Global solar coronal models driven with Alfvén and kink waves

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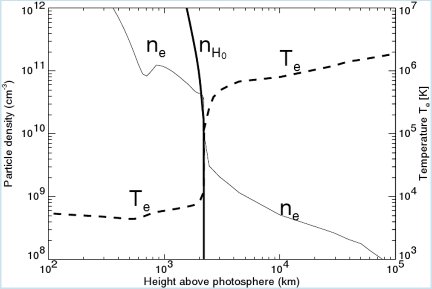
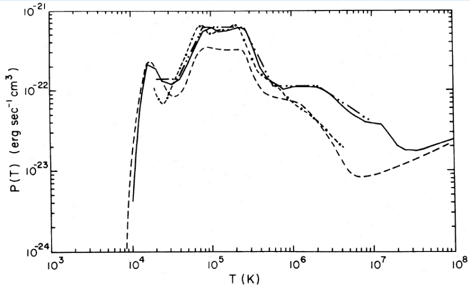
Coronal heating



Aschwanden (2004)



Rosner et al. (1978)

Low density → corona is optically thin: energy losses as $n^2 P(t)$

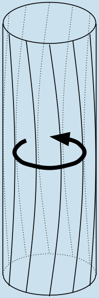
$T = 10^6 K, n = 10^{15}/m^3 \rightarrow$ energy loss $\approx 10^{-5} J/m^3 s$
internal energy $\approx 2\ 10^{-2} J/m^3$

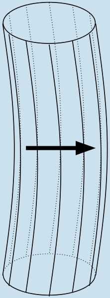
Time scale for cooling $\approx 2000s = 33min$

Need quick heat replenishing! → coronal heating problem

Alfvén vs. kink

VD et al. (2008): kink and Alfvén waves in coronal loops



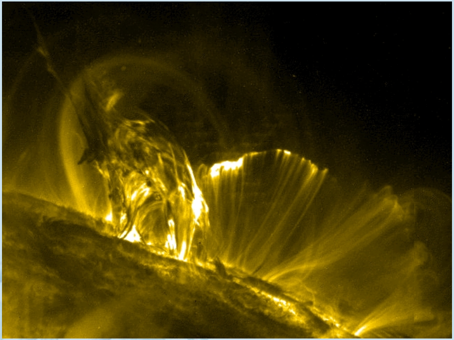


- incompressible
- transverse to $\vec{B}$
- torsional
- $V_A^2 = \dfrac{B^2}{\mu\rho}$
- homogeneous OK

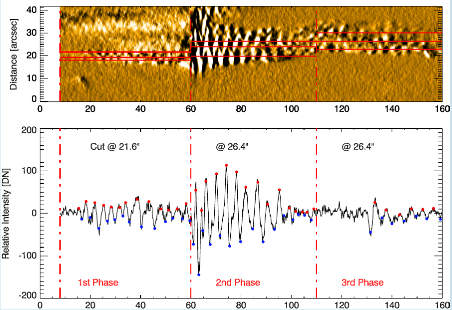
- incompressible
- transverse to $\vec{B}$
- kinking
- $V_k^2 = \dfrac{2B^2}{\mu(\rho_i + \rho_e)}$
- density (or $\vec{V}$ or $\vec{B}$) structure essential

Kink waves in loops

Decaying:
- excited after flare
- strong damping

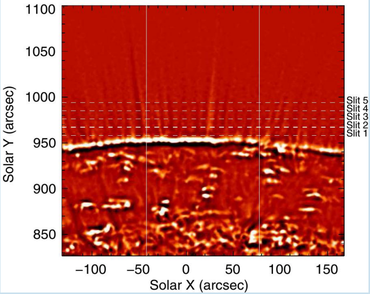


Decayless:
- no apparent excitation
- no apparent damping

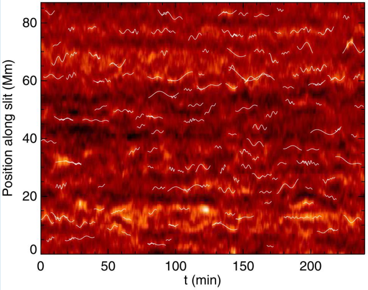


Nisticò et al. (2013)

Kink and Alfvén waves in coronal plumes

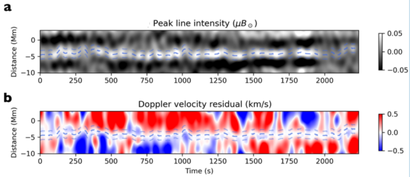


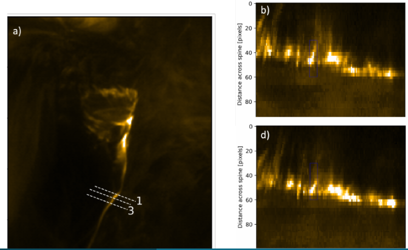
Thurgood et al. (2014) ↑



↓ Petrova et al. (2024): SolO

↓ Morton et al. (2025, subm.): DKIST



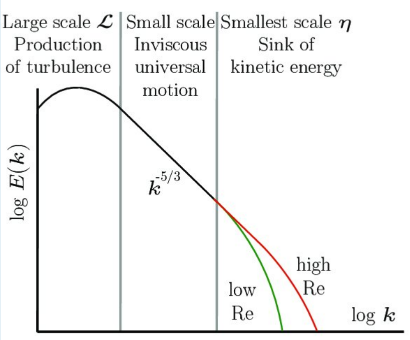


Alfvén turbulence

Propagation of Alfvén waves with Elsässer variables $\vec{z}^{\pm} = \vec{v} \pm \frac{\vec{B}'}{\sqrt{\mu\rho}}$.

Governing equations (incompressible MHD, no P'_{T} for Alfvén, $\vec{V}_0 = 0$):

$$\frac{\partial \vec{z}^{\pm}}{\partial t} \mp \vec{V}_A \cdot \nabla \vec{z}^{\pm} = -\vec{z}^{\mp} \cdot \nabla \vec{z}^{\pm}$$



Stein (2013)

AWSOM

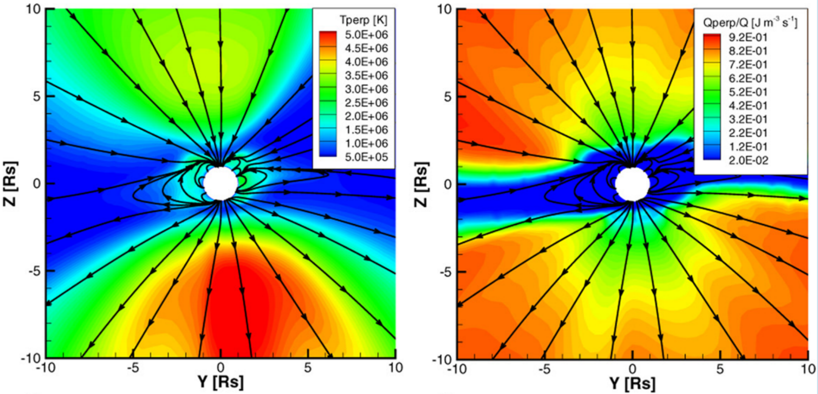
Van der Holst et al. (2014): Alfvén Wave Solar Model

- Starts from MHD/2-fluids
- two extra equations for $w_{\mathrm{A}}^{\pm}$: advection, reflection, cascade
- includes extra force due to Alfvén wave pressure P_{A}
- includes extra heating terms due to cascade

$$E = \frac{p}{\gamma - 1} + \rho\frac{v^2}{2} + \frac{B^2}{2\mu} + W_{\mathrm{A}}^{+} + W_{\mathrm{A}}^{-}$$

$$\frac{\partial W_{\mathrm{A}}^{\pm}}{\partial t} + \nabla\cdot((\vec{V} \mp \vec{V}_{\mathrm{A}})W_{\mathrm{A}}^{\pm}) + \frac{W_{\mathrm{A}}^{\pm}}{2}\nabla\cdot\vec{V} = -\frac{1}{L_\perp}\frac{1}{\sqrt{\rho}}\sqrt{W_{\mathrm{A}}^{\mp}}(W_{\mathrm{A}}^{\pm}) \pm \mathcal{R}\sqrt{W_{\mathrm{A}}^{+}W_{\mathrm{A}}^{-}}$$

AWSOM

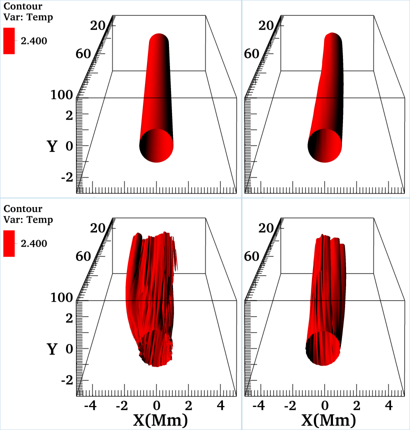


Van der Holst (private communication): problems in open-field regions, not enough driving of wind

Cooper Downs (private communication): extra heating term needed in low corona to match observations

Turbulence in transverse kink waves

Karampelas et al. (2019)



Non-linear damping of kink waves: power law and heating

How to add kink heating to AWSOM?

Q-variables

Van Doorsselaere et al. (2024): Introduce new Q-variables

$$\vec{Q}^{\pm} = \vec{V} \pm \alpha \vec{B},$$

($\vec{V}$ is velocity, $\vec{B}$ is magnetic field, α is parameter)

→ extension to Elsässer variables

$(\vec{Z} = \vec{V} \pm \vec{B}/\sqrt{\mu\rho}$, i.e. $\alpha = 1/\sqrt{\mu\rho})$

→ $\alpha\vec{B}$ is phase speed of wave

$$\frac{D^{\pm}}{Dt} = \frac{\partial}{\partial t} + \vec{Q}_0^{\pm} \cdot \nabla = \frac{\partial}{\partial t} + (\vec{V}_0 \pm \alpha \vec{B}_0) \cdot \nabla$$

Allows to track other waves than Alfvén waves

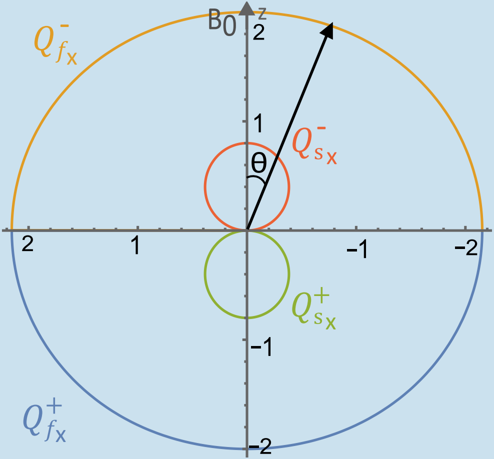
e.g. kink waves in inhomogeneous plumes

Split between propagating up or down

→ Generalisation of: characteristics, Riemann invariants, Frieman-Rotenberg, Elsässer variables

Q-variables for slow and fast waves

Numerically compute Q for slow and fast waves



Q split between propagation directions!

Induction equation

Linearised induction equation:

$$\frac{\partial \vec{B}'}{\partial t} = \nabla \times \left((\vec{V} + \vec{v}) \times \vec{B} \right)$$

Take $\vec{B} = B(x,y)\vec{e}_z$, $\vec{V} = V(x,y)\vec{e}_z$, Fourier analyse in z and t:

$$\frac{\partial \vec{B}'_\perp}{\partial t} = B(x,y)\frac{\partial \vec{v}_\perp}{\partial z} - V(x,y)\frac{\partial \vec{B}'_\perp}{\partial z}, \qquad -i\omega \vec{B}'_\perp = ik_z B(x,y)\vec{v}_\perp - ik_z V \vec{B}'_\perp$$

Rearrange using $\vec{v} = \frac{1}{2}(\delta\vec{Q}^+ + \delta\vec{Q}^-)$ and $\vec{B}' = \frac{1}{2\alpha}(\delta\vec{Q}^+ - \delta\vec{Q}^-)$:

$$(\omega - k_z V + \alpha k_z B)\delta\vec{Q}^+_\perp = (\omega - k_z V - \alpha k_z B)\delta\vec{Q}^-_\perp$$

Choice of α splits waves between directions. For kink waves:

$$V_\mathrm{k}^2 = \frac{2B^2}{\mu(\rho_i + \rho_e)}, \text{ so } \alpha = \sqrt{\frac{2}{\mu(\rho_i + \rho_e)}}$$

UAWSOM

Van Doorsselaere et al. (2025):
UAWSOM: Uniturbulence and AWSOM
$\rightarrow$ make equation for kink wave heating

- Linearise Q-MHD equations (VD et al. 2024)
- incompressible $\delta\rho = 0$
- only perp. components $\delta\vec{Q}_\perp^\pm$
- Scalar multiplication with $\rho_0\delta\vec{Q}_\perp^\pm/2$

$\rightarrow$ energy density kink $W_\mathrm{k}^\pm = \frac{\rho_0}{4}\langle(Q^\pm)^2\rangle$

- Average over cross-section
  $\rightarrow$ Remove small scales

$$\frac{\partial W_\mathrm{k}^\pm}{\partial t} + \nabla\cdot(\vec{Q}_\mathrm{k}^\mp W_\mathrm{k}^\pm) + \frac{W_\mathrm{k}^\pm}{2}\nabla\cdot\vec{V} = -\frac{1}{L_{\perp,\mathrm{VD}}}\frac{1}{\sqrt{\rho_\mathrm{e}}}(W_\mathrm{k}^\pm)^{3/2}$$

Perpendicular length $L_{\perp,\mathrm{VD}}$ scales with R, ζ and filling factor f

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Compare with Alfvén wave:

$$\mathrm{RHS}_{\text{Alfvén}} = -\frac{1}{L_{\perp,\mathrm{A}}}\sqrt{\frac{W_\mathrm{A}^\mp}{\rho_0}}W_\mathrm{A}^\pm$$

$$\frac{\partial W_\mathrm{k}^\pm}{\partial t} + \nabla\cdot(\vec{Q}_\mathrm{k}^\mp W_\mathrm{k}^\pm) + \frac{W_\mathrm{k}^\pm}{2}\nabla\cdot\vec{V} = -\frac{1}{L_{\perp,\mathrm{VD}}}\frac{1}{\sqrt{\rho_\mathrm{e}}}(W_\mathrm{k}^\pm)^{3/2}$$

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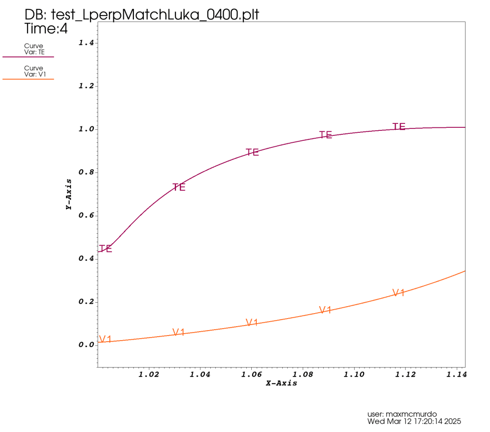
UAWSOM in MPI-AMRVAC

McMurdo et al. (2025, poster):



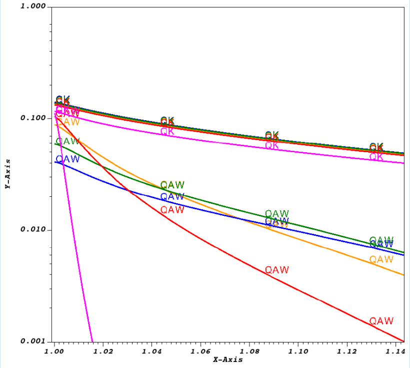
Kink wave heated equilibrium. No background heating needed.

UAWSOM in 1D

McMurdo et al. (2025, poster):

compare Alfvén and kink heating (in erg cm^{-3} s^{-1})

Equal energy in W_A and W_k



Kink wave heating larger than Alfvén wave heating

$$W_A^- \leq W_A^+ < W_k^+$$

Conclusions

- Solar coronal heating problem
- Potential wave heating contributions?
- Coronal structures $\rightarrow$ kink waves
- New wave tracking formalism: Q-variables
- UAWSOM: kink wave heating model
- kink waves heat more than Alfvén waves