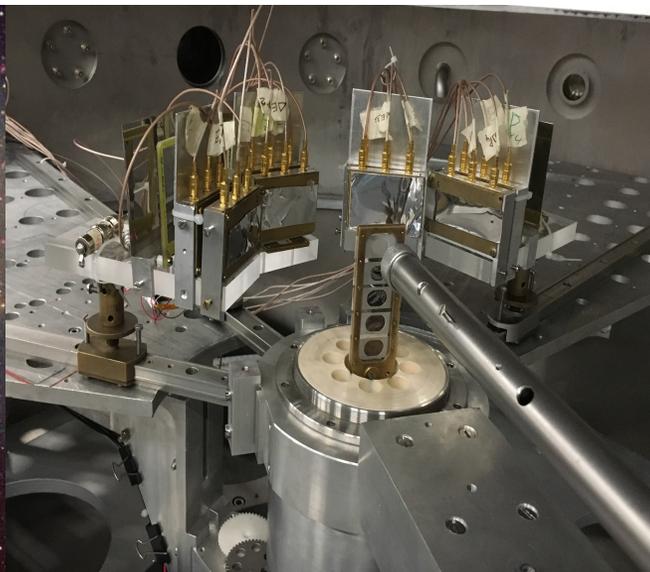




18th Russbach School on Nuclear Astrophysics

12–18 Mar 2023
Rufbach am Paß Gschütt

Introduction to Nuclear Physics in Astrophysics



Aurora Tumino

Outline of my lecture

- Nuclear physics in the abundance curve
- Features of thermonuclear reactions
- Experimental approaches
- Physics cases

Nuclear Astrophysics → Rich & Diverse Interdisciplinary Field bringing together

- Modelers
- Observers
- Nuclear physicists: Experimentalists as well as Theorists

... from the seminal **B²FH** review paper of 1957,
the basis of the modern nuclear astrophysics

this work has been considered as the greatest gift of astrophysics to modern civilization



Synthesis of the Elements in Stars*
 E. MARGARET BURBIDGE, G. R. BURBIDGE, WILLIAM A. FOWLER, AND F. HOYLE
*The first complete review of nuclear reactions explaining:
 H and He quiescent and hot burning, and of the nucleosynthesis beyond Fe.*

March
Margaret Burbidge
Geoff Burbidge
William A. Fowler
Fred Hoyle
1983 Nobel Prize

"for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe"

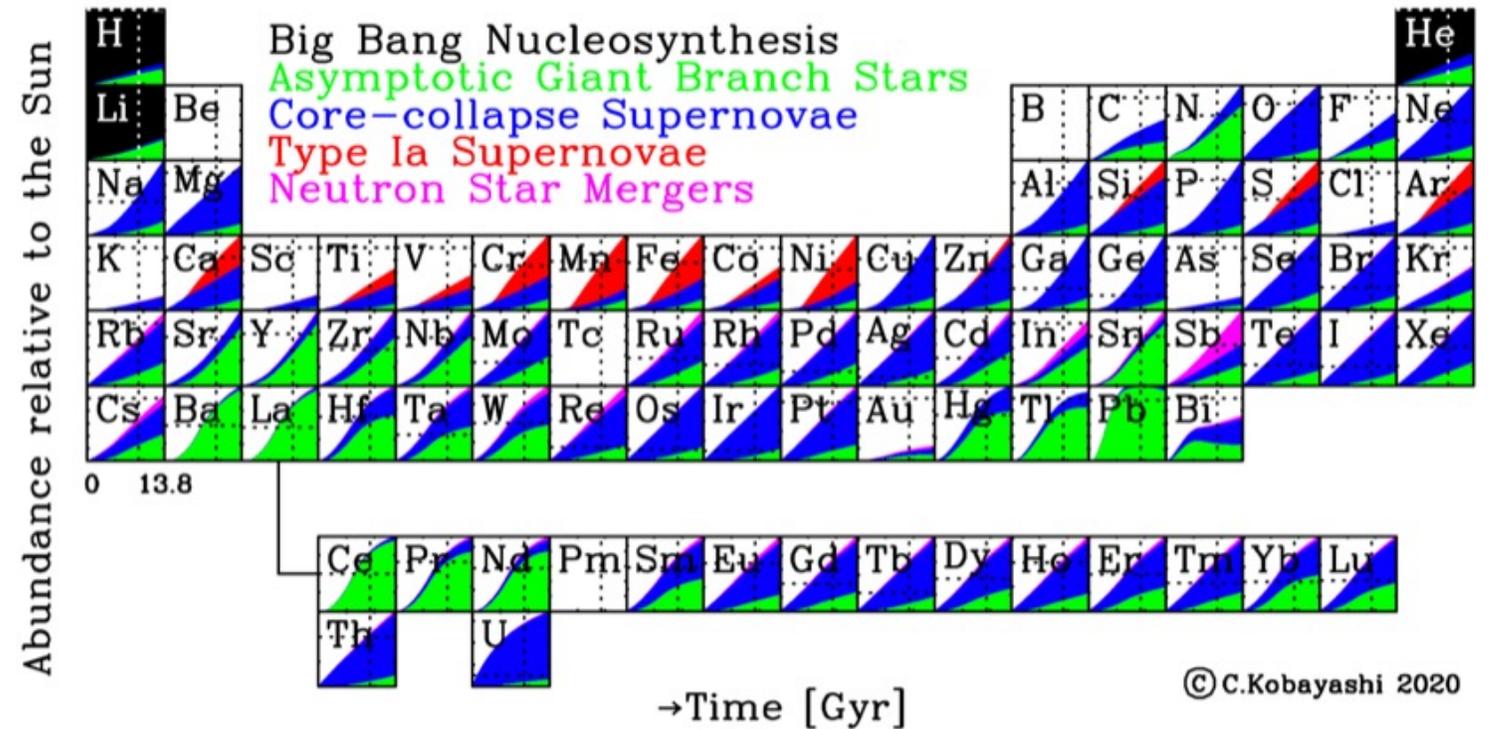
Nuclear reactions responsible for both ENERGY PRODUCTION and CREATION OF ELEMENTS in 4 ways/environments:

- **Cosmological nucleosynthesis**: creation in the Big Bang
- **Stellar nucleosynthesis**: synthesis of elements by fusion in stars
- **Explosive nucleosynthesis**: synthesis of elements by neutron and proton capture reactions in supernovae
- **Galactic nucleosynthesis**: synthesis of elements by cosmic ray spallation reactions

Where the elements are made...we WISH we knew that!

Here is the "current belief" in terms of nucleosynthetic source of elements in the Solar System

Each element in this periodic table is color-coded by the relative contribution of nucleosynthesis sources



In [astronomy](#), a "metal" is any element other than hydrogen or helium, the only elements that were produced in significant quantities in the Big Bang. Thus, the [metallicity](#) of a [galaxy](#) or other object is an indication of stellar activity after the Big Bang.

Where's the Nuclear Physics?

H burning → conversion of H to He

He burning → conversion of He to C, O ...

C, O and Ne burning → production of A: 16 to 28

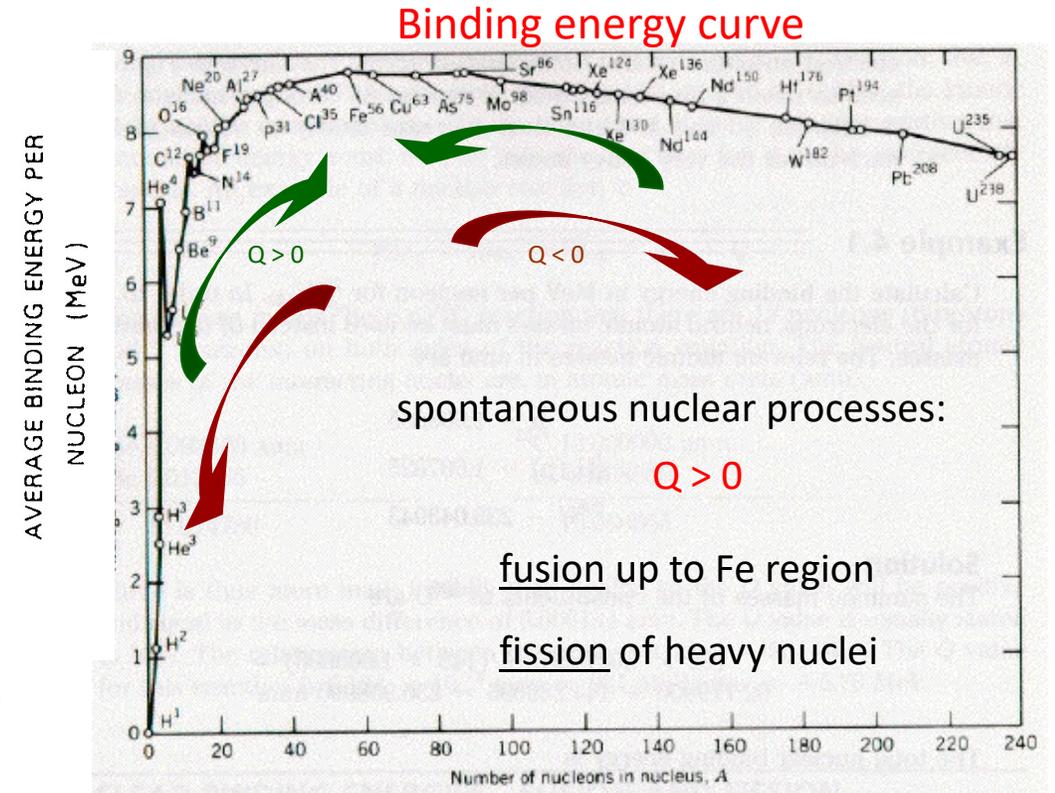
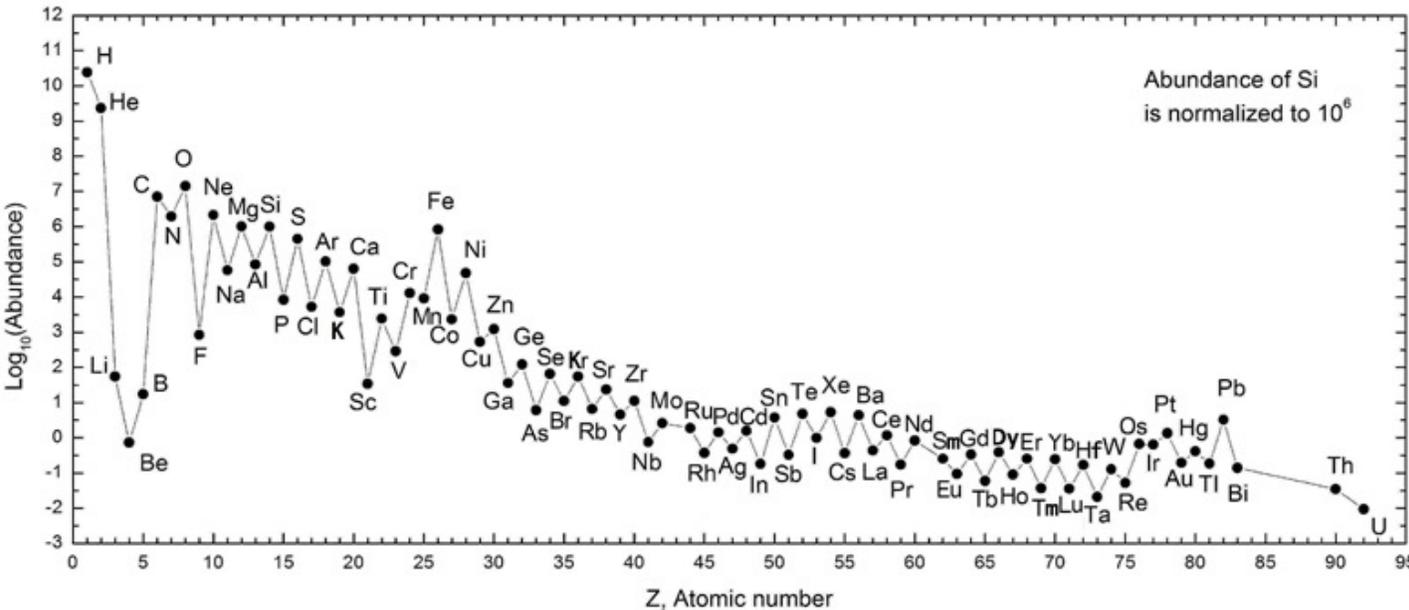
Si burning → production of A: 28 to 60

s-, r- and p-processes → production of A>60

Li, Be, and B from cosmic rays

- Big Bang Nucleosynthesis does not go beyond Li due to missing stable nuclei of mass number 5 or 8
- Odd-even staggering of abundances (Oddo-Harkins rule)
- Larger alpha-nuclei abundance, particularly those connected to particular values of Z and N (so called magic numbers, 2, 8, 20, 28, 50 ...) which are significant with regard to the structure of nuclei ... at least up to Fe
- Broad peak around Fe

Abundance of elements in the solar system



for a nuclear reaction

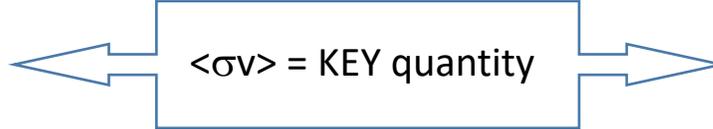


Total reaction rate: $R_{12} = (1+\delta_{12})^{-1} N_1 N_2 \langle \sigma v \rangle_{12}$ reactions $\text{cm}^{-3} \text{s}^{-1}$, N_i number density

Energy production rate: $\epsilon_{12} = R_{12} Q_{12}$ reaction Q-value: $Q_{12} = [(m_1+m_2)-(m_3+m_4)]c^2$

Nuclear physics information

energy production
as star evolves



change in abundance
of nuclei

to be determined from experiments and/or theoretical considerations



stars = cooking pots of the Universe

a) velocity distribution

interacting nuclei in plasma are in **thermal equilibrium** at temperature T

also assume **non-degenerate** and **non-relativistic** plasma

\Rightarrow **Maxwell-Boltzmann velocity distribution**

b) cross section

no nuclear theory available to determine reaction cross section a priori and can vary by orders of magnitude, depending on the interaction

cross section depends sensitively on:

- the **properties of the nuclei** involved
- the **reaction mechanism**

examples:

1 barn = $10^{-24} \text{ cm}^2 = 100 \text{ fm}^2$

Reaction	Force	σ (barn)	E_{proj} (MeV)
$^{15}\text{N}(p,\alpha)^{12}\text{C}$	strong	0.5	2.0
$^3\text{He}(\alpha,\gamma)^7\text{Be}$	electromagnetic	10^{-6}	2.0
$p(p,e^+\nu)d$	weak	10^{-20}	2.0

in practice, need **experiments** AND **theory** to determine stellar reaction rates

Reaction mechanisms in short

Consider reaction:



(y = particle or photon)

Non-resonant process

One-step process leading to final nucleus B

$$\sigma \propto |\langle y+B | H | x+A \rangle|^2$$

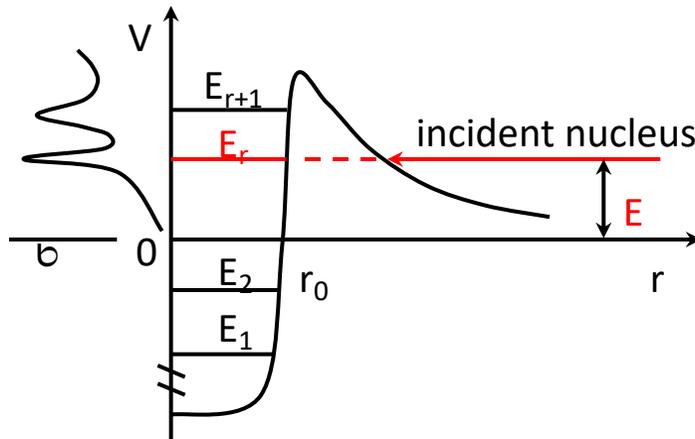
single matrix element

→ occurs at all interaction energies

→ cross section has WEAK energy dependence

Resonant process

Two-step process: 1) compound nucleus formation $x + A \rightarrow C^*$
 2) decay of compound nucleus $C^* \rightarrow y + B$



$$\sigma \propto \underbrace{|\langle y+B | H' | C^* \rangle|^2}_{\text{compound decay probability } \propto \Gamma_y} \underbrace{|\langle C^* | H | x+A \rangle|^2}_{\text{compound formation probability } \propto \Gamma_x}$$

two matrix elements

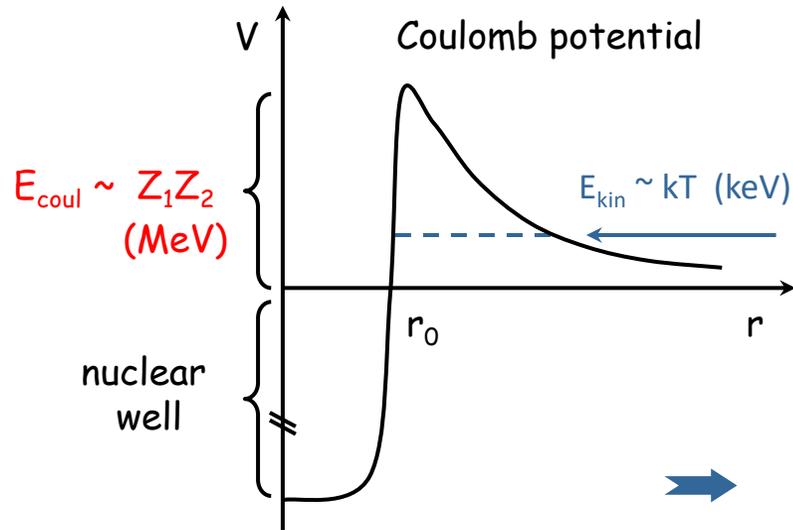
→ occurs at specific energies

→ cross section has STRONG energy dependence

Nuclear reactions between charged particles

charged particles → **Coulomb barrier**

energy available: from **thermal motion**

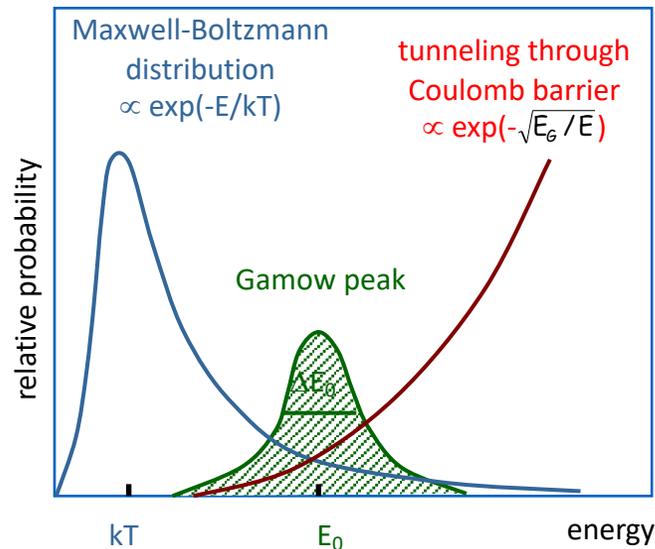
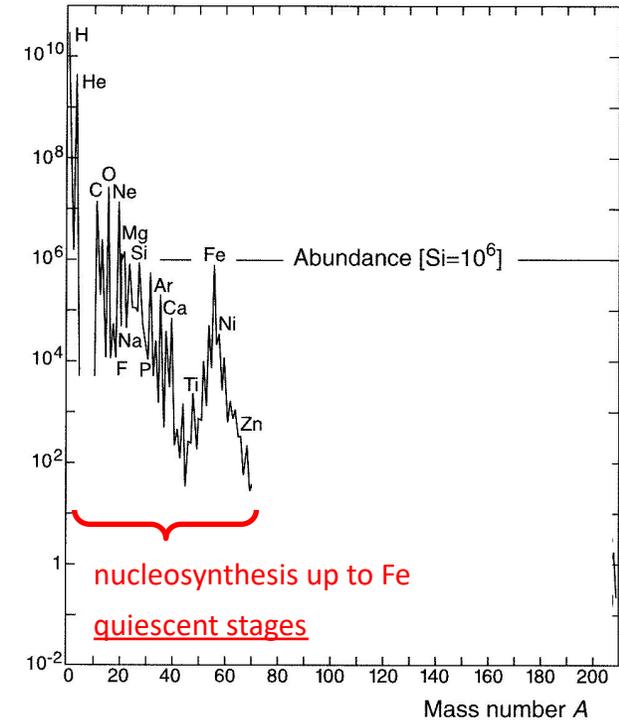


during static burning: $kT \ll E_{Coul}$

$T \sim 15 \times 10^6$ K (e.g. our Sun) $\Rightarrow kT \sim 1$ keV

reactions occur through **TUNNEL EFFECT**

tunneling probability $P \propto \exp(-2\pi\eta)$



Gamow peak: **most effective energy region** for thermonuclear reactions

It is where measurements should be carried out

Gamow energy:

$$E_0 = f(Z_1, Z_2, T)$$



varies depending on reaction and/or temperature

Examples: $T \sim 15 \times 10^6 \text{ K}$ ($T_6 = 15$)

reaction	Coulomb barrier (MeV)	E_0 (keV)	area under Gamow peak $\sim \langle \sigma v \rangle$
p + p	0.5	5.9	7.0×10^{-6}
$\alpha + {}^{12}\text{C}$	2.242	56	5.9×10^{-56}
${}^{16}\text{O} + {}^{16}\text{O}$	10.349	237	2.5×10^{-237}

$$kT \ll E_0 \ll E_{\text{coul}}$$

$10^{-18} \text{ barn} < \sigma < 10^{-9} \text{ barn}$ major experimental challenges



STRONG sensitivity
to Coulomb barrier



separate stages:

H-burning
He-burning
C/O-burning ...

Neutron captures

NO Coulomb barrier

neutrons produced in stars are quickly **thermalised**

$E_0 \sim kT = \text{relevant energy}$ (e.g. $T \sim 1-6 \times 10^8 \text{ K} \Rightarrow E_0 \sim 30 \text{ keV}$)

transmission probability:

$$P_\ell \propto E^{1/2} \quad \text{for } l = 0 \quad \text{and hence: } \sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

$$P_\ell \propto E^{1/2+\ell} \quad \text{for } l \neq 0 \quad \text{and hence: } \sigma \propto E^{\ell-1/2}$$

consequences:

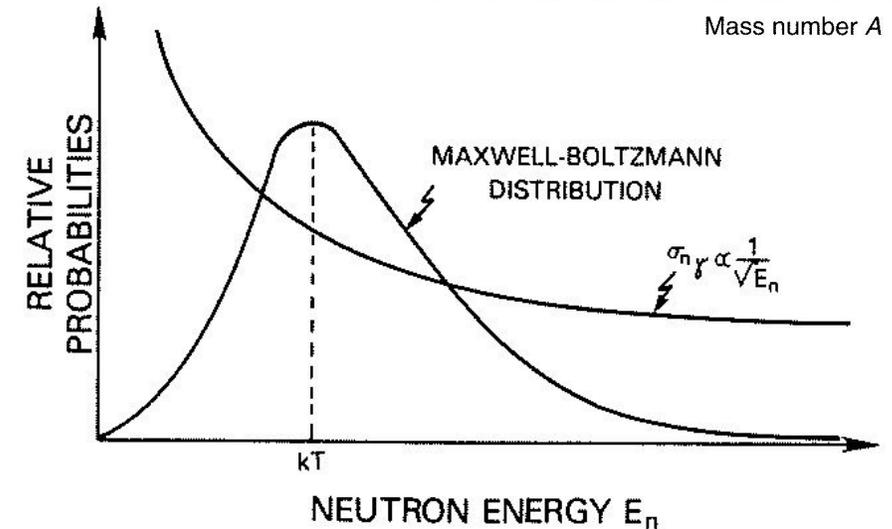
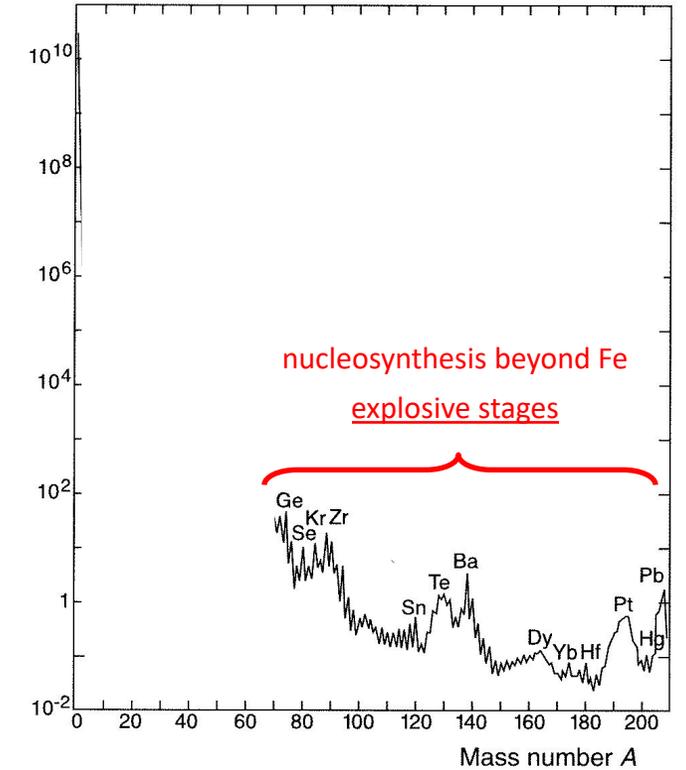
s-wave neutron capture usually dominates at **low energies**
(except if hindered by selection rules)

higher l neutron capture only plays role at **higher energies**
(or if $l=0$ capture suppressed)

For s-wave neutrons:

$$\rightarrow \langle \sigma v \rangle \sim \text{const} = \langle \sigma_T v_T \rangle \rightarrow$$

accounts for **almost flat** abundance
distribution beyond iron peak



neutron-capture cross sections can be measured **directly** at the relevant energies

A few details on cross section expressions

Cross section expression for low-energy non-resonant reactions



$$\sigma = \pi \lambda_x^2 \left| \langle B | H | x + A \rangle \right|^2 P_\ell(E)$$

“geometrical factor”
de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

matrix element
contains nuclear
properties of interaction

probability for barrier penetrability
depends on relative orbital angular
momentum l and energy E

$$\sigma = (\text{weak energy dependence}) \times (\text{strong energy dependence})$$

need expression for $P_l(E)$

factors affecting transmission probability:

- centrifugal barrier (both for charged particles and neutrons)
- Coulomb barrier (for charged particles only)

$$V_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}$$

Cross section expression for **low-energy resonant reactions**: single isolated resonance

resonant cross section given by Breit-Wigner expression

$$\sigma(E) = \pi D^2 \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1 \Gamma_2}{(E - E_R)^2 + (\Gamma/2)^2}$$

for reaction: $1 + T \rightarrow C \rightarrow F + 2$

geometrical factor
 $\propto 1/E$

spin factor ω

J = spin of CN's state
 J_1 = spin of projectile
 J_T = spin of target

strongly energy-dependent term

Γ_1 = partial width for decay via emission of particle 1
= probability of compound formation via entrance channel

Γ_2 = partial width for decay via emission of particle 2
= probability of compound decay via exit channel

Γ = total width of compound's excited state
= $\Gamma_1 + \Gamma_2 + \Gamma_\gamma + \dots$

E_R = resonance energy w.r.t. entrance channel threshold

what about penetrability considerations? \Rightarrow look for energy dependence in partial widths!

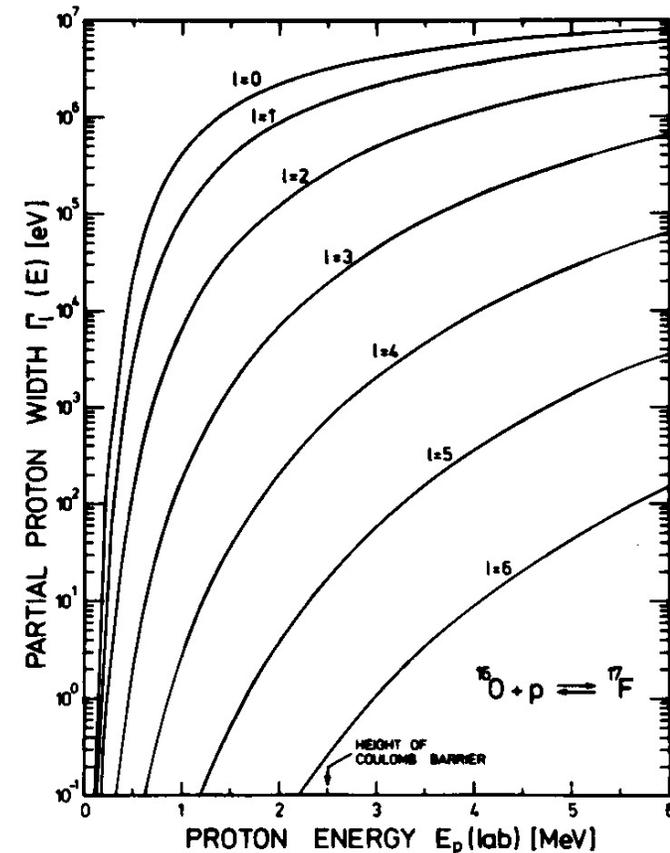
partial widths are NOT constant but energy dependent!

particle widths

$$\Gamma_1 = \frac{2\hbar}{R} P_l(E_1) \theta_l^2$$

θ_l = "reduced width" (contains nuclear physics info)

P_l gives strong energy dependence



Corresponding reaction rate for resonant processes

$$\langle \sigma v \rangle = \int \sigma(v) \phi(v) v dv = \int \sigma(E) \exp(-E/kT) E dE$$



here Breit-Wigner cross section

$$\sigma(E) = \pi D^2 \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_1 \Gamma_2}{(E - E_R)^2 + (\Gamma/2)^2}$$

integrate over appropriate energy region

$E \sim kT$	for neutron induced reactions
$E \sim$ Gamow window	for charged particle reactions

if compound nucleus has an excited state (or its wing) in this energy range

⇒ **RESONANT** contribution to reaction rate (if allowed by selection rules)

typically:

- resonant contribution dominates reaction rate
- reaction rate critically depends on resonant state properties

reaction rate for:

- narrow resonances
- broad resonances/sub-threshold states

Narrow resonance case

$$\Gamma \ll E_R$$

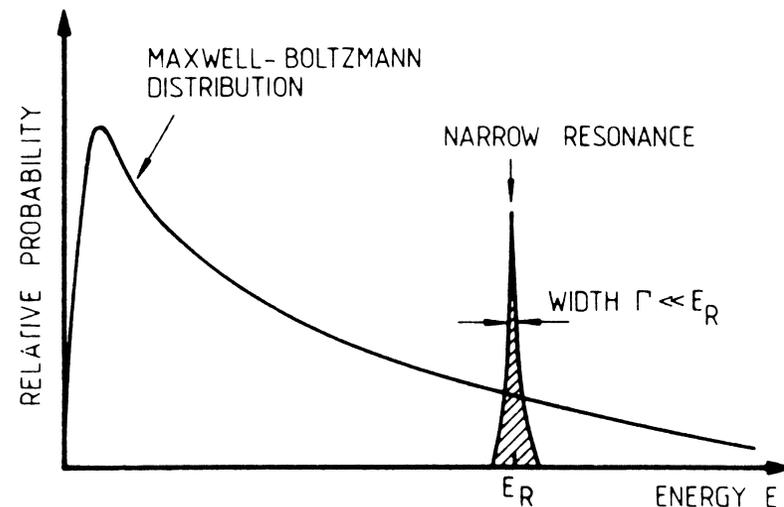
reaction rate for a single narrow resonance

$$\langle \sigma v \rangle_{12} = \left(\frac{2\pi}{\mu_{12} kT} \right)^{3/2} h^2 (\omega \gamma)_R \exp\left(-\frac{E_R}{kT}\right)$$

resonance strength

(= integrated cross section over resonant region)

$$\omega \gamma = \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1 \Gamma_2}{\Gamma} \quad (\Gamma_i \text{ values at resonant energies})$$



- resonance must be **near** relevant energy range ΔE_0 to contribute to stellar rate
- MB distribution assumed **constant** over resonance region
- partial widths also **constant**, i.e. $\Gamma_i(E) \cong \Gamma_i(E_R)$

often

$$\Gamma = \Gamma_1 + \Gamma_2$$

$$\begin{aligned} \Gamma_1 \ll \Gamma_2 &\longrightarrow \Gamma \approx \Gamma_2 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_1 \\ \Gamma_2 \ll \Gamma_1 &\longrightarrow \Gamma \approx \Gamma_1 \longrightarrow \frac{\Gamma_1 \Gamma_2}{\Gamma} \approx \Gamma_2 \end{aligned}$$

NOTE

exponential dependence on energy means:

- rate strongly dominated by **low-energy resonances** ($E_R \rightarrow kT$) if any
- small uncertainties in E_R (even a few keV) imply large uncertainties in reaction rate

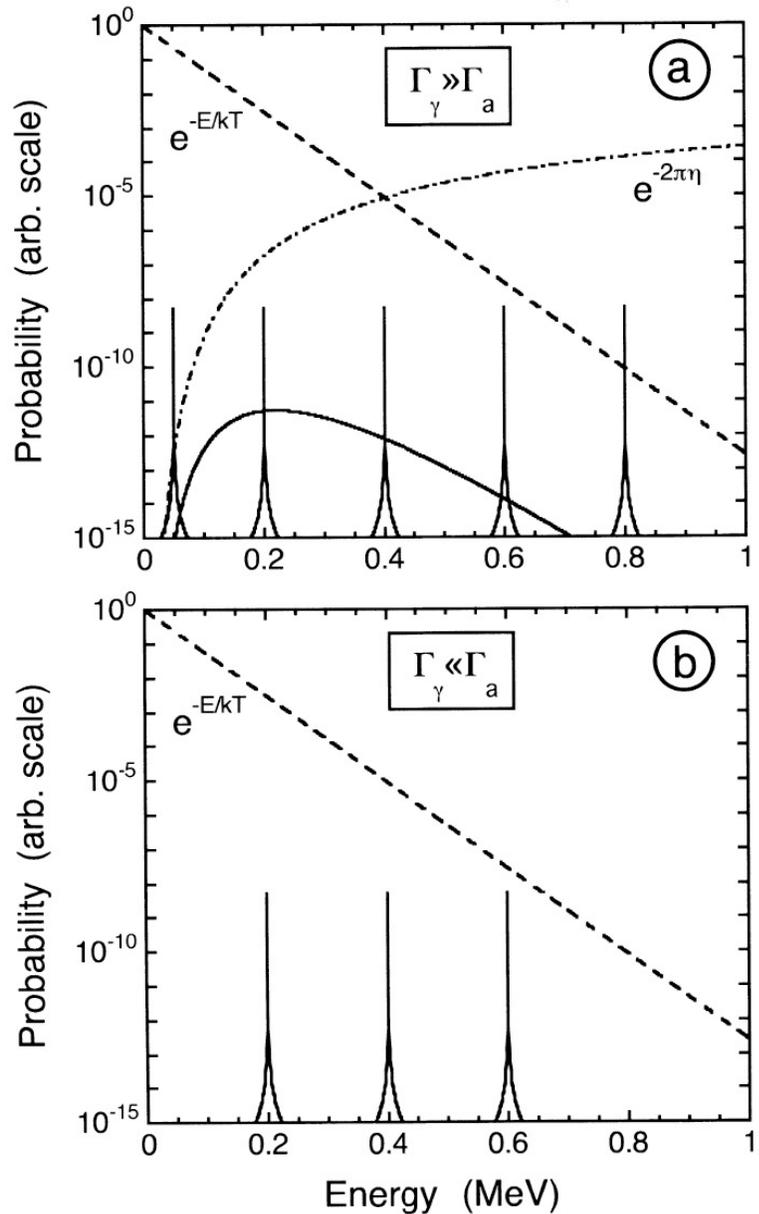
reaction rate is determined by the **smallest** width!

experimental info needed:

- partial widths Γ_i
- spin **J**
- energy E_R

note: for many unstable nuclei most of these parameters are

UNKNOWN!



resonant strength dominated by **particle width**

$$\omega\gamma = \omega\Gamma_a \quad (\text{typically for } E_R \leq 0.5 \text{ MeV})$$

- strong energy dependence through Coulomb barrier penetration
- only resonances in Gamow window are relevant to reaction rate

resonant strength dominated by **gamma width**

$$\omega\gamma = \omega\Gamma_\gamma \quad (\text{typically for } E_R > 0.5 \text{ MeV})$$

- lowest energies dominate rate because of $\exp(-E_R/kT)$ term
- no Gamow peak exists!
- effect most important at high temperatures

Broad resonance case

$$\Gamma \sim E_R$$

Breit-Wigner formula

+

energy dependence of partial and total widths

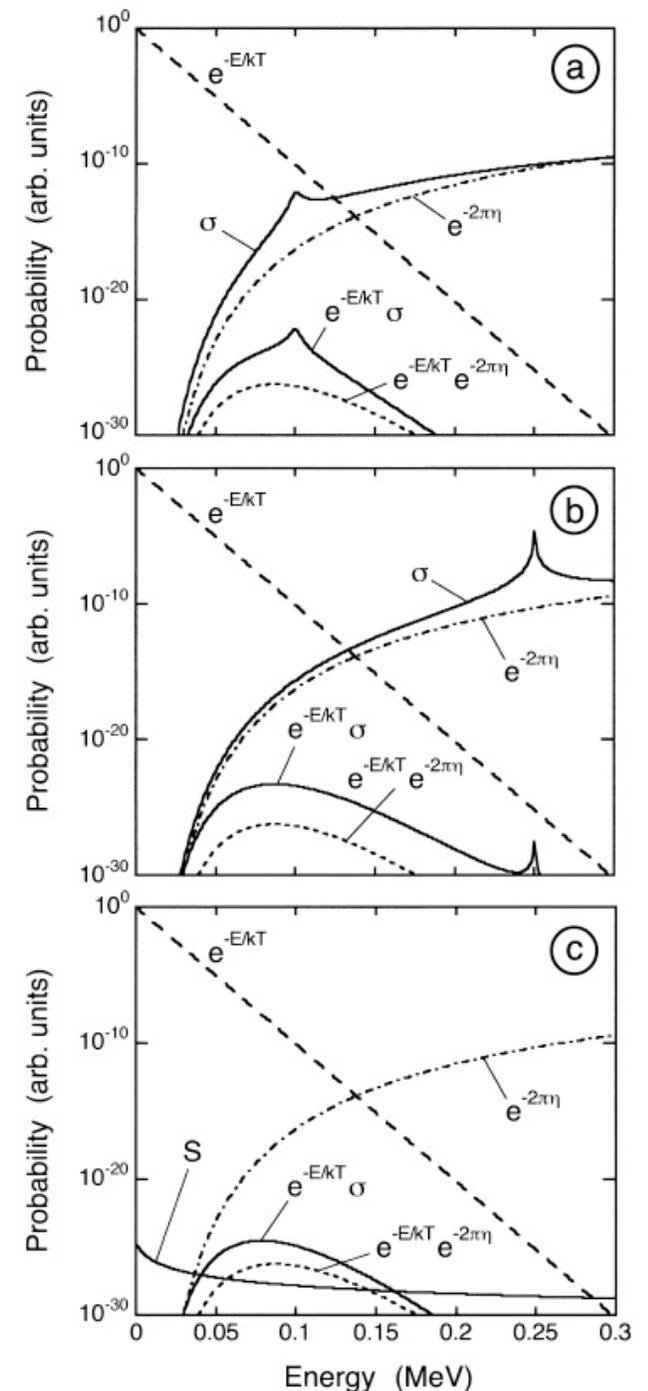
The product of **Maxwell–Boltzmann distribution and cross section** is now a complicated function of energy (lower solid line) and can no longer be integrated analytically. Instead, the reaction rates have to be calculated numerically.

broad resonance located within
Gamow peak dominates rate

broad resonance located outside
Gamow peak
low-energy wing dominates rate

broad sub-threshold resonance
high-energy wing contributes to rate

N.B. overlapping broad resonances of same $J^\pi \rightarrow$ **interference effects**

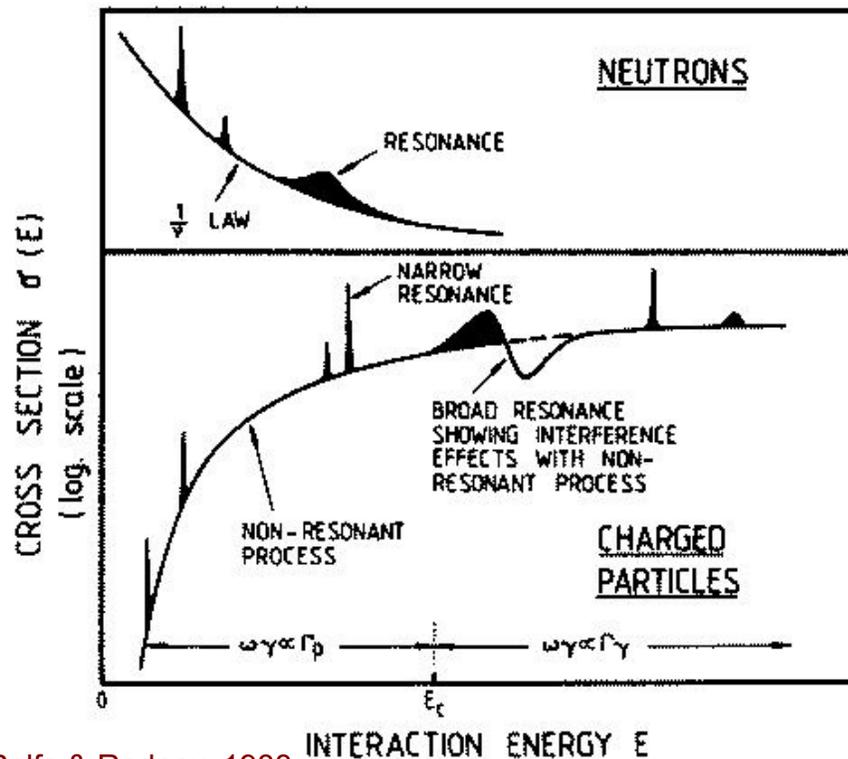


To summarize ... stellar reaction rates include contributions from

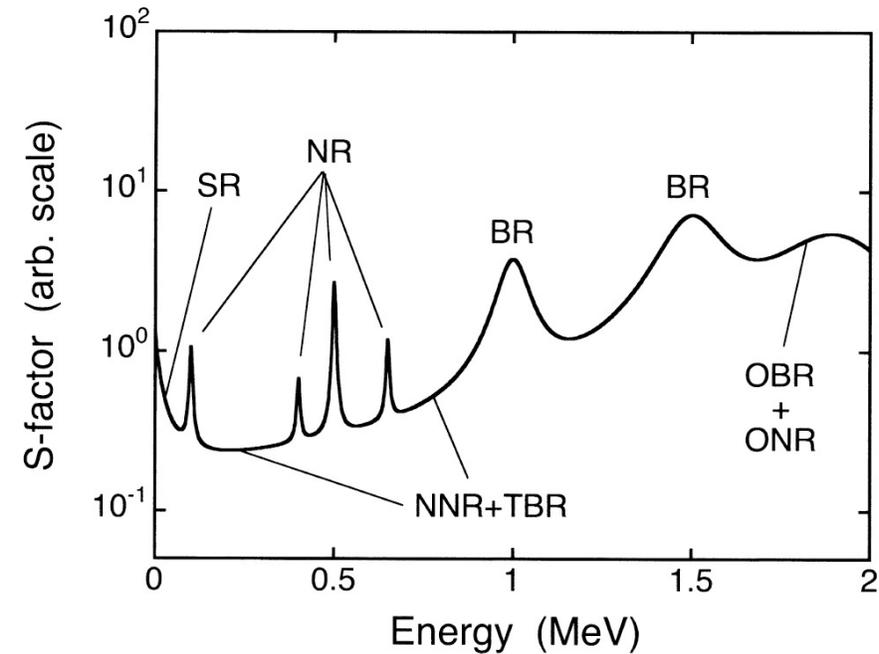
- direct transitions to the various bound states
- all narrow resonances in the relevant energy window
- broad resonances (tails) e.g. from higher lying resonances
- any interference term

total rate

$$\langle \sigma v \rangle = \sum_i \langle \sigma v \rangle_{\text{DCi}} + \sum_i \langle \sigma v \rangle_{\text{Ri}} + \langle \sigma v \rangle_{\text{tails}} + \langle \sigma v \rangle_{\text{int}}$$



Rofls & Rodney, 1988



Iliadis, 2007

Features - General Overview

Quiescent burning stages

$$T \sim 10^6 - 10^8 \text{ K} \Rightarrow E_0 \sim 10 \text{ keV} - 1 \text{ MeV} \ll E_{\text{coul}}$$

$$\Rightarrow 10^{-18} \text{ barn} < \sigma < 10^{-9} \text{ barn}$$

$$\Rightarrow \text{average interaction time } \tau \sim \langle \sigma v \rangle^{-1} \sim 10^9 \text{ y}$$

unstable species DO NOT play significant role

Explosive burning stages

$$T > 10^8 \text{ K} \Rightarrow E_0 \sim \text{MeVs} \leq E_{\text{coul}}$$

$$\Rightarrow 10^{-6} \text{ barn} < \sigma < 10^{-3} \text{ barn}$$

\Rightarrow Extrapolation may not be needed

\Rightarrow average interaction time $\tau \sim \langle \sigma v \rangle^{-1} \sim \text{seconds}$

\Rightarrow unstable species DO play significant role

How to approach experimentally

Main Issues

- poor signal-to-noise ratio

- unknown nuclear properties

- low beam intensities (**several o.d.m. lower** than for stable beams)

Requirements

Extrapolation procedure (?)

long measurements

-ultra pure targets

-high beam intensities

-high detection efficiency

...

RIBs production and acceleration

large area detectors

high detection efficiency

Recoil separators

Storage rings

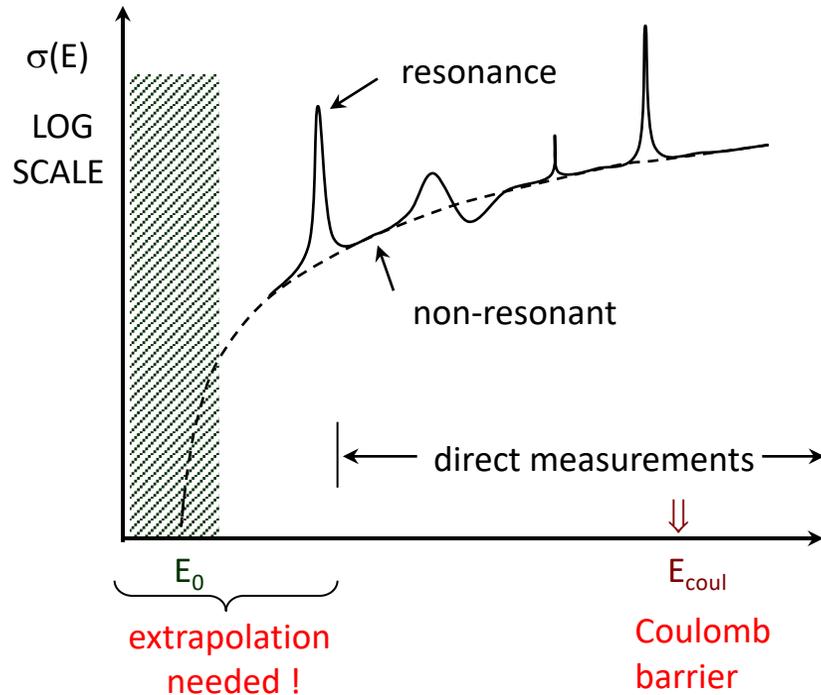
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Experimental approach: extrapolation

measure $\sigma(E)$ over as wide a range as possible, then extrapolate down to E_0 !

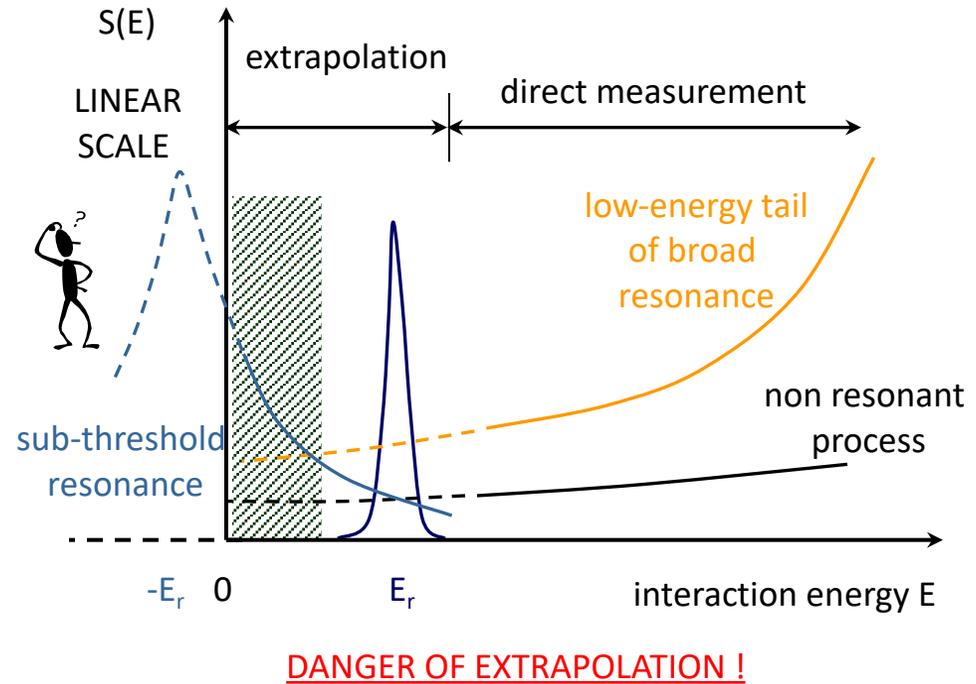
CROSS SECTION

$$\sigma(E) = \frac{1}{E} \exp(-2\pi\eta) S(E)$$



S-FACTOR

$$S(E) = E\sigma(E) \exp(2\pi\eta)$$



Experimental approach: alternative solutions

- Underground experiments to reduce (cosmic) background: LUNA (LNGS Italy), Felsenkeller (Germany), CASPAR (USA), JUNA (China), particularly suited to perform gamma spectroscopy

- Surface experiments:

inverse kinematics;

coincidence experiments (g-g, g-particle, ...);

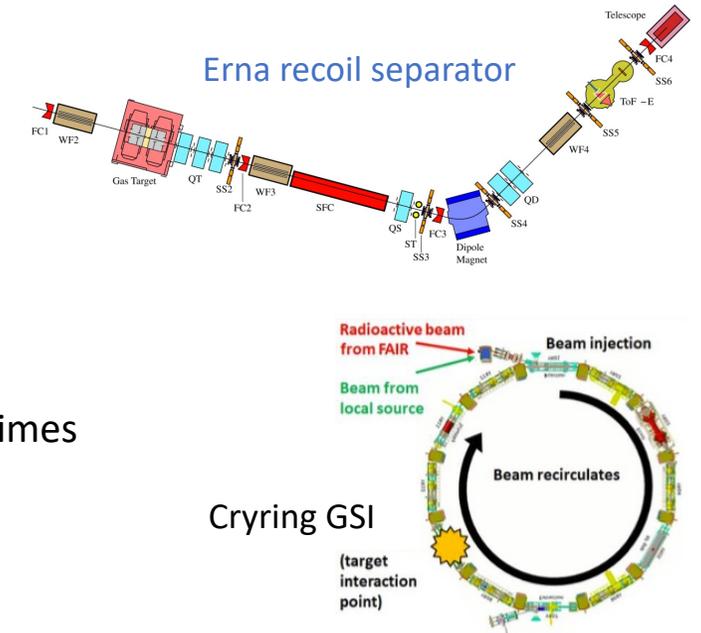
recoil separators, separate **reaction products** from unreacted beam and

disperse them according to their **mass-to-charge-state ratio**;

storage rings: to overcome beam intensity limitations. The beam is recirculated many times and therefore has repeated chances to interact with the target;

...

- Use indirect methods: Coulomb Dissociation (CD), Asymptotic Normalization Coefficients (ANC), Trojan Horse Method (THM)



These topics will be the subject of several lectures in the next days ...