18TH RUSSBACH SCHOOL ON NUCLEAR ASTROPHYSICS

Studying plasma induced variation of bound state β-decays in PANDORA

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INFN – LNS and DFA – UNICT

Under the supervision of *David Mascali*

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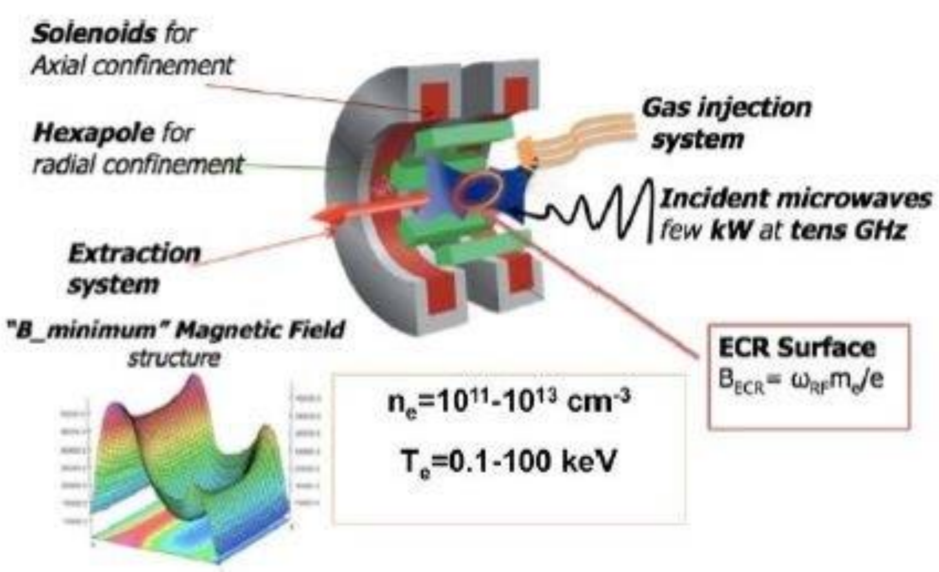
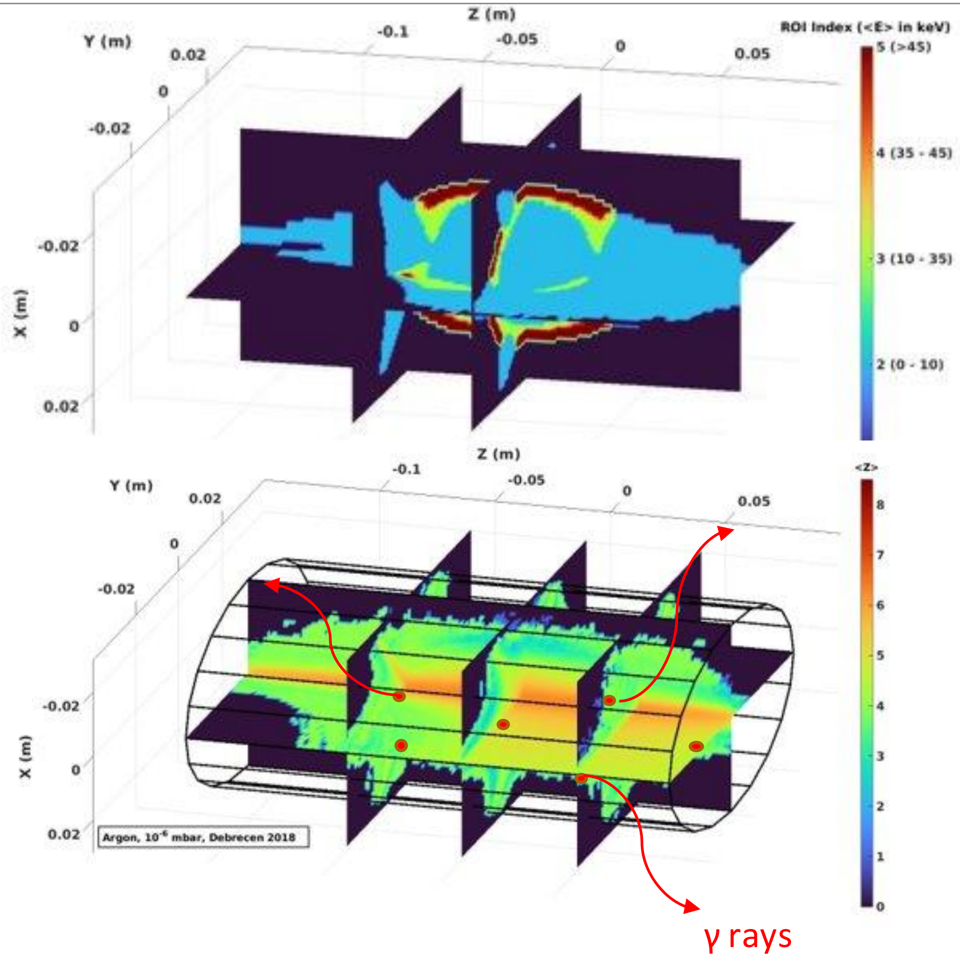
Plasmas for Astrophysics Nuclear Decay Observation and Radiation for Archaeometry

Stars in a Bottle: PANDORA

Details covered in Domenico's talk



Schematic of ECRIS and expected electron parameters [1]



Non-uniform distribution of *ne* and k_BT in the system

Non-uniform distribution of ion CSD/LPD in the system

Varying decay rates of radio-isotopes in the system

[1] D. Mascali, A. Musumarra, F. Leone, F.P. Romano, A. Galatá, S. Gammino and C. Massimi, Eur. Phys. J. A 53, 145 (2017).

Modeling In-Plasma Decay Rates

What we need & What we have

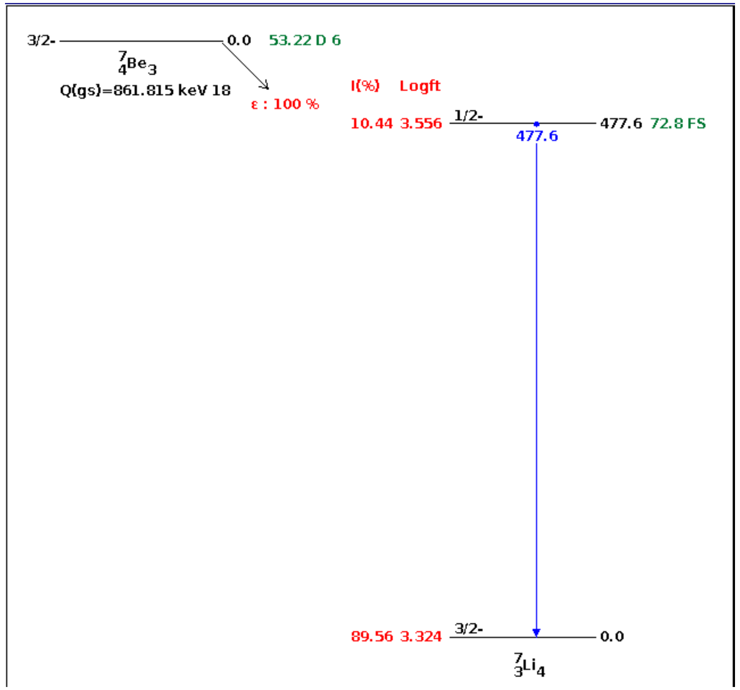
- **A nice formula to calculate decay rates – Got it ✓**
- **A nicer formula to calculate decay rates *in-plasma* [2] – Got it ✓**
- **A way to use that formula – this presentation → Focus on ^{7}Be**
- **A way to calculate all plasma stuff needed for that formula [3,4] – Got it somehow ✓**

[2] K. Takahashi and K. Yokoi, Nucl. Phys. A404 (1983) 578
[3] B. Mishra et al, Frontiers in Physics, special issue (2022)
[4] B. MIshra, EPJ Web of Conferences 275 (2023) 02001

Modeling In-Plasma Decay Rates

Steps involved

Step 1: Select isotope and transitions



Step 2: Calculate lepton phase volume

Continuum decay

$$f^*_{IF(m)} = \sum_{ij} p_{ij} \int_1^{W_{max}(ij)} (W^2-1)^{1/2} W (W_{max}(ij)-W)^2 F_0 S_{(m)}(ij) f_d(ij) \mathrm{d}W$$

Continuum capture

$$f^*_{IF(m)} = \sum_{ij} p_{ij} \int_{W_{min}(ij)}^{\infty} (W^2-1)^{1/2} W (Q(ij)/m_e c^2)^2 F_0 S_{(m)}(ij) f_c(ij) \mathrm{d}W$$

Bound decay/ Bound capture

$$f^*_{IF(m)} = \sum_{ij} p_{ij} \sum_{x(ij)} \sigma_x \frac{\pi}{2} [g_x \, or \, f_x]^2 (Q(ij)/m_e c^2)^2 S_{(m)x(ij)}$$

Step 3: Calculate decay rates

$$\lambda_{tot} = \sum \frac{\ln 2}{f_{IF(m)} t_{1/2}} f^*_{IF(m)}$$

Summation over all possible decay channels

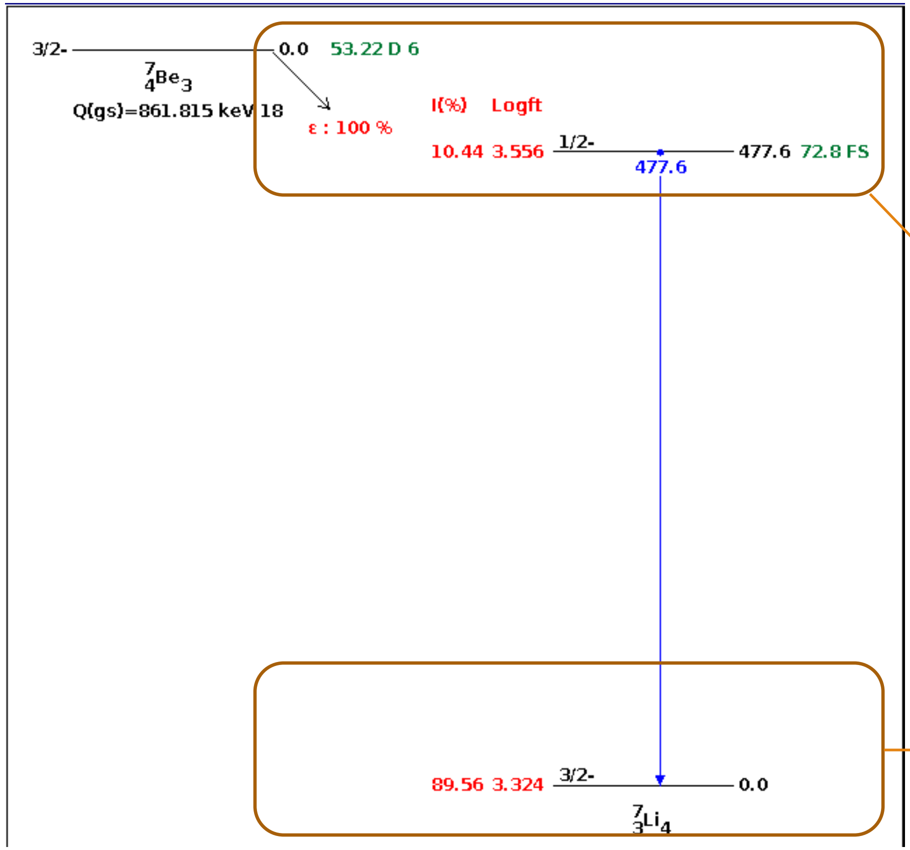
^{7}Be undergoes electron capture to ^{7}Li
$3/2^- \to 3/2^-$ (gs -> gs transition, allowed)
$3/2^- \to 1/2^-$ (gs -> es transition, allowed)

BR = 10.44 %

[2] K. Takashi and K. Yokoi, Nuclear Physics A404 (1983) 578

Modeling In-Plasma Decay Rates

Step 1: Selecting isotope and transitions



^{7}Be decay scheme [4]

^{7}Be undergoes electron capture to ^{7}Li

Contribution from both continuum and bound state electron capture

Information needed: log ft values

Correlated with the strength of the decay and the nuclear matrix element

$3/2^- \rightarrow 1/2^-$ (gs -> es transition)
Allowed transition
Log ft = 3.556

$3/2^- \rightarrow 3/2^-$ (gs -> gs transition)
Allowed transition
Log ft = 3.324

NO OTHER DECAY CHANNEL CONSIDERED

Modeling In-Plasma Decay Rates

Step 2: Calculating lepton phase volume

The lepton phase volume quantifies the number of ways a decay can occur. The phase volume changes with variations in atomic configuration, depending on type of decay

Information needed: level probability distribution (LPD), orbital occupancy, orbital electron wavefunction, decay energy and shape factor

$$f^*_{IF(m)} = \boxed{\sum_{ij} p_{ij}} \sum_{x(ij)} \boxed{\sigma_x} \frac{\pi}{2} \boxed{[g_x \, or \, f_x]^2} \boxed{(Q(ij)} / m_e c^2)^2 \boxed{S_{(m)x(ij)}}$$

(b) Shape factor describing compatibility between nuclear and lepton wavefunction

(e) Probability distribution of various charge states and levels of isotope

(d) Occupancy of orbital contributing to certain decay within relevant ionic configuration

(c) Larger of squared radial component of electron wavefunction evaluated on nuclear surface

(a) Q-value of decay when isotope in (i,j) charge state and level configuration

[2] K. Takashi and K. Yokoi, Nuclear Physics A404 (1983) 578

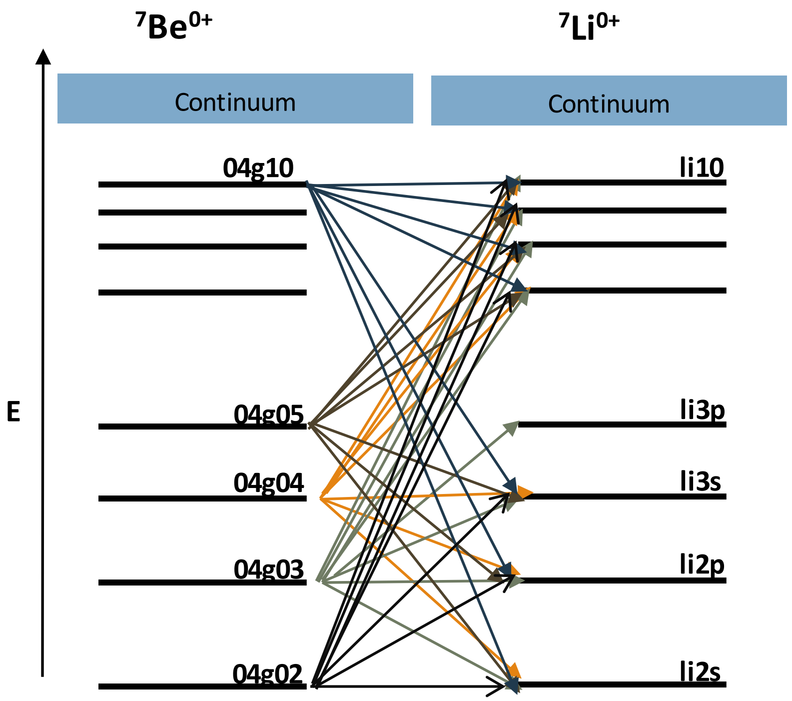
Modeling In-Plasma Decay Rates

Step 2 (a): Decay energetics

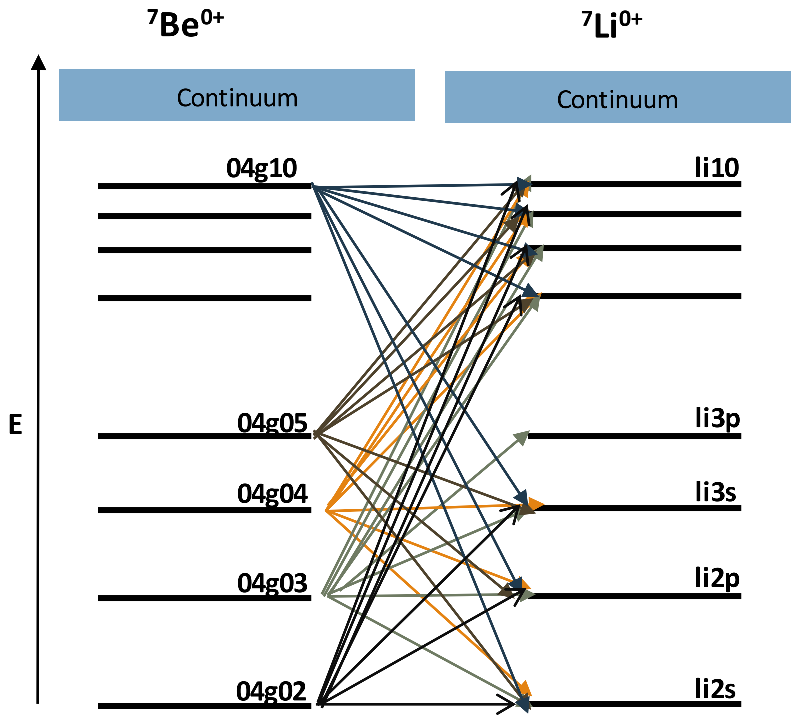
$$Q = Q_0 + (E^*_{X,K} - E^*_{Y,K'}) + (\epsilon^{i,j} - \epsilon^{i',j'}) + (\Delta_X - \Delta_Y)$$

The decay energy depends on not just the difference in nuclear masses, but on the overall system energy which includes atomic/ionic energy

Information needed: energy of different atomic configurations of parent system and coupling with daughter system



^7Be - ^7Li level coupling schematic for K- and L-shell capture (neutral ion)



All configurations should have at least one K-shell electron

No selection on daughter configuration K-shell vacant states autoionising

All configurations should have at least one L-shell electron

No selection on daughter configuration L-shell vacant states autoionising

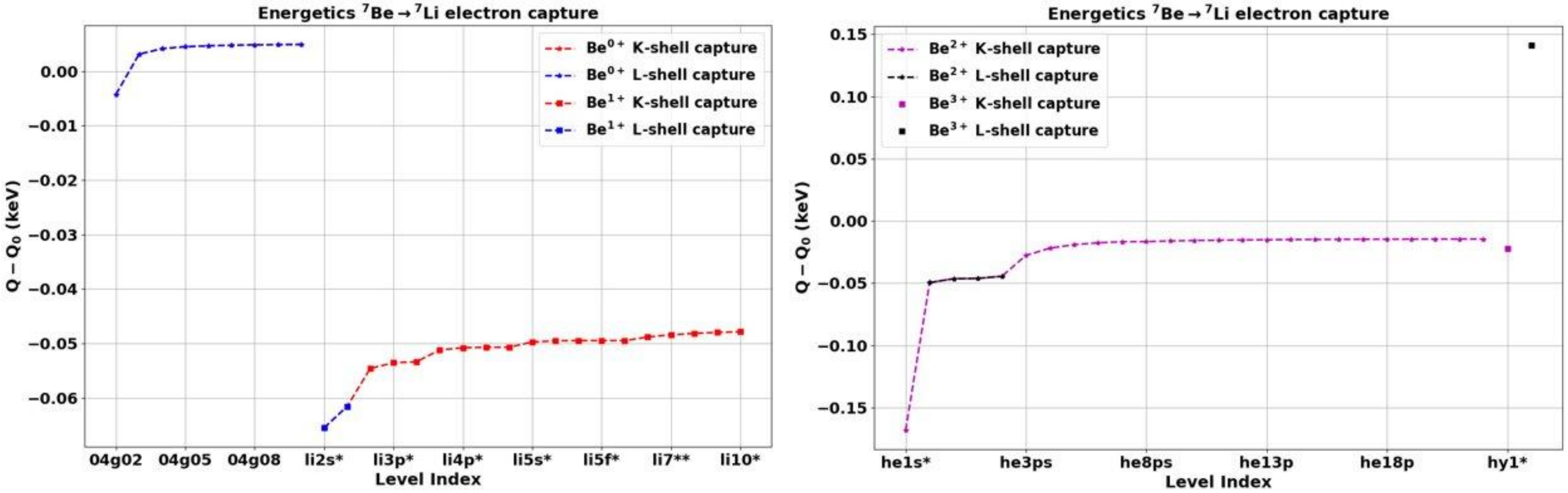
K-SHELL CAPTURE

L-SHELL CAPTURE

Modeling In-Plasma Decay Rates

Step 2 (a): Decay energetics

$$Q = Q_0 + (E^*_{X,K} - E^*_{Y,K'}) + (\epsilon^{i,j} - \epsilon^{i',j'}) + (\Delta_X - \Delta_Y)$$



^{7}Be decay Q-value as a function of charge state and level for neutral and 1+ (left and 2+ and 3+ (right)

Modeling In-Plasma Decay Rates

Step 2 (b): Shape factor

Conservation of total angular momentum implies that only certain electron orbitals can interact with the nucleus, depending on the spin and parity of the decay

Information needed: spin-parity of electron orbitals and decay transition

$$S_{(m)x} = \begin{cases} 1 & \text{for } m = a, nu \text{ and } x = ns_{1/2}, np_{1/2} \\ q^2 & \text{for } m = u \text{ and } x = ns_{1/2}, np_{1/2} \\ 9/R^2 & \text{for } m = u \text{ and } x = np_{3/2}, nd_{3/2} \\ 0 & \text{otherwise.} \end{cases}$$

[2] K. Takashi and K. Yokoi, Nuclear Physics A404 (1983) 578

Modeling In-Plasma Decay Rates

Step 2 (c): $[f_x \text{ or } g_x]^2$

The probability of electron capture from bound states depends on the square of the radial component of the orbital wavefunction evaluated on the nuclear surface

Information needed: formalism for radial wavefunctions of different orbitals

Radial component of Dirac equation – Coupled differential equations

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{\kappa}{r}P(r) - \left(2c + \frac{V-\epsilon}{c}\right)Q(r) \qquad \frac{\mathrm{d}Q(r)}{\mathrm{d}r} = \frac{\kappa}{r}Q(r) + \left(\frac{V-\epsilon}{c}\right)P(r)$$

For V = Z/r (in atomic units)

$$P(r) = \left(1 - \frac{\epsilon}{c^2}\right)^{1/2} \xi \left(\frac{\rho}{N}\right)^{\gamma} \mathrm{e}^{-\rho/2N}[-n_r F_1 + (N-\kappa)F_2]$$

$$Q(r) = \left(\frac{\epsilon}{c^2}\right)^{1/2} \xi \left(\frac{\rho}{N}\right)^{\gamma} \mathrm{e}^{-\rho/2N}[n_r F_1 + (N-\kappa)F_2]$$

- ***N = apparent principal quantum number***
- ***n_r = number of nodes in orbital spatial distribution***
- ***ξ = normalisation***
- ***ρ = 2Zr = radial function***
- ***ϵ = quantised electron energy***
- ***κ = -(j+1/2)a, a=±1***
- ***F_1, F_2 = confluent hypergeometric functions***

[5] V. M. Burke and I. P. Grant, Proceedings of the Physical Society 90 (1967) 297

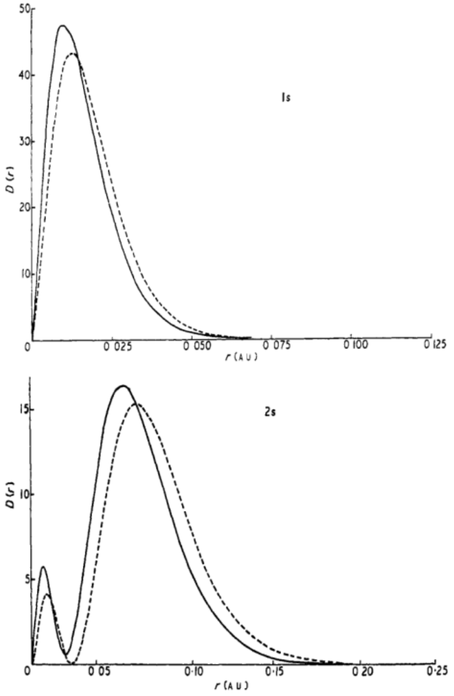
Modeling In-Plasma Decay Rates

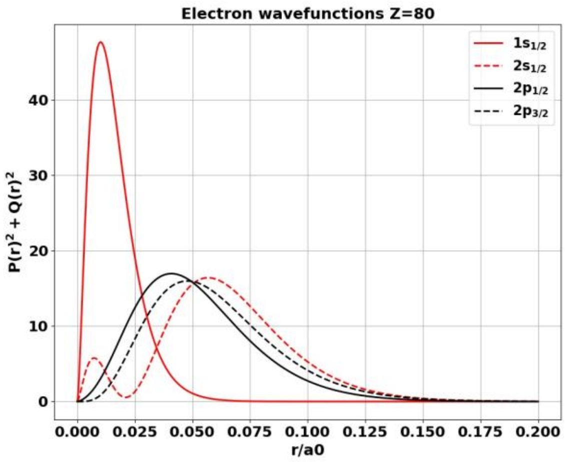
Step 2 (c): $[f_x \text{ or } g_x]^2$ (benchmarking)

Physics verification

$$D(r) = P(r)^2 + Q(r)^2$$

Radial probability density



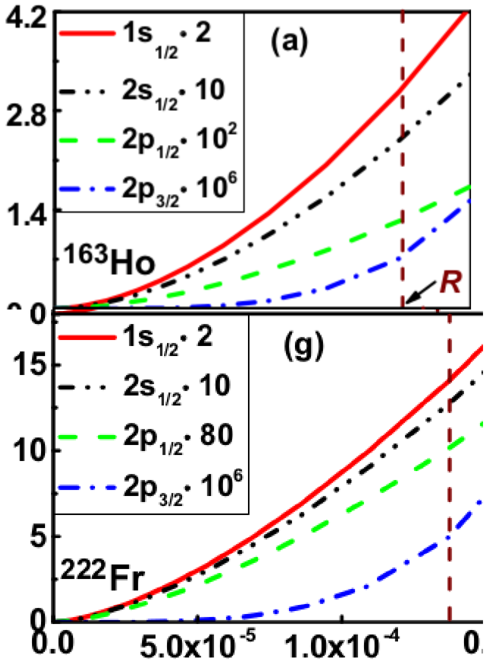


Radial probability density of 1s, 2s and 2p orbitals for Z = 80 as calculated in [5] and as calculated here

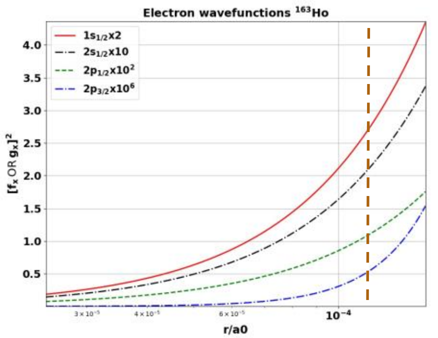
Literature benchmark

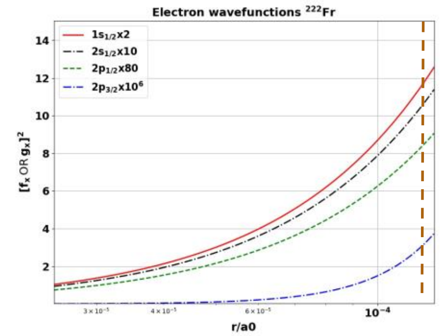
$$f_x = \left(\frac{\bar{\lambda}_c}{\hbar}\right)^{3/2} \frac{P(r)}{r} \qquad g_x = \left(\frac{\bar{\lambda}_c}{\hbar}\right)^{3/2} \frac{Q(r)}{r}$$

Effective radial wavefunction [6]



^{163}Ho and ^{222}Fr radial wavefunctions as calculated with RADIAL in [6] compared with analytical formalism

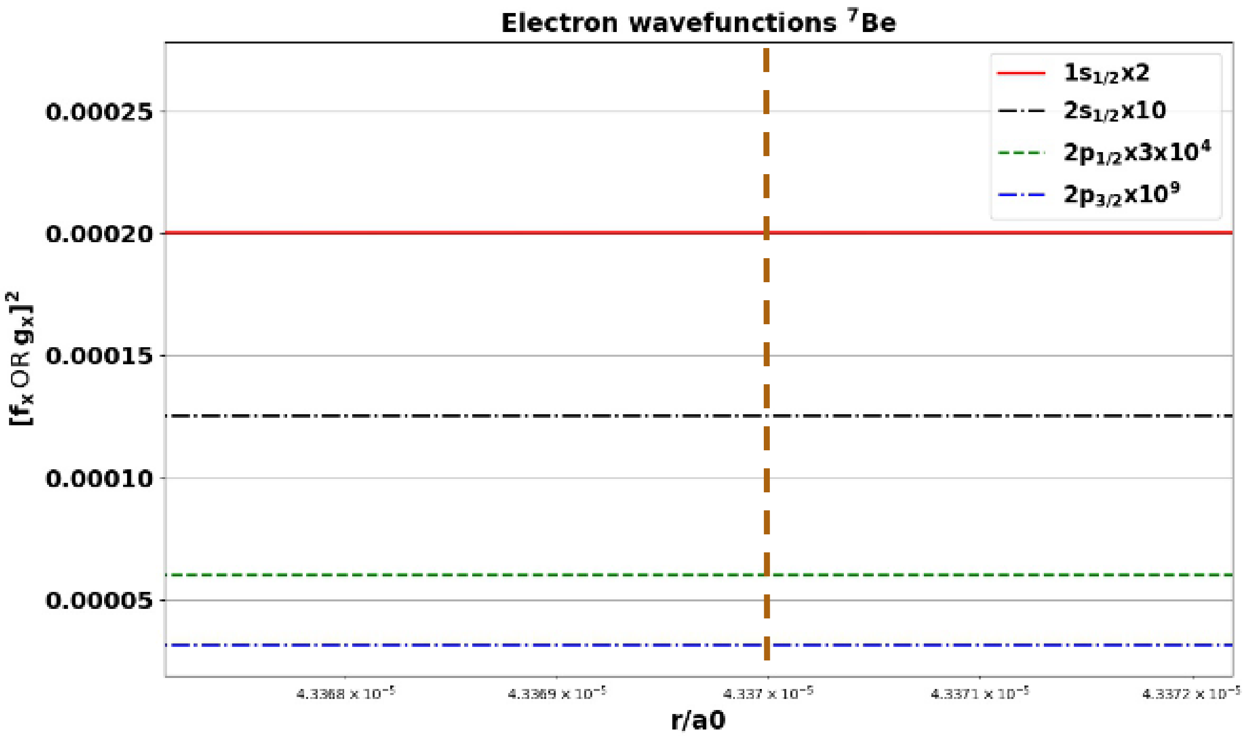




[5] V. M. Burke and I. P. Grant, Proceedings of the Physical Society 90 (1967) 297
[6] S. Liu, C. Gao and C. Xu, Physical Review C 104 (2021), 024304

Modeling In-Plasma Decay Rates

Step 2 (c): $[f_x$ or $g_x]^2$



Larger of f_x^2 or g_x^2 as calculated for ^{7}Be taking $R = R_0A^{1/3}$

Wavefunctions evaluated on nuclear surface have small values on account of small nuclear size

Only 1s and 2s contributions may be considered

Modeling In-Plasma Decay Rates

Step 2 (d): Occupancy and Level Dependent Lepton Phase Volumes

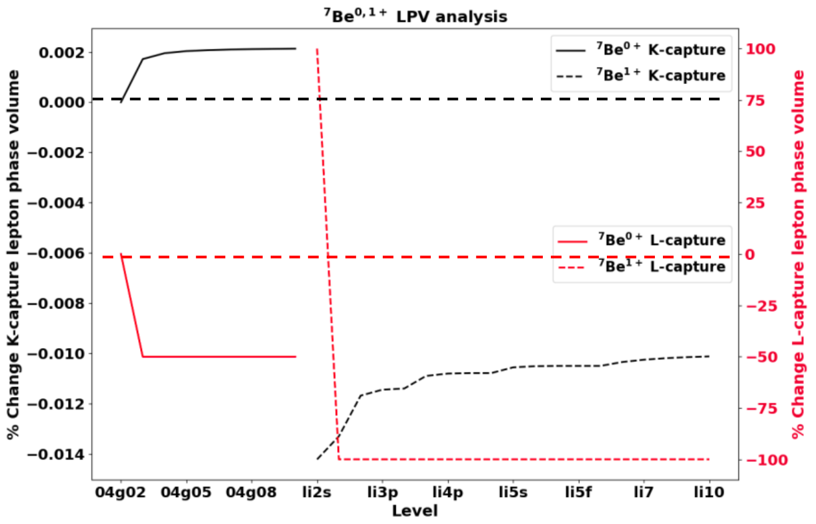
The probability of electron capture from bound states also depends on the occupancy of the relevant orbital

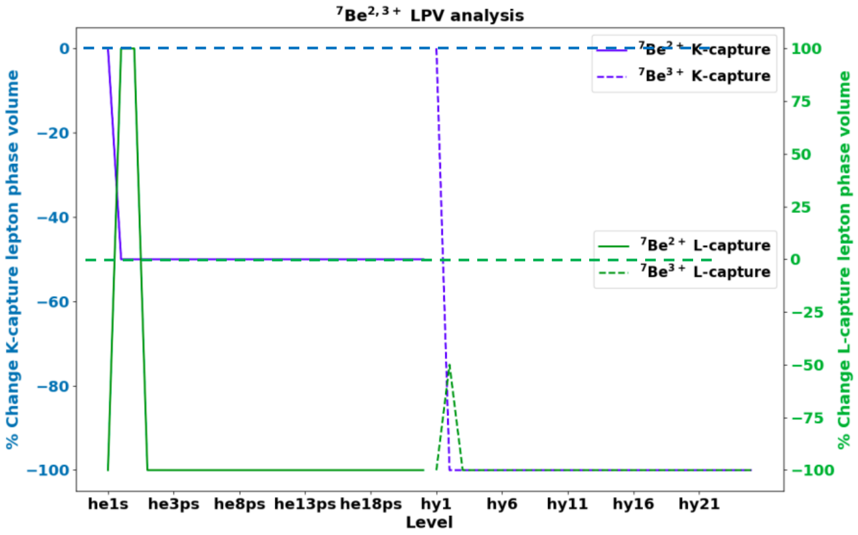
Information needed: Occupancy of relevant orbitals as a function of the level configuration and ion charge state

li2s level in $^7Be^{1+}$ - 2e$^-$ in *1s*, 1e$^-$ in *2s*
$1s_{1/2}$ occupancy = 1
$2s_{1/2}$ occupancy = 0.5
$2p_{1/2}$ occupancy = $2p_{3/2}$ occupancy = 0

Suppressed L-capture

li2p level in $^7Be^{1+}$ - 2e$^-$ in *1s*, 1e$^-$ in *2p*
$1s_{1/2}$ occupancy = 1
$2s_{1/2}$ occupancy = 0
$2p_{1/2}$ occupancy = $2p_{3/2}$ occupancy = 0.167





% change in lepton phase volume can be quite pronounced for some configurations which are accessible by plasma

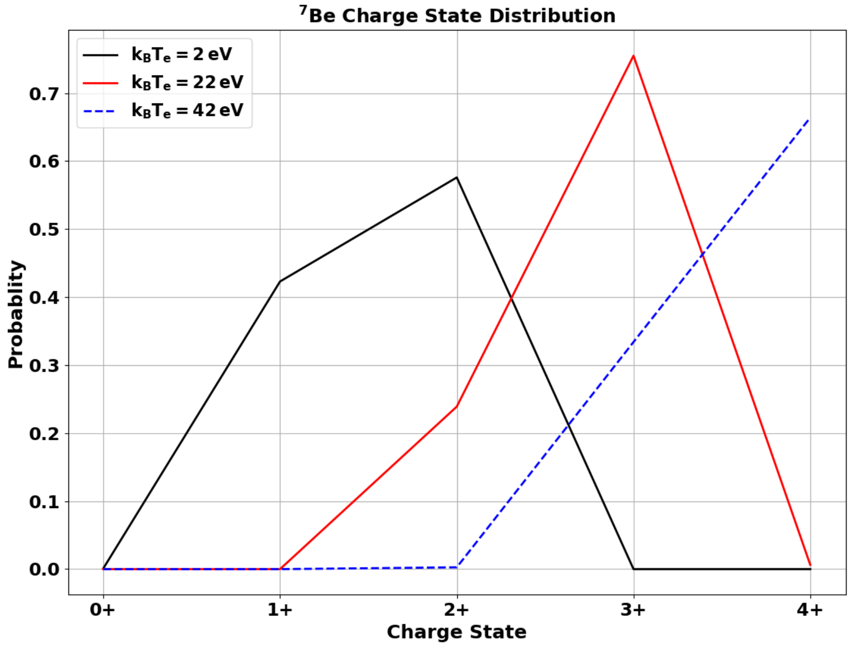
Levels can be combined into super-groups based on similarity in lepton phase volume – computational advantage

Modeling In-Plasma Decay Rates

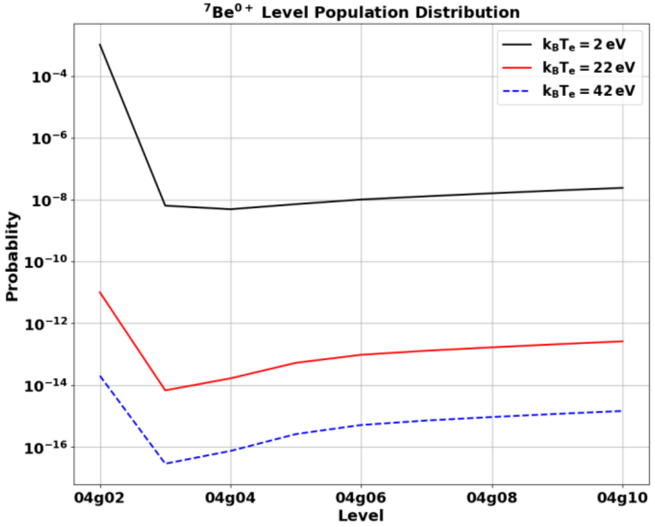
Step 2 (e): Level probability distributions

The ion CSD and LPD strongly depends on electron density and temperature

Information needed: CSD and LPD of ^{7}Be for various n_e and T_e (calculated using FLYCHK)

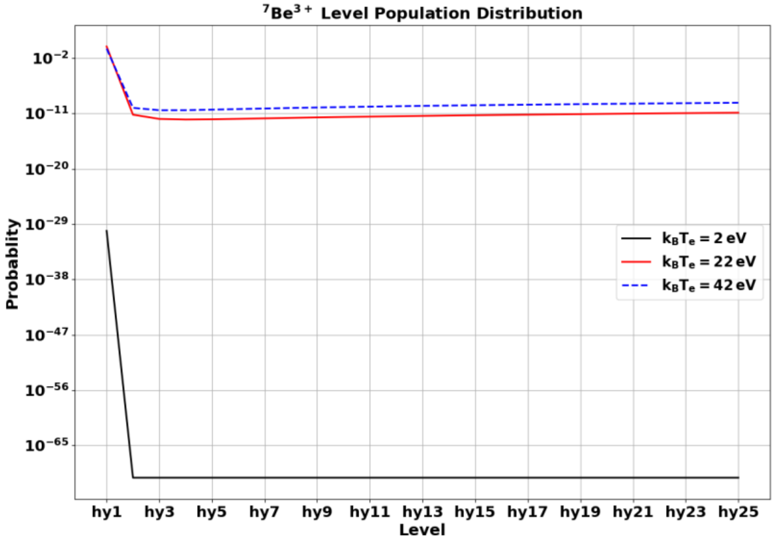


CSD of ^{7}Be for different temperatures as calculated by FLYCHK (no effect of density)



LPD of $^7Be^{0+}$ and $^7Be^{3+}$ for different temperatures as calculated by FLYCHK (no effect of density)

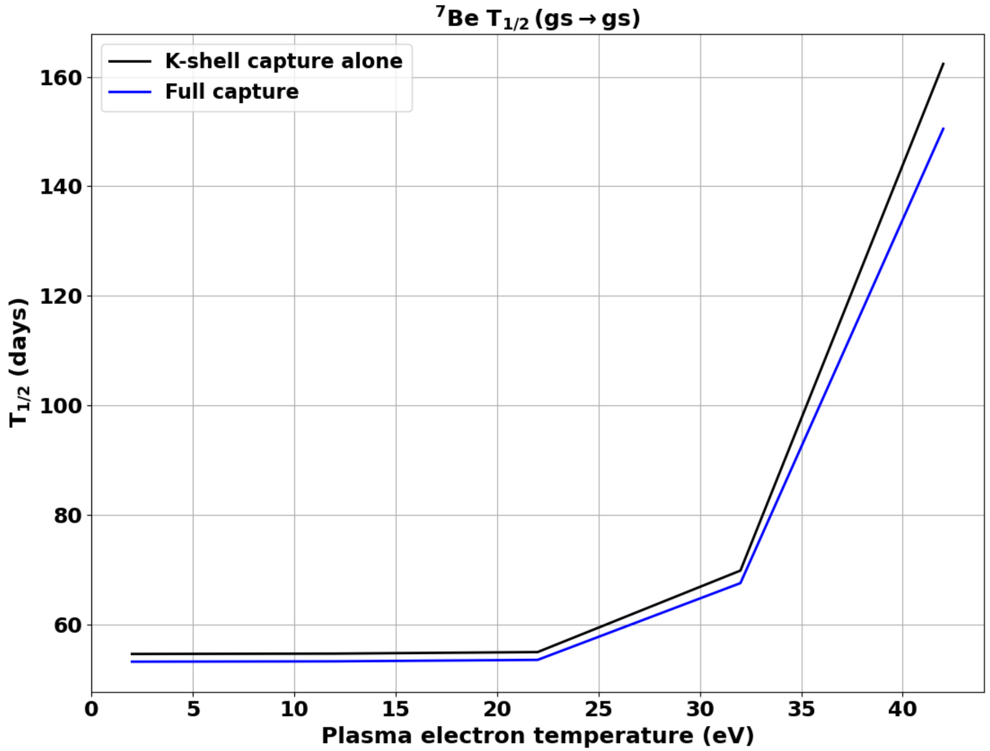
CSD and LPD calculated using grid of density and temperature values in FLYCHK under LTE approximation

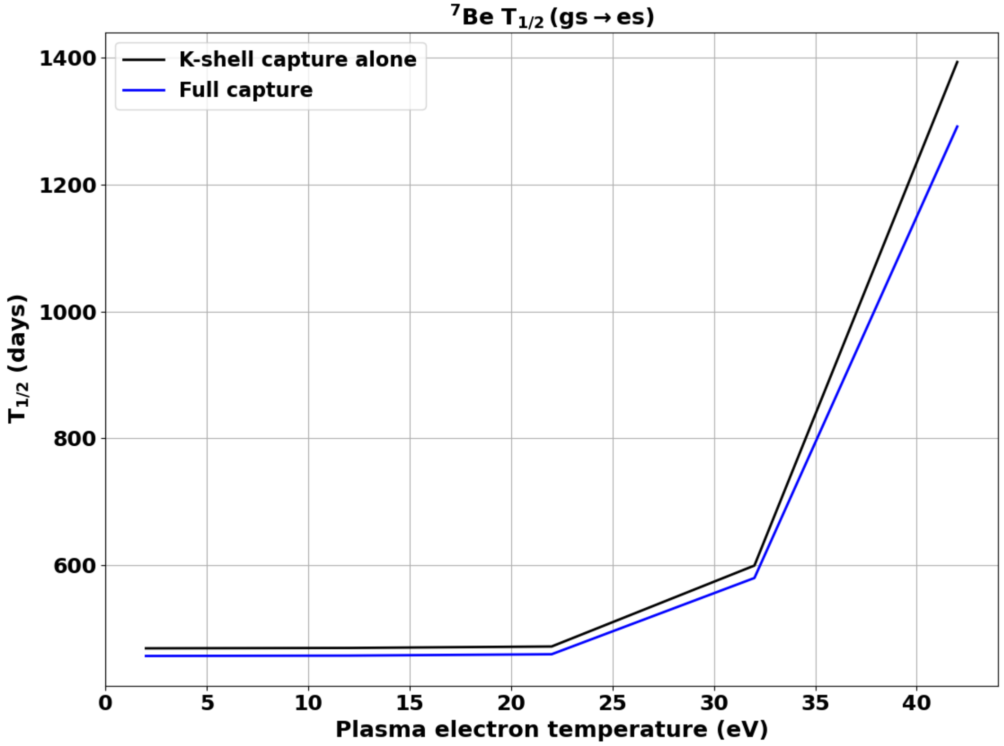


Modeling In-Plasma Decay Rates

Step 3: Calculating decay rates

^{7}Be is expected to show a rise in $T_{1/2}$ with increase in T_{e^-} high temperatures lead to greater ionisation and fewer electrons left to capture



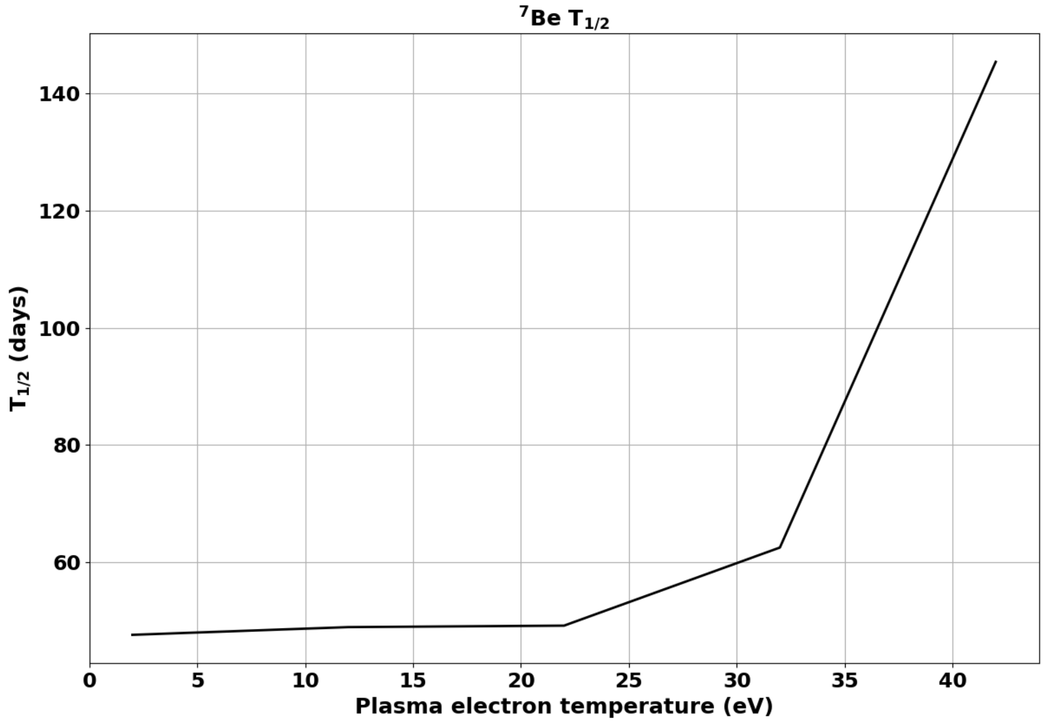


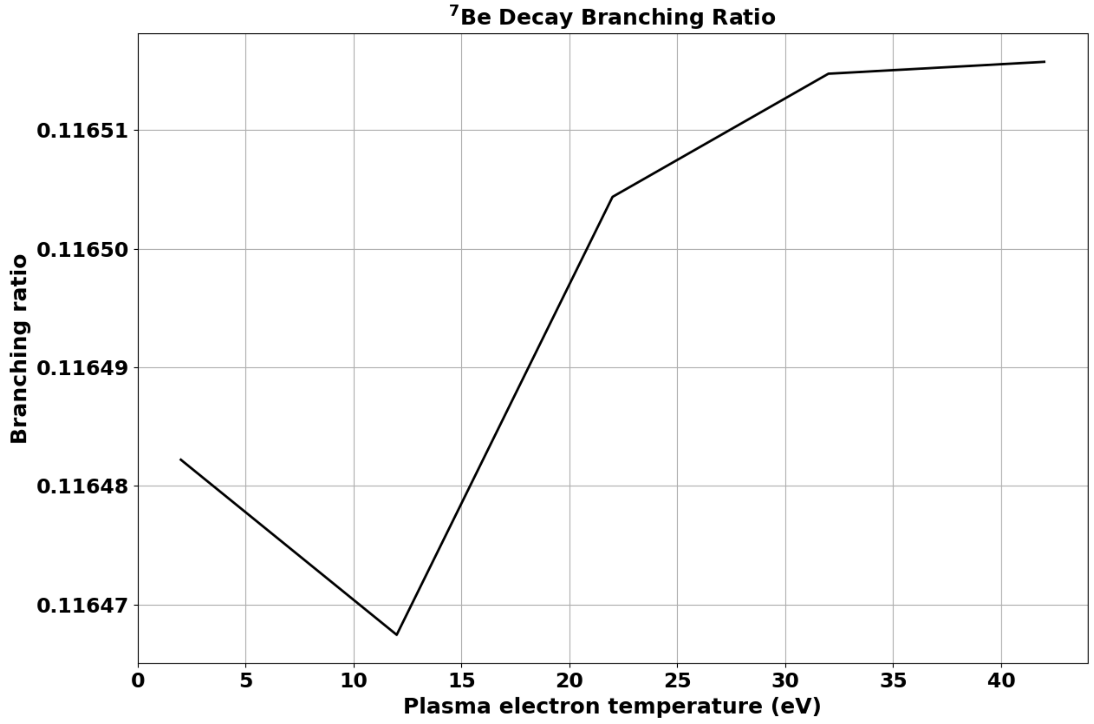
The methodology also benchmarked with decay rates in fully stripped ^{166}Dy in Liu et al

[6] S. Liu, C. Gao and C. Xu, Physical Review C 104 (2021), 024304

Modeling In-Plasma Decay Rates

Results





^{7}Be ground state (neutral and 04g02 level config)

$T_{1/2}$ calculated = 47.4 days
Branching ratio = 11.6 %

$T_{1/2}$ measured [7] = 53 ± 6 days
Branching ratio = 10.7 ± 0.2%

[7] G.J. Matthews, R.C. Haight, R.G. Lanier and R.M White, Physical Review C 28 (1983) 2

Conclusions and Next Steps

- **Radio-isotopes like ^{7}Be can show modification of lifetimes when ionised or excited**
- **Results show increase in lifetime due to absence of bound electrons for capture at eV temperatures, much lower than stellar temperatures**
- **7Be in stellar interior proceeds by continuum capture [8]**
- **The recipe for calculating in-plasma bound state decay and bound state capture is ready and tested**

$$f^{*}_{IF(m)} = \boxed{\sum_{ij} p_{ij}} \sum_{x(ij)} \sigma_x \frac{\pi}{2} [g_x \, or \, f_x]^2 (Q(ij)/m_e c^2)^2 S_{(m)x(ij)}$$

- **CSD and LPD calculated under LTE approximation here, but can be calculated under NLTE approximation in ECR plasma**
- **3D Particle-in-Cell Monte Carlo code ready for the purpose**
- **Number of levels *ij* can be reduced by level grouping based on similarity in lepton phase volume contribution**

[8] S. Simonucci, S. Taioli, S. Palmerini and M. Busso, The Astrophysical Journal 764:118 (2013)

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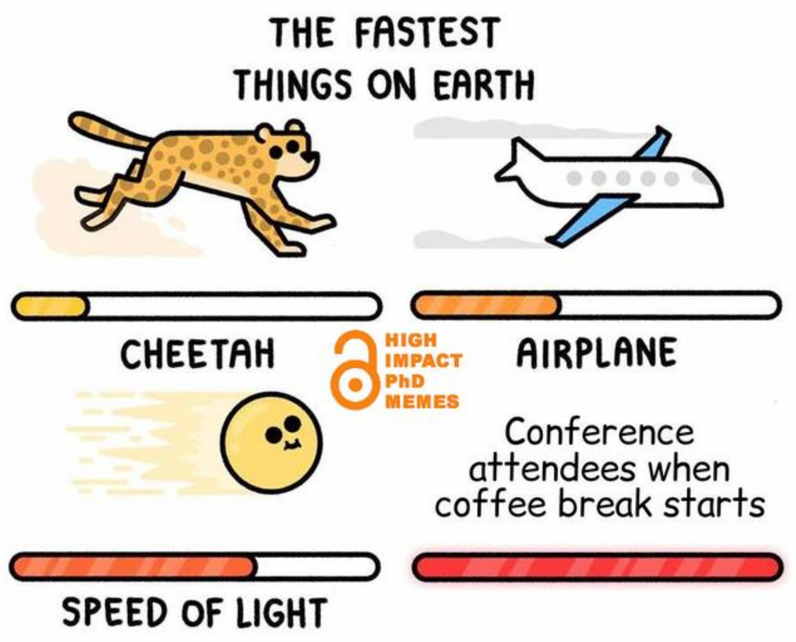
Alberto Mengoni

...and the PANDORA collaboration

THANK YOU FOR YOUR ATTENTION!



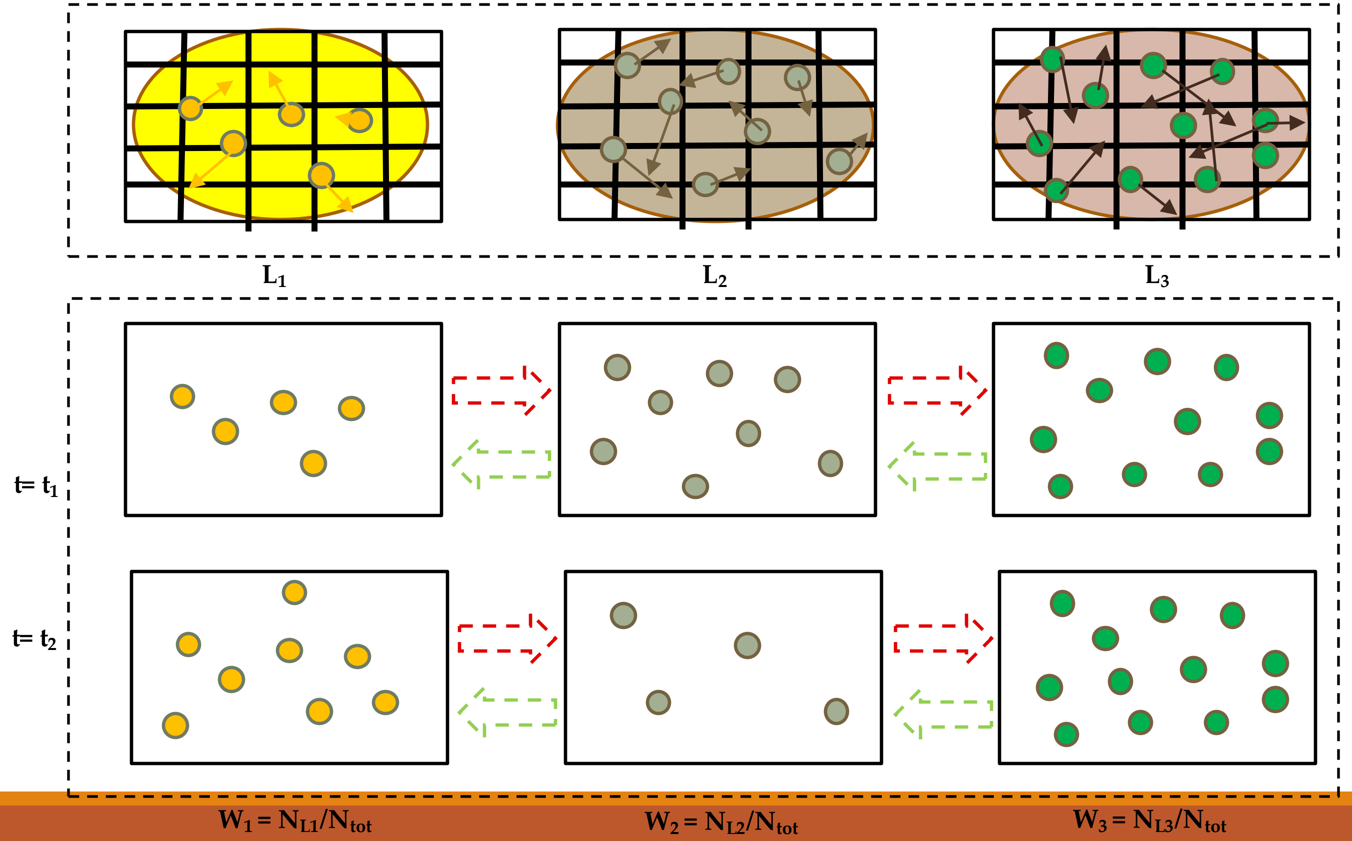
Additional Content

BUFFER IONS BALANCE EQUATION:

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \boxed{n_{i-1} n_e \gamma_{i-1,i}} - \boxed{n_i n_e \gamma_{i,i+1}} + \boxed{n_{i+1} n_0 E_{i+1,i}} - \boxed{n_i n_0 E_{i,i-1}} - \frac{n_i}{\tau_i}$$

KEYWORD(S): Steady state

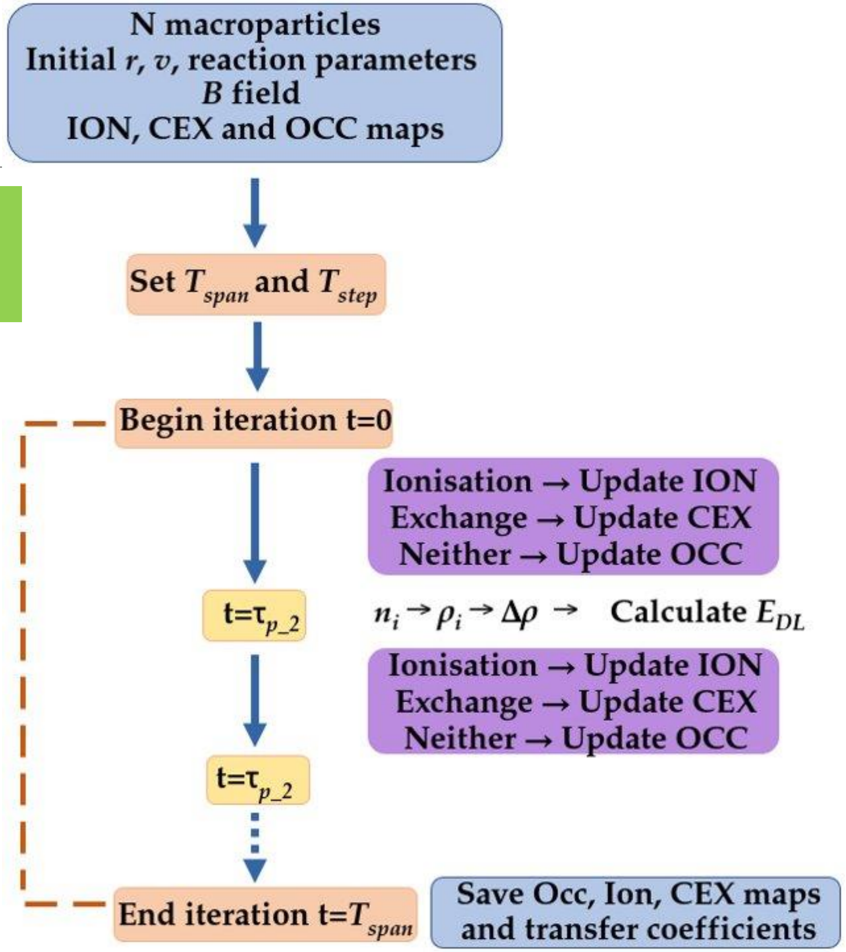
The contribution from each process can be evaluated independently to construct CSD



Particle transport under EM fields with collision to generate 3D spatial distribution (occupation maps)

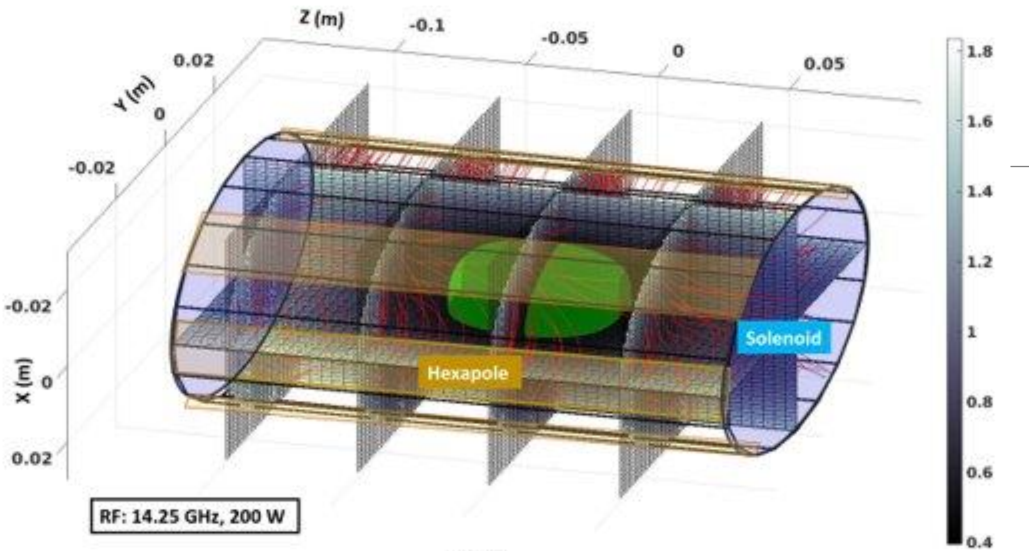
Constant monitoring of loss/gain of macroparticles between various levels to calculate relative weight of each level

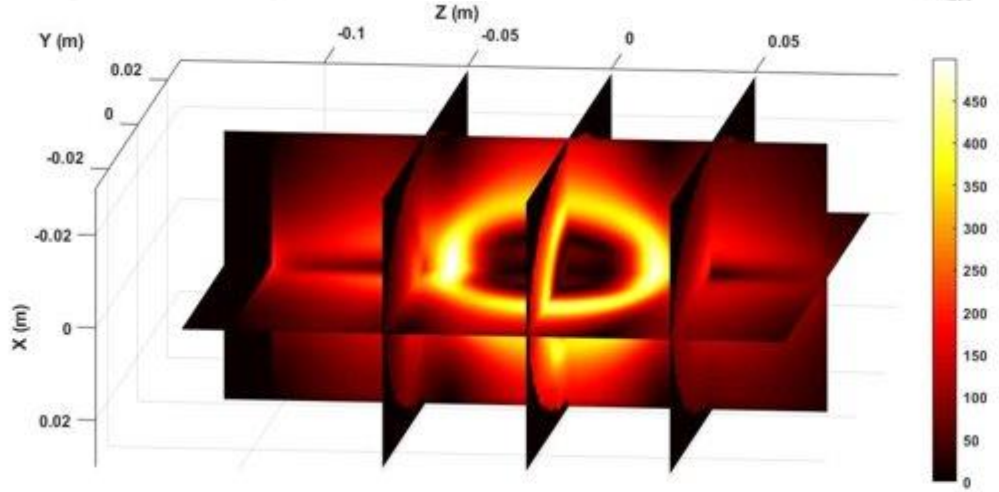
Forward reactions – ionisation, excitation, absorption
Backward reactions – charge exchange, de-excitation, spontaneous emission

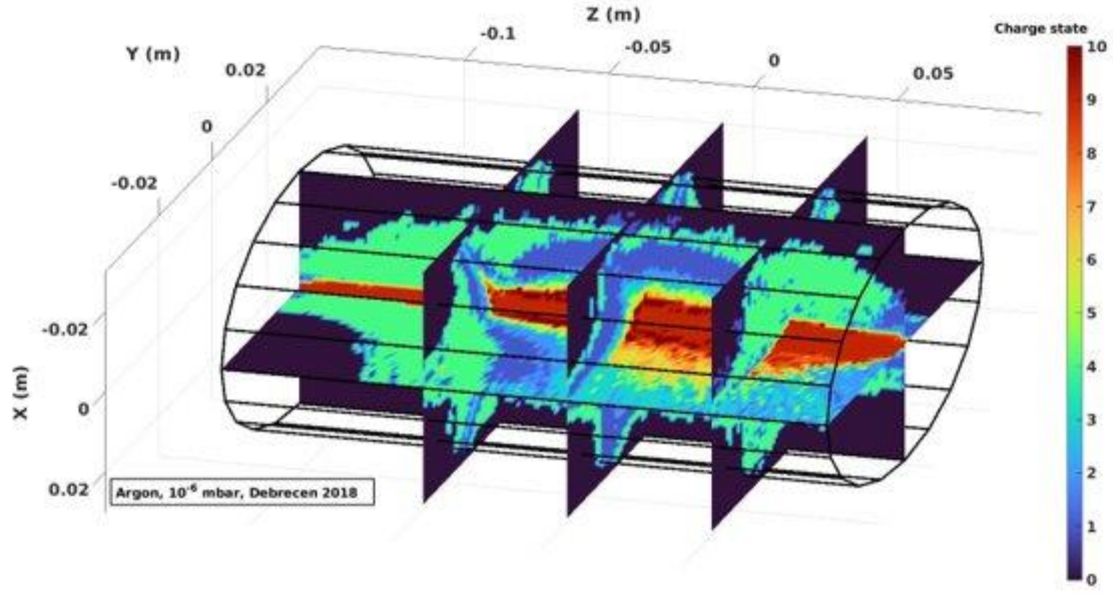


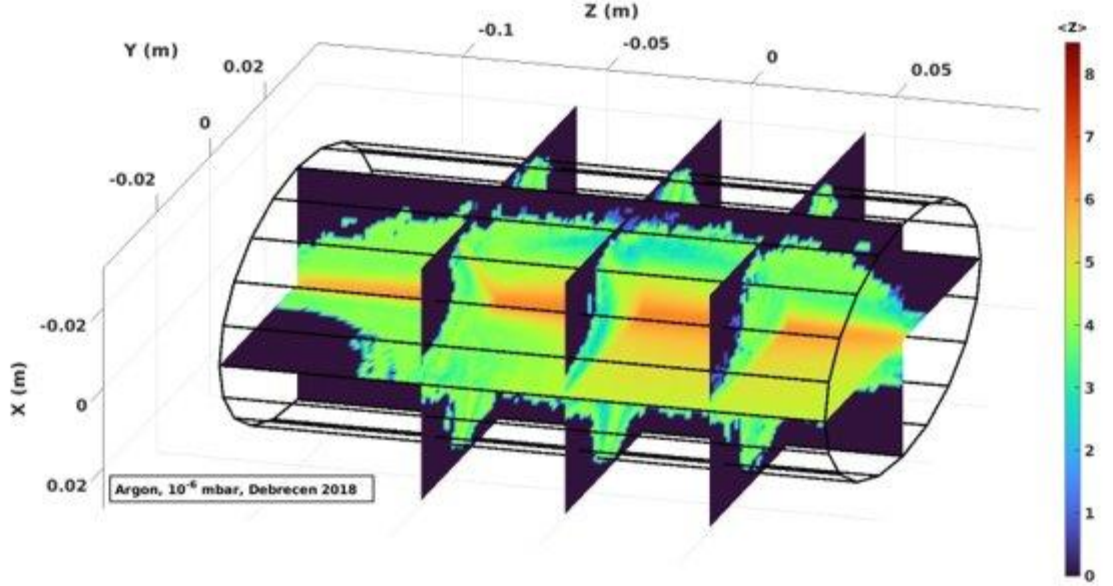
[10] B. Mishra *et al*, Frontiers in Physics 10:932448

Additional Content









Additional Content

$$\frac{\mathrm{d}n_i^R}{\mathrm{d}t} = \left[n_{i-1}^R n_e \gamma_{i-1,i} - n_i^R n_e \gamma_{i,i+1}\right] + \left[n_{i+1}^R \sum_{j=0}^{N-1} n_j E_{i+1,j} + n_{i-1}^R \sum_{j=1}^{N} n_j E_{i-1,j}\right] - \left[n_i^R \sum_{j=0}^{N-1} n_j E_{i,j-1} + n_i^R \sum_{j=1}^{N} n_j E_{i,j+1}\right] - \frac{n_i^R}{\tau_i^R}$$

$$\frac{\mathrm{d}n_{ii'}^R}{\mathrm{d}t} = \left[\sum_{ij'<ii'} n_{ij'}^R n_e C_{ij'ii'} - n_{ii'}^R \sum_{ij'>ii'} n_e C_{ii'ij'}\right] + \left[\sum_{ij'>ii'} n_{ij'}^R A_{ij'ii'} - n_{ii'}^R \sum_{ij'<ii'} A_{ii'ij'}\right]$$