“Interdisciplinary Physics of the Sun”
Spanish-German WE-Heraeus-Seminar
Bad Honnef, July 2025

Standard Solar Model and variations with energy-loss kernels

Yago Herrera
Aldo Serenelli

Special thanks to F. Villante

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Solar interior structure modelling

(a simple overview)

Structure and evolution equations

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\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\mathrm{d}P}{\mathrm{d}m} &= -\frac{Gm}{4\pi r^4} \\
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\frac{\mathrm{d}T}{\mathrm{d}m} &= \begin{cases} -\dfrac{3\kappa}{4a\tilde{c}T^3}\dfrac{L}{16\pi^2 r^4} \\ \dfrac{\Gamma_2 - 1}{\Gamma_2}\dfrac{T}{P}\dfrac{\mathrm{d}P}{\mathrm{d}m} \end{cases} \\
\frac{\mathrm{d}X}{\mathrm{d}t} &= r_X \,.
\end{aligned}
$$

Solar boundary conditions

At $m = M_\odot, t = \tau_\odot$
$L = L_\odot, r = R_\odot, T = T_\odot^{\mathrm{eff}}, \ldots$

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Constant parameters

- Elemental abundances (X_i)
  - MB22 $\rightarrow$ $\mathrm{Z/X} = 0.02249$
  - AAG21 $\rightarrow$ $\mathrm{Z/X} = 0.01866$
- Nuc. reaction rates $\rightarrow$ S-factors

$$S_{i,j}(0),\quad S'_{i,j}(0),\quad S''_{i,j}(0)$$

Physical quantities relations

- EoS $\rightarrow$ $\rho(P,T,\ldots)$
- **Opacity** $\rightarrow$ $\boldsymbol{\kappa_R(\rho,T,X_i)}$

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Solar structure and observables

- Radial profiles ✔
- Sound speed profile
  $\leftrightarrow$ Helioseismology
- Neutrino outflows
  pp, pep, hep, ^{7}Be, ^{8}B, ...
- Calibrated parameters
  α_{MLT}, $\mathrm{R_{bcz}}$, $\mathrm{Y_{ini}}$, ...

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Hard-coded in Fortran
→ recompile each time

Few hours per thread
(GARching STellar Evolution Code)

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Sun as lab for new physics?

- Exotic particles → energy-loss

$$\varepsilon \rightarrow \varepsilon - \delta\varepsilon$$

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! $\rightarrow$? **Uncertainties**

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Standard Solar Model variations

Input parameters (const.)	Solve equations	Output (parameters or functions)
$(p_1, \ldots, p_n)$	$\longrightarrow$	$Q_{j[p_1,\ldots,p_n]}(x); \quad j = 1, \ldots, N; \quad x = r/R_\odot$

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Linear approximation: Small enough $\Delta p_i \implies \Delta Q_j(x) \simeq \sum_{i=1}^{n} \left.\frac{\partial Q_j(x)}{\partial p_i}\right|_{[p1,\ldots,p_n]} \Delta p_i + \mathcal{O}(\Delta^2)$

Much simpler linear system

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Much simpler linear system

Small Δp_i one at a time (numerical approach)* $\xrightarrow[n \text{ times}]{}$ $\left.\frac{\partial Q_j(x)}{\partial p_i}\right|_{[p1,\ldots,p_n]} \simeq \frac{\Delta Q_j(x)}{\Delta p_i}$

(*) See F. Villante's talk for the analytical approach (LSM)

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SSM theoretical uncertainties

$$p_i = \bar{p}_i \pm \sigma_{p_i} \qquad ! \longrightarrow ? \qquad Q_j = \bar{Q}_j \pm \sigma_{Q_j}$$

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$$p_i = \bar{p}_i \pm \sigma_{p_i}$$

! → ?

$$Q_j = \bar{Q}_j \pm \sigma_{Q_j}$$

n-dim normal distribution of samples (M-C)

$$(p_1^{[k]}, \dots, p_n^{[k]});$$

$$k = 1, \dots, 10^5$$

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n-dim normal distribution of samples (M-C)

$$(p_1^{[k]}, \dots, p_n^{[k]}); \quad k = 1, \dots, 10^5 \longrightarrow \Delta Q_j(x)^{[k]} = \sum_{i=1}^{n} \frac{\partial Q_j(x)}{\partial p_i} \Delta p_i^{[k]}$$

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Standard deviation

$$Q_j = \bar{Q}_j \pm \sigma_{Q_j} \qquad \mathrm{Corr}\,\langle Q_j, Q_{j'} \rangle$$

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! → **?**

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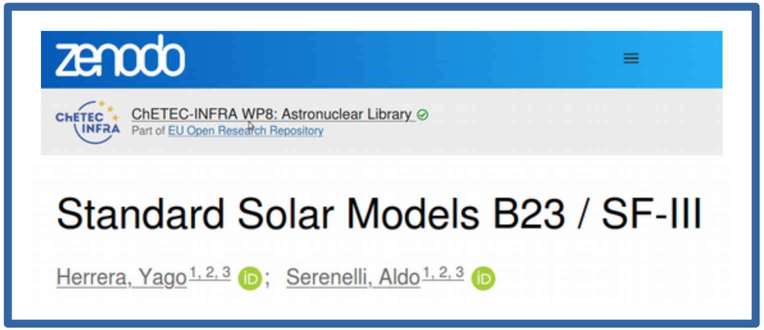
Input uncertainties (SF-III)

- Nuclear S-factors: $S_{11}, S_{33}, S_{34}, S_{e7}, S_{17}, S_{hep}, S_{114}, S_{116}$
- Abundances: C, N, O, Ne, Mg, Si, S, Ar, Fe,
- Other parameter: τ, diff. coef., L, κ_A, κ_B

Neutrino flux prediction uncertainties

	$\Phi[\nu\ \mathrm{s}^{-1}\mathrm{cm}^{-2}]$ (MB22)	$\Phi[\nu\ \mathrm{s}^{-1}\mathrm{cm}^{-2}]$ (AAG21)
pp	$5.950 \cdot 10^{10}\ (1 \pm 6.408 \cdot 10^{-03})$	$6.000 \cdot 10^{10}\ (1 \pm 5.644 \cdot 10^{-03})$
pep	$1.422 \cdot 10^{08}\ (1 \pm 1.312 \cdot 10^{-02})$	$1.447 \cdot 10^{08}\ (1 \pm 1.089 \cdot 10^{-02})$
hep	$7.934 \cdot 10^{03}\ (1 \pm 3.047 \cdot 10^{-01})$	$8.161 \cdot 10^{03}\ (1 \pm 3.051 \cdot 10^{-01})$
7Be	$4.875 \cdot 10^{09}\ (1 \pm 8.128 \cdot 10^{-02})$	$4.517 \cdot 10^{09}\ (1 \pm 7.346 \cdot 10^{-02})$
8B	$5.074 \cdot 10^{06}\ (1 \pm 1.456 \cdot 10^{-01})$	$4.309 \cdot 10^{06}\ (1 \pm 1.254 \cdot 10^{-01})$
13N	$3.102 \cdot 10^{08}\ (1 \pm 1.509 \cdot 10^{-01})$	$2.221 \cdot 10^{08}\ (1 \pm 1.315 \cdot 10^{-01})$
15O	$2.304 \cdot 10^{08}\ (1 \pm 1.795 \cdot 10^{-01})$	$1.577 \cdot 10^{08}\ (1 \pm 1.586 \cdot 10^{-01})$
17F	$4.699 \cdot 10^{06}\ (1 \pm 1.686 \cdot 10^{-01})$	$3.401 \cdot 10^{06}\ (1 \pm 1.573 \cdot 10^{-01})$

Barcelona 2023 SSM



DOI:10.5281/zenodo.10822316

Input data

- S-factors:
  Solar Fusion III
- Solar abundances:
  GS98, AGSS09, C11, AAG21, MB22...

Output data

- Radial profiles (physical, chemical, neutrino, etc)
- Neutrino outflows at 1AU + uncert + corr
- Partial derivatives (power laws)

"GS98" :: Grevesse & Sauval (1998), Space Sci. Rev., 85, 161.
"AGSS09" :: Asplund et al. (2009), ARA&A, 47, 481.
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"MB22p" :: Magg et al. (2022), A&A 661, A140. (Photospheric)

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Barcelona 2023 SSM + parameter variation routine

zenodo

ChETEC-INFRA WP8: Astronuclear Library
Part of EU Open Research Repository

Standard Solar Models B23 / SF-III

Herrera, Yago[1,2,3]; Serenelli, Aldo[1,2,3]

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Δp_i [dex] →

output Φ_ν

~ 2 s →

```
Input variations (in dex) for nuclear reactions S-factors,
elements abundances, and other solar parameters:
(Edit these values in the script INPUT section)

 S11   S33   S34   Se7   S17  Shep  S114  S116   Age  Diff  Lumi OpacA OpacB
 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
   C     N     O    Ne    Mg    Si     S    Ar    Fe
 0.0   0.1   0.1   0.0   0.0   0.0   0.0   0.0   0.0

Computing output variations...

 Results written to: RESULTS/nu_fluxes_new.dat
 Results written to: RESULTS/prof_Csound_new.dat
 Results written to: RESULTS/var_Csound_new.dat
 Results written to: RESULTS/deltas_Csound_new.dat
 Plots saved to: RESULTS/Plots_Csound_new.pdf

Neutrino generation rates, surface He, and convective zone base radius
           GS98        AGSS09           C11         AAG21         MB22m         MB22p
pp  5.95177e+10 6.00170e+10 5.98173e+10 5.99172e+10 5.94179e+10 5.94179e+10
pep 1.42074e+08 1.44800e+08 1.43735e+08 1.44334e+08 1.41840e+08 1.41840e+08
hep 7.90221e+03 8.17415e+03 8.05369e+03 8.11598e+03 7.87795e+03 7.89001e+03
be7 4.92039e+09 4.48958e+09 4.66381e+09 4.57668e+09 4.96372e+09 4.93940e+09
b8  5.18345e+06 4.25678e+06 4.61589e+06 4.43553e+06 5.27796e+06 5.22332e+06
n13 2.94498e+08 2.14920e+08 2.61127e+08 2.33856e+08 3.28304e+08 3.26619e+08
o15 2.22147e+08 1.54906e+08 1.90477e+08 1.69134e+08 2.49036e+08 2.47106e+08
f17 6.92202e+06 4.24153e+06 5.34028e+06 4.39166e+06 6.11717e+06 6.06683e+06
ys  2.43943e-01 2.33503e-01 2.37925e-01 2.35757e-01 2.45446e-01 2.44818e-01
rcz 7.06758e-01 7.16665e-01 7.10821e-01 7.14221e-01 7.06814e-01 7.07119e-01

real    0m2.445s
user    0m1.324s
sys     0m0.024s
```

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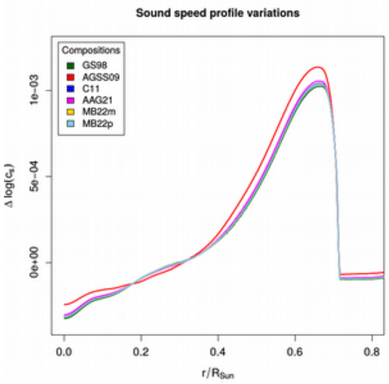

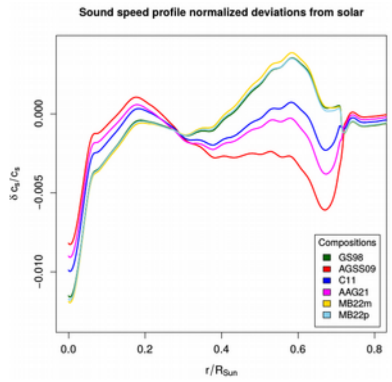

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SSM function variations

Sun as lab for new physics?

Exotic particles $\rightarrow$ extra energy-loss
(axions, hidden photons, etc)

Discrete set of parameters $\longleftrightarrow$ **Function of continuous variable(s)**

$\{\tilde{p}_i = p_i + \Delta p_i\}_{i=1,\dots,n}$

$\tilde{\varepsilon} = \varepsilon - \delta\varepsilon$ $\quad$ $\delta\varepsilon := \lambda f(Q_j(x))$ $\quad$ "small" λ

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Partial derivatives ⟷ **Functional derivatives (Kernels)**

$$\Delta Q_j(x) \simeq \sum_{i=1}^{n} \frac{\partial Q_j(x)}{\partial p_i}\bigg|_{[p1,\ldots,p_n]} \Delta p_i$$

$$\Delta Q(x) \simeq \int_D \frac{\delta Q(x)}{\delta\varepsilon(x')}\, \delta\varepsilon(x') + \mathcal{O}(\lambda^2)$$

$$\Delta Q(x) \simeq \int_D K_Q(x,x')\ f(x')\ dx'$$

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$\{\tilde{p}_i = p_i + \Delta p_i\}_{i=1,\dots,n}$		$\tilde{\varepsilon} = \varepsilon - \delta\varepsilon \qquad \delta\varepsilon := \lambda f(Q_j(x))$ "small" λ
Partial derivatives	$\longleftrightarrow$	**Functional derivatives (Kernels)**
$\Delta Q_j(x) \simeq \sum_{i=1}^{n} \left.\frac{\partial Q_j(x)}{\partial p_i}\right\vert_{[p1,\dots,p_n]} \Delta p_i$		$\Delta Q(x) \simeq \int_D \frac{\delta Q(x)}{\delta\varepsilon(x')}\, \delta\varepsilon(x') + \mathcal{O}(\lambda^2)$ $\Delta Q(x) \simeq \int_D K_Q(x,x')\; f(x')\; dx'$
PD calculation	$\longleftrightarrow$	**Kernel calculation**
Small Δp_i one at a time		Localized perturbation: $f(x') = C_o\delta(x'-x_o) \simeq C_o g_\sigma(x'-x_o)$ $\quad \sigma << 1$
$\left.\frac{\partial Q_j(x)}{\partial p_i}\right\vert_{[p1,\dots,p_n]} \simeq \frac{\Delta Q_j(x)}{\Delta p_i}$		$K_Q(x, x_o^{[k]}) \simeq C_o^{-1}\Delta Q(x)^{[k]}$ $k = 1,\dots,n \sim \sigma^{-1}$ Careful with borders! $\left\vert x_o - \begin{matrix}0\\1\end{matrix}\right\vert \sim \sigma$

SSM energy-loss Kernels

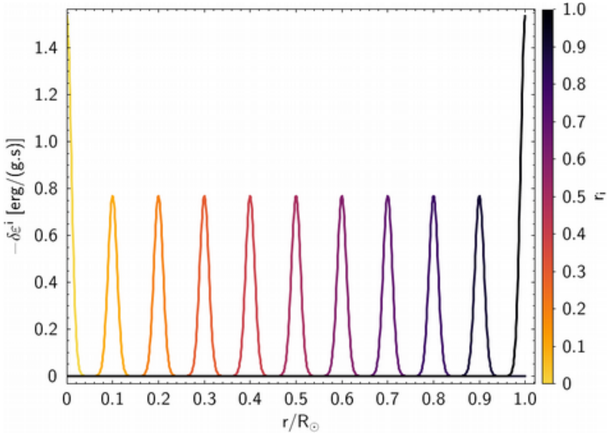
Localized perturbations

$$\delta\varepsilon := \lambda f(Q_j(x))$$

$$f(x') = C_o\, g_\sigma(x' - x_o), \quad \sigma = 0.01$$

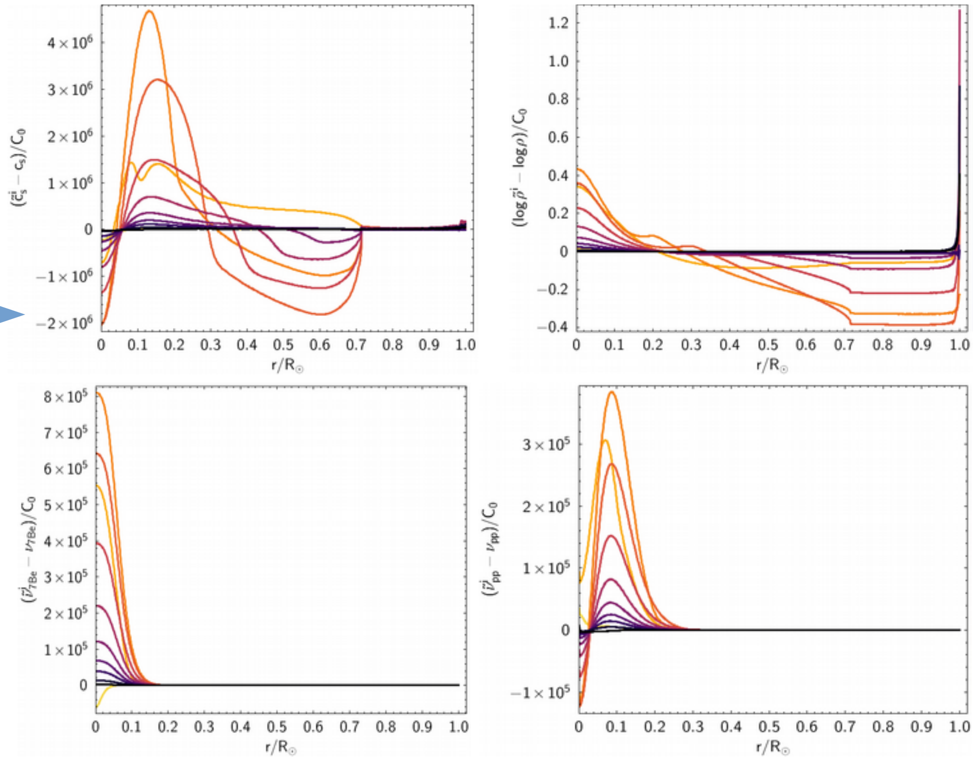
$$n = 100 + 10 \text{ (border correction)}$$

$$C_o = 0.01 \frac{L_\odot}{M_\odot} \implies \lambda < 0.04$$



n times

Kernels



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SSM + γ-Axions energy-loss with Kernels method

γ - Axions energy-loss

$$\boxed{\epsilon_{a\gamma} = \frac{g_{a\gamma}^2}{4\pi}\frac{T^7}{\rho}F(\kappa^2),}$$

$$F(\kappa^2) = 1.842(\kappa^2/12)^{0.31}. \qquad \kappa^2 = \pi\alpha\frac{n_B}{T^3}\left(\mathrm{Y_e} + \sum_j \mathcal{Z}_j^2\,\mathrm{Y}_j\right)$$

in theory: $\Delta Q(x) \simeq \int_D K_Q(x,x')\ \epsilon_{a\gamma}(x')\ dx'$

Numerically $\rightarrow$ discrete sum + iteration [i]

$$\Delta Q^{[i]}(x) \simeq \sum_{k=1}^{n} K_Q(x,x_k)\ \epsilon_{a\gamma}\big(Q^{[i-1]}(x_k)\big)\ \Delta x_k$$

$$Q^{[i]} = Q^{[i-1]} + \Delta Q^{[i]}, \qquad Q^{[0]} = Q^{\mathrm{SSM}}$$

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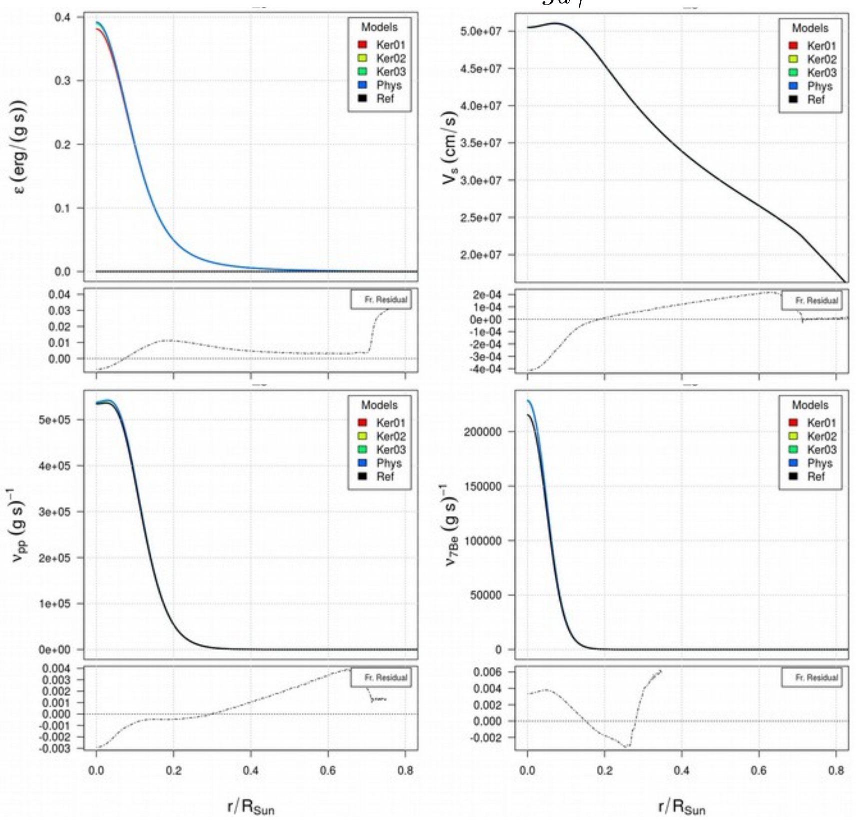
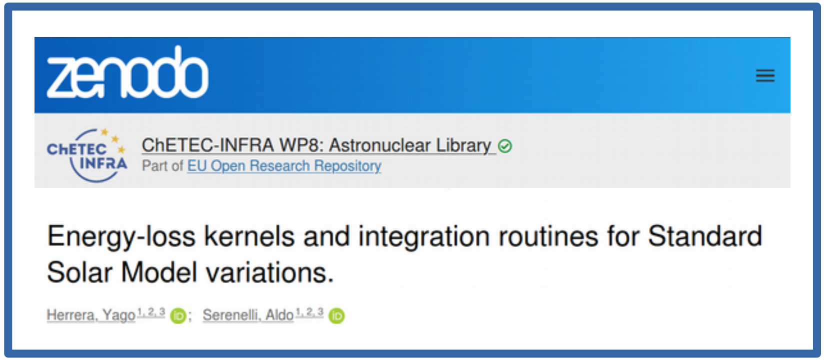
$g_{a\gamma} = 4.0 \cdot 10^{-10}$ eV



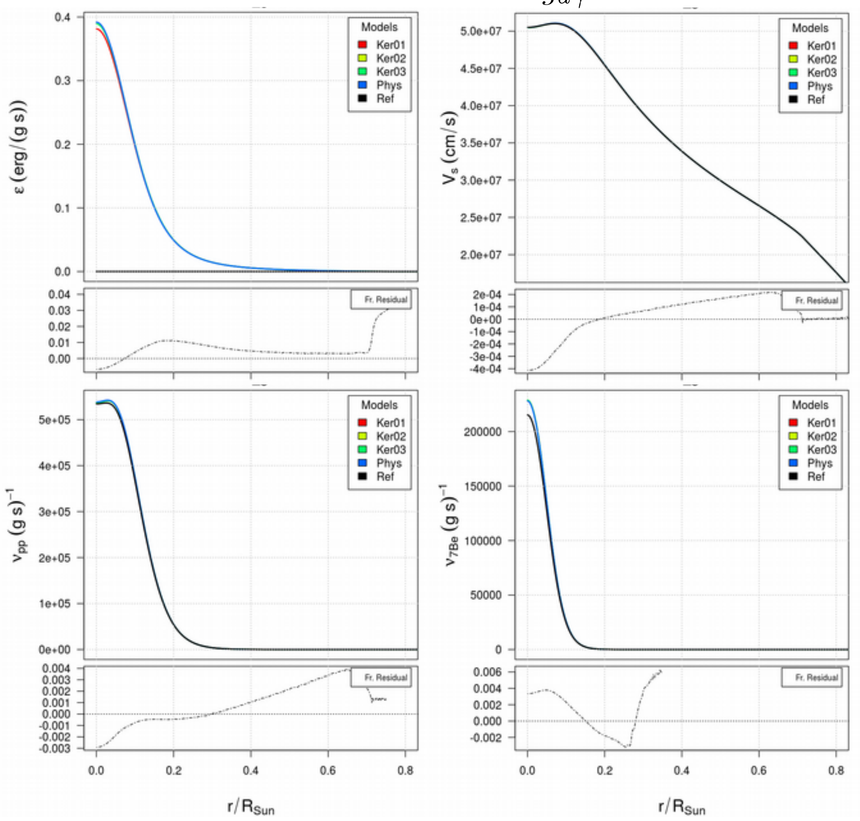
SSM energy-loss Kernels + integration routine (beta)



DOI:10.5281/zenodo.15624062

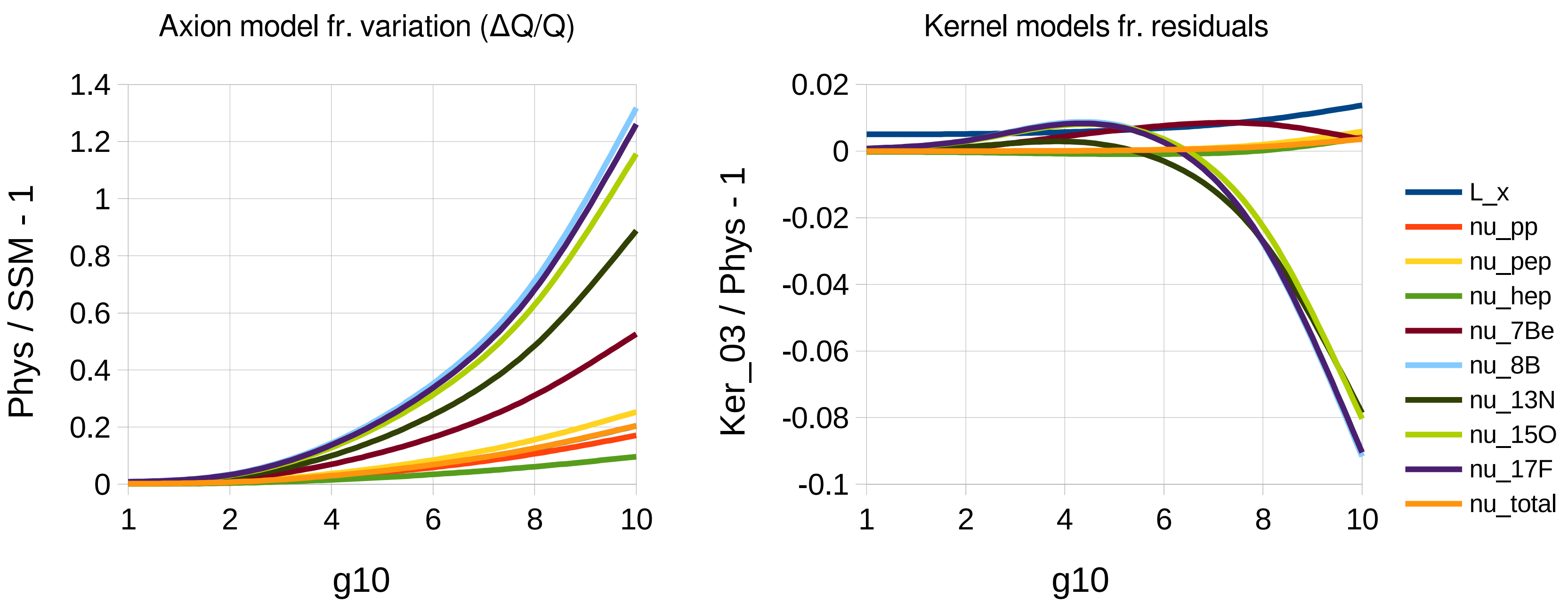
- **Scripts:**
  - Iterative Kernel * E-loss integration, ~ 10 sec per iteration,
  - Neutrino outflows integration
  - Plotting scripts
- **Model/Kernels data:**
  - B23 SSM_MB22 (expanded for Kernels)
  - Kernels
  - SSM+gAxion (g10=1–10)
  - SSM+hidden photons (hp=1–6)
- **Several examples:**
  Energy-loss functions, input, output, etc

$$g_{a\gamma} = 4.0 \cdot 10^{-10}\ \mathrm{eV}$$



SSM + γ-Axions with Kernels method

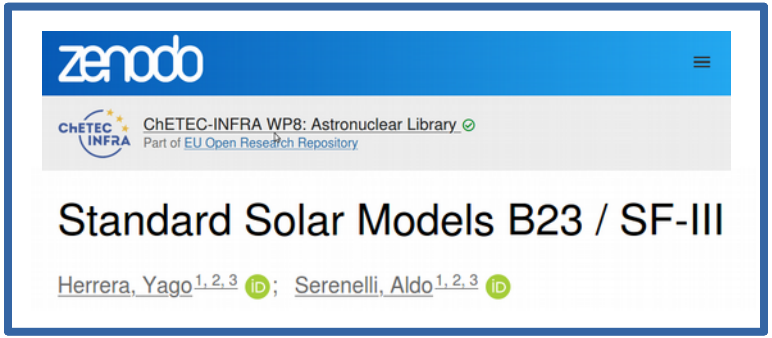
Integrated energy-loss and neutrino outflows



Y.Herrera, F. Villante, A. Serenelli, M. Maltoni?, C. Sanchez-García? (article in preparation)

EXCELENCIA MARÍA DE MAEZTU

Thank you!



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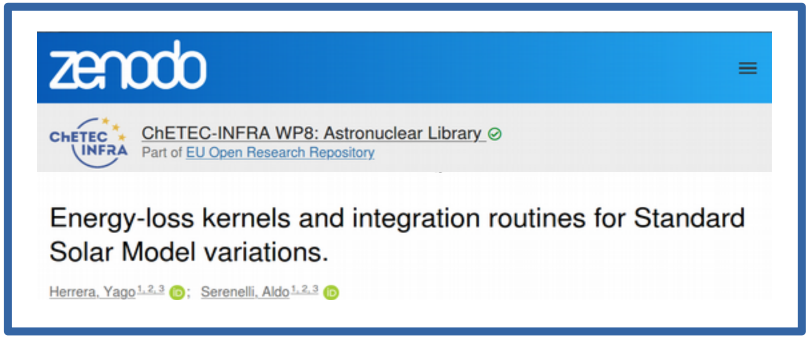

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I'm for hire!!

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