (inputs)**  
(e.g. EoS, nuclear processes)

**Astrophysical (macrophysics)  
hydrodynamic/static models**  
(simulations)

**Nuclear theory** (with model parameters)

**Nuclear physics Experiments**  
e.g. nuclear masses, resonances, decay rates, ...

**Astrophysical observations**  
(e.g. GW, NS masses, light curves,...)



# EoS modelling: different approaches

## ▪ Ab-initio (“microscopic”) approaches

based on quantum many-body theories from realistic nuclear interactions (variational methods, (D)BHF, chiral EFT, Monte-Carlo, Green’s func., ...) → usually restricted to homogeneous matter (core)

## ▪ Phenomenological approaches

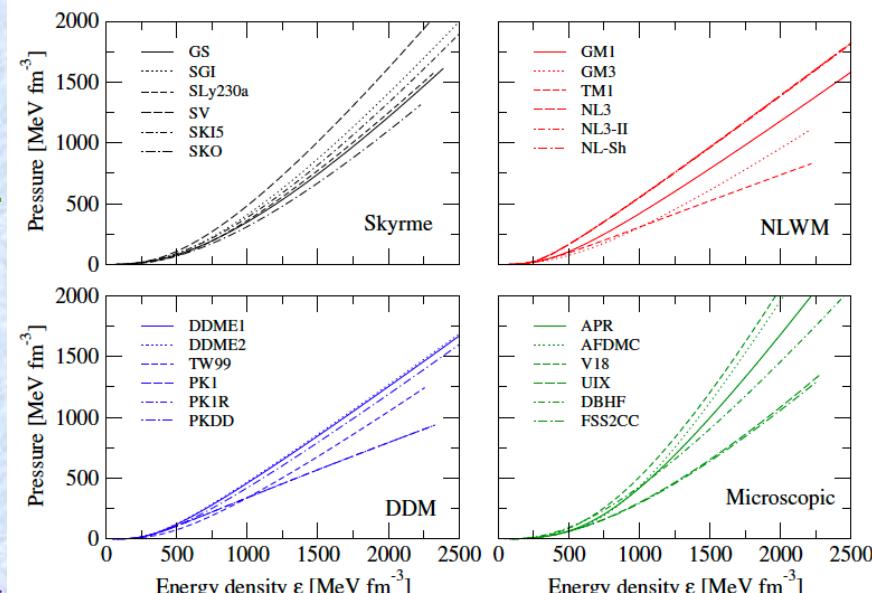
based on effective interactions with parameters adjusted to reproduced nuclear properties (EDF e.g. Skyrme/Gogny, meta-models, ...)

→ also applicable for inhomogeneous matter (crust + core)

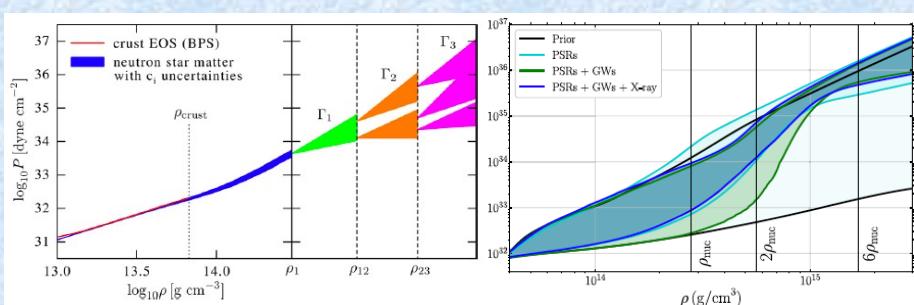
## ▪ Agnostic (non-parametric) approaches

- Piecewise polytropes (PP)
- Speed-of-sound models (CSM)
- Spectral functions (SF)
- Gaussian processes (GP)

→ but what about info on composition?



Burgio & Vidana, Universe 6, 119 (2020)



Hebeler et al., ApJ 773, 11 (2013) Landry et al. PRD 101, 123007 (2020)

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018)

Agnostic approaches, e.g. PP: Reed et al. PRD 2009, Hebeler et al. ApJ 2013, Annala et al. PRL 2018, ...; CSM: Tews et al.

ApJ 2018, Tan et al. PRL 2020; Somasundaram et al., PRC 2023; SF: Lindblom 2010, Lindblom & Indik 2014, ...; GP: Landry et al. PRD 2020, Essick et al. PRD 2020; Legred et al. PRD 2021, PRD 2022, ...



# A choice: meta-model (nucleons)

- **Meta-model** approach for nucleons : flexible functional (“quasi” agnostic)  
→ expansion in density and asymmetry around  $n_{\text{sat}}$  and  $\delta = 0$  (with  $m_q^*$  included)

$$\epsilon_B(n) \approx n (e_B(n, \delta = 0) + e_{\text{sym}}(n) \delta^2) \quad x = (n - n_{\text{sat}})/3n_{\text{sat}}$$

$$\approx n \sum_{m=0}^4 \frac{1}{m!} \left( \left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m \quad \delta = (n_n - n_p)/n$$

Empirical parameters (bulk)  $X_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$

$$E_{\text{sat}} = e_{\text{sat}}$$

$$E_{\text{sym}} = J = e_{\text{sym}}$$

$$K_{\text{sat}} = 9n_{\text{sat}}^2 \left. \frac{d^2 e_{\text{sat}}}{dx^2} \right|_{n=n_{\text{sat}}, \delta=0}$$

$$L_{\text{sym}} = 3n_{\text{sat}} \left. \frac{de_{\text{sym}}}{dx} \right|_{n=n_{\text{sat}}, \delta=0}$$

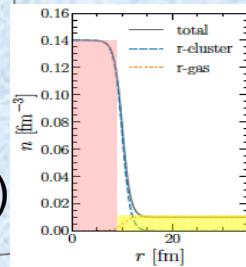
$$Q_{\text{sat}} = 27n_{\text{sat}}^3 \left. \frac{d^3 e_{\text{sat}}}{dx^3} \right|_{n=n_{\text{sat}}, \delta=0}$$

$$K_{\text{sym}} = 9n_{\text{sat}}^2 \left. \frac{d^2 e_{\text{sym}}}{dx^2} \right|_{n=n_{\text{sat}}, \delta=0}$$

...

$$Q_{\text{sym}} = 27n_{\text{sat}}^3 \left. \frac{d^3 e_{\text{sym}}}{dx^3} \right|_{n=n_{\text{sat}}, \delta=0}$$

~ 15 – 20  
parameters



- If one wants to model the crust → + surface and Coulomb term (CLDM)  
→ surface parameters  
(plus some additional modifications:  $m^*$ , low-density corrections, ...)

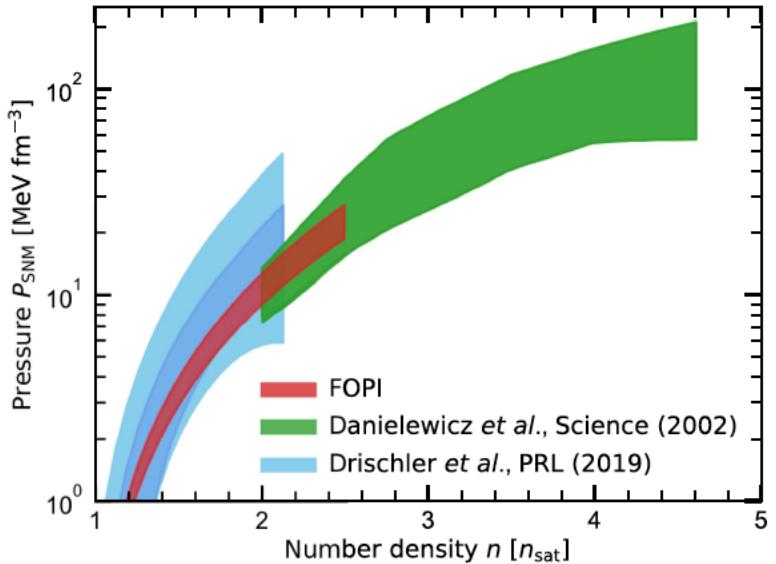


# How can we get constraints?

## Nuclear physics exp./ theory

- Measure of **nuclear properties**:
  - masses and radii of nuclei
  - collective modes, polarizability
  - neutron skins, HIC, flows
  - etc ...
- **ab-initio calculations**

→ “low” density (better in nucleonic sector)

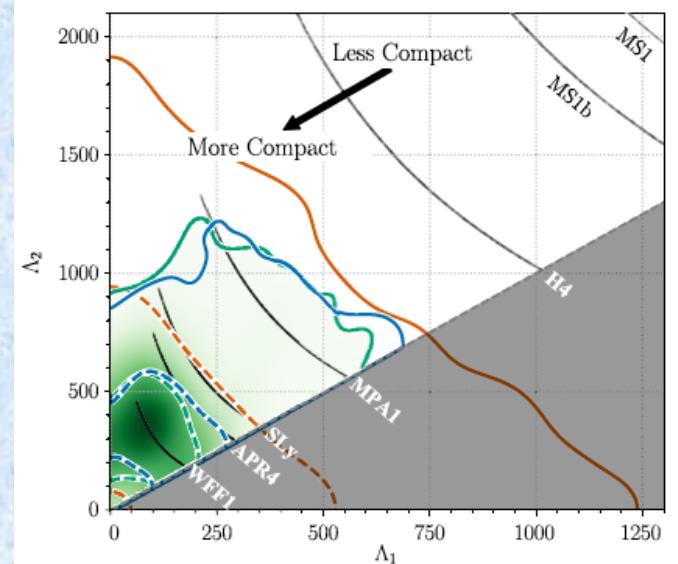


Huth et al., Nature 606, 276 (2022)

## Astrophysical observations

- Measure of **NS properties**:
  - NS masses and radii
  - rotational frequency, oscillation modes
  - cooling, moment of inertia
  - etc ...
- **Gravitational waves**

→ “high” density



Abbott et al., PRL 121, 161101 (2018)

A. F. Fantina

see A. Raduta's talk for  
astro constraints



# How can we get constraints?

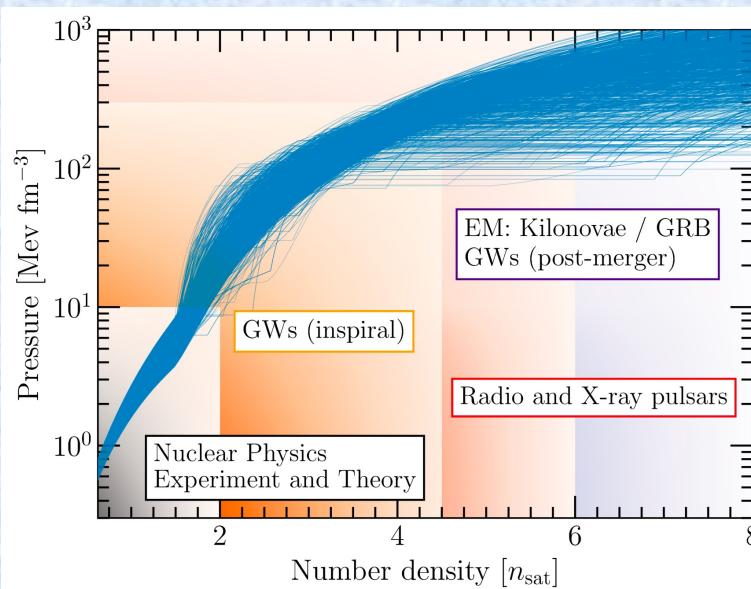
## Nuclear physics exp./ theory

- Measure of **nuclear properties**:
  - masses and radii of nuclei
  - collective modes, polarizability
  - neutron skins, HIC, flows
  - etc ...
- **ab-initio calculations**

→ “low” density (better in nucleonic sector) → “high” density

## Astrophysical observations

- Measure of **NS properties**:
  - NS masses and radii
  - rotational frequency, oscillation modes
  - cooling, moment of inertia
  - etc ...
- **Gravitational waves**

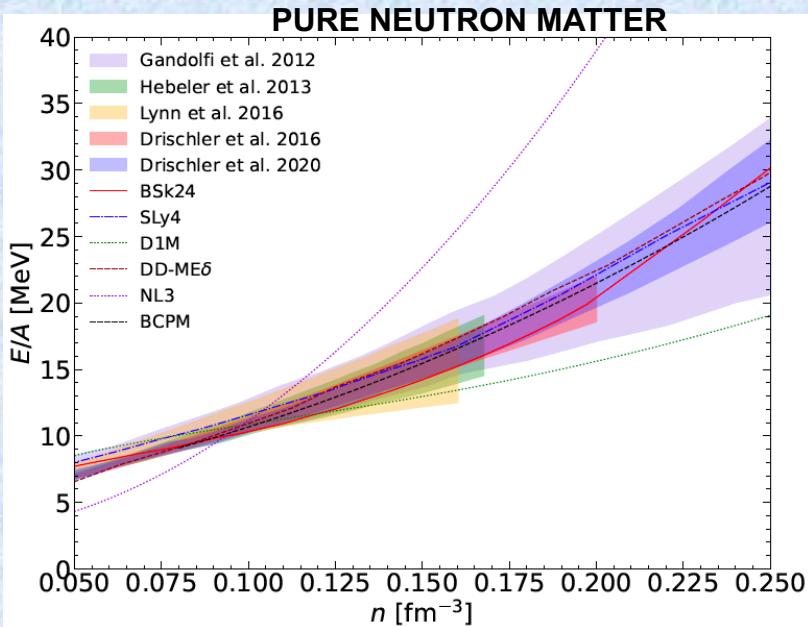


N.B.: rectangles only qualitative,  
→ EoS dependence !

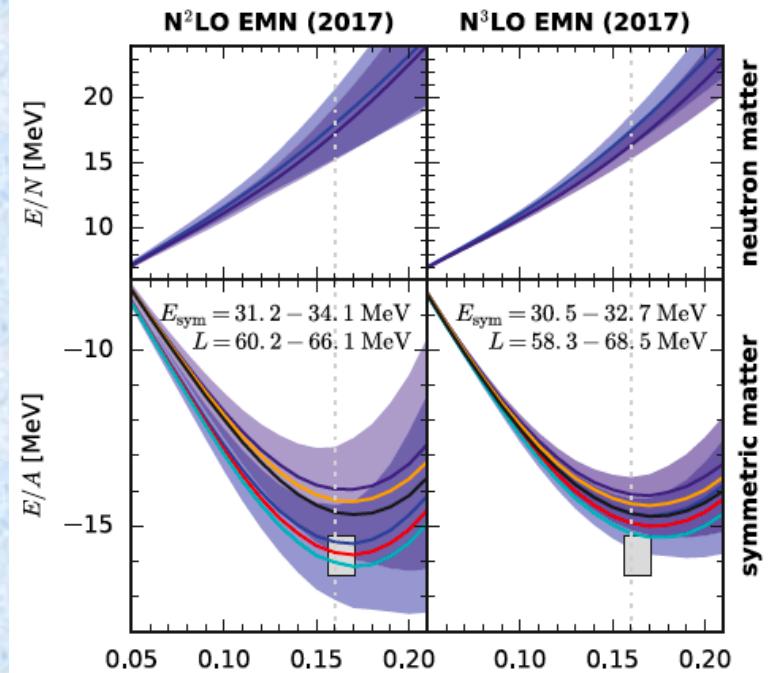
Pang et al., arXiv:2205.08513 (2022)



# Constraints from nucl. phys.: theo



Fantina & Gulminelli, J.Phys. Conf. Ser. (submitted 2022);  
see also Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)



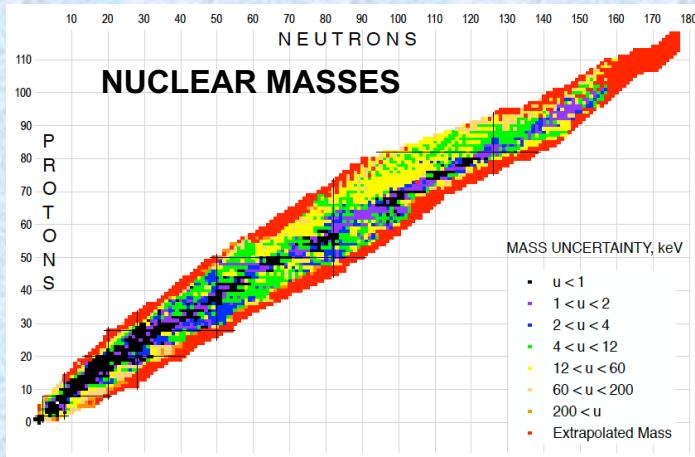
Drischler et al., PPNP 121, 103888 (2021)

- Not all popular models agree with ab-initio constraints!
- Reasonable agreement of ab-initio (PNM) up to ~ saturation density  
→ PNM calculations benchmark for phenomenological models

N.B.: for symmetric matter (ab-initio): (i) saturation point difficult to obtain ;  
(ii) larger uncertainties ; (iii) cluster formation at subsaturation

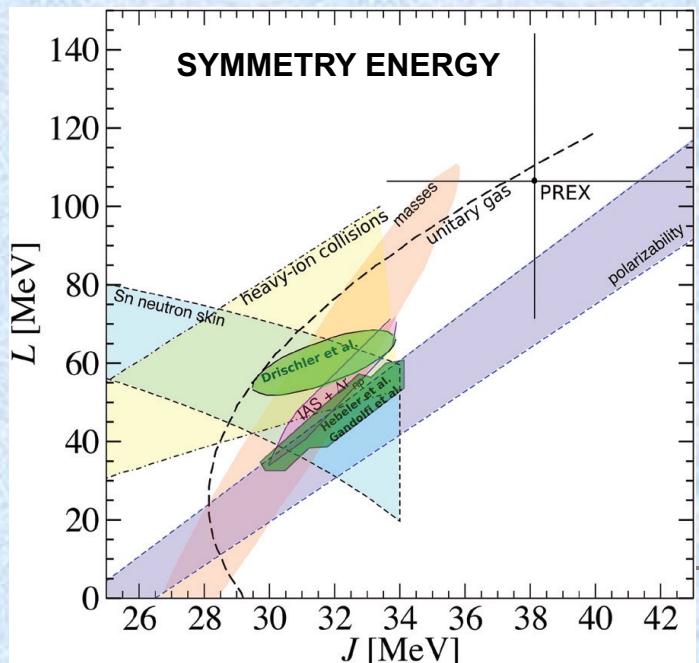


# Constraints from nucl. phys.: exp (1)



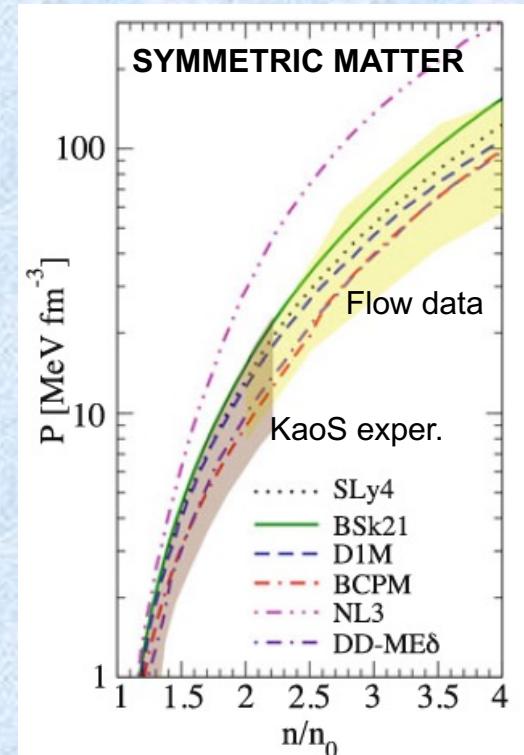
Kondev et al., Chin. Phys. C 45, 030001 (2021)

- Constraints at “low” densities
- low-order parameters
- Constraints more on “symmetric” matter



Gulminelli & Fantina, Nucl. Phys. News 31, 9 (2021)

- Not always “clear” constraints
- “tension”



Burgio & Fantina, ASSL 457, 255 (2018)  
(Flow: Danielewicz et al., Science 2002  
KaoS: Lynch et al., PPNP 2009)

N.B.: deduced constraints are often *not* raw data,  
but combined with models  
→ model dependence of constraints !



# Constraints from nucl. phys.: exp (2)

Model	Ref.	$E_{\text{sat}}$ (MeV)	$n_{\text{sat}}$ ( $\text{fm}^{-3}$ )	$K_{\text{sat}}$ (MeV)	$E_{\text{sym}}$ (MeV)	Model	Ref.	$Q_{\text{sat}}$ (MeV)	$L_{\text{sym}}$ (MeV)	$K_{\text{sym}}$ (MeV)	$K_{\tau}$ (MeV)
El. scatt.	Wang-99 [55]		0.1607	235 ±15							
LDM	Myers-66 [56]	-15.677	0.136 <sup>a</sup>	295	28.06	DF-Skyrme	Berdichevsky-88 [71]	30	0		
LDM	Royer-08 [57]	-15.5704	0.133 <sup>a</sup>		23.45	DF-Skyrme	Farine-97 [72]	-700 ±500			
LSD	Pomorski-03 [58]	-15.492	0.142 <sup>a</sup>		28.82	DF-Skyrme	Alam-14 [31]	-344 ±46	65 ±14	-23 ±73	-322 ±34
DM	Myers-77 [59]	-15.96	0.145 <sup>a</sup>	240	36.8	DF-Skyrme	McDonnell-15 [66]	40 ±20			
FRDM	Buchinger-01 [60]		0.157			DF-NLRMF	NL3* [67]	124	123	106	-690
			±0.004			DF-NLRMF	PK [68]	-25	116	55	-630
INM	Satpathy-99 [61]	-16.108	0.1620	288 ±20		DF-DDRMF	DDME1,2 [69,70]	400 ±80	53 ±3	-94 ±7	-500 ±7
DF-Skyrme	Tondeur-86 [62]		0.158			DF-DDRMF	PK [68]	-119	79.5	-50	-491
DF-Skyrme	Klupfel-09 [63]	-15.91 ±0.06	0.1610 ±0.0013	222 ±8	30.7 ±1.4	Correlation	Centelles-09 [73]	70 ±40			-425 ±175
DF-BSK2	Goriely-02 [64]	-15.79	0.1575	234	28.0	DF-RPA	Carbone-10 [74]	60 ±30			
DF-BSK24, 28,29	Goriely-15 [65]	-16.045 ±0.005	0.1575 ±0.0004	245	30.0	Correlation	Danielewicz-14 [75]	53 ±20			
DF-Skyrme	McDonnell-15 [66]	-15.75 ±0.25	0.160 ±0.005	220 ±20	29 ±1	Correlation	Newton-14 [76]	70 ±40			
DF-NLRMF	NL3* [67]	-16.3	0.15	258	38.7	Correlation	Lattimer-14 [77]	53 ±20			
DF-NLRMF	PK [68]	-16.27	0.148	283	37.7	GMR	Sagawa-07 [78]	-500 ±50			
DF-DDRMF	DDME1,2 [69,70]	-16.17 ±0.03	0.152 ±0.00	247 ±3	32.7 ±0.4	GMR	Patel-14 [79]	-550 ±100			
DF-DDRMF	PK [68]	16.27	0.150	262	36.8	Present Estimation		300 ±400	60 ±15	-100 ±100	-400 ±100
Present Estimation		-15.8 ±0.3	0.155 ±0.005	230 ±20	32 ±2						

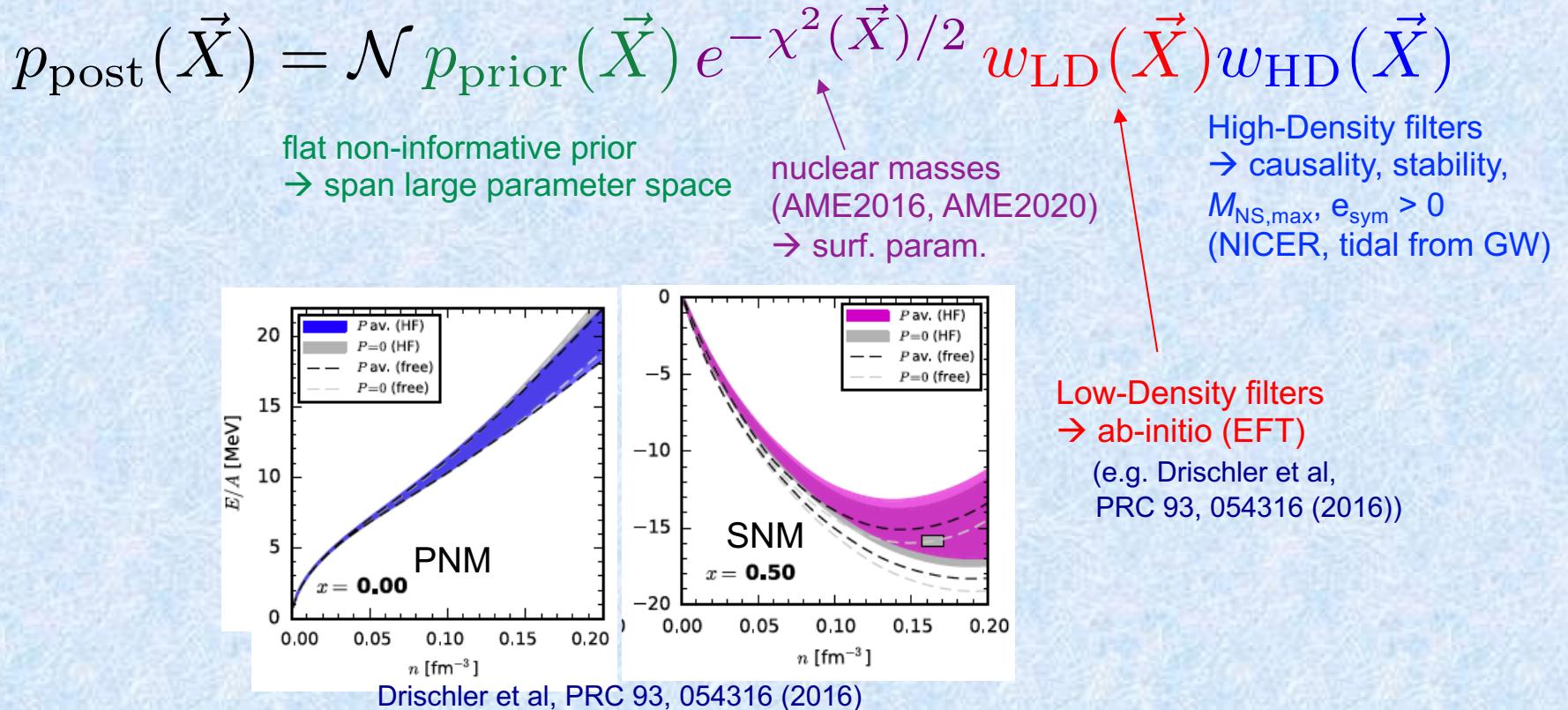
Margueron et al., PRC 97, 025805 (2018)  
 see also Stone et al., PRC 89, 044316 (2014)

N.B.: parameter estimation from various analysis  
 of experimental data  
 → but through different models  
 → not straightforward nor unambiguous extraction



# Bayesian study

- Explore many models ! → One can vary parameters (independently)  
Empirical parameters (bulk)  $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$   
 $\text{prob}(\text{hp}|\text{data}) \propto \text{prob}(\text{hp}) \times \text{prob}(\text{data}|\text{hp})$





# Crustal properties and correlations

Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021)

## CRUST-CORE TRANSITION

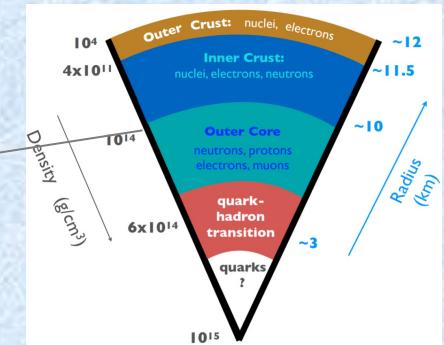


Image Credit: 3G Science White Paper

$n_{CC}$

	$E_{sat}$	$n_{sat}$	$K_{sat}$	$Q_{sat}$	$Z_{sat}$	$E_{sym}$	$L_{sym}$	$K_{sym}$	$Q_{sym}$	$Z_{sym}$	$\sigma_0$	$b_s$	$\sigma_{0c}$	$\beta$	$p$
LD+HD ( $n \geq 0.02 \text{ fm}^{-3}$ )	-0.04	-0.07	0.11	-0.05	-0.02	-0.30	-0.57	-0.15	0.45	-0.15	0.05	0.52	-0.15	-0.04	0.51
LD+HD ( $n \geq 0.1 \text{ fm}^{-3}$ )	-0.06	-0.06	0.33	-0.46	0.17	-0.15	-0.29	-0.10	0.39	-0.16	0.06	0.34	-0.11	-0.08	0.33
Prior	0.14	0.09	0.13	-0.18	0.02	0.08	-0.56	0.11	0.20	-0.05	-0.17	0.07	0.29	0.18	0.18

$P_{CC}$

	$E_{sat}$	$n_{sat}$	$K_{sat}$	$Q_{sat}$	$Z_{sat}$	$E_{sym}$	$L_{sym}$	$K_{sym}$	$Q_{sym}$	$Z_{sym}$	$\sigma_0$	$b_s$	$\sigma_{0c}$	$\beta$	$p$
LD+HD ( $n \geq 0.02 \text{ fm}^{-3}$ )	-0.05	-0.03	0.08	-0.04	-0.02	-0.08	-0.53	-0.30	0.34	-0.07	0.04	0.47	-0.04	-0.03	0.52
LD+HD ( $n \geq 0.1 \text{ fm}^{-3}$ )	-0.05	-0.06	0.27	-0.34	0.12	0.02	-0.31	-0.41	0.45	-0.16	0.05	0.19	-0.03	-0.07	0.22
Prior	0.10	0.00	0.08	-0.06	0.02	0.22	-0.06	-0.44	0.42	-0.07	-0.13	-0.07	0.23	0.11	0.10

bulk

surface

- importance of parameters (both *bulk* and *surface*) on observables
- importance of higher-order parameters

## CRUSTAL MOMENT OF INERTIA

Carreau et al., PRC 100, 055803 (2019)

$I_{\text{crust},1.4}/I_{1.4}$

	$n_{sat}$	$E_{sat}$	$K_{sat}$	$Q_{sat}$	$Z_{sat}$	$E_{sym}$	$L_{sym}$	$K_{sym}$	$Q_{sym}$	$Z_{sym}$	$m^*/m$	$\Delta m^*/m$	$\nu$	$\beta$	$\delta \theta$	$\nu_s$
LD p=3	0.01	-0.05	0.20	-0.06	-0.06	-0.24	-0.38	0.42	0.77	-0.32	0.00	0.07	-0.04	0.00	0.04	0.19
HD p=3	-0.03	0.24	0.26	-0.22	0.07	0.26	-0.08	-0.33	0.46	-0.10	-0.03	0.04	-0.10	0.00	-0.23	-0.02
LD	0.04	-0.21	0.16	-0.05	-0.04	-0.09	-0.23	0.26	0.52	-0.20	-0.01	0.04	-0.02	0.56	0.46	0.58
HD	-0.02	0.14	0.24	-0.21	0.08	0.25	-0.08	-0.31	0.42	-0.09	-0.02	0.04	-0.09	0.25	-0.09	0.18
Prior	0.02	0.12	0.21	-0.11	0.03	0.25	-0.03	-0.46	0.48	-0.11	-0.04	-0.14	-0.15	0.11	-0.08	0.10

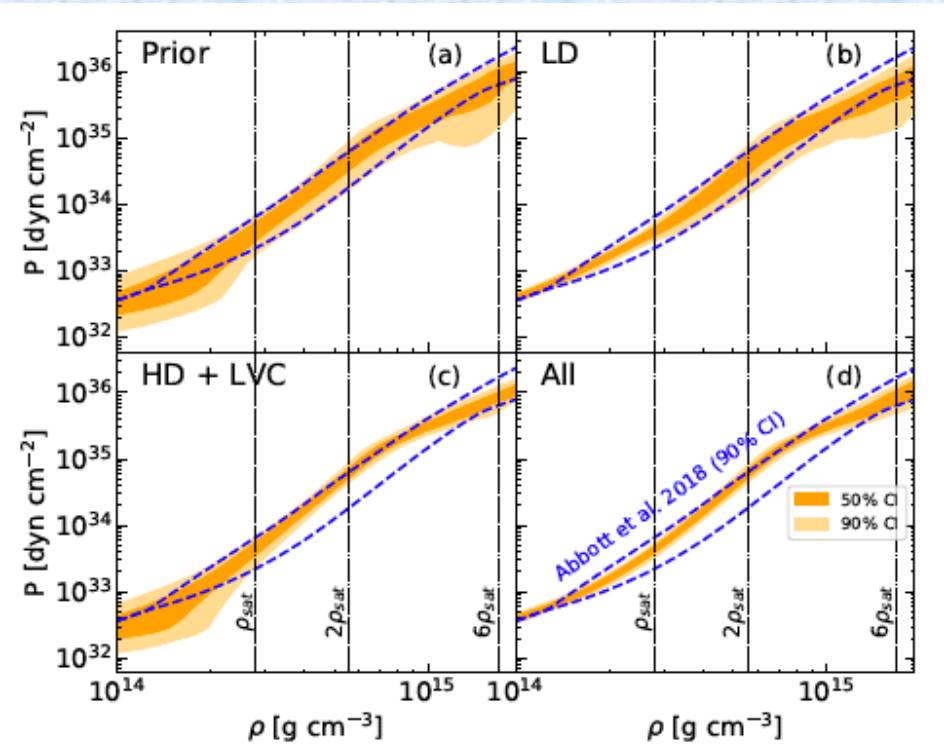
bulk

surface 35

see also Balliet et al., ApJ 918, 79 (2021)



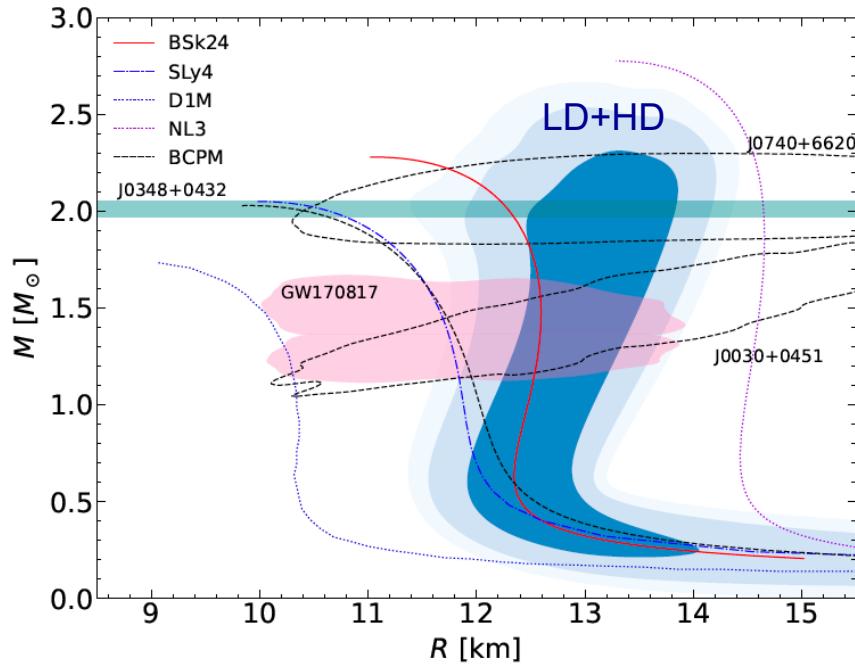
# EoS : effect of LD/HD constraints



Dinh Thi et al., Universe 7, 373 (2021)

- filters reduce uncertainties
- posterior compatible with observations  
but: some popular models are not !
  
- nucleonic hp compatible with observations  
→ observations not yet enough constraining!

LD (EFT calc.) → low-density (nucl.phys.)  
HD → high-density (astro)  
(causality,  $M_{\max}$ , NICER, GW)

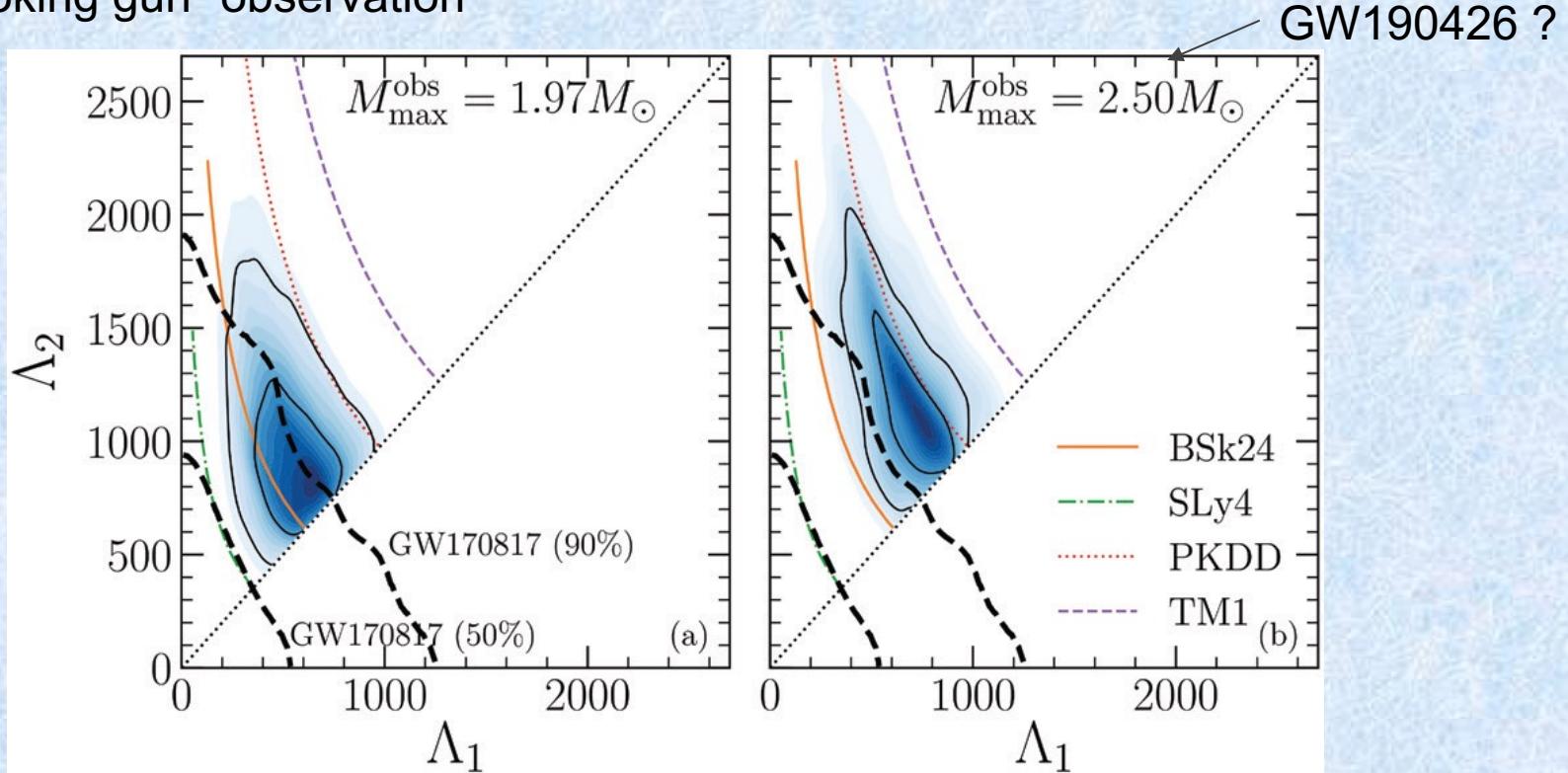


Fantina & Gulminelli, submitted,  
see also Dinh Thi et al., A&A 654, A114 (2021)



# How to discriminate models ? (1)

- “Smoking gun” observation



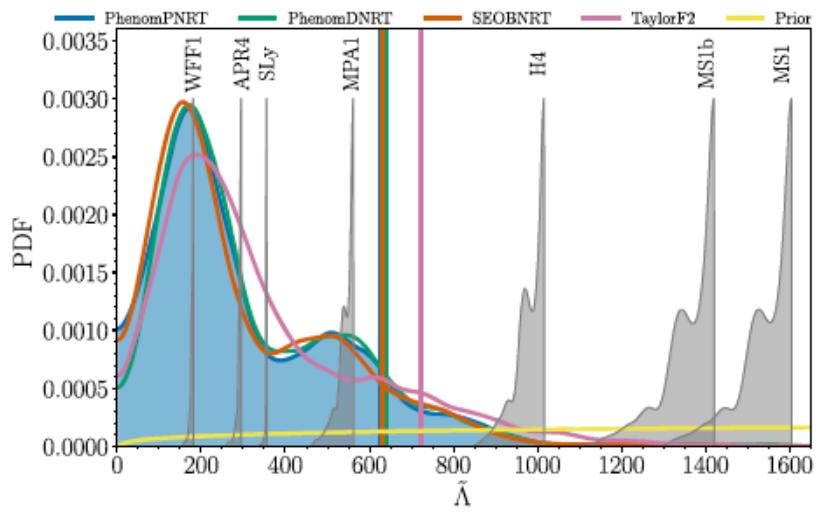
Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021); T. Carreau, PhD Thesis (2020)

- posterior (nucleonic matter) compatible with observations
- but: if  $M_{\text{max}} \sim 2.5 M_{\odot}$  → challenge for nucleonic hypothesis ! → exotica !
- meta-model (nucleonic) can be used as null hp



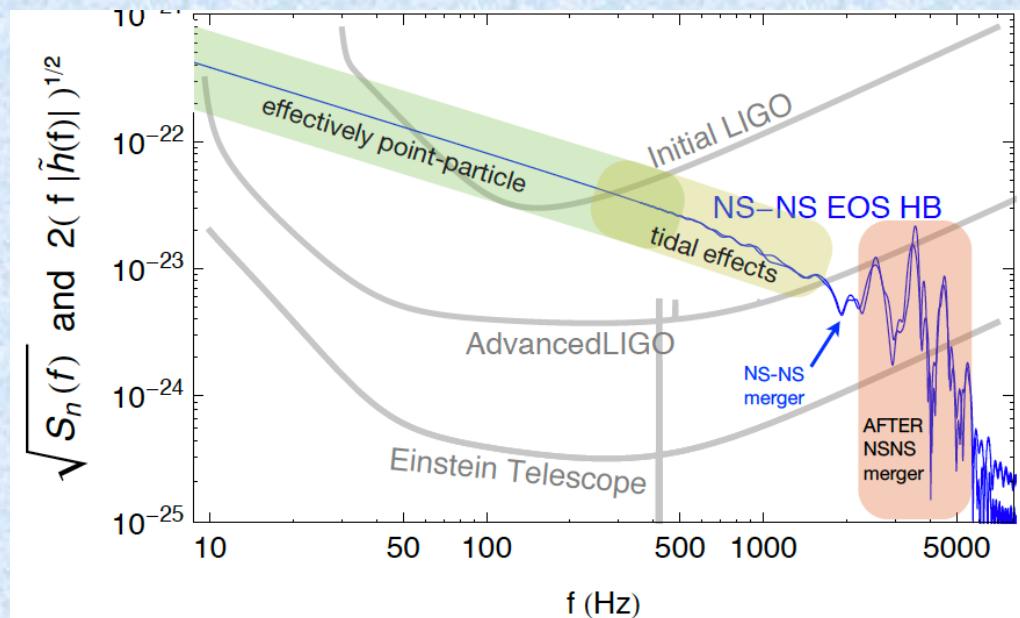
# How to discriminate models ? (2)

- More precise determination of observables (e.g.  $\Lambda$ )
- More sensitive detectors → new generation (e.g. ET)



Abbott et al., Phys. Rev. X 9, 011001 (2019)

current constraint:  $\tilde{\Lambda} \sim [0-800]$

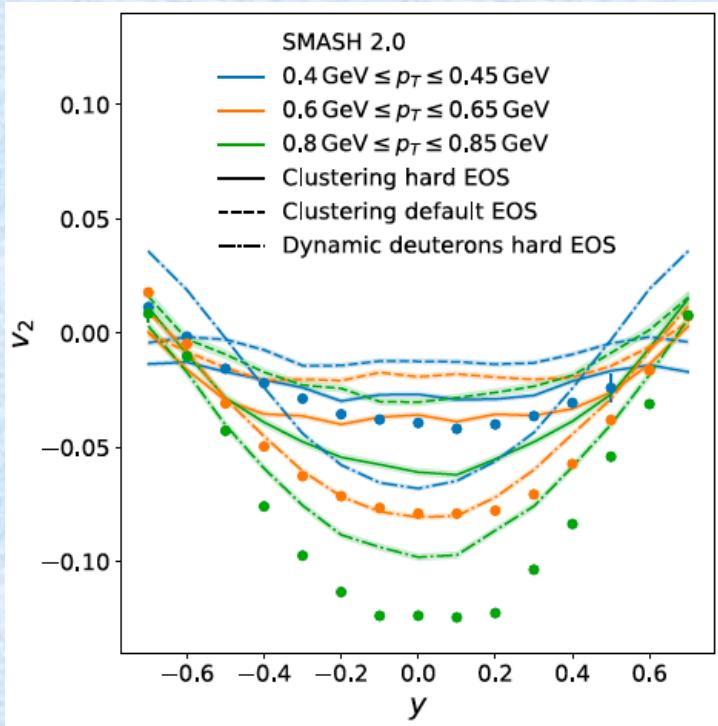


Read, CGWAS lecture (2015)

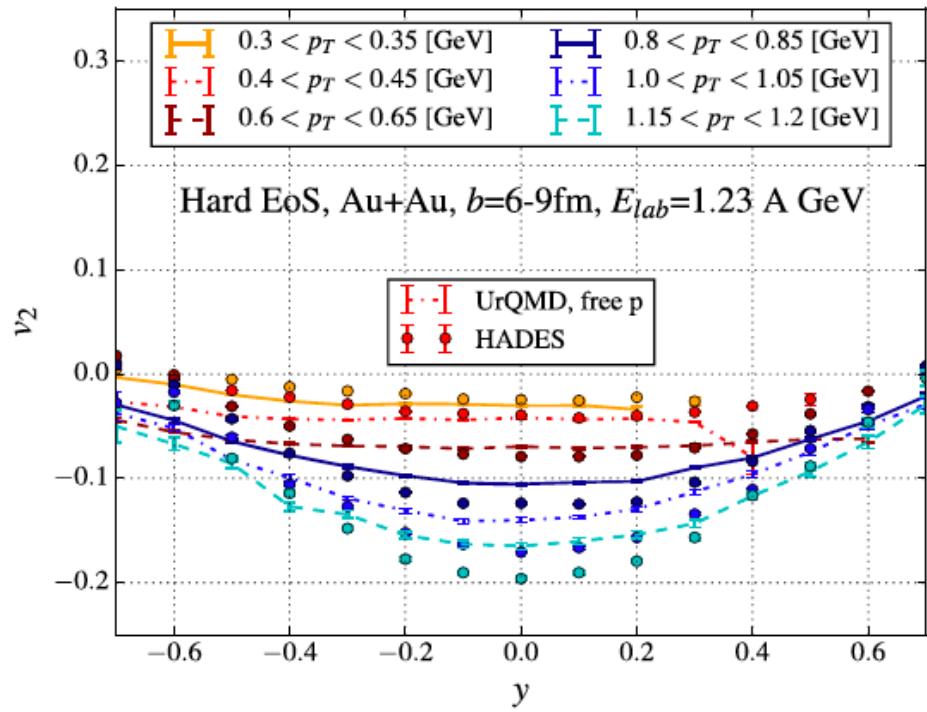


# How to discriminate models ? (3)

- More constraints from nuclear physics at high density ( $\sim 2 n_{\text{sat}}$ )
  - e.g. HADES collaboration; elliptic flow: transport model vs data



Mohs et al., PRC 105, 034906 (2022)



Hillman et al., J. Phys. G 47, 055101 (2020)



# Conclusions & open questions

- ❖ Nuclear inputs needed for neutron-star modelling  
→ extrapolations of data & theoretical models needed
- ❖ Nuclear physics + astrophysics → constraints on EoS
- ❖ Uncertainties in nuclear data → impact astro observables
  - ✓ need of (microscopic) reliable theoretical model when no data
  - ✓ need of experimental data to calibrate the models
  - ✓ need of (more precise) astrophysical observations

- 
- Extrapolation from raw data → **model dependence of the constraints**
  - Lab. exper. mostly “low” density (~ saturation density), low  $T$  probed; matter in astro sites different from lab → **extrapolation to astro conditions (high  $T$  and density, asymmetry, charge neutral)** ?
  - Astro simulations vs microphysics inputs → **uncertainties in nuclear / astro, consistency of inputs** and relative **effects of microphysics inputs in astro modelling** ? → systematic studies / bayesian analysis needed
-