The response of the Sun to modifications of its internal properties

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PRIN PANTHEON

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(*) partly based on work done in collaboration with **A. Serenelli and Y. Herrera (ICE and IEEC, BCN)**

Standard Solar Models

Our comprehension of the Sun is based on the Standard Solar Model (SSM).
Stellar structure equations are solved, starting from a ZAMS model to present solar age (we neglect rotation, magnetic fields, etc.):

$$
\begin{aligned}
\frac{\partial m}{\partial r} &= 4\pi r^2 \rho \\
\frac{\partial P}{\partial r} &= -\frac{G_{\rm N} m}{r^2}\rho \\
P &= P(\rho, T, X_i) \\
\frac{\partial l}{\partial r} &= 4\pi r^2 \rho\, \epsilon(\rho, T, X_i) \\
\frac{\partial T}{\partial r} &= -\frac{G_{\rm N} m T \rho}{r^2 P}\nabla
\end{aligned}
$$

$$
\nabla = \mathrm{Min}(\nabla_{\rm rad}, \nabla_{\rm ad}) \rightarrow \left\{
\begin{aligned}
\nabla_{\rm rad} &= \frac{3}{16\pi a c\, G_{\rm N}} \frac{\kappa(\rho, T, X_i)\, l\, P}{m\, T^4} \\
\nabla_{\rm ad} &= (d\ln T/d\ln P)_{\rm s} \simeq 0.4
\end{aligned}
\right.
$$

Chemical evolution driven by nuclear reaction, diffusion and gravitational settling, convection

Standard input physics for EoS, nuclear reactions, opacity, etc.

Free-parameters (mixing length, Y_{ini}, Z_{ini}) adjusted to match the observed properties of the Sun (radius, luminosity, Z/X).

Note that equations are non-linear → Iterative method to determine mixing length, Y_{ini}, Z_{ini}

The Standard Solar Model (SSM)

The predictions of SSMs can be **falsified** by other observations. e.g.:

- *Solar neutrinos:*

Hydrogen fusion in the solar core produce a huge amount of neutrinos that can be measured in suitable detectors (Davis 1964, Bahcall 1964)

$4H + 2e^{-} \rightarrow {}^{4}He + 2\nu_e + \text{energy}$

Solar Neutrino Problem
Nuclear energy generation (cross sections, etc.)

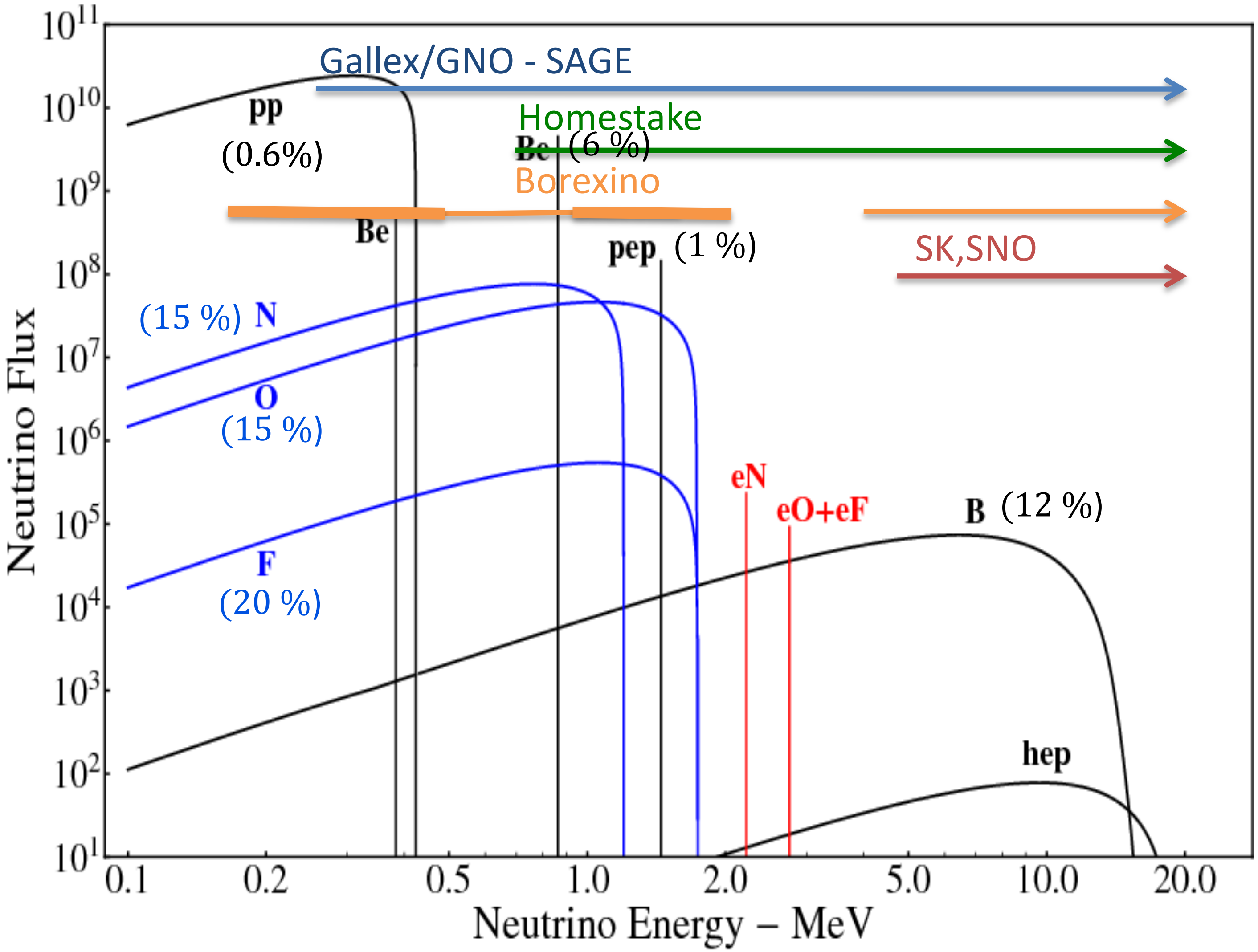
- *Helioseismology:*

Solar oscillations originally discovered by Leighton at al. 1962 and interpreted as standing acoustic waves

Elemental Diffusion
Opacity, EoS, …

Constant improvement in SSM constitutive physics was triggered during last decades by solar neutrino and helioseismic data.

The solar neutrino spectrum



Recent Milestones from **Borexino**:

- ^{7}Be (and ^{8}B) neutrino direct detection [PRL 2008]
- pp (and pep) neutrinos direct detection [Nature 2014, 2018]
- CNO neutrinos signal identification [Nature 2020, PRL 2022, PRD 2023]

Status of direct determination of solar neutrino fluxes after Borexino

[Gonzales-Garcia et al, JHEP 2024]

Implementing the **solar luminosity constraint**: $L_\odot = 4\pi D^2 \sum_i \left(\frac{Q}{2} - \langle E_\nu \rangle_i\right) \Phi_i$

$$
\begin{aligned}
f_{\rm pp} &= 0.9969^{+0.0041}_{-0.0039}\,[^{+0.0095}_{-0.0092}]\,, & \Phi_{\rm pp} &= 5.941^{+0.024}_{-0.023}\,[^{+0.057}_{-0.055}] \times 10^{10}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^7\rm Be} &= 1.019^{+0.020}_{-0.017}\,[^{+0.047}_{-0.041}]\,, & \Phi_{^7\rm Be} &= 4.93^{+0.10}_{-0.08}\,[^{+0.23}_{-0.20}] \times 10^{9}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{\rm pep} &= 1.000^{+0.016}_{-0.018}\,[^{+0.041}_{-0.042}]\,, & \Phi_{\rm pep} &= 1.421^{+0.023}_{-0.026}\,[^{+0.058}_{-0.060}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{13}\rm N} &= 1.25^{+0.17}_{-0.14}\,[^{+0.47}_{-0.40}]\,, & \Phi_{^{13}\rm N} &= 3.48^{+0.47}_{-0.40}\,[^{+1.30}_{-1.10}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{15}\rm O} &= 1.22^{+0.17}_{-0.14}\,[^{+0.46}_{-0.39}] & \Phi_{^{15}\rm O} &= 2.53^{+0.34}_{-0.29}\,[^{+0.94}_{-0.80}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{17}\rm F} &= 1.03^{+0.20}_{-0.20}\,[^{+0.47}_{-0.48}]\,, & \Phi_{^{17}\rm F} &= 5.51^{+0.75}_{-0.63}\,[^{+2.06}_{-1.75}] \times 10^{7}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^8\rm B} &= 1.036^{+0.020}_{-0.020}\,[^{+0.047}_{-0.048}]\,, & \Phi_{^8\rm B} &= 5.20^{+0.10}_{-0.10}\,[^{+0.24}_{-0.24}] \times 10^{6}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{\rm hep} &= 3.8^{+1.1}_{-1.2}\,[^{+2.7}_{-2.7}]\,, & \Phi_{\rm hep} &= 3.0^{+0.9}_{-1.0}\,[^{+2.2}_{-2.1}] \times 10^{4}\ \text{cm}^{-2}\ \text{s}^{-1}\,.
\end{aligned}
$$

The different comp. of the solar neutrinos flux have been **directly** determined with accuracy level:

pp: $\sim$ 8%
pep: $\sim$ 10%
^{7}Be: $\sim$ 2 %
^{8}B: $\sim$ 2 %
CNO: $\sim$ 15%

Not implementing the **solar luminosity constraint**:

$$
\begin{aligned}
f_{\rm pp} &= 1.038^{+0.076}_{-0.066}\,[^{+0.18}_{-0.16}]\,, & \Phi_{\rm pp} &= 6.19^{+0.45}_{-0.39}\,[^{+1.1}_{-1.0}] \times 10^{10}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^7\rm Be} &= 1.022^{+0.022}_{-0.018}\,[^{+0.051}_{-0.042}]\,, & \Phi_{^7\rm Be} &= 4.95^{+0.11}_{-0.089}\,[^{+0.25}_{-0.22}] \times 10^{9}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{\rm pep} &= 1.039^{+0.082}_{-0.065}\,[^{+0.19}_{-0.16}]\,, & \Phi_{\rm pep} &= 1.48^{+0.11}_{-0.09}\,[^{+0.26}_{-0.22}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{13}\rm N} &= 1.16^{+0.19}_{-0.19}\,[^{+0.50}_{-0.45}]\,, & \Phi_{^{13}\rm N} &= 3.32^{+0.53}_{-0.54}\,[^{+1.40}_{-1.24}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{15}\rm O} &= 1.16^{+0.19}_{-0.19}\,[^{+0.49}_{-0.44}] & \Phi_{^{15}\rm O} &= 2.41^{+0.38}_{-0.39}\,[^{+1.02}_{-0.90}] \times 10^{8}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^{17}\rm F} &= 1.01^{+0.16}_{-0.16}\,[^{+0.45}_{-0.38}]\,, & \Phi_{^{17}\rm F} &= 5.25^{+0.84}_{-0.85}\,[^{+2.21}_{-1.97}] \times 10^{6}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{^8\rm B} &= 1.034^{+0.020}_{-0.021}\,[^{+0.052}_{-0.051}]\,, & \Phi_{^8\rm B} &= 5.192^{+0.10}_{-0.11}\,[^{+0.26}_{-0.26}] \times 10^{6}\ \text{cm}^{-2}\ \text{s}^{-1}\,,\\
f_{\rm hep} &= 3.6^{+1.2}_{-1.1}\,[^{+3.0}_{-2.6}]\,, & \Phi_{\rm hep} &= 2.9^{+1.0}_{-0.9}\,[^{+2.4}_{-2.1}] \times 10^{4}\ \text{cm}^{-2}\ \text{s}^{-1}\,.
\end{aligned}
$$

$$
\frac{L_\odot(\text{neutrino-inferred})}{L_\odot} = 1.038^{+0.069}_{-0.060}\,[^{+0.17}_{-0.15}]\,.
$$

Helioseismology

The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R dr\, K^{nl}_{u,Y}(r)\,\frac{\delta u}{u}(r) + \int_0^R dr\, K^{nl}_{Y,u}(r)\,\delta Y + \frac{F(\nu_{nl})}{\nu_{nl}}$$

squared isothermal sound speed

Related to temperature stratification in the sun

surface helium abundance

See Basu & Antia 07 for a review

Impressive agreement with SSM predictions …

Surface helium abundance

$$Y_{\rm b} = 0.2485 \pm 0.0035$$

$$Y_{\rm b} = 0.243 \quad \textit{(GS98)}$$

Inner radius of the solar convective envelope

$$R_{\rm b}/R_\odot = 0.713 \pm 0.001$$

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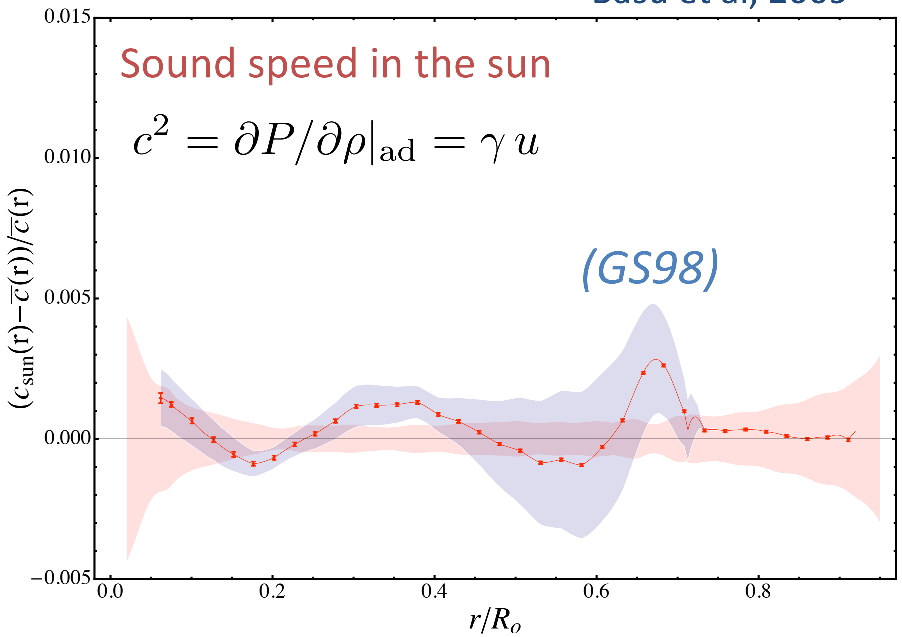
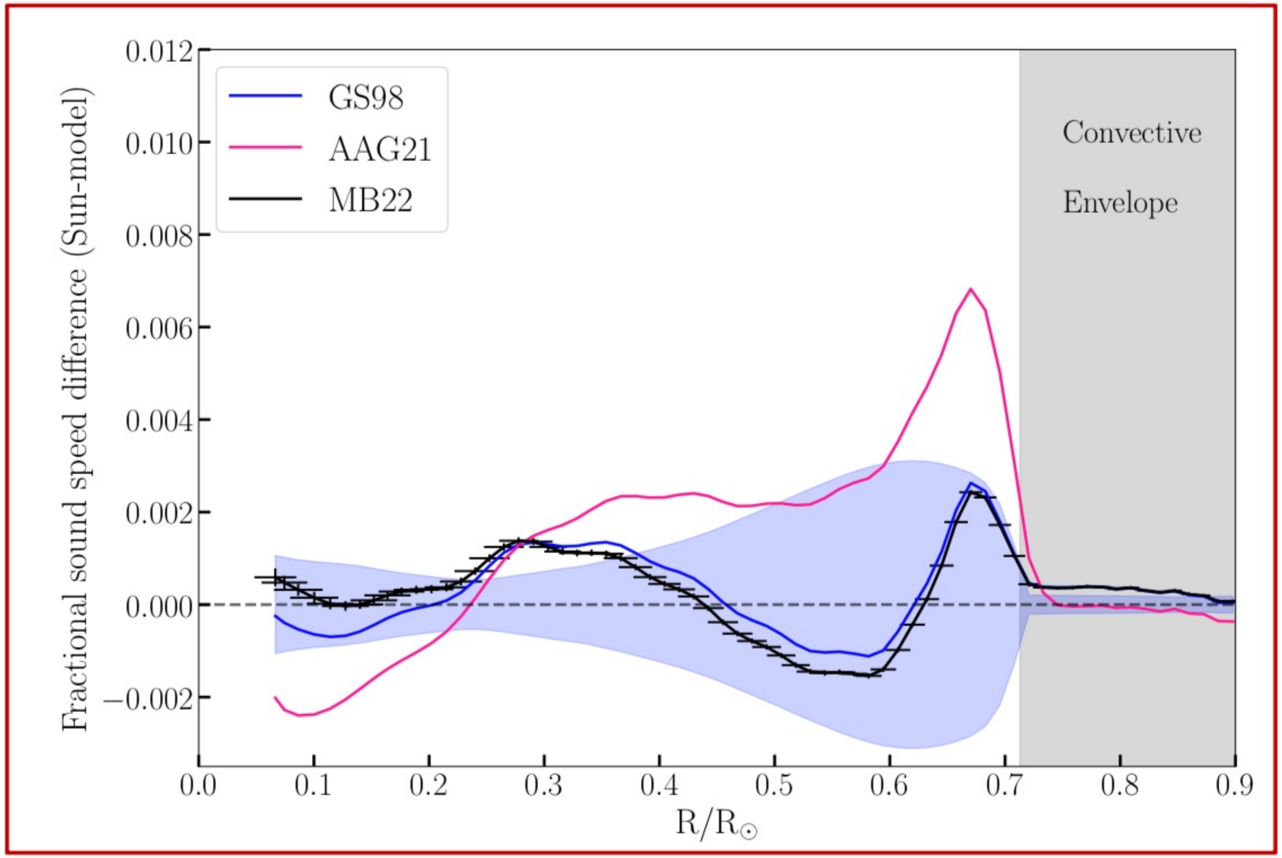
Basu et al, 2009



*... even if a new puzzle, i.e. the **solar composition problem**, came out in the new millennium*

Model	$R_{CZ}/R_\odot$	Y_S
MB22-phot	0.7123	0.2439
MB22-met	0.7120	0.2442
AAG21	0.7197	0.2343
AGSS09-met	0.7231	0.2316
GS98	0.7122	0.2425
C11	0.7162	0.2366

Situation in 2022



Helioseismic determinations

$$R_b/R_\odot = 0.713 \pm 0.001$$
$$Y_b = 0.2485 \pm 0.0035$$

Magg et al. 2022

HZ surface composition provide a better description of helioseismic data

Response of the Sun to:

Modifications of:

1. Nuclear cross sections in the pp chain and CNO cycle;

2. Energy transfer, i.e. Opacity of the solar plasma;

3. Energy production/losses (see talk of Yago Herrera)

The role of nuclear cross sections

Hydrogen Burning: PP chain and CNO cycle

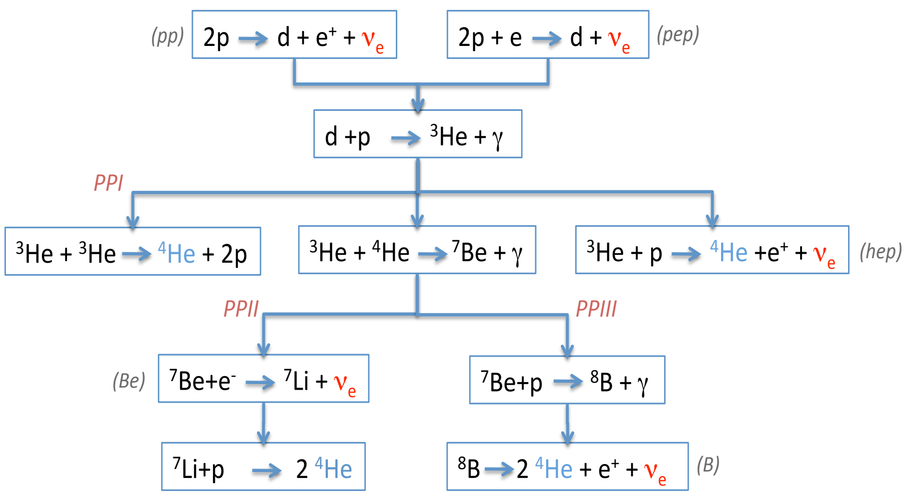
The Sun is powered by nuclear reactions that transform H into ^{4}He:

$$4H + 2e^- \rightarrow {}^4He + 2\nu_e + \text{energy}$$

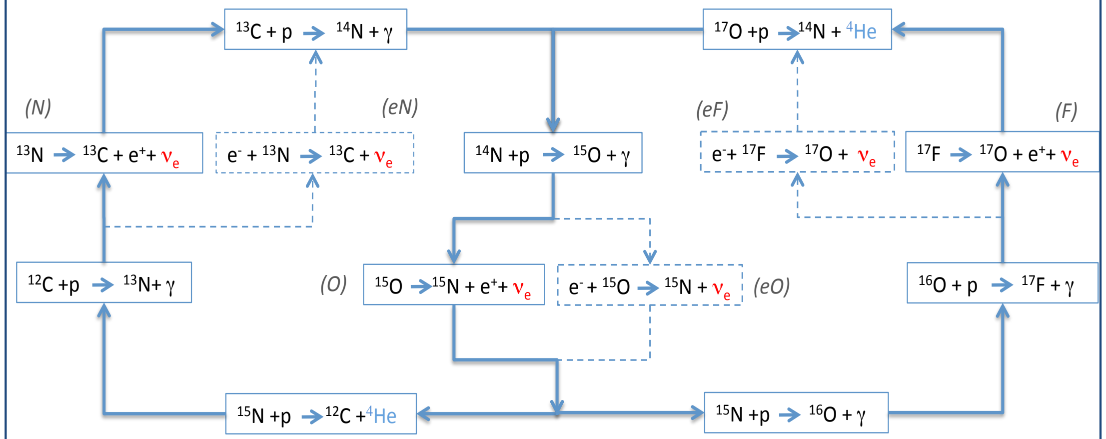
Q = 26,7 MeV (globally)

Free stream – 8 minutes to reach the earth
Direct information on the energy producing region.

The PP-chain



The CN-NO (bi-)cycle



The pp chain is responsible for about 99% of the total energy (and neutrino) production.

C, N and O nuclei are used as catalysts for hydrogen fusion.

CNO (bi-)cycle is responsible for about 1% of the total neutrino (and energy) budget. Important for more advanced evolutionary stages

Neutrino fluxes in the pp-chain

The pp and ^{7}Be neutrino fluxes are linked by the solar luminosity constraint:

$$\Phi(\mathrm{pp}) \simeq \frac{1}{4\pi D^2}\frac{2\mathrm{L}_\odot}{Q_\mathrm{I}} - \Phi(^7\mathrm{Be})$$

$L_\odot \;=\; Solar\ luminosity$

$D \;=\; Sun - Earth\ distance$

$Q_I \;\simeq 26\ MeV$

See e.g.:

Bahcall and Ulmer (1996)

Degl'Innocenti et al. (1997)

Villante and Serenelli (2021)

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$L_\odot$ = *Solar luminosity*
D = *Sun – Earth distance*
$Q_I \simeq 26\ MeV$

The ^{7}Be neutrino flux is determined by the integrated rate of ^{3}He+^{4}He reaction:

$$\Phi(^7\mathrm{Be}) = \frac{\lambda_{34}}{4\pi D^2}$$

$$\lambda_{34} = \int d^3r\, \frac{\rho^2}{m_\mathrm{u}^2}\frac{X_3\, Y}{12}\langle\sigma v\rangle_{34}$$

See e.g.:
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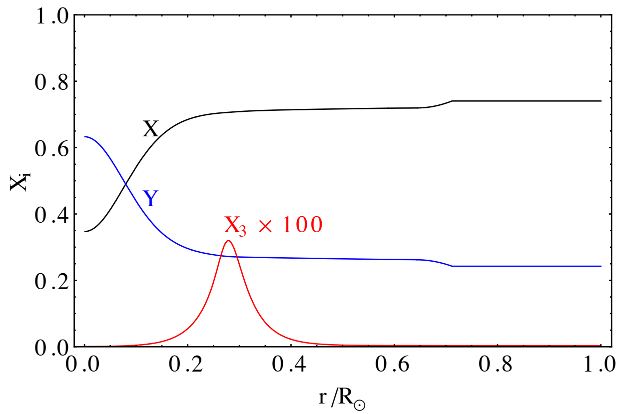
$$\begin{aligned} L_\odot &= \text{Solar luminosity} \\ D &= \text{Sun} - \text{Earth distance} \\ Q_I &\simeq 26\ MeV \end{aligned}$$



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$$X_3 \simeq X_{3,\mathrm{eq}} = 3\, X\sqrt{\frac{\langle\sigma v\rangle_{11}}{2\langle\sigma v\rangle_{33}}}$$

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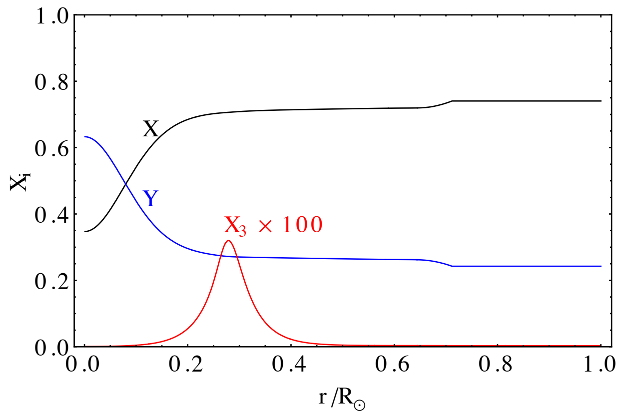
$L_\odot$ = *Solar luminosity*
D = *Sun − Earth distance*
$Q_I \simeq 26$ *MeV*



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^{8}B neutrinos are produced when a proton (instead of an electron) is captured by ^{7}Be:

$$\Phi(^8\mathrm{B}) = r\,\Phi(^7\mathrm{Be})$$

$$r \equiv \frac{\lambda_{17}}{\lambda_{e7}} \sim \frac{\langle\sigma v\rangle_{17}}{\langle\sigma v\rangle_{e7}}$$

See e.g.:
Bahcall and Ulmer (1996)
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Villante and Serenelli (2021)

Neutrino fluxes in the pp-chain

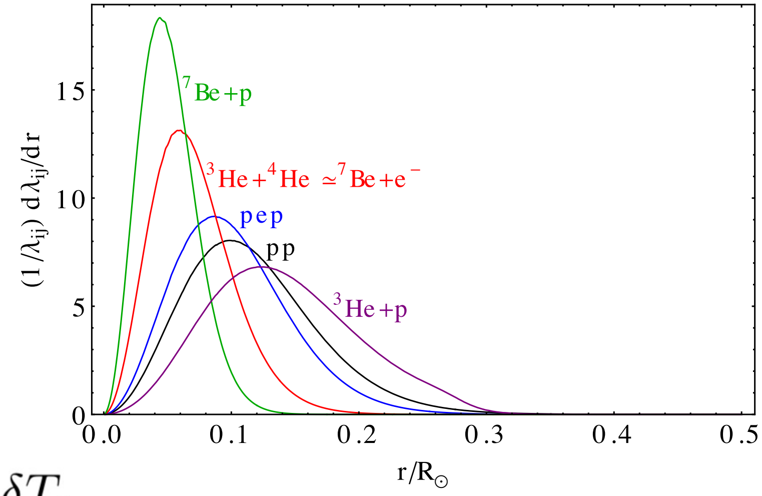
Following the previous discussion and considering that: $\langle \sigma v \rangle_{ij} \propto S_{ij}\, T_{\rm c}^{\gamma_{ij}}$

$$\delta\Phi(\mathrm{pp}) = -\eta\,\delta S_{34} - \frac{\eta}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{pp}}\,\delta T_{\rm c}$$

$$\delta\Phi(\mathrm{pep}) = -\eta\,\delta S_{34} - \frac{\eta}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{pep}}\,\delta T_{\rm c}$$

$$\delta\Phi(^7\mathrm{Be}) = \delta S_{34} + \frac{1}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{Be}}\,\delta T_{\rm c}$$

$$\delta\Phi(^8\mathrm{B}) = (\delta S_{17} - \delta S_{e7}) + \delta S_{34} + \frac{1}{2}\,(\delta S_{11} - \delta S_{33}) + \beta_{\mathrm{B}}\,\delta T_{\rm c}$$



where:

- δQ is the fractional variation of the generic quantity Q
- $\eta \equiv \Phi(Be)/\Phi(pp) \simeq 0.08$
- $\beta_{Be} = \gamma_{34} + (\gamma_{11} - \gamma_{33})/2 \simeq 11$
- $\beta_{pp} = \eta\, \beta_{Be} \simeq -0.9$
- $\beta_{pep} = \beta_{pp} - 1/2 \simeq -1.4$
- $\beta_B = \beta_{Be} + \gamma_{17} + 0.5 \simeq 24$

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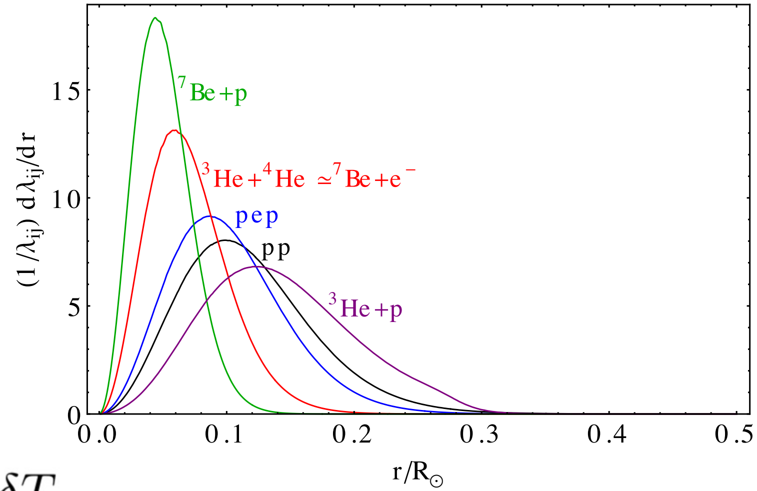

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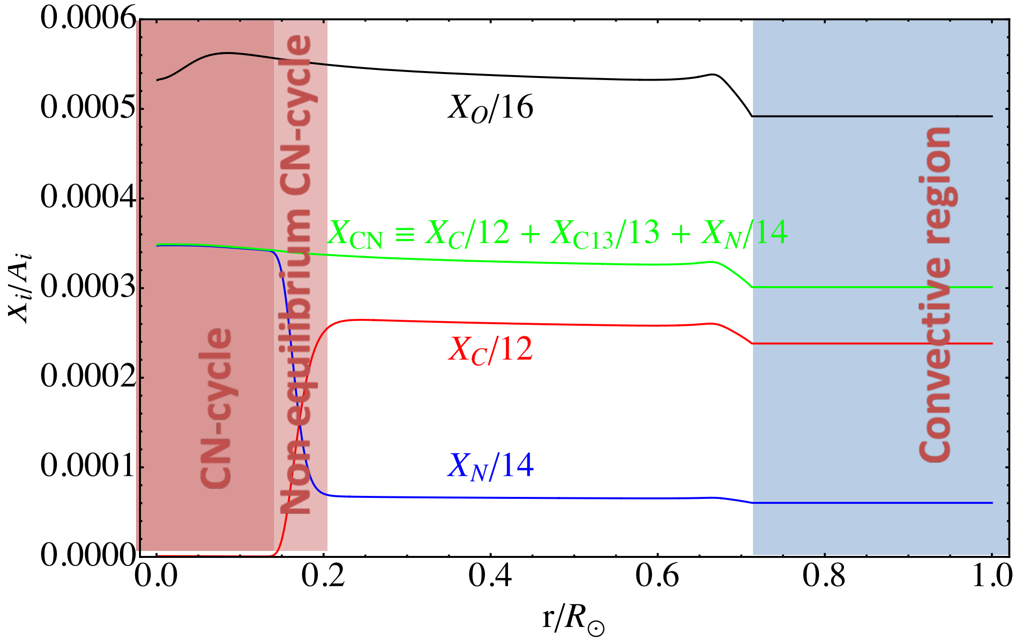
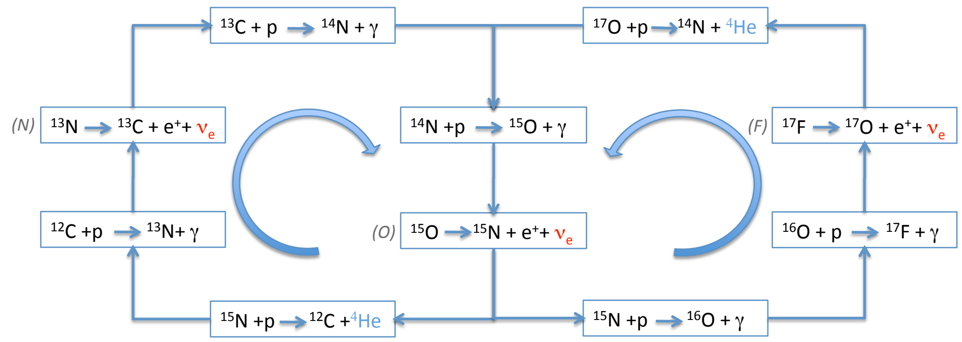
Neutrino fluxes in the CN-cycle





^{15}O neutrinos are produced in the equilibrium region:

$$\Phi(^{15}\mathrm{O}) = \lambda_{114}/(4\pi D^2) \qquad \lambda_{114} = \int d^3r\, \frac{\rho^2}{m_u^2}\, \frac{X\, X_{14}}{14}\, \langle \sigma v \rangle_{114}$$

^{13}N neutrinos are also produced in the non-equilibrium region (where a non-negligible amount of ^{12}C is still present):

$$\Phi(^{13}\mathrm{N}) = \lambda_{112}/(4\pi D^2) \qquad \lambda_{112} = \lambda_{114} + \lambda_{112}^{(\mathrm{ne})}$$

Villante and Serenelli (2021)

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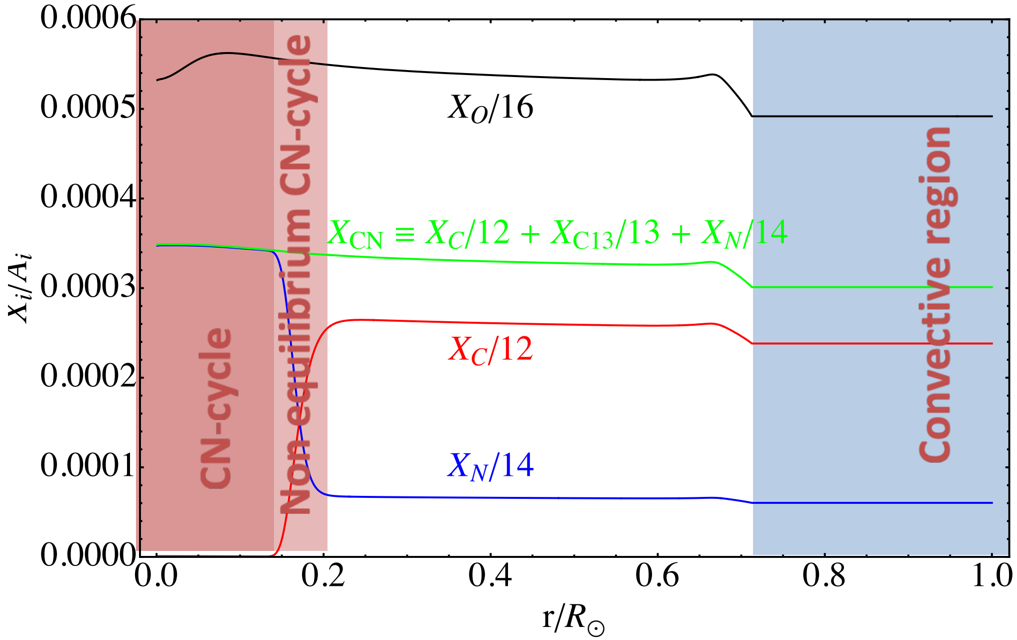

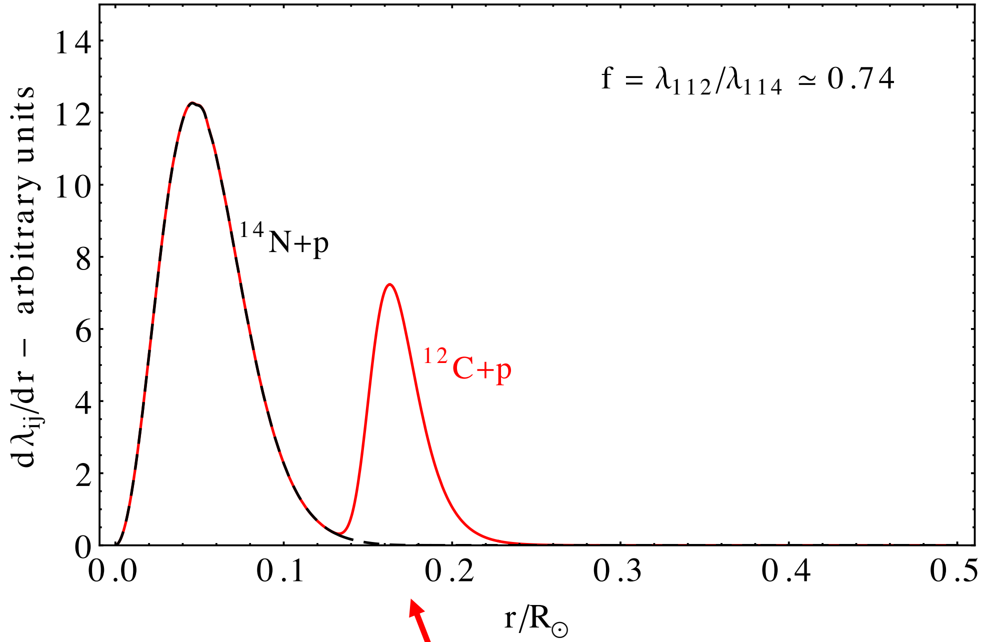

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Villante and Serenelli (2021)

Neutrino fluxes in the CN-cycle

Villante and Serenelli (2021)

By expanding previous relationships, we obtain the CNO neutrino fluxes as a function:
- cross sections, central temperature and chemical composition

(^{14}**N in the core** and the ^{12}**C in the non-equilibrium region**)

$$\delta\Phi(^{15}\mathrm{O}) \simeq \delta X_{14,\mathrm{c}} + \delta S_{114} + \beta_{\mathrm{O}}\,\delta T_{\mathrm{c}}$$

$$\delta\Phi(^{13}\mathrm{N}) = f\,\left[\delta X_{14,\mathrm{c}} + \delta S_{114} + \gamma_{114}\,\delta T_{\mathrm{c}}\right] + (1-f)\,\left[\delta X_{12}(r_{\mathrm{ne}}) + \delta S_{112} + \gamma_{112}\,\delta T_{\mathrm{c}}\right]$$

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C+N elements are transformed to ^{14}N in the core:

$$\delta X_{14,\mathrm{c}} = a\, \delta X_{14,\,\mathrm{s}} + (1-a)\, \delta X_{12,\,\mathrm{s}} + b\, \left(\Delta^{(\mathrm{cs})} - 0.16\right)$$

Surface abundances *Effect of diffusion*

$$b = 1/(1 + 0.16) = 0.86$$
$$a = 6\xi/(6\xi + 7) = 0.20$$
$$\xi = X_{14,s}/X_{12,s} = 0.30$$

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Villante and Serenelli (2021)

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$$a\ =\ 6\xi\,/(6\xi + \ 7)\ =\ 0.20$$
$$\xi\ = X_{14,s}/X_{12,s} =\ 0.30$$

The **residual ^{12}C in non-equilibrium region** anticorrelates with **^{12}C+p reaction rate**

$$\delta X_{12}(r_{\mathrm{ne}}) = \delta X_{12,\,\mathrm{s}} + b\ \left(\Delta^{(\mathrm{cs})} - 0.16\right) - \delta S_{112} - \gamma_{112}\, \delta T_{\mathrm{c}}$$

Neutrino fluxes in the CN-cycle

Villante and Serenelli (2021)

By expanding previous relationships, we obtain the CNO neutrino fluxes as a function:
- cross sections, central temperature and chemical composition

(^{14}**N in the core** and the ^{12}**C in the non-equilibrium region**)

$$\delta\Phi(^{15}\mathrm{O}) \simeq \delta X_{14,\mathrm{c}} + \delta S_{114} + \beta_{\mathrm{O}}\,\delta T_{\mathrm{c}}$$

$$\delta\Phi(^{13}\mathrm{N}) = f\,\left[\delta X_{14,\mathrm{c}} + \delta S_{114} + \gamma_{114}\,\delta T_{\mathrm{c}}\right] + (1-f)\,\left[\delta X_{12}(r_{\mathrm{ne}}) + \cancel{\delta S_{112}} + \cancel{\gamma_{112}\,\delta T_{\mathrm{c}}}\right]$$

C+N elements are transformed to ^{14}N in the core:

$$\delta X_{14,\mathrm{c}} = a\,\delta X_{14,\mathrm{s}} + (1-a)\,\delta X_{12,\mathrm{s}} + b\,\left(\Delta^{(\mathrm{cs})} - 0.16\right)$$

Surface abundances *Effect of diffusion*

$$b = 1/(1+0.16) = 0.86$$
$$a = 6\xi/(6\xi+7) = 0.20$$
$$\xi = X_{14,s}/X_{12,s} = 0.30$$

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$$f = \Phi(^{15}O)/\Phi(^{13}N) = 0.74$$
$$\beta_O = \gamma_{114} \simeq 20$$
$$\beta_N = f\;\beta_O \simeq 15$$

The **final results** are:

$$\boxed{\begin{aligned}\delta\Phi(^{15}\mathrm{O}) &= \beta_{\mathrm{O}}\,\delta T_{\mathrm{c}} + (1-a)\,\delta X_{12,\mathrm{s}} + a\,\delta X_{14,\mathrm{s}} + b\;\left(\Delta^{(\mathrm{cs})} - 0.16\right) + \delta S_{114}\\ \delta\Phi(^{13}\mathrm{N}) &= \beta_{\mathrm{N}}\,\delta T_{\mathrm{c}} + (1-a')\,\delta X_{12,\mathrm{s}} + a'\,\delta X_{14,\mathrm{s}} + b\;\left(\Delta^{(\mathrm{cs})} - 0.16\right) + f\,\delta S_{114}\end{aligned}}$$

How to take advantage of the described correlations?

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition.** *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using ^{8}**B-neutrinos as solar thermometer;**
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^{8}B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013
See also (application to BX obs. rate):
Agostini et al, EPJ 2021
Villante & Serenelli, Frontiers 2021

$$\delta\Phi(^{15}\mathrm{O}) - x\,\delta\Phi(^{8}\mathrm{B}) \simeq \boxed{\delta X_{\mathrm{CN}}^{\mathrm{core}}} + \delta S_{114} - x\left(\delta S_{17} - \delta S_{e7} + \delta S_{34} + \frac{\delta S_{11}}{2} - \frac{\delta S_{33}}{2}\right)$$

$$x = \frac{\beta_{\mathrm{O}}}{\beta_{\mathrm{B}}} \sim 0.8$$

Linear Solar Models:
a tool to investigate the properties of the Sun

Linear Solar Models

- We wish to investigate response of the Sun to generic modifications of its internal properties.

$$\begin{aligned}
\frac{\partial m}{\partial r} &= 4\pi r^2 \rho \\
\frac{\partial P}{\partial r} &= -\frac{G_{\rm N} m}{r^2}\rho \\
P &= P(\rho, T, X_i) \\
\frac{\partial l}{\partial r} &= 4\pi r^2 \rho\, \epsilon(\rho, T, X_i) \\
\frac{\partial T}{\partial r} &= -\frac{G_{\rm N} m T \rho}{r^2 P}\nabla
\end{aligned}$$

$$\nabla = \mathrm{Min}(\nabla_{\rm rad}, \nabla_{\rm ad}) \rightarrow \begin{cases} \nabla_{\rm rad} = \dfrac{3}{16\pi ac\, G_{\rm N}}\dfrac{\kappa(\rho, T, X_i)\, l\, P}{m\, T^4} \\ \nabla_{\rm ad} = (d\ln T/d\ln P)_{\rm s} \simeq 0.4 \end{cases}$$

Linear Solar Models

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- Giving for granted mass conservation (i.e. continuity equation) and hydrostatic equilibrium and assuming that EoS is under control, we consider modification of **energy transfer** (i.e. opacity or equivalent effects) and **energy production/absorption** in the Sun.

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\end{aligned}
\right.
}
$$

- These modification can be highly nontrivial.
- In order to discuss their effect in the most general terms, we developed the so-called **Linear Solar Models (LSM).**

F.L. Villante and B. Ricci - ***Astrophys.J.714:944-959,2010***
F.L. Villante - ***J.Phys.Conf.Ser.203:012084,2010***

Linear Solar Models: the basic idea

- The starting point:

SSMs provide a good approximation of the real sun. Small modifications are likely to explain disagreement with helioseismology.

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- The method:

We write:

$$\begin{aligned} h(r) &= \overline{h}(r)[1+\delta h(r)] \\ X_i(r) &= \overline{X}_i(r)[1+\delta X_i(r)] \\ Y(r) &= \overline{Y}(r) + \Delta Y(r) \end{aligned} \qquad h = l,\ m,\ \rho,\ P,\ T$$

where $\overline{h}(r)$, $\overline{X}_i(r)$ are the SSMs predicted values, and we expand linearly in $\left\{ \begin{array}{l} \delta h(r) \\ \delta X_i(r) \\ \Delta Y(r) \end{array} \right.$

Assumption: the variation of the *present* solar composition (i.e. the $\delta X_i(r),\ \ \Delta Y(r)$) can be deduced with sufficient accuracy from the variation of the nuclear reaction efficiency and diffusion velocities in the *present* sun (i.e. the $\delta h(r)$)

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- <u>The result:</u>

A linear system of ordinary differential equations that can be used to study the response of the sun to an arbitrary modification input parameters.

Linear Solar Models – Final set of equations equations

$$\begin{aligned}
\frac{d\delta m}{dr} &= \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right]\\
\frac{d\delta P}{dr} &= \frac{1}{l_P}\left[(\gamma_P - 1)\,\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right]\\
\frac{d\delta l}{dr} &= \frac{1}{l_l}\left[\beta'_P\,\delta P + \beta'_T\,\delta T - \delta l + \beta'_Y\,\Delta Y_{\rm ini} + \beta'_C\,\delta C + \beta'_\epsilon\,\delta\epsilon\right]\\
\frac{d\delta T}{dr} &= \frac{1}{l_T}\left[\alpha'_P\,\delta P + \alpha'_T\,\delta T + \delta l + \alpha'_Y\,\Delta Y_{\rm ini} + \alpha'_C\,\delta C + \delta\kappa + \alpha'_\epsilon\,\delta\epsilon\right]
\end{aligned}$$

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System of four linear differential equations in *δm, δP, δl, δT*

Note that:

EOS: we assumed perfect gas scaling and neglect the role of metals

$$\delta\rho(r) = \delta P(r) - \delta T(r) - P_Y \Delta Y(r)$$

$$P_Y(r) = -\frac{\partial \ln\mu}{\partial Y} = -\frac{5}{8 - 5Y(r) - 6Z(r)}$$

Linear Solar Models – Final set of equations equations

$$\frac{d\delta m}{dr} = \boxed{\frac{1}{l_m}}[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon]$$

$$\frac{d\delta P}{dr} = \frac{1}{l_P}[(\gamma_P - 1)\,\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon]$$

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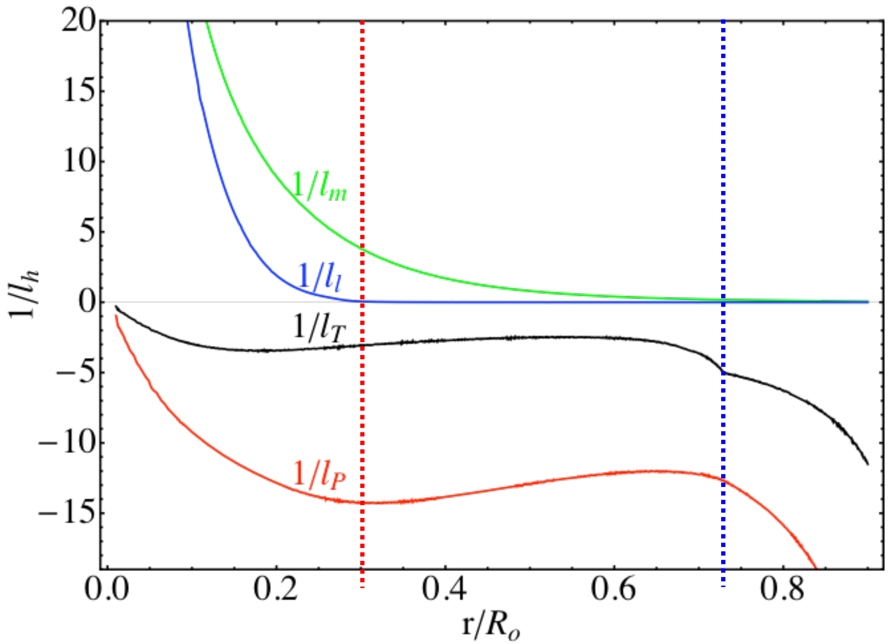
$$\frac{d\delta T}{dr} = \frac{1}{l_T}[\alpha'_P\,\delta P + \alpha'_T\,\delta T + \delta l + \alpha'_Y\,\Delta Y_{\rm ini} + \alpha'_C\,\delta C + \boxed{\delta\kappa} + \alpha'_\epsilon\,\delta\epsilon]$$

$\delta\varepsilon$, $\delta\kappa$ = fractional variations of the input params → provide the source terms

Inverse scale height of *h* in SSM

$$l_h = \left[\frac{d\ln(\bar{h})}{dr}\right]^{-1}$$



Linear Solar Models – Final set of equations equations

$$\frac{d\delta m}{dr} = \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini} + \gamma_\epsilon\,\delta\epsilon\right]$$

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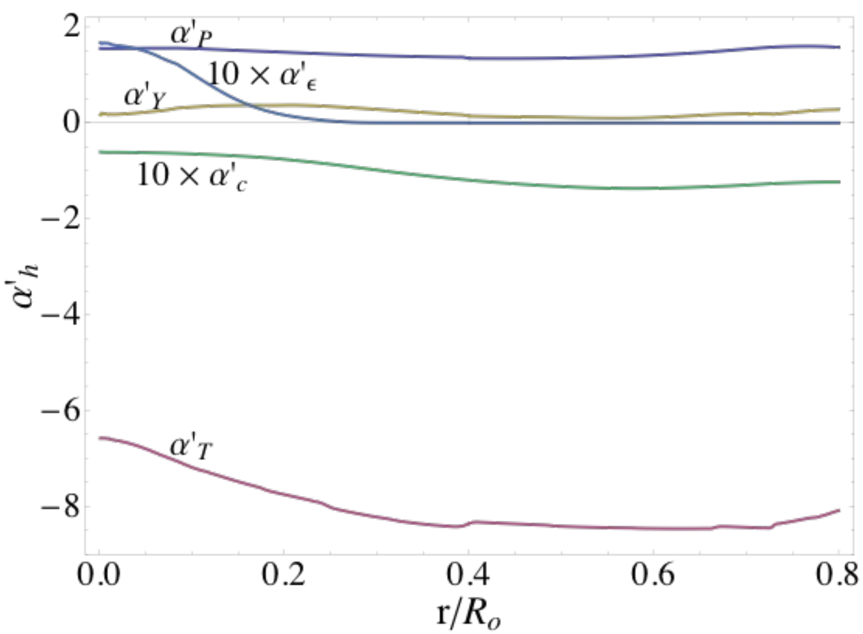
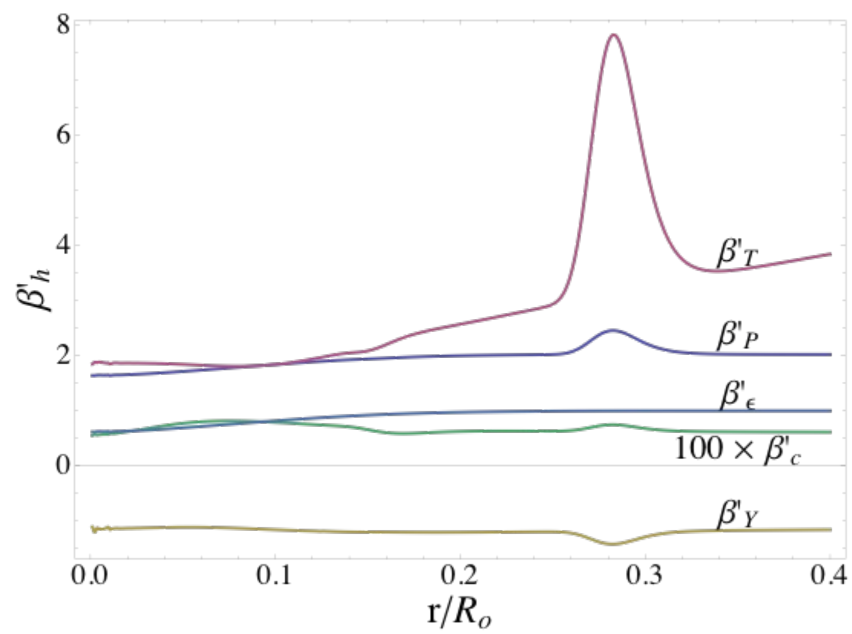
The coefficients γ_h, β'_h and α'_h describes the response of the plasma (EOS, energy generations and radiative transfer) to variation of structural (δm, δP, δL, δT) and chemical properties.





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\end{aligned}
$$

To be solved between with the boundary conditions

At the center of the sun (r = 0)

$$
\begin{aligned}
\delta m &= \gamma_{P,0}\,\delta P_0 + \gamma_{T,0}\,\delta T_0 + \gamma_{Y,0}\,\Delta Y_{\rm ini} + \gamma_{\epsilon,0}\,\delta\epsilon_0\\
\delta P &= \delta P_0\\
\delta T &= \delta T_0\\
\delta l &= \beta'_{P,0}\,\delta P_0 + \beta'_{T,0}\,\delta T_0 + \beta'_{Y,0}\,\Delta Y_{\rm ini} + \beta'_{C,0}\,\delta C + \beta'_{\epsilon,0}\,\delta\epsilon_0
\end{aligned}
$$

At the convective boundary ($r = \overline{R}_b$)

$$
\begin{aligned}
\delta m &= -\overline{m}_{\rm conv}\,\delta C\\
\delta P &= \delta C\\
\delta T &= A'_Y\,\Delta Y_{\rm ini} + A'_C\,\delta C\\
\delta l &= 0
\end{aligned}
$$

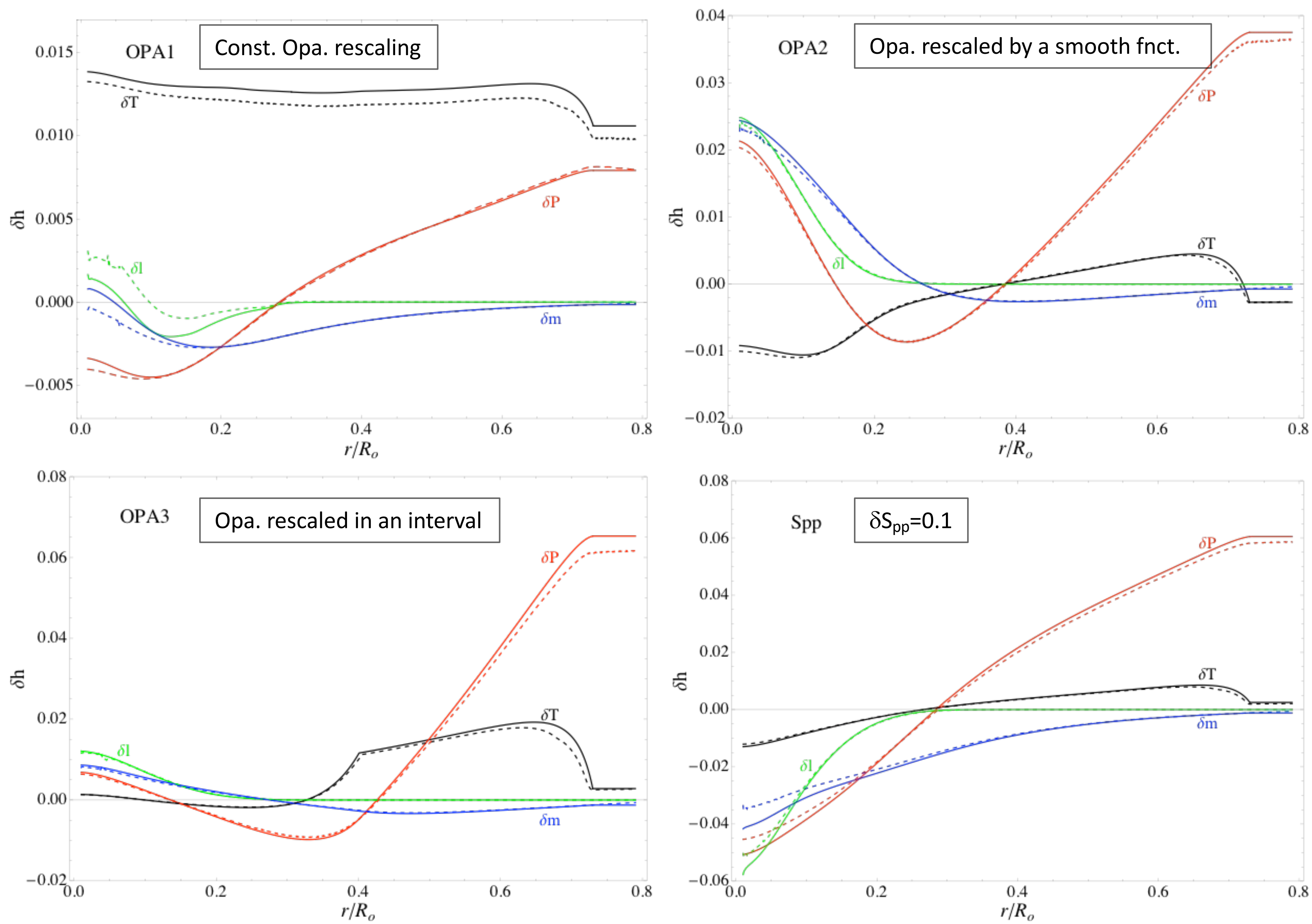
Univocally determine the parameters δP_0, δT_0, ΔY_{ini}, δC

F.L. Villante and B. Ricci, 2010
F.L. Villante , 2010

Linear Solar Models – Validation

F.L. Villante and B. Ricci, 2010, F.L. Villante 2010

Solid - Linear solar models
Dotted - SSMs



The role of opacity

The relation between opacity and metals

$$\frac{d\delta m}{dr} = \frac{1}{l_m}\left[\gamma_P\,\delta P + \gamma_T\,\delta T - \delta m + \gamma_Y\,\Delta Y_{\rm ini}\right]$$
$$\frac{d\delta P}{dr} = \frac{1}{l_P}\left[(\gamma_P - 1)\;\delta P + \gamma_T\,\delta T + \delta m + \gamma_Y\,\Delta Y_{\rm ini}\right]$$
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The **source term** that is responsibile for the modification of the sun (and that can be bounded from obs. data) can be decomposed as:

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \delta\kappa_{\rm Z}(r)$$

Intrinsic opacity change

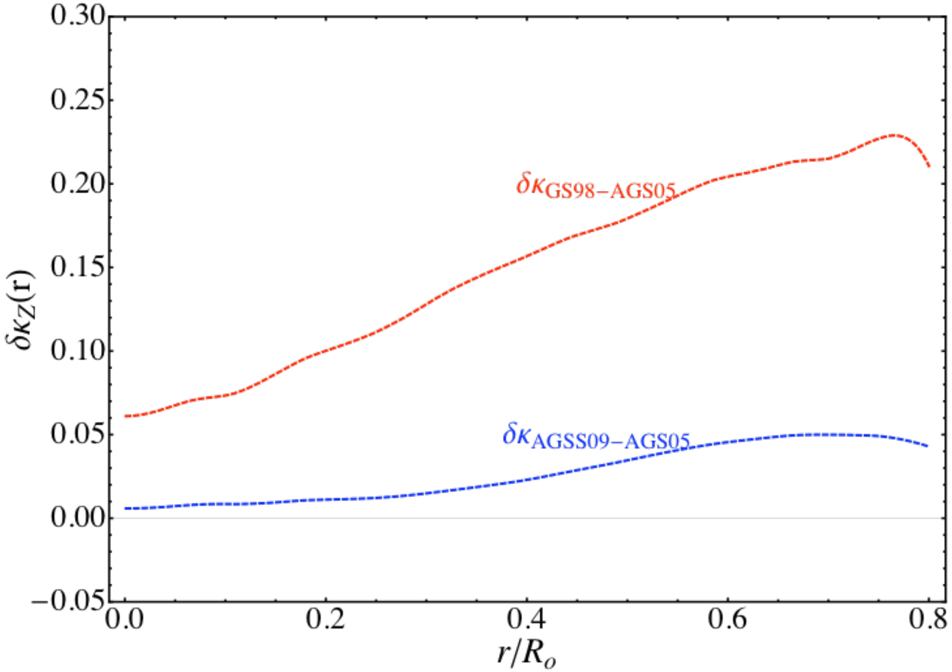
$$\delta\kappa_{\rm I}(r) = \frac{\kappa(\overline{\rho}(r),\overline{T}(r),\overline{Y}(r),\overline{Z}_i(r))}{\overline{\kappa}(\overline{\rho}(r),\overline{T}(r),\overline{Y}(r),\overline{Z}_i(r))} - 1$$

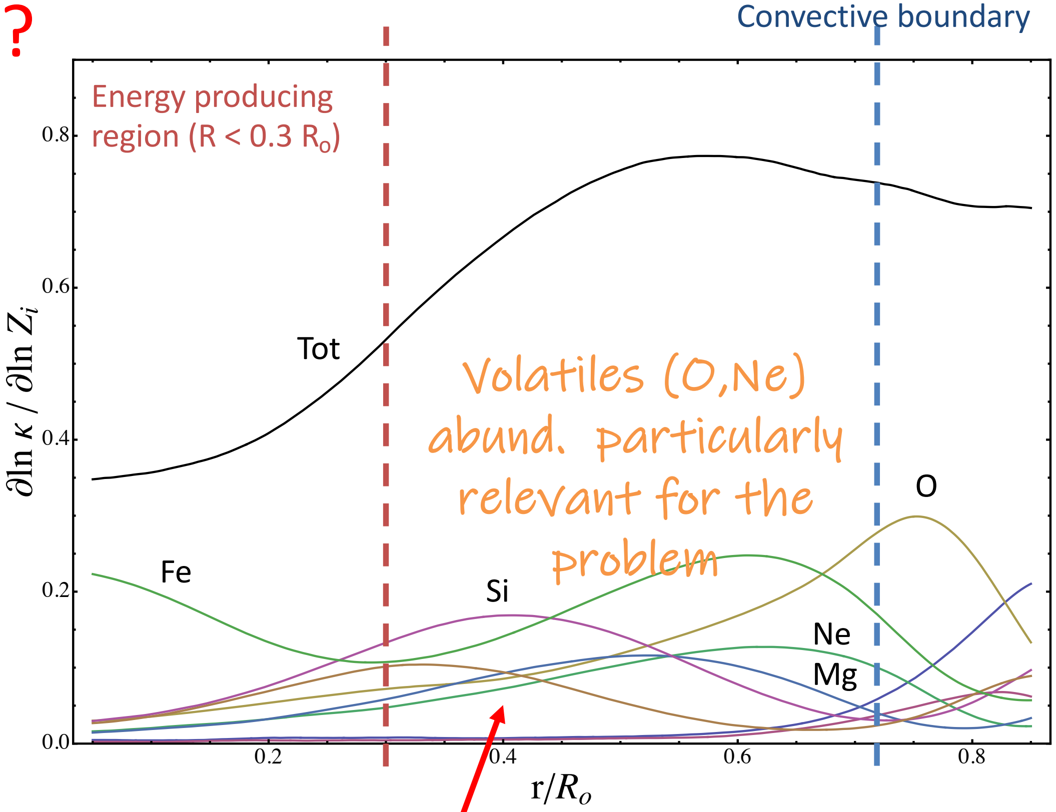
Composition opacity change

$$\delta\kappa_{\rm Z}(r) = \frac{\overline{\kappa}(\overline{\rho}(r),\overline{T}(r),\overline{Y}(r),Z_i(r))}{\overline{\kappa}(\overline{\rho}(r),\overline{T}(r),\overline{Y}(r),\overline{Z}_i(r))} - 1$$

F.L. Villante – Astrophys.J.724:98-110,2010

Why metals are so important?





A change of the solar composition affects the efficiency of radiative energy transfer in the core of the Sun

$$\delta\kappa_Z(r) = \frac{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), Z_i(r))}{\overline{\kappa}(\overline{\rho}(r), \overline{T}(r), \overline{Y}(r), \overline{Z}_i(r))} - 1 \simeq \sum_i \frac{\partial \ln \overline{\kappa}}{\partial \ln Z_i}\, \delta z_i$$

$$\delta z_i = \frac{(Z_{i,b}/X_b) - (\overline{Z}_{i,b}/\overline{X}_b)}{(\overline{Z}_{i,b}/\overline{X}_b)}$$

F.L. Villante – Astrophys.J.724:98-110,2010

The opacity kernels

We study the response of the sun to arbitrary opacity variations:

$$\delta\kappa(r) = \delta\kappa_{\rm I}(r) + \delta\kappa_{\rm Z}(r)$$

If we consider a small variation of the opacity, the sun respond linearly. The variation of a generic quantity Q is then given by:

$$\delta Q = \int dr \boxed{K_Q(r)} \delta\kappa(r)$$

The kernel $K_Q(r)$ are the fundamental tools to discuss, in the **most general terms**, the informations on opacity profile and/or composition provided by present observational data

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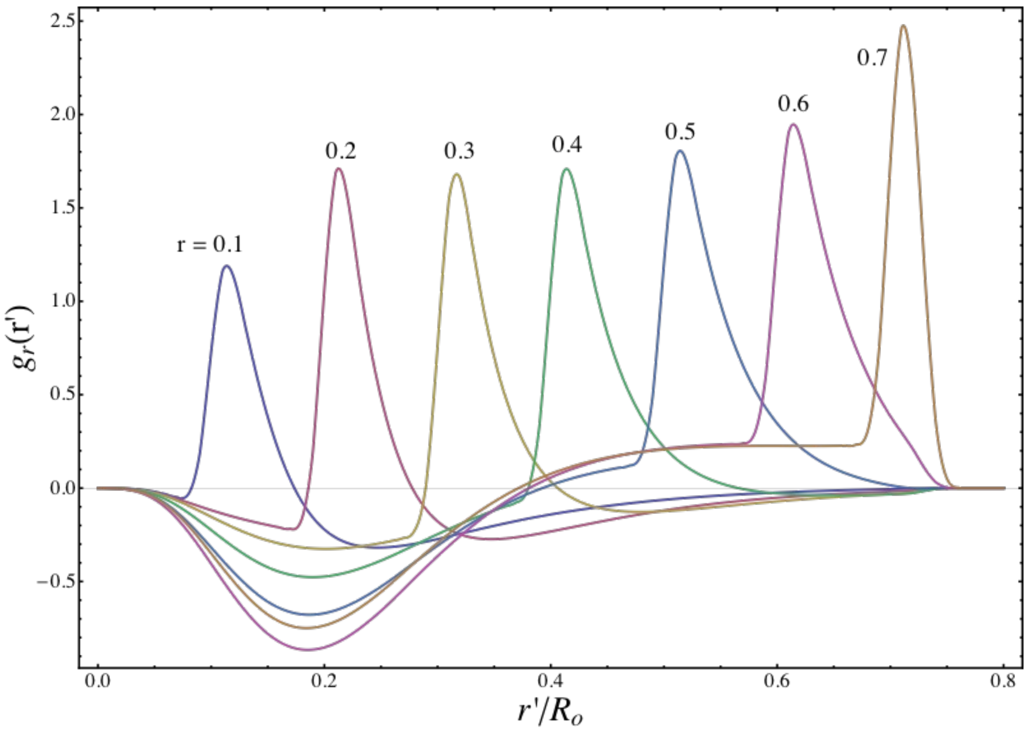
The opacity kernels for the sound speed

$$\delta u(r) = \int dr' \; K_u(r, r') \; \delta\kappa(r')$$



The kernels are not positive definite → **compensating effects** can occur …

$$\delta u_0(r) = \int dr' K_u(r, r') \simeq 0$$

The sound speed is *insensitive to a global rescaling of opacity*

F.L. Villante – Astrophys.J.724:98-110,2010

But what other observables tell us?

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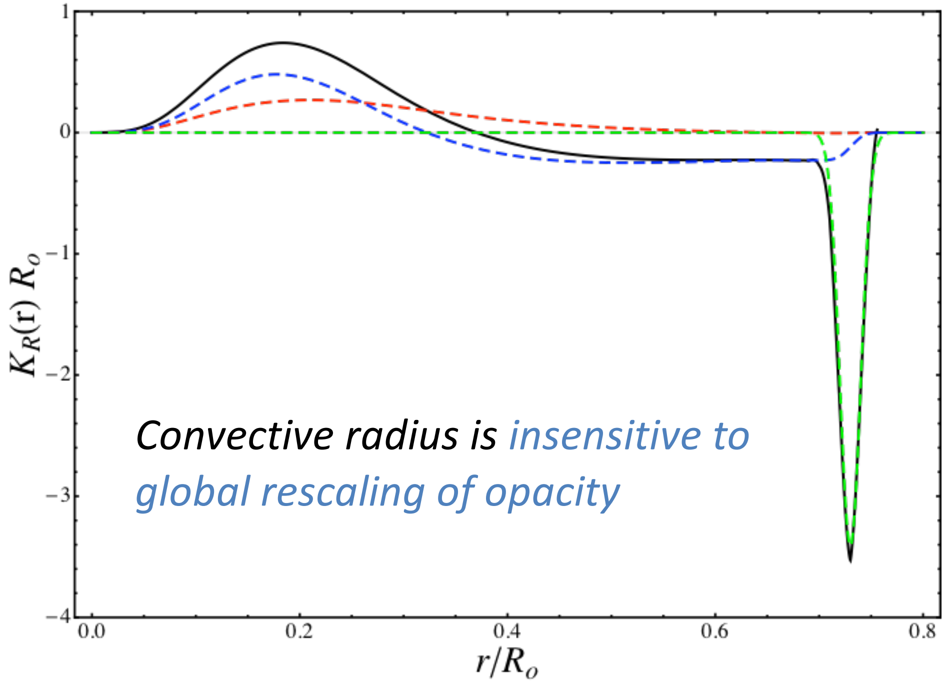
Convective radius:

$$\delta R_{\rm b} = \int dr\ K_{\rm R}(r)\ \delta\kappa(r)$$

$$\delta R_{\rm b} = \Gamma_Y\ \Delta Y_{\rm ini} + \Gamma_C\ \delta C + \Gamma_\kappa\ \delta\kappa_{\rm b}$$

$$\left\{\begin{array}{lcl} \Gamma_Y & = & 0.449 \\ \Gamma_C & = & -0.117 \\ \Gamma_\kappa & = & -0.085 \end{array}\right.$$

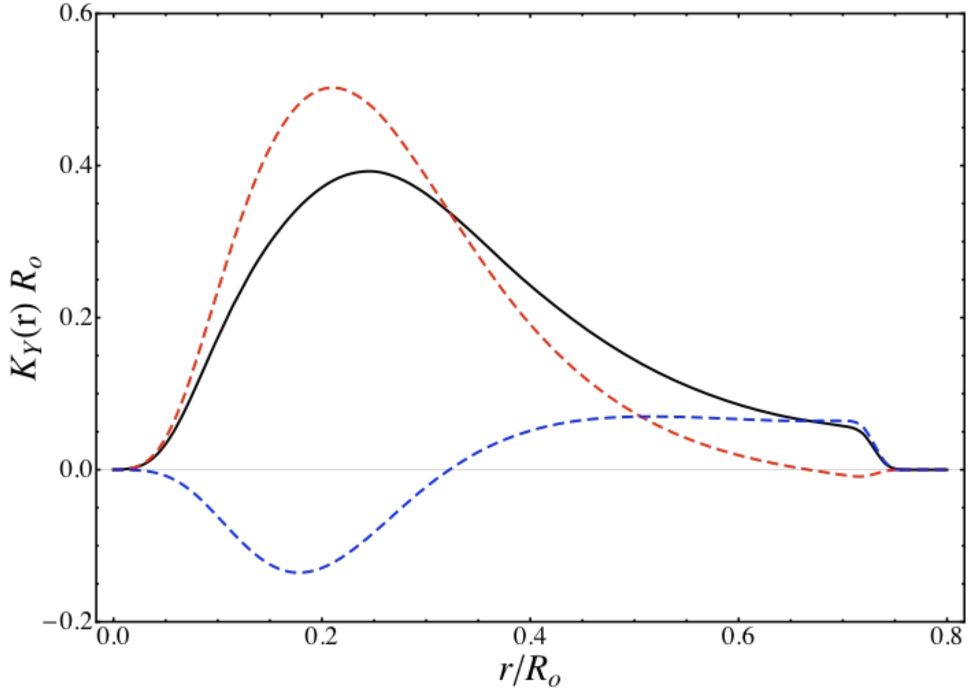


Surface helium:

$$\Delta Y_{\rm b} = \int dr\ K_{\rm Y}(r)\ \delta\kappa(r)$$

$$\Delta Y_{\rm b} = A_{\rm Y}\ \Delta Y_{\rm ini} + A_{\rm C}\ \delta C$$

$$\left\{\begin{array}{lcl} A_Y & = & 0.838 \\ A_C & = & 0.033 \end{array}\right.$$



A model independent relation for $\delta\kappa_b$

F.L. Villante – Astrophys.J.724:98-110,2010

We have that:

$$\Delta Y_{\rm b} = A_{\rm Y}\, \Delta Y_{\rm ini} + A_{\rm C}\, \delta C$$

$$\begin{cases} A_Y &= 0.838 \\ A_C &= 0.033 \end{cases}$$

$$\delta R_{\rm b} = \Gamma_Y\, \Delta Y_{\rm ini} + \Gamma_C\, \delta C + \Gamma_\kappa\, \delta\kappa_{\rm b}$$

$$\begin{cases} \Gamma_Y &= 0.449 \\ \Gamma_C &= -0.117 \\ \Gamma_\kappa &= -0.085 \end{cases}$$

Remenber that: $\delta C = \delta P_{\rm b} = \delta\rho_{\rm b}$

We eliminate ΔY_{ini} from equations and obtain:

$$\delta\kappa_{\rm b} = C_Y\, \Delta Y_{\rm b} + C_{\rm R}\, \delta R_{\rm b} + C_\rho\, \delta\rho_{\rm b}$$

$$\begin{cases} C_Y &= -\dfrac{\Gamma_Y}{A_Y\, \Gamma_\kappa} = 6.27 \\ C_{\rm R} &= \dfrac{1}{\Gamma_\kappa} = -11.71 \\ C_\rho &= \dfrac{1}{\Gamma_\kappa}\left[\dfrac{A_C\, \Gamma_Y}{A_Y} - \Gamma_C\right] = -1.58 \end{cases}$$

Model independent:
no parametrization for $\delta\kappa(r)$ is needed

By using $\delta R_{\rm b} = -0.0205 \pm 0.0015$, $\Delta Y_{\rm b} = 0.0195 \pm 0.0034$ and $\delta\rho_{\rm b} = 0.08$:

$$\delta\kappa_{\rm b} \simeq 0.24 \pm 0.03$$

Solar neutrino fluxes - opacity kernels

F.L. Villante – **Astrophys.J.724:98-110,2010**

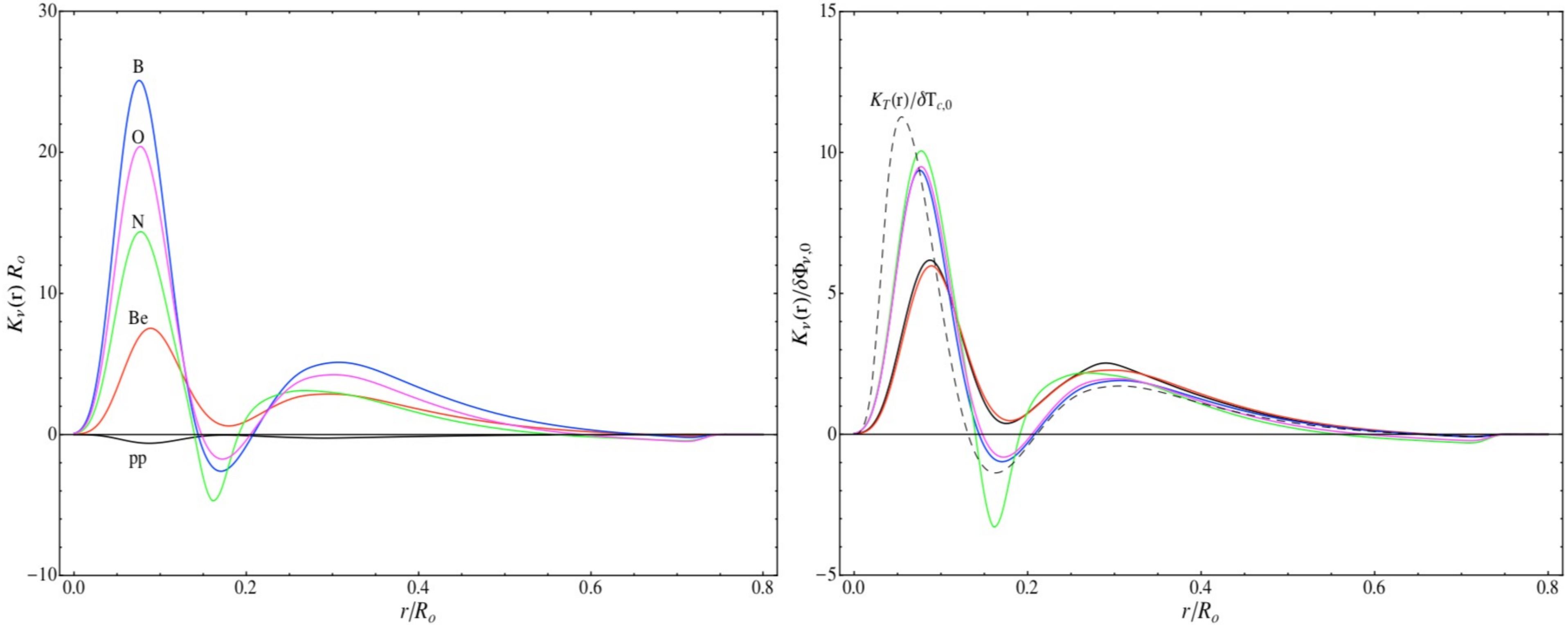


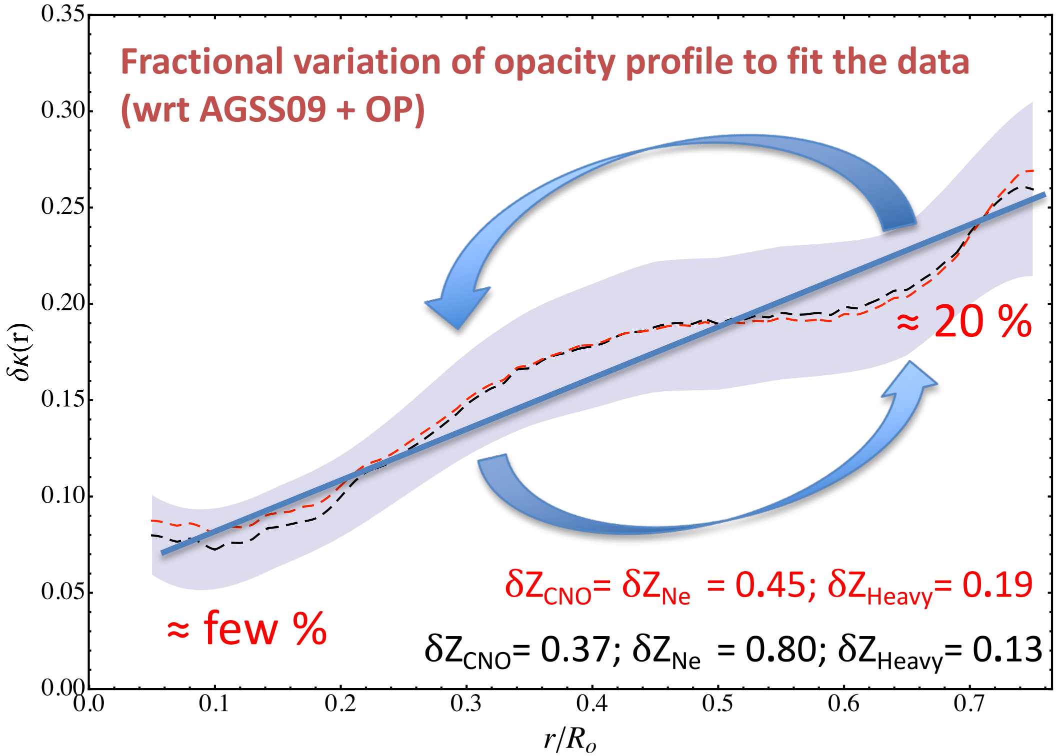
Figure 6. Left panel: the solar neutrino kernels $K_\nu(r)$ defined in Equation (32). Right panel: the solid lines are the normalized solar neutrino kernels $K_\nu(r)/\delta\Phi_{\nu,0}$. The dashed line shows the normalized kernel $K_T(r)/\delta T_{c,0}$ defined in Equation (36), that describes the response of the solar central temperature to localized opacity modifications.

The solar opacity profile

The **"optimal" opacity profile** of the Sun appears to be well-constrained by the **combination of all solar observational data**

Note that:

- The **sound speed** and the **convective radius** determine the tilt of $\delta\kappa(r)$ (but not the scale)
- The **surface helium** and the **neutrino fluxes** determine the scale for $\delta\kappa(r)$ but not the tilt



*F.L. Villante and B. Ricci - **Astrophys.J.714:944-959,2010***
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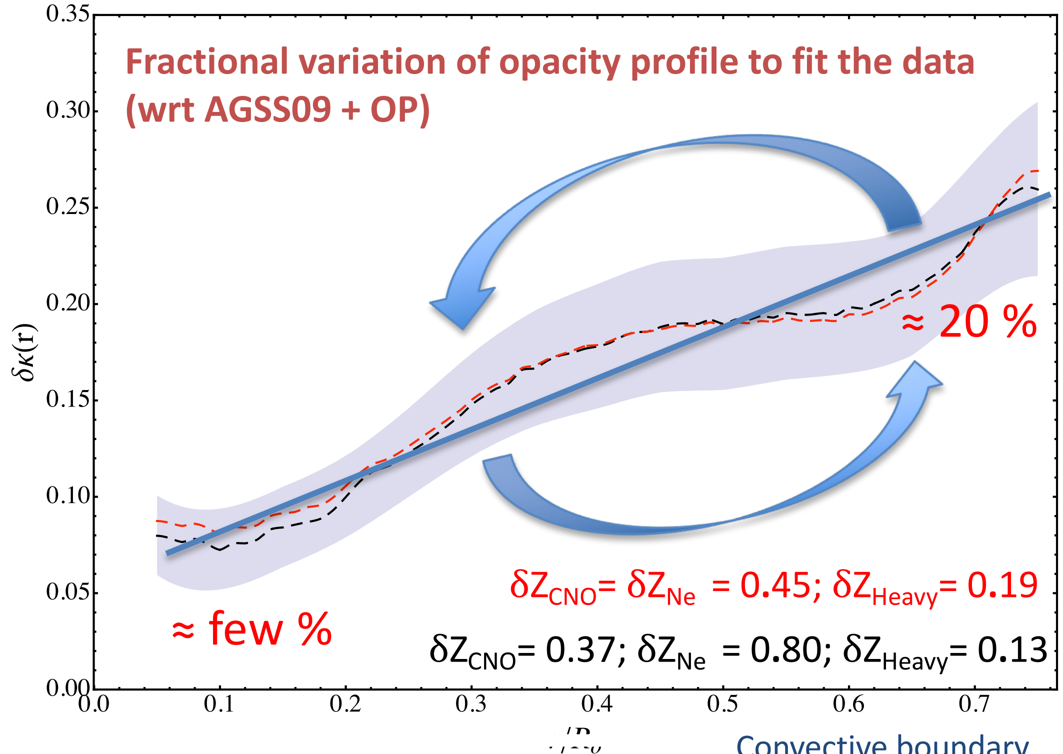
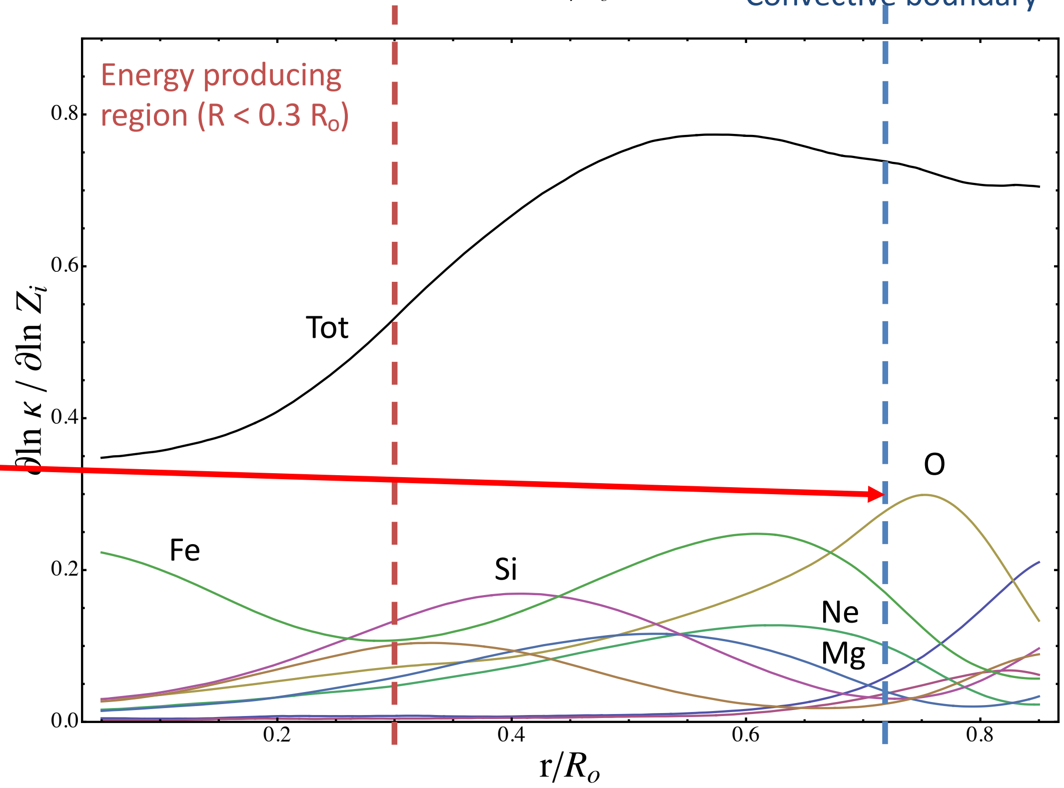
… but this can be equally obtained by **intrinsic** or **composition** opacity changes (being, in particular, **oxygen** the most «effective» element)





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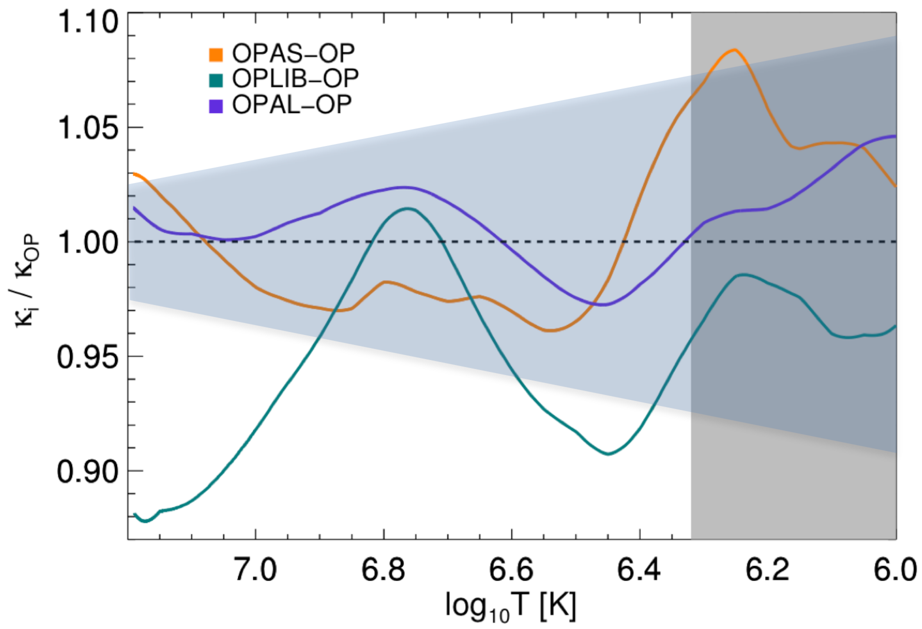
Paramaterizing uncertainty in opacity calculations …

[Vinyoles et al, *Astrophys.J.* 835 (2017) 2, 202]

Opacity uncertainty in B16-SSMs is parameterized as:

$$\delta\kappa(T) = \kappa_a + (\kappa_b/\Delta)\ \ln(T/T_C)$$

κ_a, κ_b = random variables
(means equal to 0 and variances σ_a= 0.02 and σ_b= 0.067)



This prescription is motivated by:

- Opacity calculations more accurate at the solar core (~2%) than at the base of the convective envelope (~7%);

- It avoids underestimating the opacity error contribution to sound speed and convective radius (sensitive to tilt and not to scale of opacity)

… but **it still remains** a very simplified description of the real situation

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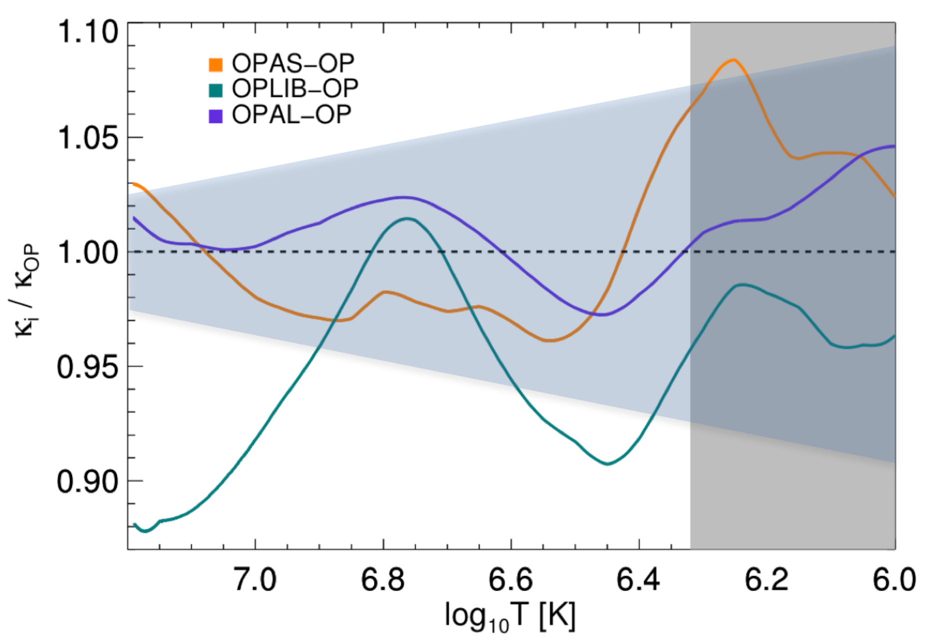

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… but **it still remains** a very simplified description of the real situation

Since the shape is as important as the magnitude, it would be important to have **physically motivated parameterizations** of **radiative opacity uncertainties** (and their **correlations**) along the solar structure.

Conclusions

- SSM is a fully predictive framework (that brilliantly survived the solar neutrino problem). It can now be tested with high precision by helioseismology and solar neutrino data

- Solar neutrino physics entered the precision era and Borexino has opened the way to CNO neutrino detection

- Some unsolved puzzles could be addressed in the future → (Present and future) CNO neutrino measurements, combined with precise determinations of ^{8}B and ^{7}Be fluxes, can shed light on the solar abundance problem

- To exploit the full potential of future measurements and to go beyond the SSM framework → improvements in constitutive physics are still needed [nuclear cross sections and radiative opacities]

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Finanziato dall'Unione europea
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Thank you

Probing solar composition with neutrinos

By considering

$$\frac{R_{\rm CNO}^{\rm Bx}}{R_{\rm CNO}^{\rm SSM}} = \frac{R_{^{15}\rm O}^{\rm Bx}}{R_{^{15}\rm O}^{\rm SSM}} = \frac{\Phi_{^{15}\rm O}^{\rm Bx}}{\Phi_{^{15}\rm O}^{\rm SSM}} = 1.35\,^{+0.24}_{-0.16}$$

Borexino CNO neutrino signal (scaled to GS98 prediction)
[Borexino: PRL 2022, arXiv: 2307.14636]

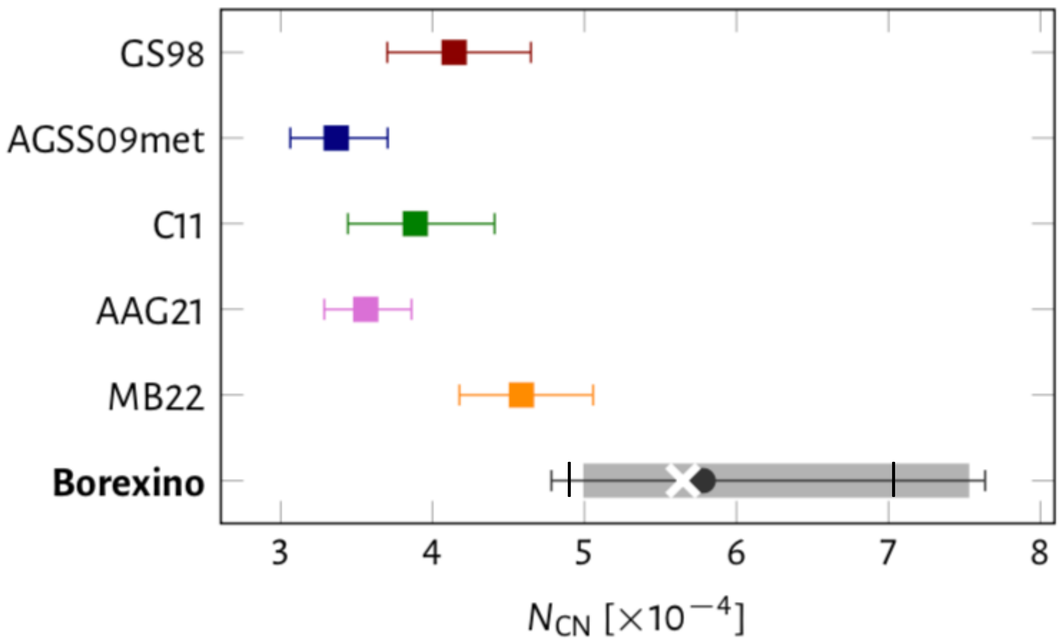
$$\frac{\Phi_{^{8}\rm B}^{\rm global}}{\Phi_{^{8}\rm B}^{\rm SSM}} = 0.96 \pm 0.027$$

^{8}B flux determined from global analysis (scaled to GS98 prediction)

One obtains:

$$\frac{(N_{\rm C}+N_{\rm N})/N_{\rm H}}{[(N_{\rm C}+N_{\rm N})/N_{\rm H}]^{\rm SSM}} = 1.35 \times (0.96)^{-0.769} \times$$
$$\times\left[1 \pm \left(\,^{+0.18}_{-0.12}({\rm CNO}) \pm 0.097({\rm nucl}) \pm 0.023(^{8}{\rm B}) \pm 0.005({\rm env}) \pm 0.027({\rm diff}) \pm 0.022({\rm O/N})\right)\right]$$

Note: reduced error wrt Borexino, PRL 2022



N.B.
This determination is robust wrt to environmental parameters variations (including opacity).

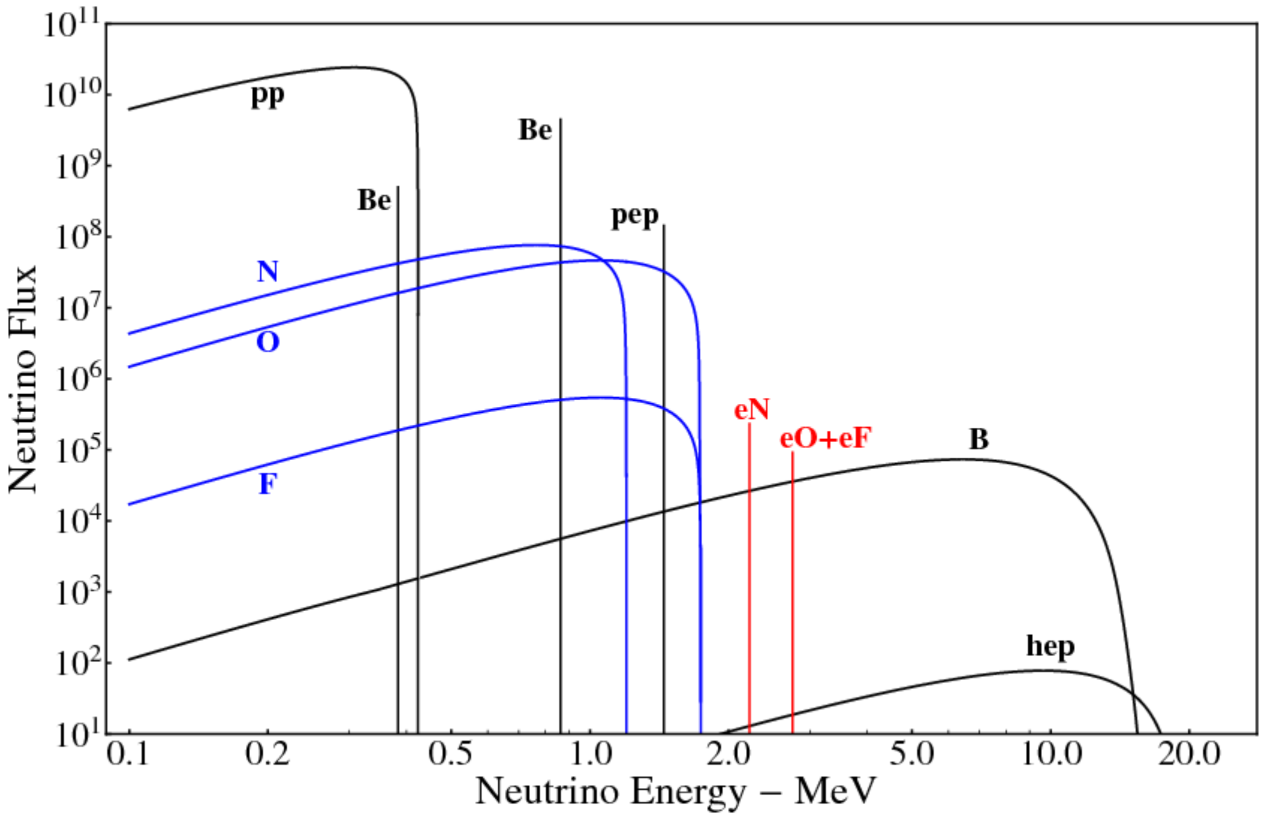
Only limited by nuclear reaction uncertainties:

$S_{114} \rightarrow 7.6\,\%$
$S_{17} \rightarrow 3.5\,\%$
$S_{34} \rightarrow 3.4\,\%$

ecCNO neutrinos

In the CN-NO cycle, besides the conventional CNO neutrinos (blue lines), monochromatic ecCNO neutrinos (red lines) are also produced by **electron capture** reactions:

$$
\begin{array}{llll}
{}^{13}\mathrm{N} + e^- & \rightarrow & {}^{13}\mathrm{C} + \nu_e & E_\nu = 2.220\ \mathrm{MeV} \\
{}^{15}\mathrm{O} + e^- & \rightarrow & {}^{15}\mathrm{N} + \nu_e & E_\nu = 2.754\ \mathrm{MeV} \\
{}^{17}\mathrm{F} + e^- & \rightarrow & {}^{17}\mathrm{O} + \nu_e & E_\nu = 2.761\ \mathrm{MeV}
\end{array}
$$



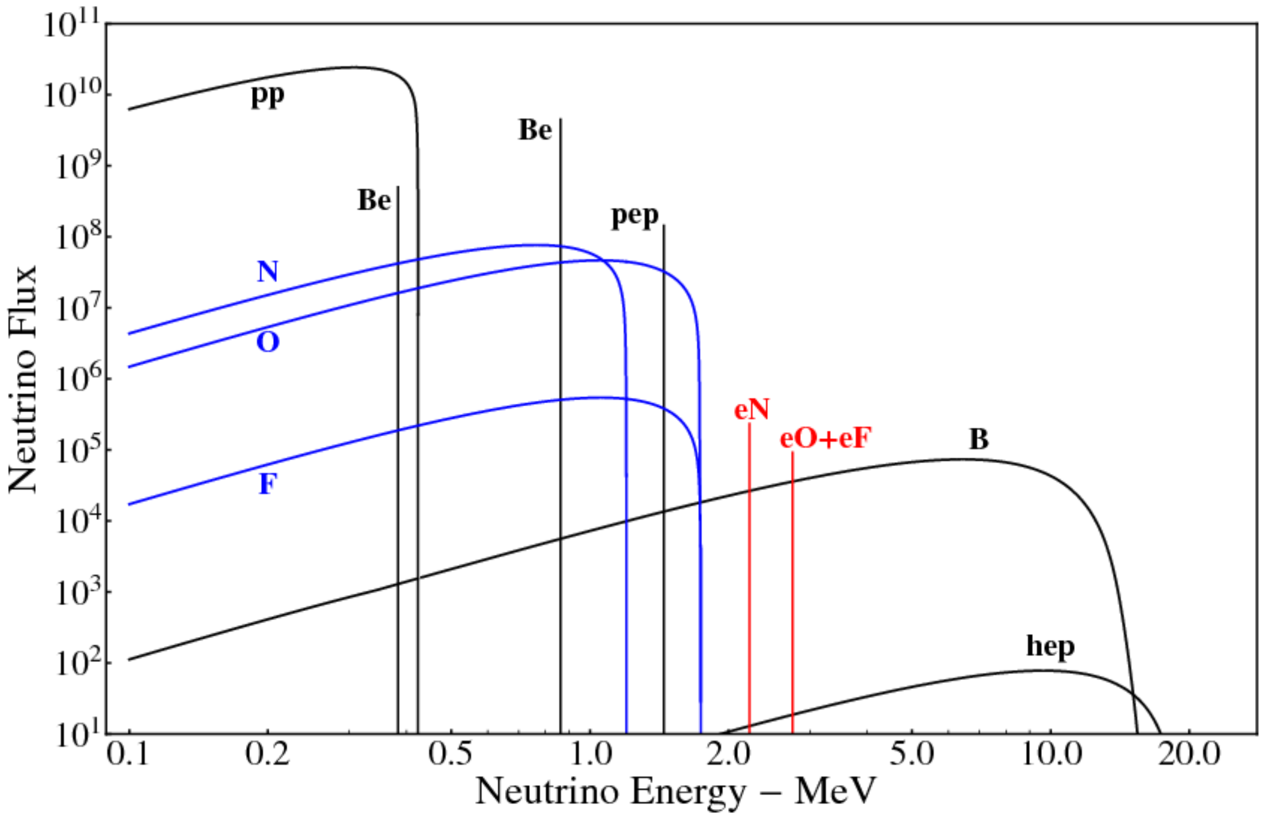
F.L. Villante, PLB 742 (2015) 279-284
L.C. Stonehill et al, PRC 69, 015801 (2004)
J.N. Bahcall, PRD 41, 2964 (1990).

ecCNO neutrinos

The ecCNO fluxes are extremely low: $\Phi_{ecCNO} \approx (1/20)\ \Phi_B$. Detection is extremely difficult but could be rewarding. Indeed:

- ecCNO neutrinos are sensitive to the **metallic content of the solar core** (same infos as CNO neutrinos);

- Being monochromatic, they probe the solar neutrino **survival probability** at specific energies ($E_\nu \cong 2.5$ MeV) exactly **in the transition region**.



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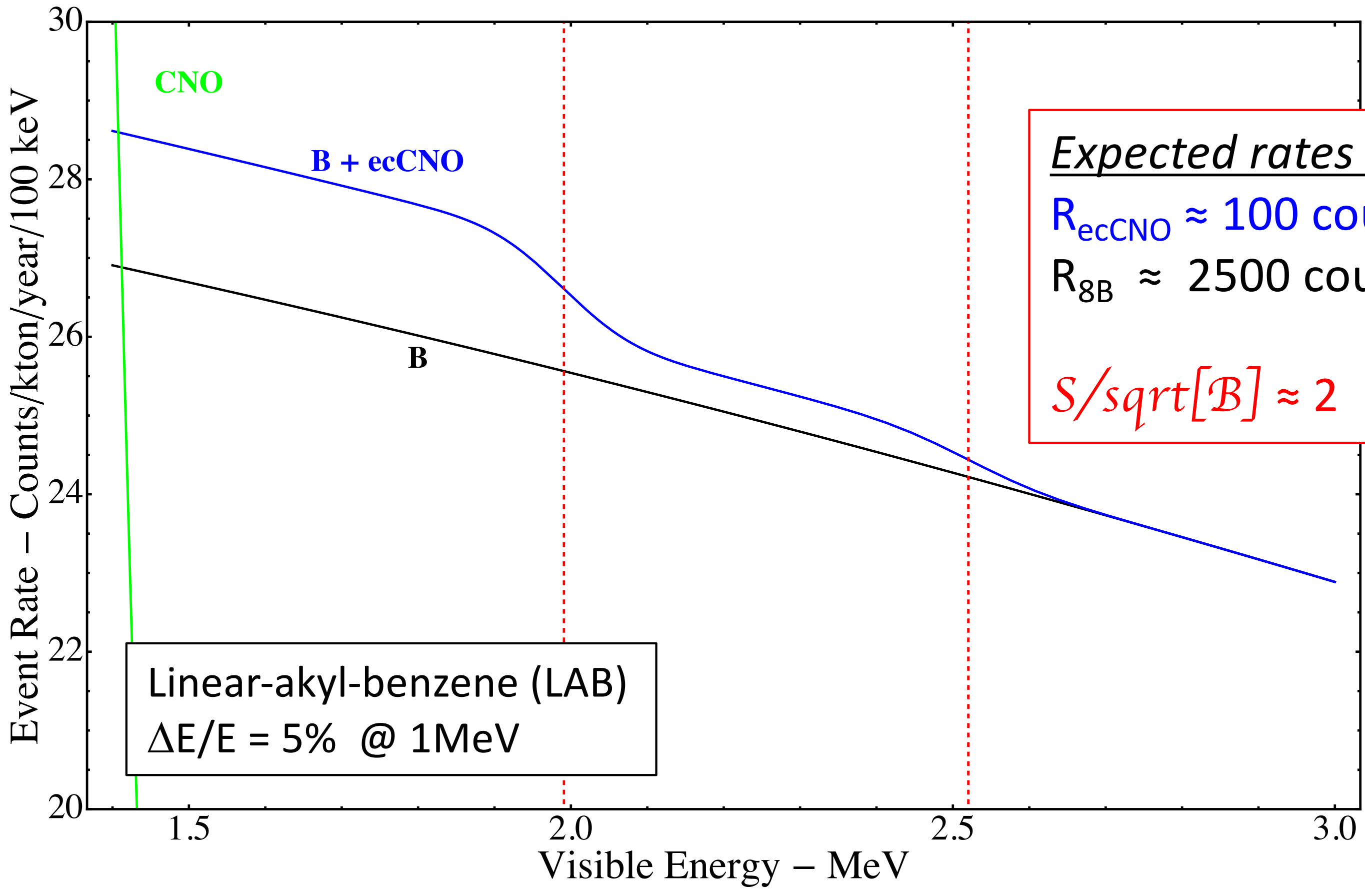
Expected rates in Liquid Scintillators

- $\nu - e$ elastic scattering of ecCNO neutrinos produces Compton shoulders (smeared by energy resolution) at 2.0 and 2.5 MeV;

- ecCNO neutrino signal has to be extracted statistically from the (irreducible) ^{8}B neutrino background.



<u>*Expected rates [1.5 MeV, 2.5 MeV]*</u>

$R_{ecCNO} \approx$ 100 counts/10 kton/year

$R_{8B} \approx$ 2500 counts/10kton/year

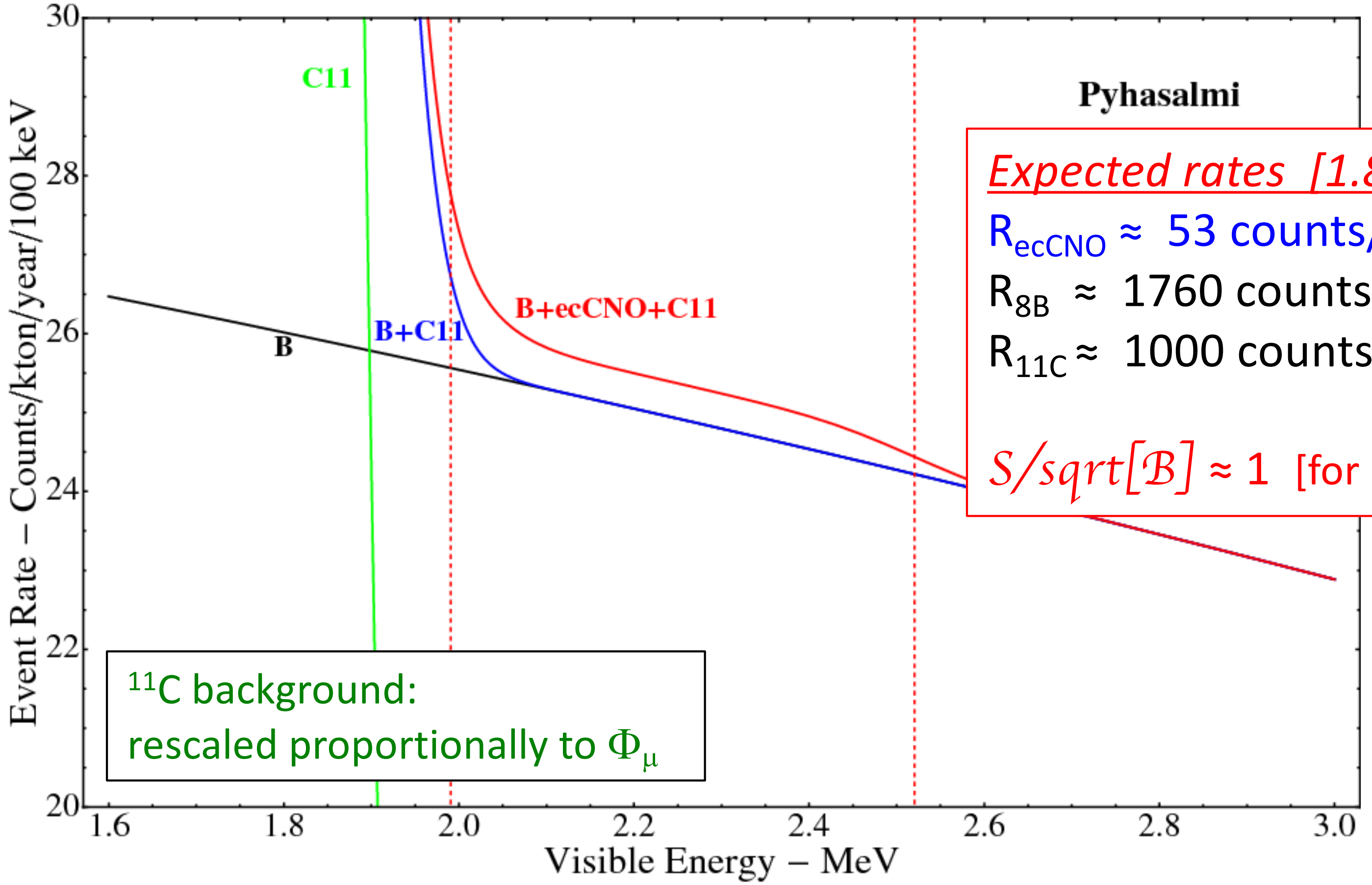
$S/sqrt[\mathcal{B}] \approx 2$ [for 10kton × year exposure]

F.L. Villante, Phys.Lett. B742 (2015) 279-284

Expected rates in Liquid Scintillators

Additional background sources:

- **Intrinsic:** negligible/tagged (with Borexino Phase-I radio-purity levels);
- **External:** reduced by self-shielding (Fid. mass reduced from 50 to ≈20 kton in LENA);
- **Cosmogenic:** ^{11}C overlap with the observation window.



Expected rates [1.8 MeV, 2.5 MeV]

$R_{ecCNO} \approx$ 53 counts/10 kton/year

$R_{8B} \approx$ 1760 counts/10kton/year

$R_{11C} \approx$ 1000 counts/10kton/year

$S/sqrt[\mathcal{B}] \approx 1$ [for 10kton × year exposure]

Signal comparable to stat. fluctuations for exposures 10 kton × year or larger.

100 counts / year above 1.8 MeV in 20 kton detector → 3σ detection in 5 year in LENA

F.L. Villante, Phys.Lett. B742 (2015) 279-284

Removing composition-opacity degeneracy

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition.** *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using **^{8}B-neutrinos as solar thermometer;**
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^{8}B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013
See also (application to BX obs. rate):
Agostini et al, EPJ 2021
Villante & Serenelli, Frontiers 2021

$$
\begin{aligned}
\varphi_{^{15}\mathrm{O}} / \varphi_{^{8}\mathrm{B}}^{0.769} = & \boxed{x_{\mathrm{C}}^{0.802}\, x_{\mathrm{N}}^{0.204}}\, x_{D}^{0.181} \\
& \times \left[x_{S_{11}}^{-0.866}\, x_{S_{33}}^{0.345}\, x_{S_{34}}^{-0.689}\, x_{S_{e7}}^{0.769}\, x_{S_{17}}^{-0.791}\, x_{S_{hep}}^{0.000}\, x_{S_{114}}^{1.046}\, x_{S_{116}}^{0.001} \right] \quad \text{(nucl)} \\
& \times \left[x_{\mathrm{Age}}^{0.313}\, x_{L_{\odot}}^{0.602}\, x_{\kappa_a}^{0.018}\, x_{\kappa_b}^{-0.050} \right] \quad \text{(solar)} \\
& \times \left[x_{\mathrm{O}}^{0.006}\, x_{\mathrm{Ne}}^{-0.003}\, x_{\mathrm{Mg}}^{-0.003}\, x_{\mathrm{Si}}^{0.001}\, x_{\mathrm{S}}^{0.001}\, x_{\mathrm{Ar}}^{0.001}\, x_{\mathrm{Fe}}^{0.005} \right] \quad \text{(met)}
\end{aligned}
$$

Probing solar composition with neutrinos

By considering

$$\frac{R^{Bx}_{CNO}}{R^{SSM}_{CNO}} = \frac{R^{Bx}_{^{15}O}}{R^{SSM}_{^{15}O}} = \frac{\Phi^{Bx}_{^{15}O}}{\Phi^{SSM}_{^{15}O}} = 1.35^{+0.24}_{-0.16}$$

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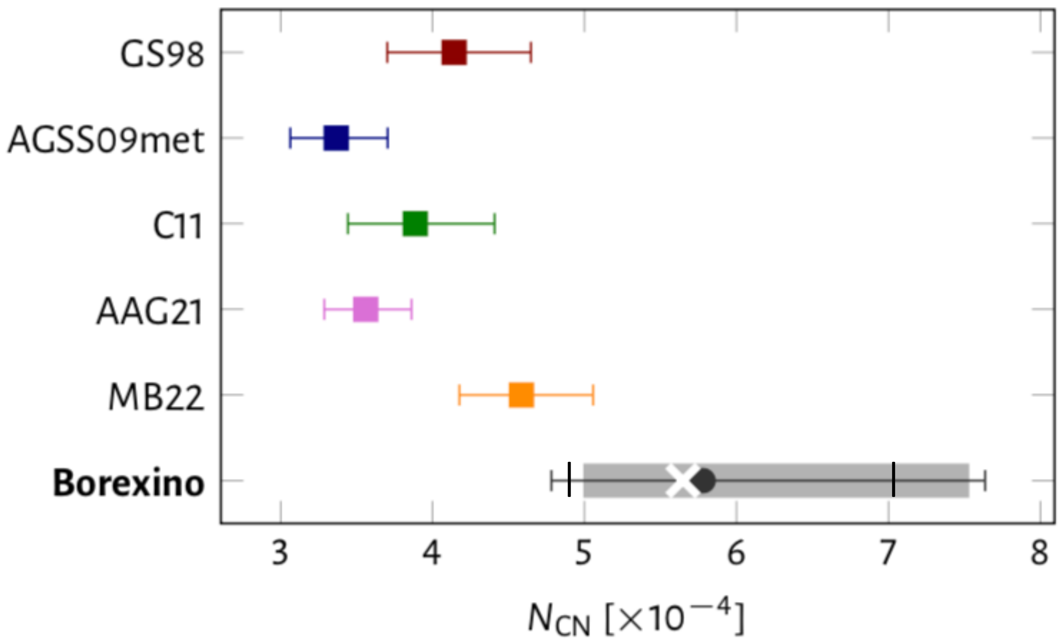

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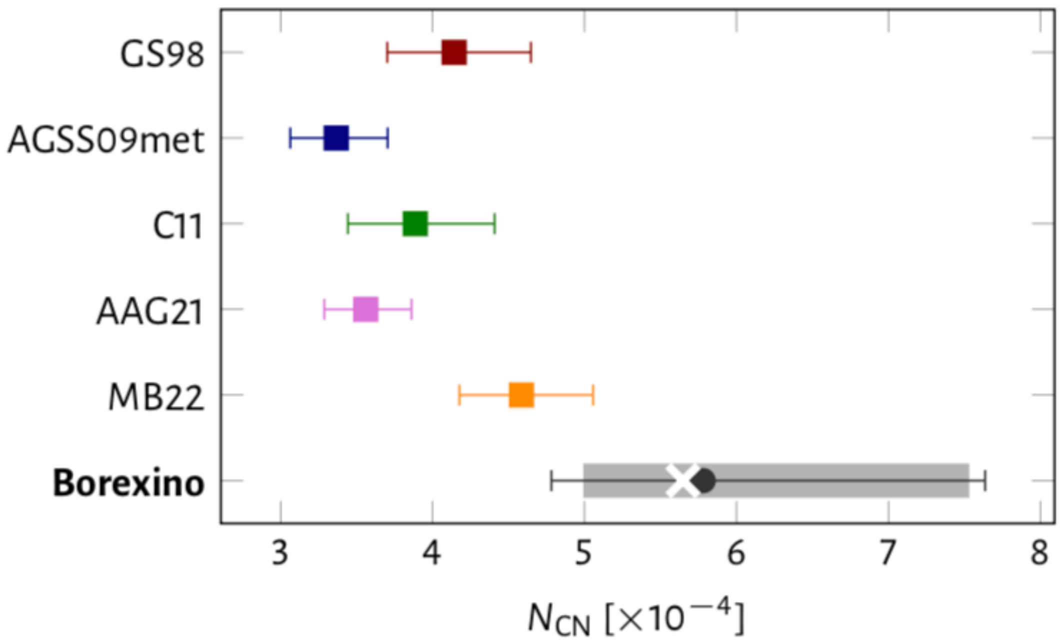

Error contributions