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f-mode Travel-Time Signature of Sunspot Models and Plages

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Abstract

Two main models have been proposed to describe the magnetic subsurface structure of sunspots: the monolithic model and the cluster model. However, despite its relevance, this problem remains an important open question and challenge in current solar physics. In this work, we investigate the travel-time signature of two sunspot models such as the above, using numerical simulations of surface gravity (*f*-mode) wave propagation. We have shown that the travel-time shift and caustics patterns can be used to distinguish between a monolithic and a clustered model, as well as to differentiate between a compact and a loose cluster configuration [1]

Motivation

Sunspots and Plages are a very important solar structures. They are a direct manifestation of the magnetic field on the surface of the Sun. Furthermore, they play a key role in energy transfer from the solar interior to the chromosphere and corona through magneto-acoustic-gravity wave propagation. Consequently, accurate knowledge of these features is essential for understanding solar activity.

Simulations setup

Simulations were done using the SLiM code which solves the 3D linear and ideal MHD equations in a stratified atmosphere [2] [3] . The incoming wave consists of surface gravity wave packet (*f* -mode). Simulations were performed with different *f* -mode central frequencies: ν =2 mHz, ν =3 mHz, ν =4 mHz, and ν = 5 mHz.

Sunspot Models:

1-The monolithic model:

The initial monolithic tube is superimposed on the polytropic background atmosphere and along the *z*-axis with the profile $B(r) = B_0 \exp(-r^4/R^4)$, where $\boldsymbol{R}$ is the tube radius and B_0 = 4820 G **[4].**

2-The cluster model:

The compact cluster model consists of seven identical flux tubes in a hexagonal compact configuration. Two cases are studied:

- Small size compact model: individual tube radius is set to be 200 km, the radius of the equivalent monolithic tube of the cluster is R = 880 km.

- Large size compact model: individual tube radius is set to be 400 km, the radius of the equivalent monolithic tube is R = 1.76 Mm.

The open (loose) cluster model consists of 6 and 9 small identical magnetic-flux tubes of 200 km radius in a loose configuration. The radius of the equivalent monolithic tube is R = 1.2 Mm.

Travel Time measurement:

Simulations provide the perturbed vertical velocity $V_z(x, y, t)$ related to the sunspot model, as well as the unperturbed vertical velocity $V_{z0}(x, y, t)$. The phase travel-time shift $\delta t(x, y)$ is determined by least-squares fitting (V_{z0} to V_z) of a parabola, where X is the function to be minimized:

$$X(x, y, t) = \int [v_z(x, y, \tau) - v_{z0}(x, y, \tau - t)]^2 \mathrm{d}\tau$$

A negative travel-time shift, $\delta t < 0$, corresponds to an advance in the arrival of the perturbed vertical velocity $V_z(x, y, t)$, relative to the unperturbed wave packet V_{z0} (x, y, t), while a positive anomaly, $\delta t > 0$, corresponds to a delay in the arrival of the perturbed wave packet.

Results

- The amplitude of the travel-time shift for the **monolithic** model shows a strong dependence on the ratio R/λ. As this ratio increases, the negative travel-time amplitude behind the tube also increases (Figure 1) . For R/λ between 0.018 (R =200 km, ν =2 mHz) and 0.457 (R =880 km, ν =5 mHz), this result means that the waves propagate faster when traveling the vertical **monolithic** magnetic field (Figure 2). However, for R/λ =0.685 (R =1.2 Mm, ν =5 mHz) and larger, the interpretation of the travel-time shift becomes more difficult and ambiguous due to the contamination of the y = 0 region beyond the tubes by the caustic pair, especially for the high-frequencies incoming wave (Figures 4, 5).

- For the **cluster** models associated to **monolithic** tubes of radii R = 880 km ($0.073 \leq R/\lambda \leq 0.457$) and R = 1.2 Mm ($0.11 \leq R/\lambda \leq 0.44$), the maximum negative amplitude of the travel-time is concentrated only in the near field behind these models (Figures 3, 5). In contrast, for the **monolithic** equivalent models, this maximum is concentrated mainly at the pair of caustics (Figures 2, 5). This result can be verified using helioseismic measurements in order to distinguish between the **monolithic** model and the **compact** or **open cluster** model.

- For both **compact cluster** models (small and large size), the travel-time map at low *f* -mode frequencies (2 mHz and 3 mHz) fits quite good qualitatively that of the equivalent **monolithic** tube (Figures 2, 3, 4).

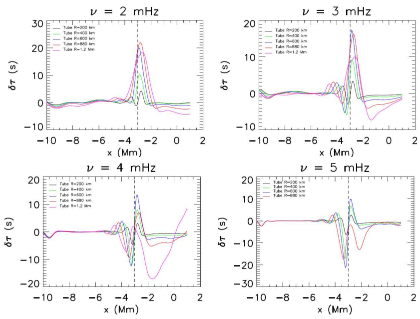

Figure 1: Travel-time shift δt (s) as a function of the distance x (Mm), measured at y = 0 for different **monolithic** tube radii R = 200 km (*black curve*), R = 400 km (*green curve*), R = 600 km (*blue curve*), R = 880 km (*red curve)* and R = 1.2 Mm (*magenta curve*), and for different frequencies from 2 mHz to 5 mHz. The *vertical dashed line* at x =−3 Mm corresponds to the position of the sunspot center.

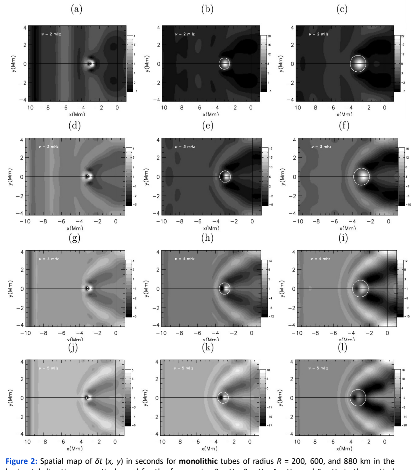

Figure 2: Spatial map of δt (x, y) in seconds for **monolithic** tubes of radius R = 200, 600, and 880 km in the horizontal direction respectively, and for the frequencies 2 mHz, 3 mHz, 4 mHz, and 5 mHz in the vertical direction respectively. The *colour bars show* the travel-time shift in seconds.

- For high-frequencies *f* -mode waves (4 mHz and 5 mHz), both **compact cluster** models are in the scattering regime or in the coherent-scattering regime. This is confirmed by observing the caustics of the individual tubes in the travel-time map (Figures 3, 4).

- The amplitude of the negative travel-time shift from the **small size compact cluster** is slightly larger than that of the equivalent **monolithic** model, which can be interpreted as a speed-up of the waves through this structure. However, contamination of the y = 0 region beyond the **cluster** by individual tube caustics makes this possibility less likely (Figure 3).

- Individual tubes within the **large compact cluster** are larger than those in the **small compact cluster** model. As a result, each pair of caustics from individual tubes within this model will converge more closely at high frequencies, similar to the case of a single **monolithic** flux tube. This configuration makes the crossing of some of these caustics just behind the **cluster** at y = 0, resulting in a large negative amplitude travel-time anomaly, making the interpretation ambiguous in this case as well (Figure 4).

- At low *f* -mode frequencies, the horizontal distance between the sunspot center and the position where the caustics emerge can be an observational indicator of whether the model is **monolithic** or **compactly clustered**. This distance is approximately equal to or slightly larger than the *Frensel distance* for the **monolithic** model, but larger for the **compact cluster** model of the same size (**Figures 2, 3, 4).**

- In contrast to the **compact cluster** model, the travel-time map of the **open cluster** of both configurations does not show much similarity to that of the **monolithic** equivalent model. In fact, this **open model** is already in the scattering regime at low frequencies *f* -mode waves, which allows more contribution of individual tubes in terms of caustics to the travel-time map. A significant amount of caustic crossing is observed behind this model in the y = 0 region, especially for high frequency waves, making the interpretation of the travel-time shift ambiguous (Figure 5).

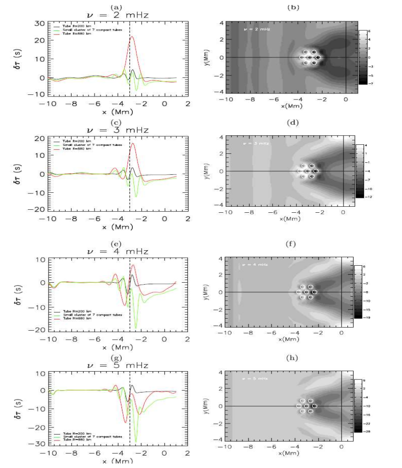

Figure 3: The *left panel* shows the plot of δt (s) as a function of x (Mm), measured at y = 0 for the **small-size compact cluster** model (*green curve*), for the **equivalent monolithic** tube of 880 km radius (*red curve*) and for the **single tube** of 200 km radius (*black curve*). The *right panel* shows the spatial map of the travel-time shift δt (x, y) for **the small compact cluster**. From *top to bottom* the *panels* show δt for the different frequencies.

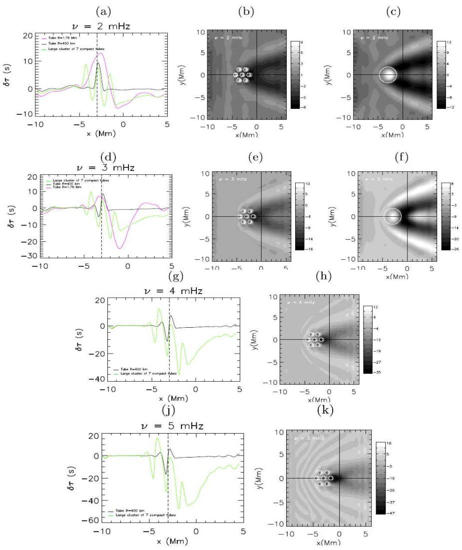

Figure 4: The left panel shows the plot of δt as a function of x (Mm), measured at y = 0 for the **large size compact cluster** (green curve) and its equivalent **monolithic** tube of 1.76 Mm radius (magenta curve), as well as for the **single** 400 km radius tube (black curve). The two right panels show δt maps of both **the large size compact cluster** and **the equivalent monolithic** tube of R =1.76 Mm, respectively. From top to bottom the panels show δt for the different frequencies.

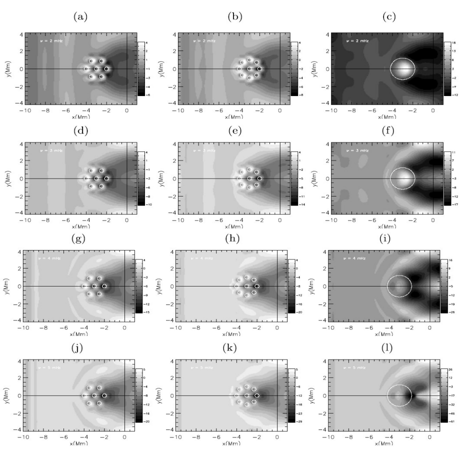

Figure 5: Spatial map of the travel-time shift δt. In the horizontal direction, the maps show the travel-time for the 7 and the 9 flux tubes **open cluster** models, and for the equivalent **monolithic** tube of 1.2 Mm radius, respectively. From top to bottom, the panels show δt maps for different frequencies from 2 mHz to 5 mHz, respectively.

References

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[3] Daiffallah et al. 2011, *Solar Phys.* **268**, 309.

[4] Cally and Bogdan, 1997, *Astrophys. J.* **486**, L67.