Initial steps in the inference of horizontal velocities

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Context & Motivations

Goal: Infer horizontal velocity fields in the solar atmosphere to improve the:

- Understanding of plasma dynamics
- Determination of electric field & currents
- Inference of gas pressure.

In the Chromosphere velocities can become important for the force balance.

$$\nabla P_{\rm g} = \rho\mathbf{g} - \rho(\mathbf{v}\nabla)\mathbf{v} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}$$

Induction equation

From the spectra and inversion codes (FIRTEZ) we can measure the LOS component of the velocity vector (v_z) and the magnetic field (**B**). Horizontal velocities are challenging because spectral lines are not sensitive to them.
To obtain the other two components of the velocity we will use the induction equation in ideal MHD.

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

This leads to an overdetermined system of equations with two unknowns (v_x,v_y) in 3D. Similar idea to that of Longcope (2004) but in three-dimensions. This will be implemented in the Stokes inversion code (FIRTEZ) in the framework on a **DFG project** (538773352**)**. Our initial tests are also in 2D:

Induction equation in (y,z) (v_y as unknown):

$$\frac{\partial B_y}{\partial t} = \frac{\partial v_y}{\partial z}B_z + v_y\frac{\partial B_z}{\partial z} - \frac{\partial v_z}{\partial z}B_y - v_z\frac{\partial B_y}{\partial z}$$
$$\frac{\partial B_z}{\partial t} = -\frac{\partial v_y}{\partial y}B_z - v_y\frac{\partial B_z}{\partial y} + \frac{\partial v_z}{\partial y}B_y + v_z\frac{\partial B_y}{\partial y}$$

Induction equation in (x,y) (v_x and v_y as unknowns):

$$\frac{\partial B_x}{\partial t} = \frac{\partial v_x}{\partial y}B_y + v_x\frac{\partial B_y}{\partial y} - \frac{\partial v_y}{\partial y}B_x - v_y\frac{\partial B_x}{\partial y}$$
$$\frac{\partial B_y}{\partial t} = -\frac{\partial v_x}{\partial x}B_y - v_x\frac{\partial B_y}{\partial x} + \frac{\partial v_y}{\partial x}B_x + v_y\frac{\partial B_x}{\partial x}$$

Numerical method

Discretization: central finite differences (1st or 2nd order)

Boundary conditions:

- Dirichlet (e.g., v=0 at boundary)
- Neumann (e.g., dv/dr = 0)

Use **ghost cells** to handle edges

Least squares method (pseudo-inverse) to solve the overdetermined system

Proof of concept in (y,z)

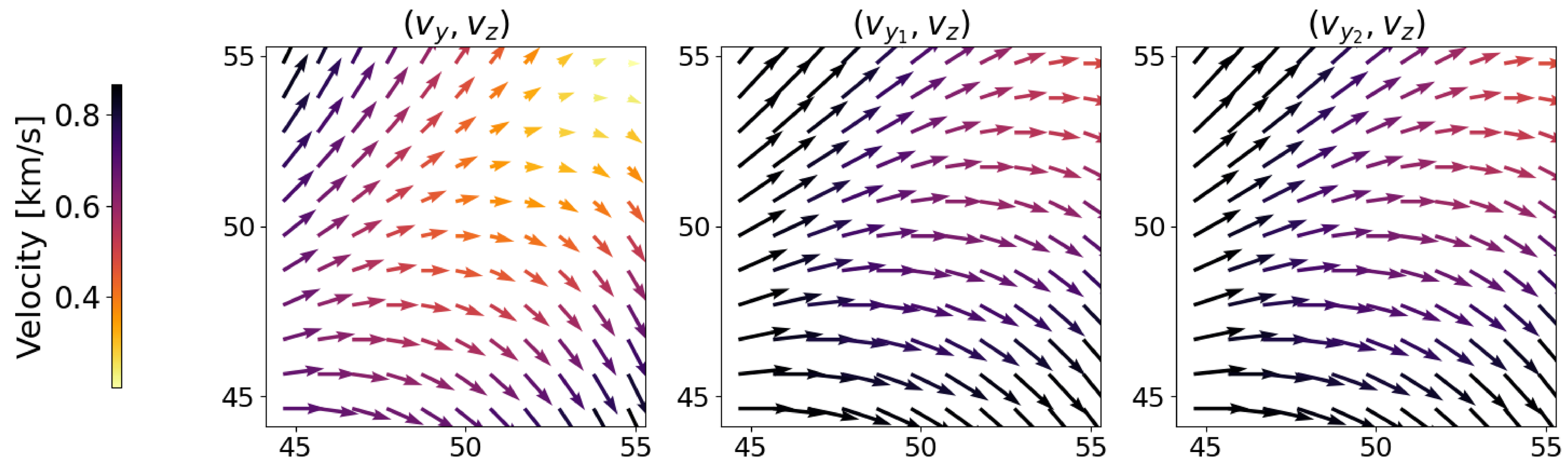

Figure 1. (left panel): Analytical velocity field in the (y,z) plane (v_y,v_z); (middle panel): inferred velocity field using Dirichlet boundary conditions (v_{y1},v_z); (right panel): inferred velocity field using Neumann boundary conditions (v_{y2},v_z).

Proof of concept in (x,y)

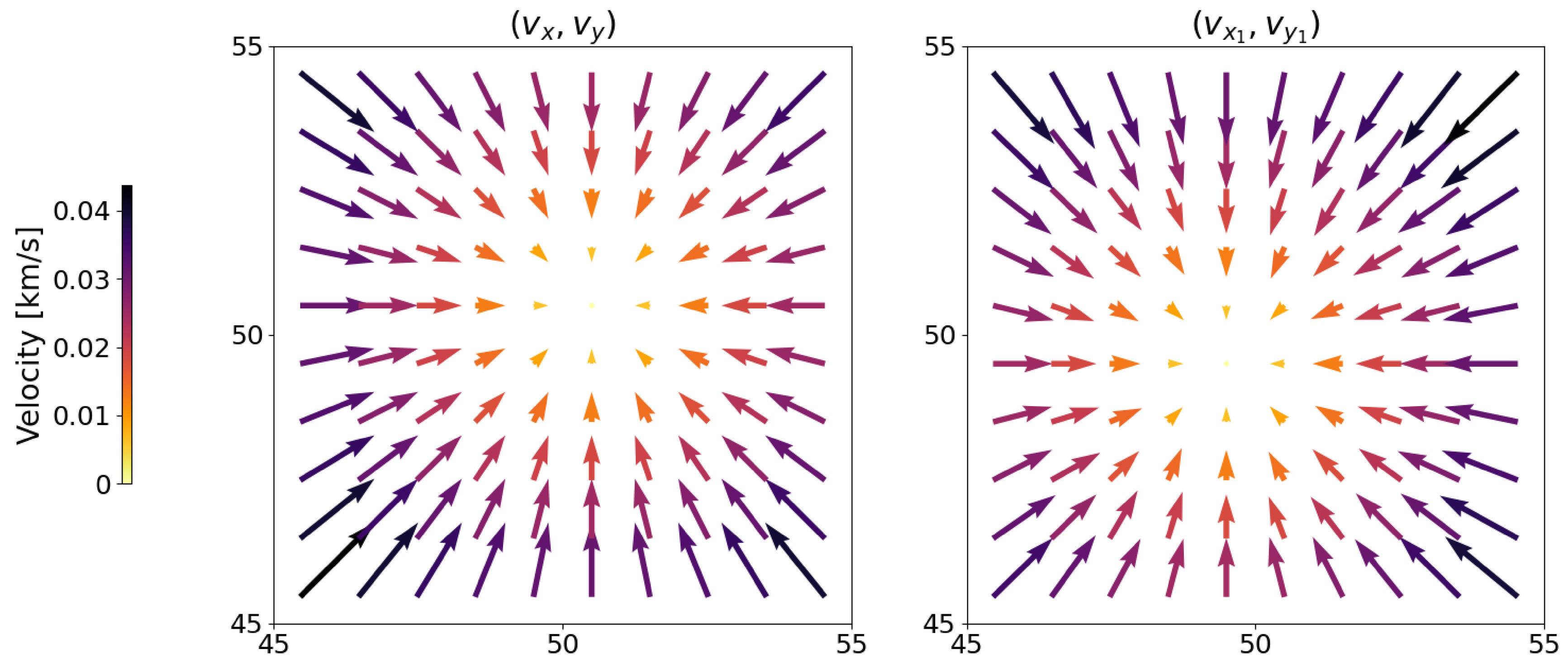

Figure 2. Example of inferred velocities in the (x,y) plane (perpendicular plane to the LOS direction) using Neumann boundary conditions. (left panel): Analytical velocity field (v_x,v_y); (right panel): inferred velocity field (v_{x1},v_{y1}).

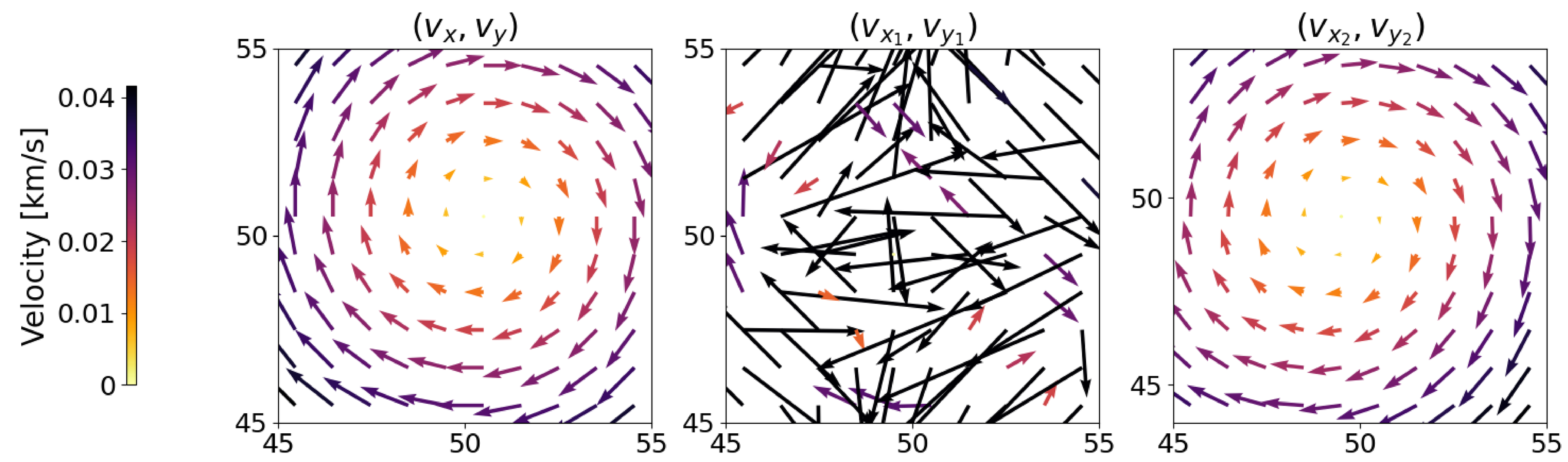

Figure 3. Example of inferred velocities in the (x,y) plane where Neumann boundary conditions yield a non-uniqueness of solution so that we define a new set-up with Dirichlet boundary conditions and apodisation on the magnetic field (**B**). (left panel): Analytical velocity field (v_x,v_y); (middle panel): inferred velocity field using Neumann boundary conditions (v_{x1},v_{y1}); (right panel): inferred velocity field using the Dirichlet boundary conditions and an apodisation in **B** (v_{x2},v_{y2}).

Future

- Extend to full 3D domain
- Test with simulation data (CO5BOLD, MuRAM)
- Test with observational data
- Integrate module to FIRTEZ