Solar neutrino flux fluctuations caused by g modes

Interdisciplinary Physics of the Sun (4th July, 2025)

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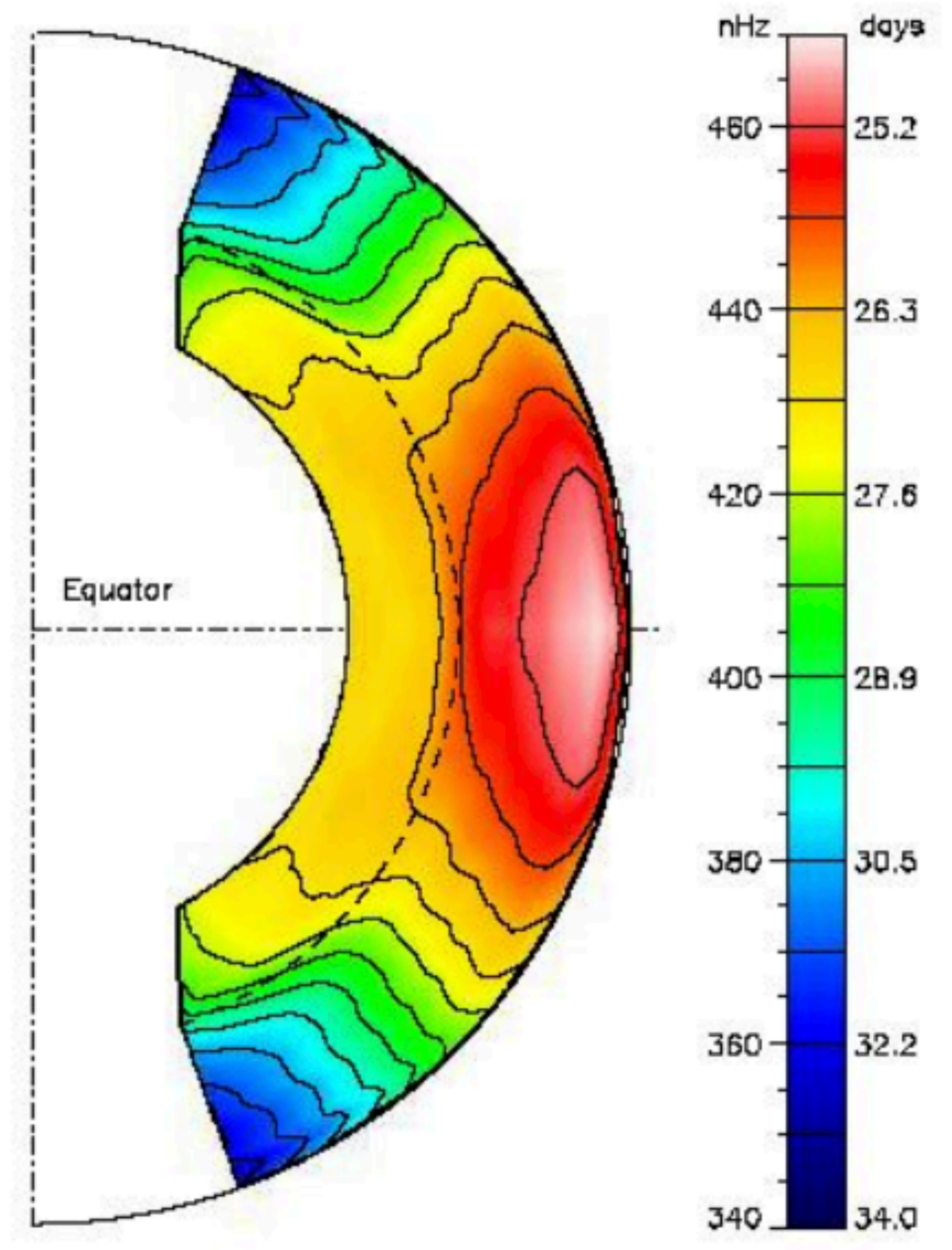
Helioseismology: the study of solar oscillations to probe its interior

	restoring force	frequency (period)	sensitive to…	Can we detect it?
p mode	pressure	~ 3000 micro Hz (~ 5 min.)	outer envelope (conv. + upper rad.)	Yes (Leighton+1962)
f mode	buoyancy	~ 1500 micro Hz (~ 10 min.)	outer envelope	Yes (Leighton+1962)
inertial mode	Coriolis force	~ 0.1 micro Hz (~ months)	outer envelope (conv. + upper rad.)	Yes (Löptien+2018, Gizon+2021)
g mode	buoyancy	< 100 micro Hz	central region	No, so far…

Ex. 2-d rotational profile of the Sun



(Thompson+1996)

(Talks by Rachel, Aldo, and Gaël)

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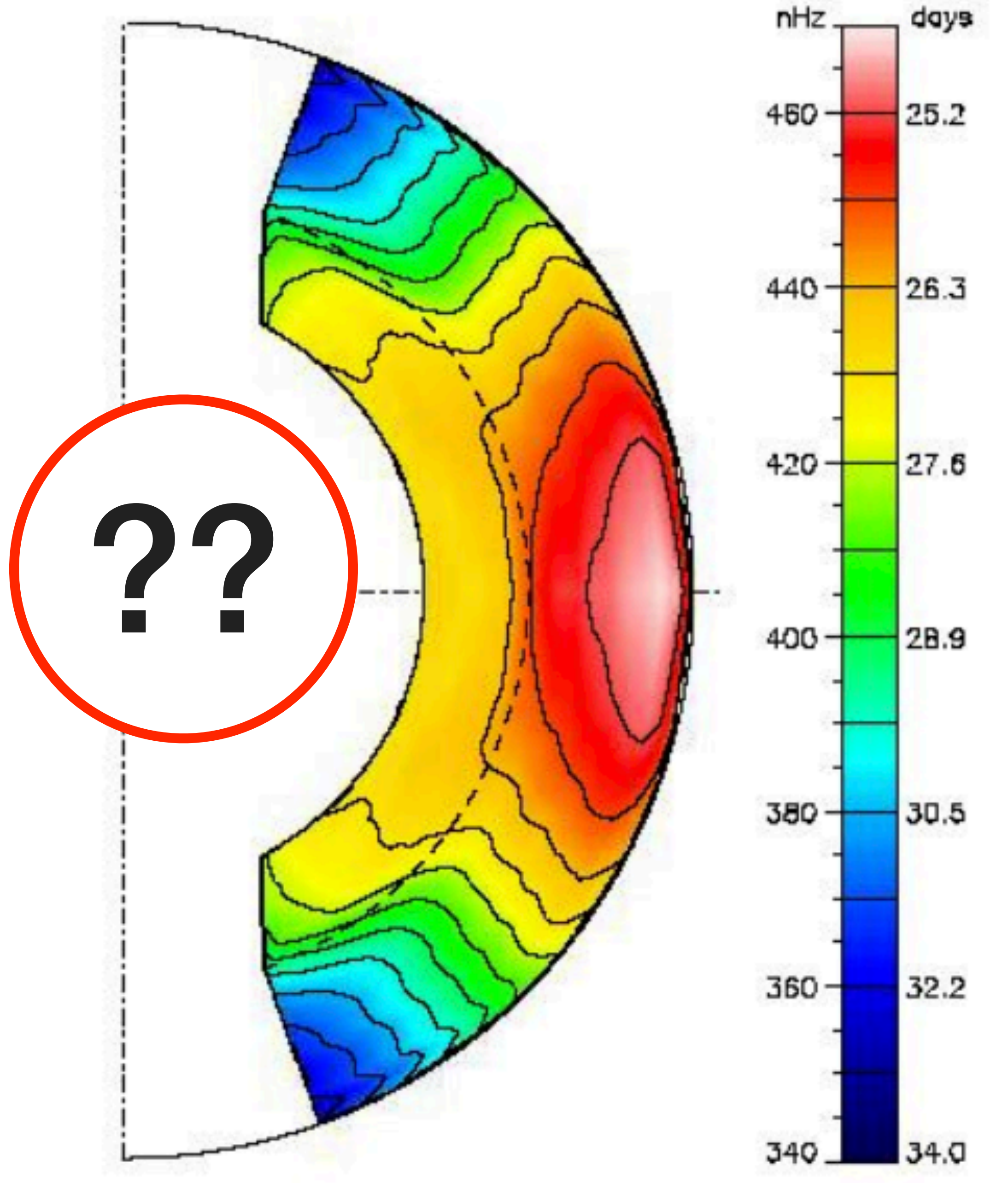

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Difficulty in solar g-mode detection

- There have been many attempts of solar g-mode detection, but …
  (see a review by Appourchaux+2010)

- The main difficulty is that amplitudes of solar g modes are very small at the surface

    - e.g., a few mm/s in line-of-sight velocity (vlos)

  * Actually, it is theoretically difficult to estimate solar g-mode amplitudes (Belkacem+2022, Pincon+2021)

- We need a few more decades observations (in vlos) to achieve signal-to-noise ratio required for the firm detection of solar g modes (Appourchaux+2010) (→ how about solar neutrinos…??)

(Discussions on solar g-mode detection by LISA)

Why solar neutrino for solar g-mode detection?

- The idea is:
  - solar g modes perturb T and ρ in the central region
  - δT and $\delta\rho$ cause the perturbation in the nuclear reaction rate $\delta\varepsilon$
  - $\delta\varepsilon$ is related to the neutrino flux density fluctuation $\delta\phi$

  (e.g. Gough 1991, Kumar and Bahcall 1992 in the context of solar neutrino problem)

- Lopes and Turck-Chièze (2014) evaluated flux fluctuations caused by g modes (in the case of $^{8}\mathrm{B}$ neutrinos), putting some constraints on solar g-mode amplitudes
  - How about other types of neutrinos, such as $^{7}\mathrm{Be}$, $^{13}\mathrm{N}$, $^{15}\mathrm{O}$, and $^{17}\mathrm{F}$ neutrinos? (suggested by a Kamiokande experimentalist, Yuuki Nakano)

(Talks by Michael and Francesco)

Goals

- To evaluate neutrino flux fluctuations caused by solar g modes (for various neutrinos such as $^{7}\mathrm{Be}$, $^{13}\mathrm{N}$, $^{15}\mathrm{O}$, and $^{17}\mathrm{F}$ neutrinos)

- Can we detect solar g modes via solar neutrino flux measurements by neutrino detectors?
(Actually, the answer is not positive one…)

Assumptions in our formulation

- Linear adiabatic oscillations:

$$\delta \ln \rho = (\Gamma_3 - 1)^{-1} \delta \ln T: \qquad \delta\rho \rightleftarrows \delta T$$

- Approximation for nuclear reaction rates (following Lopes & Turck-Chièze 2014):

$$\varepsilon \propto \rho^{\beta} T^{\eta}: \qquad \delta\varepsilon \rightleftarrows \delta\rho, \delta T$$

- A linear relation between $\delta \ln \phi$ and $\delta \ln \varepsilon$:

→ $\delta\phi$ is determined by, e.g., δT alone

- Negligence of time-delay effect:
  - All the neutrinos produced at a certain time in the solar interior are assumed to arrive at an observational point at the same time

The first-order perturbation

- The neutrino flux (of the equilibrium state) is:

$$\Phi_0 = \int \phi_0 \rho_0 \mathrm{d}V \quad (\phi_0 = \phi_0(r) \text{ is the neutrino flux density in units of } \mathrm{cm}^{-2}\, s^{-1}\, g^{-1})$$

- The first-order neutrino flux fluctuation is:

$$\Delta\Phi_{\mathrm{osc}} = \int \phi' \rho_0 \mathrm{d}V + \int \phi_0 \rho' \mathrm{d}V$$

- The eigenfunctions may be given as:

$$\phi'(r, \theta, \psi, t) = \sum_{n\ell m} \mathrm{Re}[\phi'_{n\ell}(r) Y_\ell^m(\theta, \psi) e^{-i(\omega_{n\ell m} t + \delta_{n\ell m})}]$$

$$\rho'(r, \theta, \psi, t) = \sum_{n\ell m} \mathrm{Re}[\rho'_{r,n\ell}(r) Y_\ell^m(\theta, \psi) e^{-i(\omega_{n\ell m} t + \delta_{n\ell m})}]$$

The first-order perturbation… is zero!

- The neutrino flux fluctuation is:

$$\Delta\Phi_{\rm osc} = \int \phi' \rho_0 \mathrm{d}V + \int \phi_0 \rho' \mathrm{d}V$$

$$= \sum_{n\ell m} \int_0^{R_\odot} G_{n\ell}(r)\phi_0\rho_0 r^2 \mathrm{d}r \times \int_0^{\pi} P_\ell^m(\cos\theta)\sin\theta \mathrm{d}\theta \times \int_0^{2\pi} \mathrm{Re}[\exp(-i[\omega_{n\ell m}t + \delta_{n\ell m}] - m\psi)]\mathrm{d}\psi$$

$= 0$ (except for $(\ell, m) = (0, 0)$). But no $\ell = 0$ g modes exist.)

$$\phi'(r, \theta, \psi, t) = \sum_{n\ell m} \mathrm{Re}[\phi'_{n\ell}(r) Y_\ell^m(\theta, \psi) e^{-i(\omega_{n\ell m}t + \delta_{n\ell m})}]$$

$$\rho'(r, \theta, \psi, t) = \sum_{n\ell m} \mathrm{Re}[\rho'_{r,n\ell}(r) Y_\ell^m(\theta, \psi) e^{-i(\omega_{n\ell m}t + \delta_{n\ell m})}]$$

What are other possible effects...?

Current assumptions:

- linear adiabatic oscillation
- $\varepsilon \propto \rho^{\beta} T^{\eta}$
- $\delta \ln \phi = \delta \ln \varepsilon = c_1 \delta \ln T$
- Negligence of time-delay effect:
    - All the neutrinos from the solar interior are assumed to arrive at an observational point at the same time

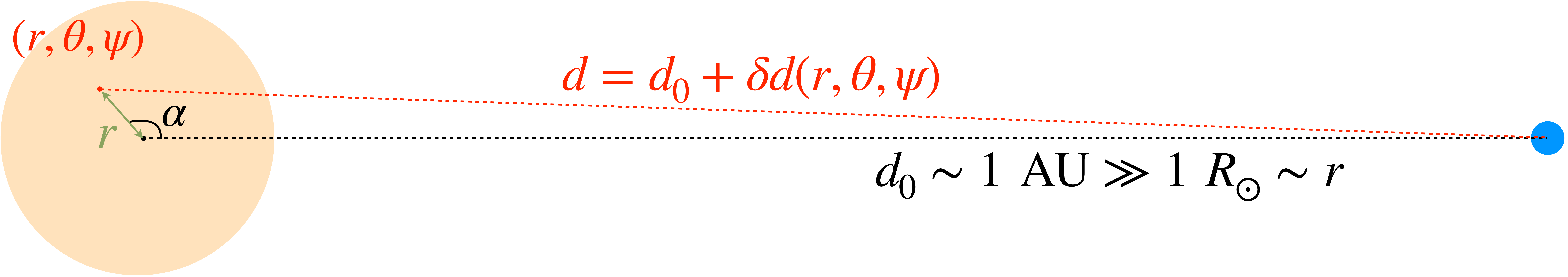
Taking into account the time-delay effects

- The neutrino flux fluctuation is:

$$\Delta\Phi_{\mathrm{osc}}(t) = \int \phi' \rho_0 \mathrm{d}V + \int \phi_0 \rho' \mathrm{d}V$$

(c: speed of light)

$$= \int \phi'(r, \theta, \psi, t - d/c)\rho_0(r)\mathrm{d}V + \int \phi_0(r)\rho'(r, \theta, \psi, t - d/c)\mathrm{d}V$$



$$\delta d \sim -r\cos\alpha = -r\sin\theta\cos\psi$$

Taking into account the time-delay effects

$$\Delta\Phi_{\rm osc}(t) = \frac{2\pi}{\sqrt{3}}\sum_n \frac{\omega_{n11}}{c}\int G_{n1}(r) r \mathrm{d}r \times \sin(\omega_{n11}t + \delta'_{n11}) - \frac{2\pi}{\sqrt{3}}\sum_n \frac{\omega_{n1-1}}{c}\int G_{n1}(r) r \mathrm{d}r \times \sin(\omega_{n1,-1}t + \delta'_{n1,-1})$$

$$\delta d \sim -r\cos\alpha = -r\sin\theta\cos\psi$$

- Only $(\ell, m) = (1, \pm 1)$ g modes can be non-zero
- We can evaluate $G_{n\ell}$ with
  - a reasonable solar model (we used solar models in Kunitomo+2022)
  - eigenfunctions that can be obtained by numerically computing linear adiabatic oscillations of the reference model (we used GYRE; Townsend & Teitler 2013)
  - and the g-mode amplitude parameter $A_{n\ell} = |\delta \ln T|_{\rm max}$ (we assumed $A_{n\ell} = 10^{-5}$)

→ The relative fluctuation in the neutrino flux thus evaluated is $< 10^{-10}$
(for ^{8}B, ^{7}Be, ^{13}N, ^{15}O, and ^{17}F neutrinos)

What are other possible effects...?

Current assumptions:

$$\Delta\Phi_{\rm osc} = \int \phi' \rho_0 {\rm d}V + \int \phi_0 \rho' {\rm d}V + \int \phi' \rho' {\rm d}V$$

- linear adiabatic oscillation

$$\delta \ln \phi = c_1 \delta \ln T + c_2 (\delta \ln T)^2$$

- $\varepsilon \propto \rho^\beta T^\eta$
- $\delta \ln \phi = \delta \ln \varepsilon = c_1 \delta \ln T$ ← can be expanded up to the second-order...??
- Negligence of time-delay effect:
  - All the neutrinos from the solar interior are assumed to arrive at an observational point at the same time

Second-order effect might be non-negligible…??

CONSTANT

$$\Delta\Phi_{\rm osc}(t) = \sum_{(n,\ell,m\neq 0)} \sum_{(n'=n,\ell'=\ell,m'=m)} \pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r$$

(MOSTLY) PERIODIC

$$+ \sum_{(n,\ell,m\neq 0)} \sum_{(n'\neq n,\ell'=\ell,m'=m)} \pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o - o')$$

$$+ \sum_{(n,\ell,m\neq 0)} \sum_{(n',\ell'=\ell,m'=-m)} \pi(-1)^m \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o + o')$$

$$+ \sum_{(n,\ell,m=0)} \sum_{(n',\ell'=\ell,m'=0)} 2\pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o)\cos(o'),$$

where $o(t) = \omega_{n\ell m} t + \delta_{n\ell m}$, **and** $o'(t) = \omega_{n'\ell' m'} t + \delta_{n'\ell' m'}$

Periodic components are still small…

CONSTANT

$10^{-10}\ldots$

Difficulty in frequency analysis??

(MOSTLY) PERIODIC

$$
\begin{aligned}
\Delta\Phi_{\rm osc}(t) = & \sum_{(n,\ell,m\neq 0)} \sum_{(n'=n,\ell'=\ell,m'=m)} \pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \\
& + \sum_{(n,\ell,m\neq 0)} \sum_{(n'\neq n,\ell'=\ell,m'=m)} \pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o-o') \\
& + \sum_{(n,\ell,m\neq 0)} \sum_{(n',\ell'=\ell,m'=-m)} \pi(-1)^m \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o+o') \\
& + \sum_{(n,\ell,m=0)} \sum_{(n',\ell'=\ell,m'=0)} 2\pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')} \mathrm{d}r \times \cos(o)\cos(o'),
\end{aligned}
$$

$A_{n\ell} = 10^{-5}$

where $o(t) = \omega_{n\ell m}t + \delta_{n\ell m}$**, and** $o'(t) = \omega_{n'\ell' m'}t + \delta_{n'\ell' m'}$

Speculation…

$10^{-10} \times 10^{5} \sim 10^{-5}$

CONSTANT

These terms are just sums of all g modes

(MOSTLY) PERIODIC

$$
\begin{aligned}
\Delta\Phi_{\rm osc}(t) = & \sum_{(n,\ell,m\neq0)} \sum_{(n'=n,\ell'=\ell,m'=m)} \pi \int_0^{R_\star} Q_{(n\ell),(n'\ell')}\mathrm{d}r \\
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\end{aligned}
$$

$A_{n\ell} = 10^{-5}$

where $o(t) = \omega_{n\ell m}t + \delta_{n\ell m}$, **and** $o'(t) = \omega_{n'\ell' m'}t + \delta_{n'\ell' m'}$

Summary and conclusions

- Solar g modes are important
- Solar neutrino flux measurements could help us to find the solar g modes
- The first-order fluctuation in the neutrino flux is zero
- We see non-zero first-order fluctuations if we take into account the time-delay effect
    - but the relative fluctuation is in the neutrino flux is $< 10^{-10}$ when we assume $A_{n\ell} = 10^{-5}$ (for ^{8}B, ^{7}Be, ^{13}N, ^{15}O, and ^{17}F neutrinos)
- We are evaluating the second-order fluctuation in the neutrino flux
    - again, the relative fluctuation caused by a single g mode is too small to detect $\sim 10^{-10}$ when we assume $A_{n\ell} = 10^{-5}$ (for ^{8}B, ^{7}Be, ^{13}N, ^{15}O, and ^{17}F neutrinos)

→ it is almost impossible to detect individual g modes via neutrino flux measurements…

- But, the non-time varying component can be non-negligible ($\sim 10^{-5}$??)
  (when we assume $A_{n\ell} = 10^{-5}$ and that the number of g modes that exist inside the Sun is $\sim 10^5$)

Vielen Dank!

Thank you very much!

ご清聴いただきありがとうございました！

おまけ

In the case of the second-order fluctuation

- What we have to compute:

$$\Delta\phi_{\rm osc} = \int \phi' \rho_0 \mathrm{d}V + \int \phi_0 \rho' \mathrm{d}V + \int \phi' \rho' \mathrm{d}V$$

Eulerian perturbation:

$$q' = q(\boldsymbol{r}, t) - q_0(\boldsymbol{r}, t)$$

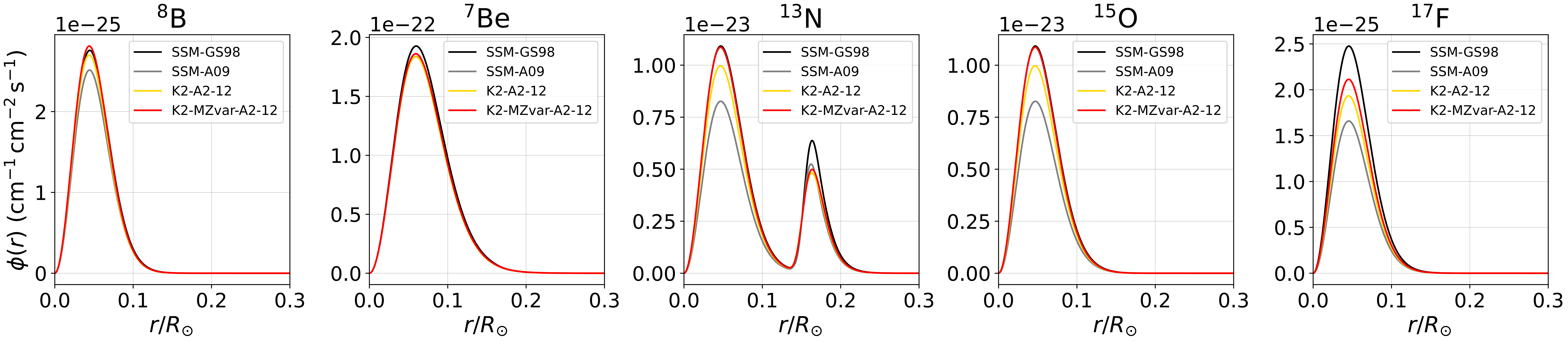
- But we have a few differences as below:
  - Lagrangian perturbation $\delta\phi$ is related to Eulerian perturbation ϕ' to second-order, namely, $\delta\phi = \phi' + (\boldsymbol{\xi} \cdot \nabla)\phi_0 + \dfrac{1}{2}\boldsymbol{\xi} \cdot \{H(\phi_0)\boldsymbol{\xi}\}$
  - we take into account the second-order terms in $\delta\phi$:

$$\delta \ln \phi = c_1(\delta \ln T) + c_2(\delta \ln T)^2$$

  - and, we have to consider couplings among modes with different mode indices

Evaluation with solar models

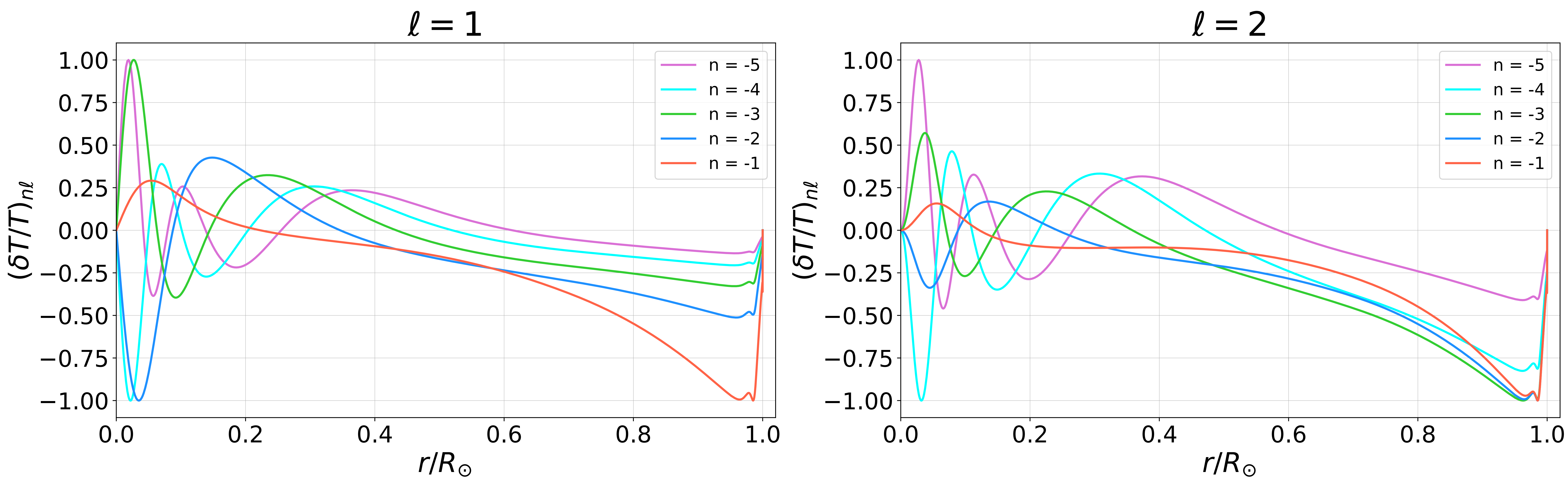
- Four 1-dimensional solar equilibrium models (Kunitomo+2022):
  - SSM-GS98: standard solar model (SSM) with the GS98 composition (high-$Z \sim 0.02$)
  - SSM-A09: SSM with the A09 composition (low-$Z \sim 0.01$)
  - K2-A2-12: a model where accretion in pre-main-sequence evolution is taken into account
  - K2-MZvar-A2-12: the same as K2-A2-12 except that chemical composition of accreted material is not constant



Flux densities inside the Sun for different neutrinos

Eigenfunctions

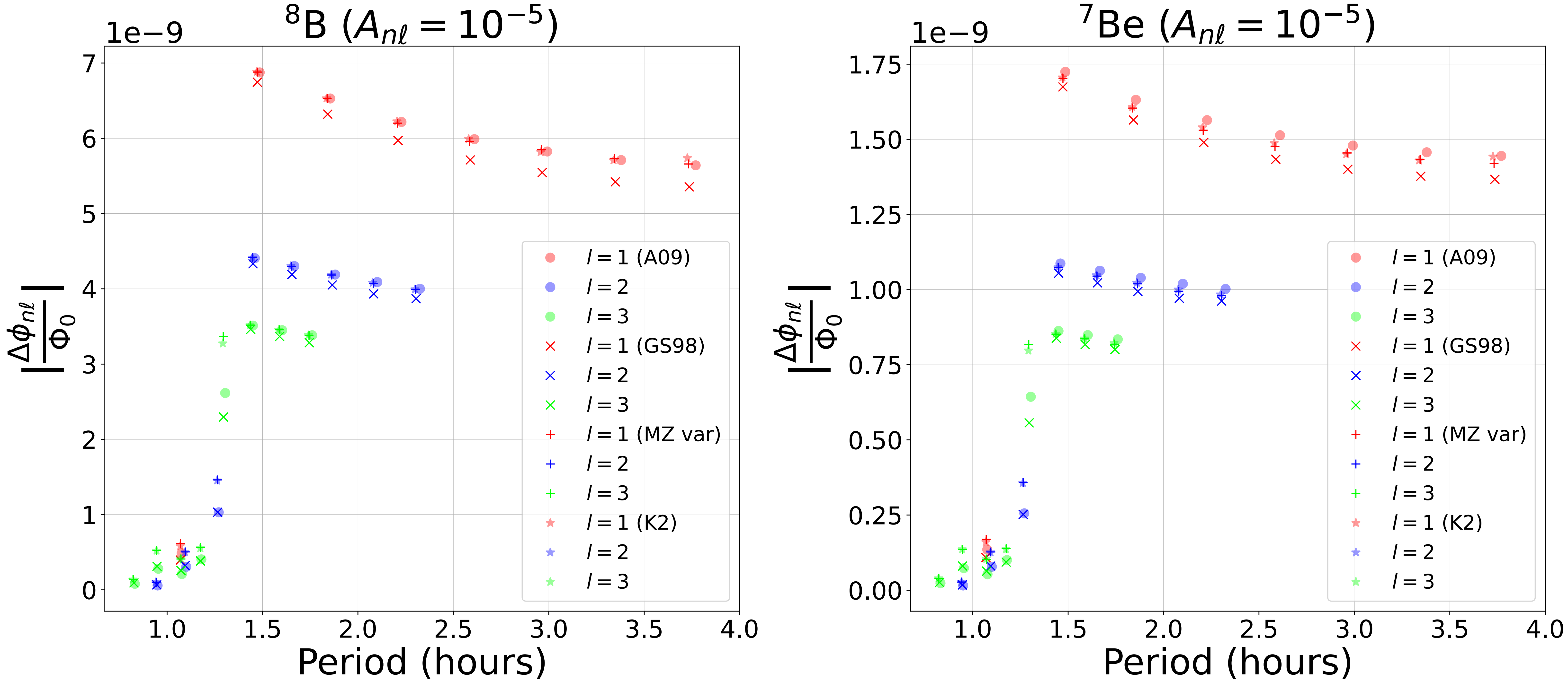
- For each solar model, we compute linear adiabatic oscillations via GYRE (Townsend+2013)
- Eigenfunctions are normalized so that the absolute value of maximum $(\delta \ln T)_{n\ell}$ is unity
  - We will multiply them later by $A_{n\ell}$ to (somehow) quantify the g-mode amplitudes



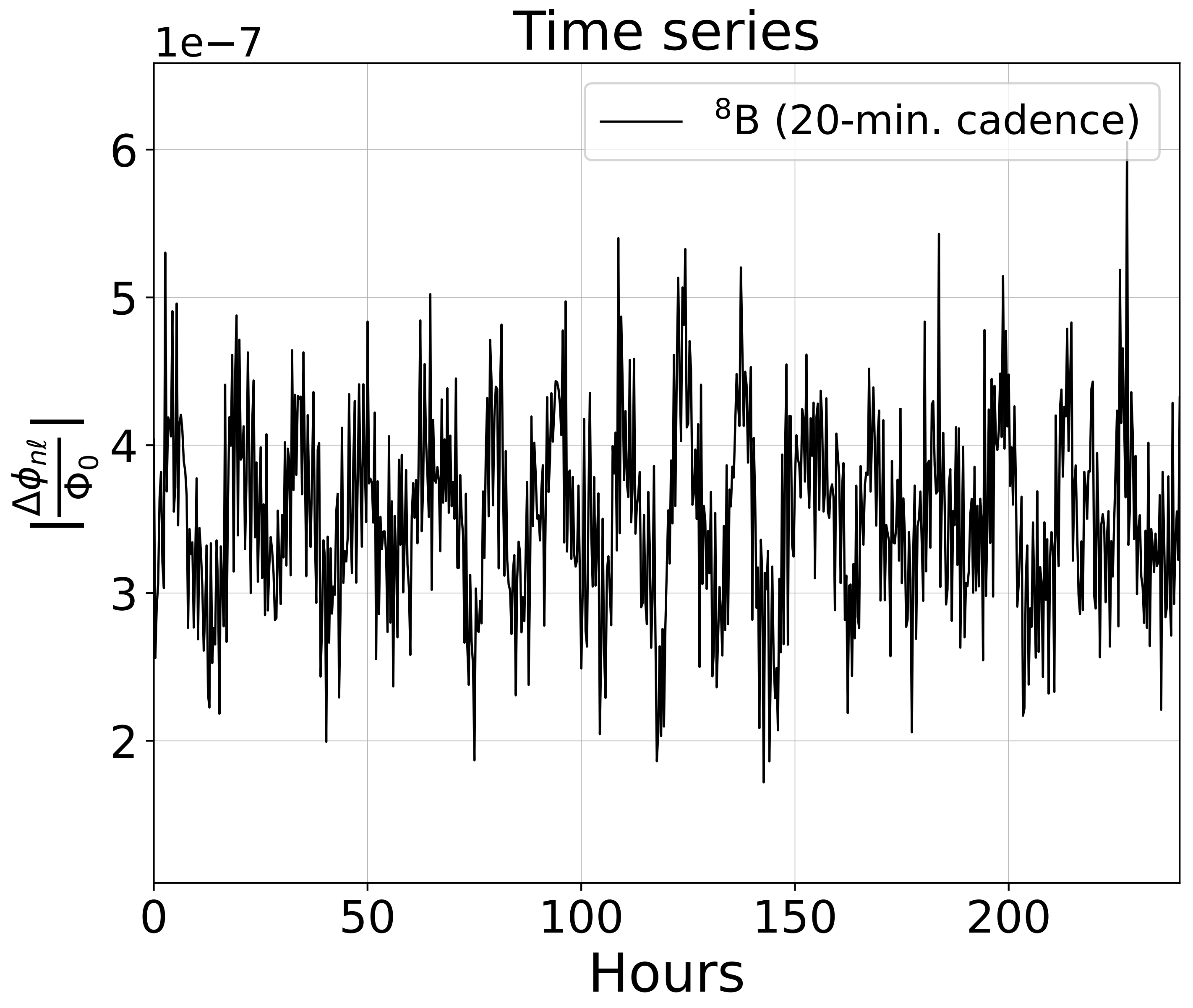
Temperature eigenfunctions $\delta T_{n\ell}(r)$ for the model K2-MZvar-A2-12

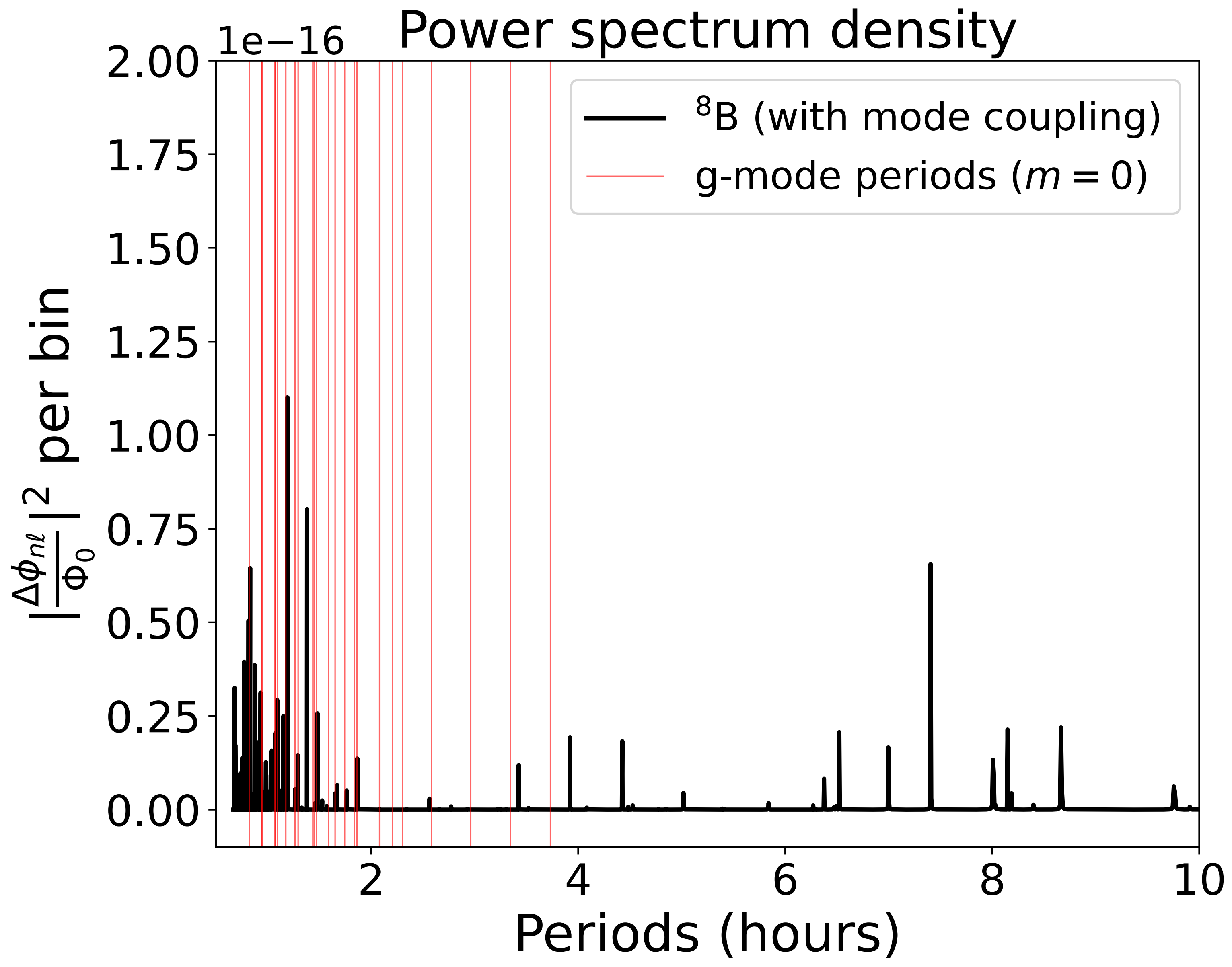
Fluctuation evaluation in the case of a single g mode

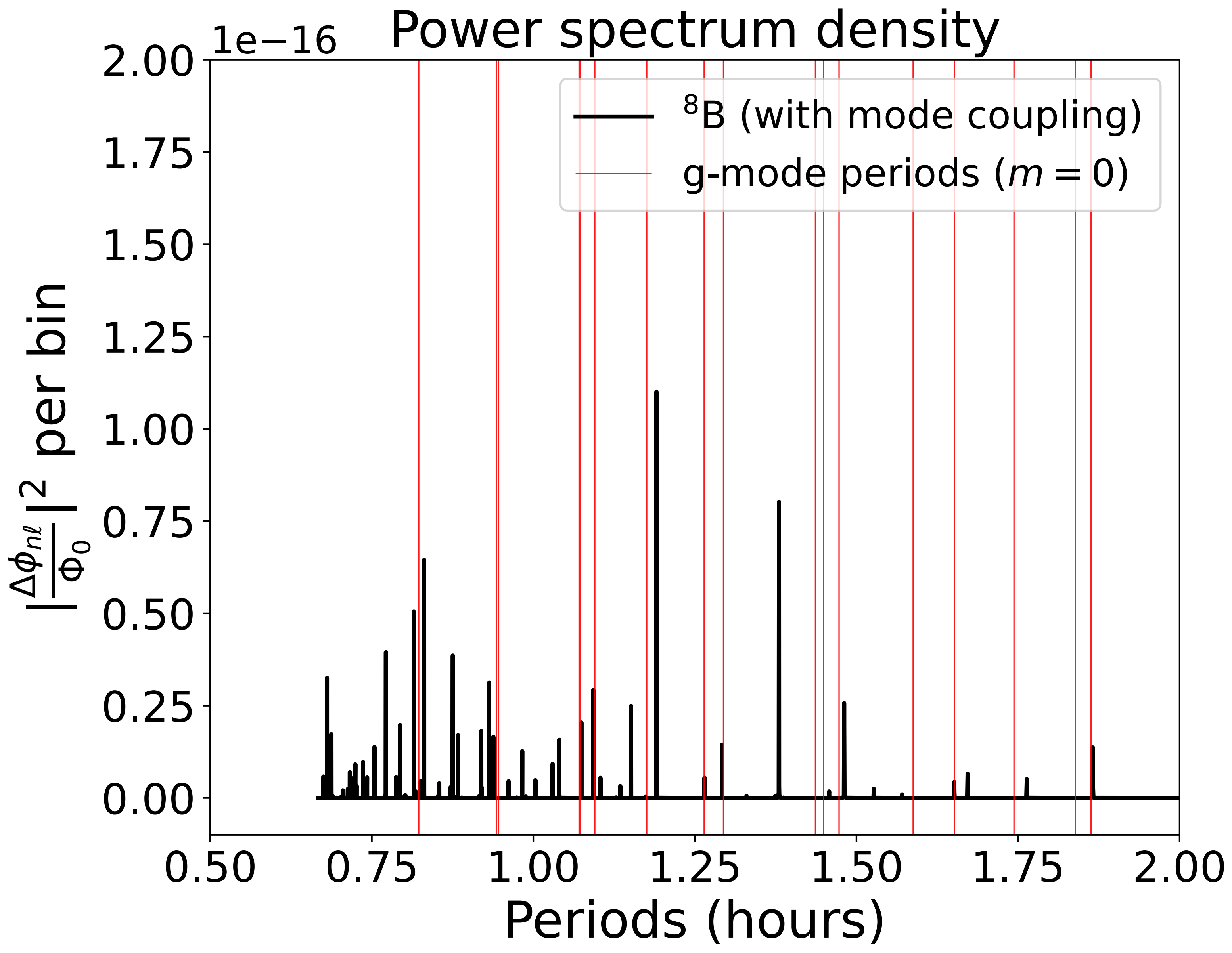
- In the case of a single g mode: $\Delta\phi_{\mathrm{osc}} = \pi \int_0^{R_\odot} Q_{(n\ell),(n\ell)} \mathrm{d}r + \pi \int_0^{R_\odot} Q_{(n\ell),(n\ell)} \mathrm{d}r \times \cos(2o)\,.$
- $A_{n\ell} = 10^{-5}$ is assumed
- Neutrino flux fluctuations caused by this g mode are of the order of 10^{-9} in relative difference



Fluctuation evaluation in the case of a few dozens of g modes







If $A_{n\ell}$ changes in accordance with the solar cycle…

- In our analysis, we assume that $A_{n\ell}$ is a constant for all the g modes
    - What determines $A_{n\ell}$, the g-mode amplitude, is turbulent convection (e.g. Belkacem+2022)
- The constancy in $A_{n\ell}$ guarantees the constancy of the non-time-varying component, but…
- If $A_{n\ell}$ changes with some periodicities, the non-time-varying component could vary with the same periods
    - One promising candidate for such periodicities is the 11-year solar cycle (??)
    - Magnetic fields in the convection zone changes → convection changes → g-mode amplitudes ($A_{n\ell}$) change → non-time-varying component in the neutrino flux fluctuation changes
- Such long-period (~ 11 years) variations in the solar neutrino flux are not confirmed yet…