

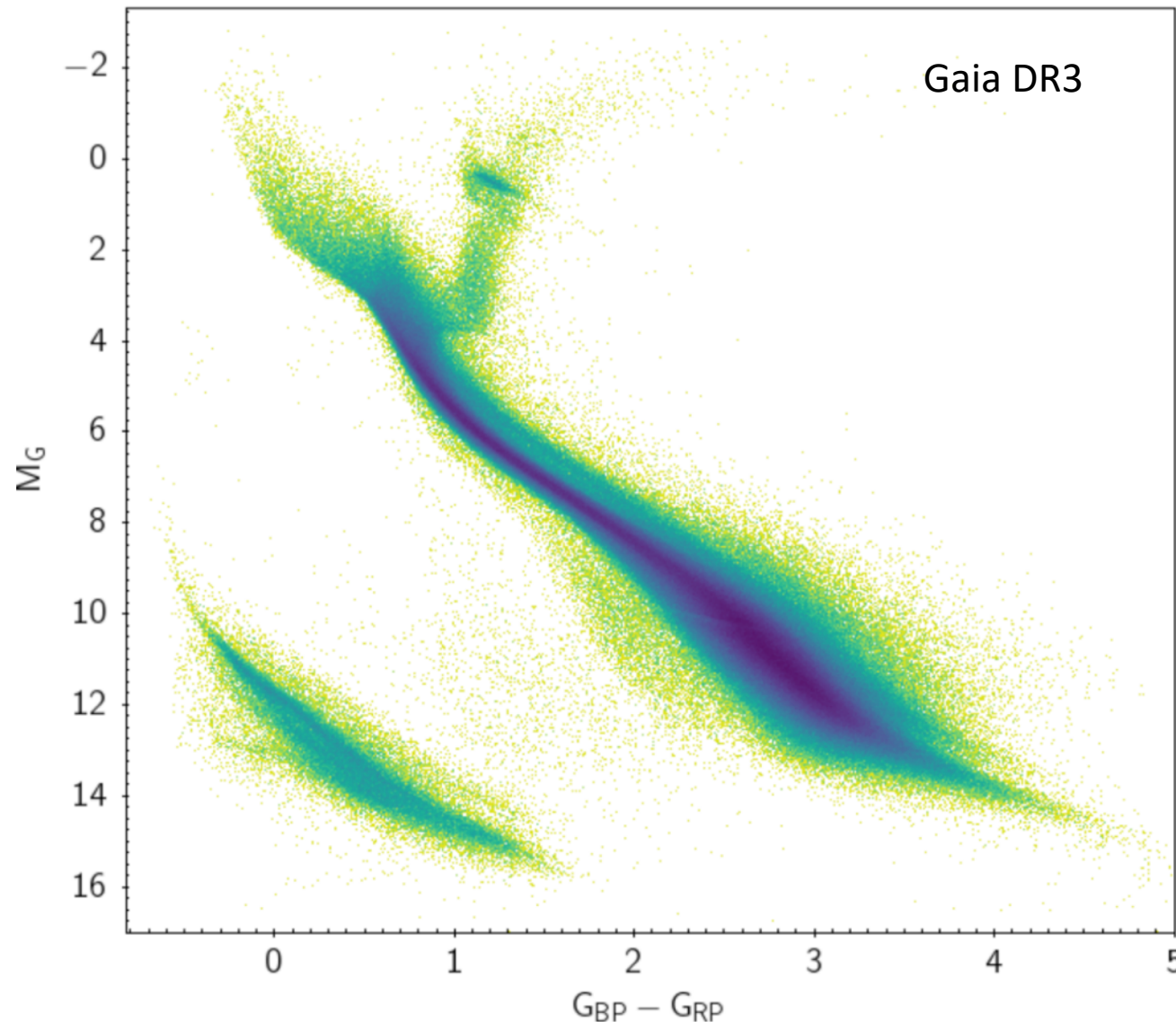
The standard solar model and related stuff

Interdisciplinary Physics of the Sun
W.-E. Heraeus – June 30th–July 4th - 2025

A. Serenelli

-

Why the Sun? It is “foundation” science

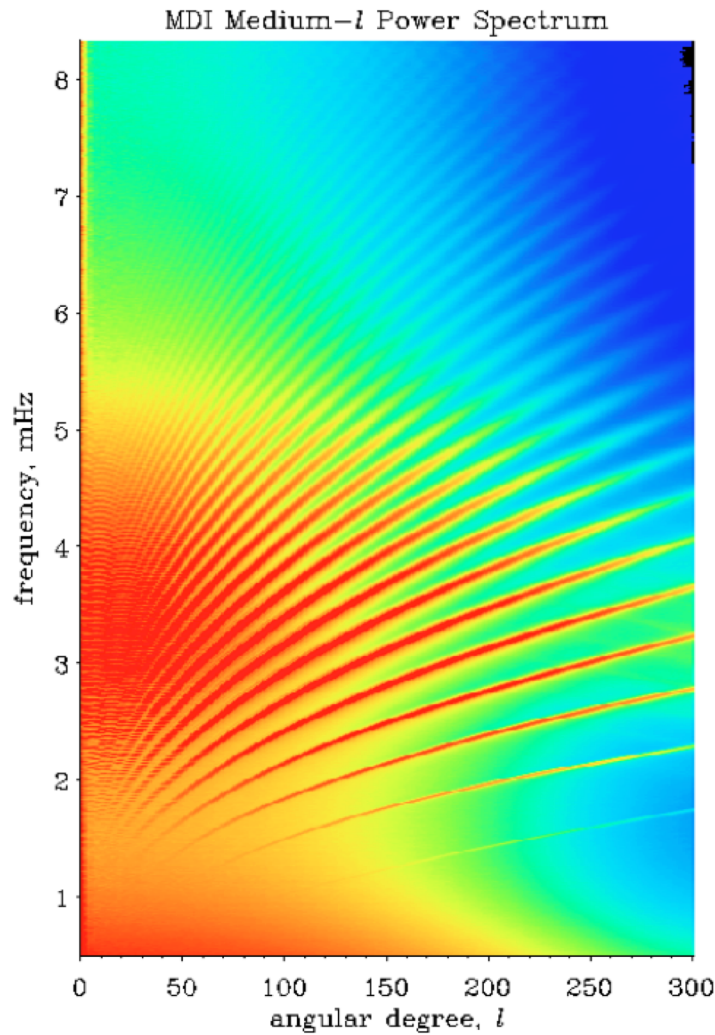


~ 10^9 individual stars with measurements
colors, temperature, luminosity, (composition)

~ 10^3 with accurate, precise, (model) independent
mass determinations
selective club: eclipsing binaries

**1 star with accurate, precise, (model) independent
age determination
meteoritic dating
+ highly accurate radius & mass**

Why the Sun? It is “foundation” science



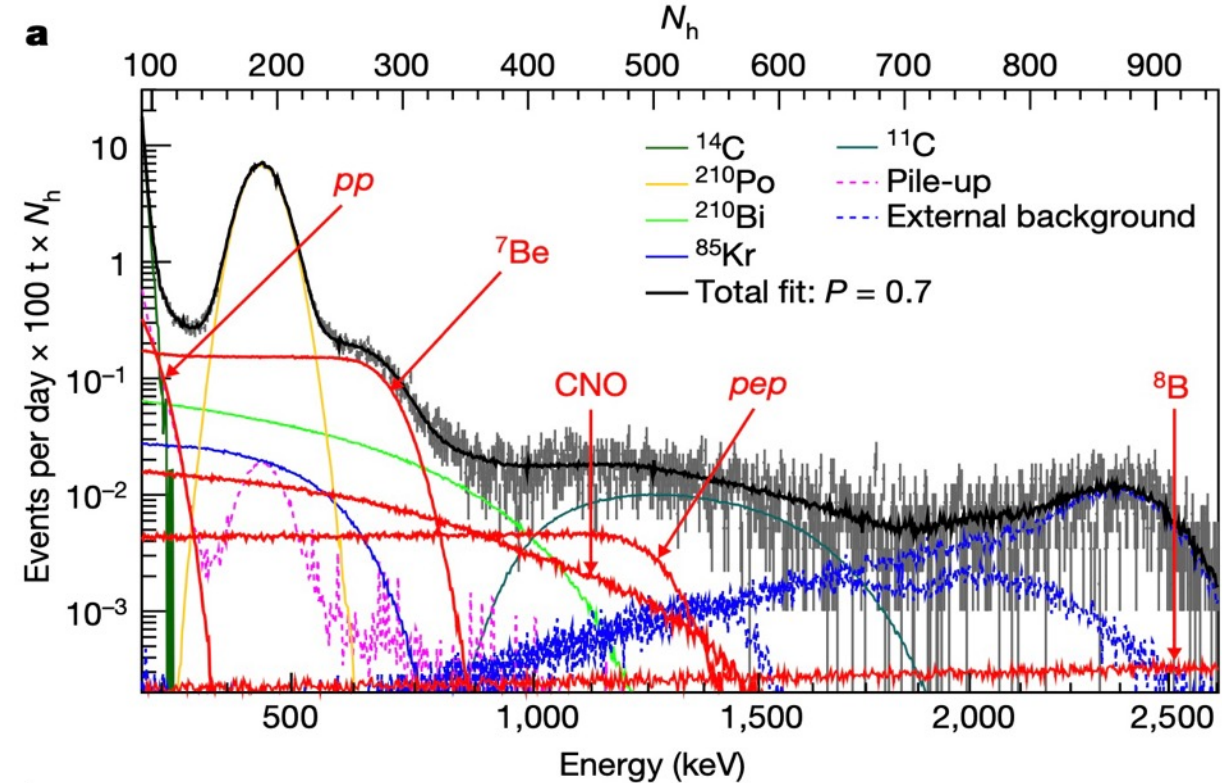
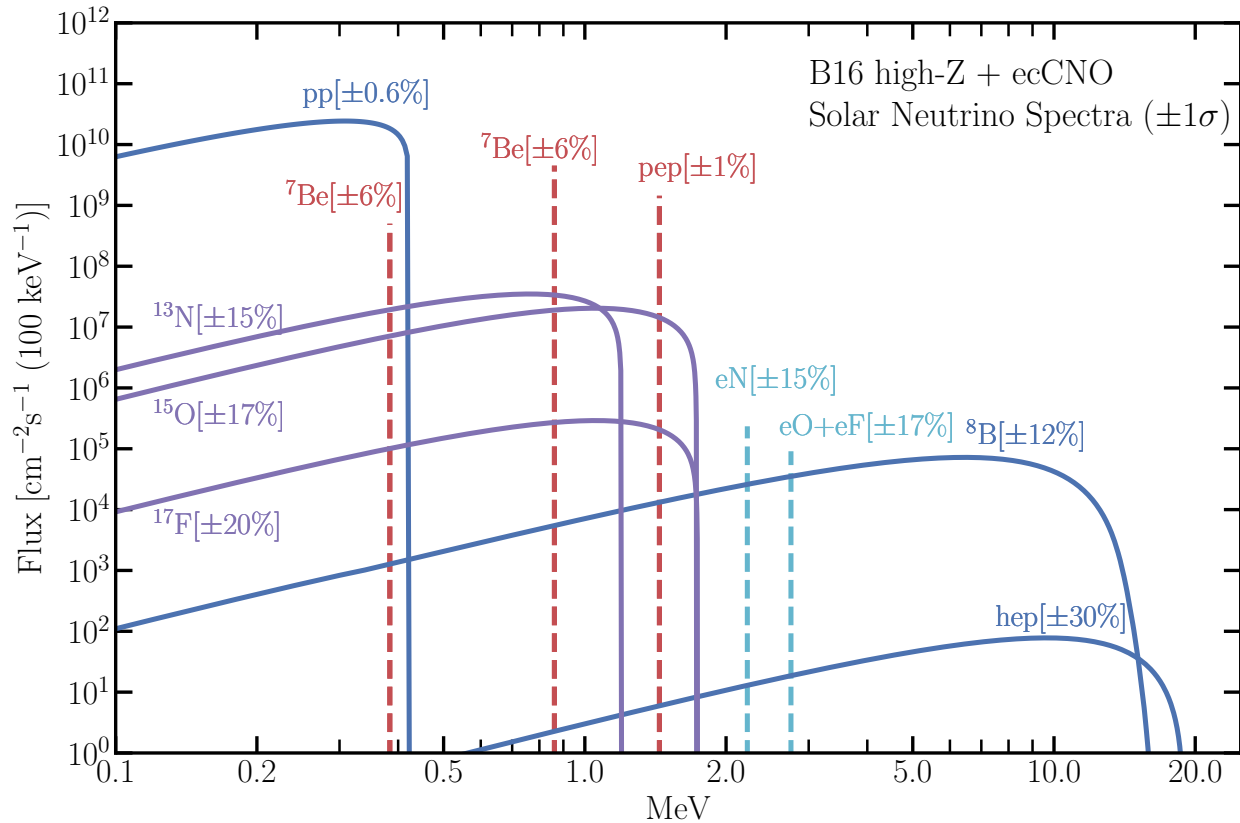
Helioseismology

- $>10^5$ eigenmodes → inversion of internal structure:
sound speed, density, adiabatic index (EoS)
- global quantities:
surface helium, depth of convective envelope
- beyond standard solar models:
internal rotation profile (depth and latitude)

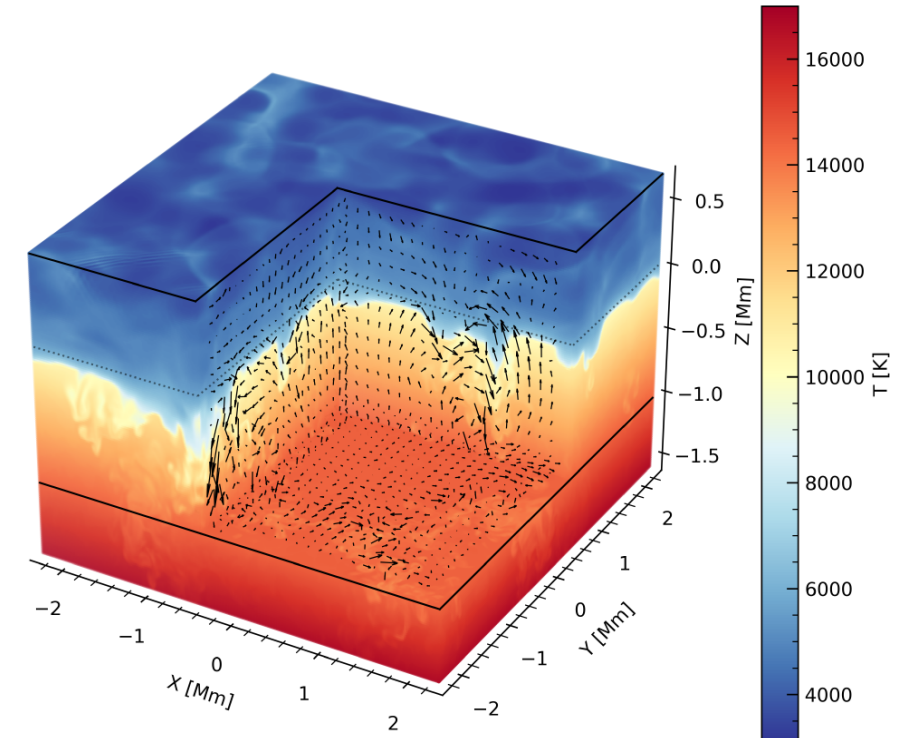
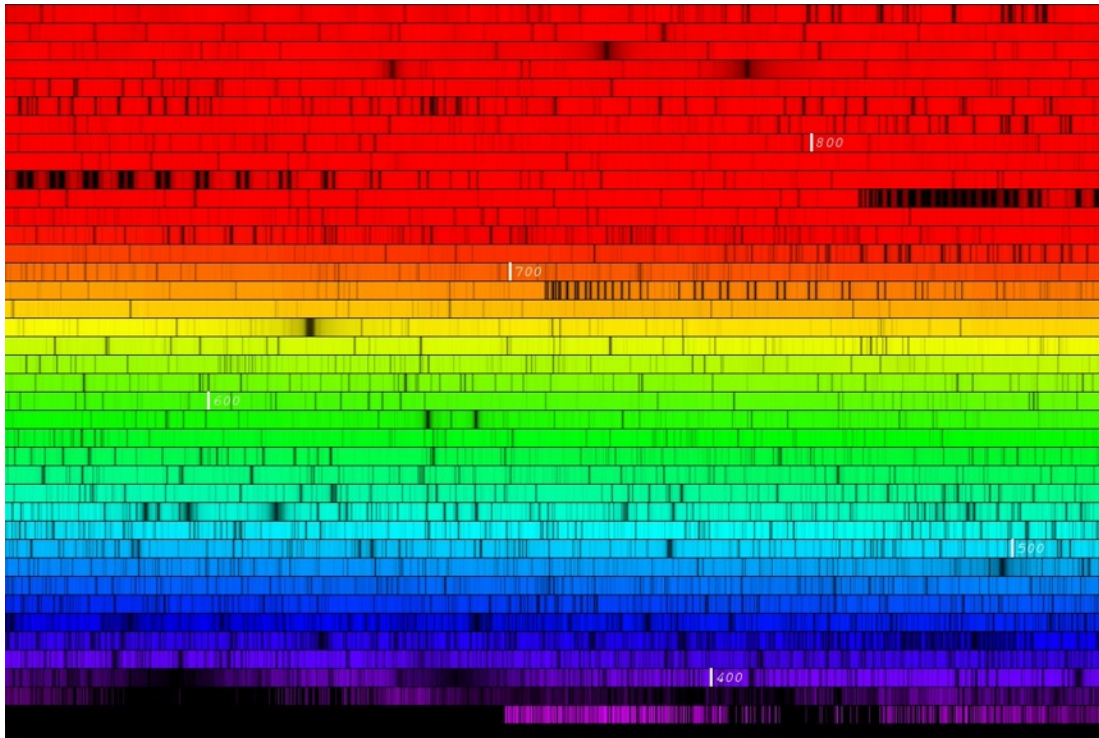
Allows testing theory of stellar evolution by looking at internal structure

Why the Sun? It is “foundation” science

Solar neutrinos → information on solar core, nuclear physics



Borexino solar neutrino spectrum
and identified solar fluxes

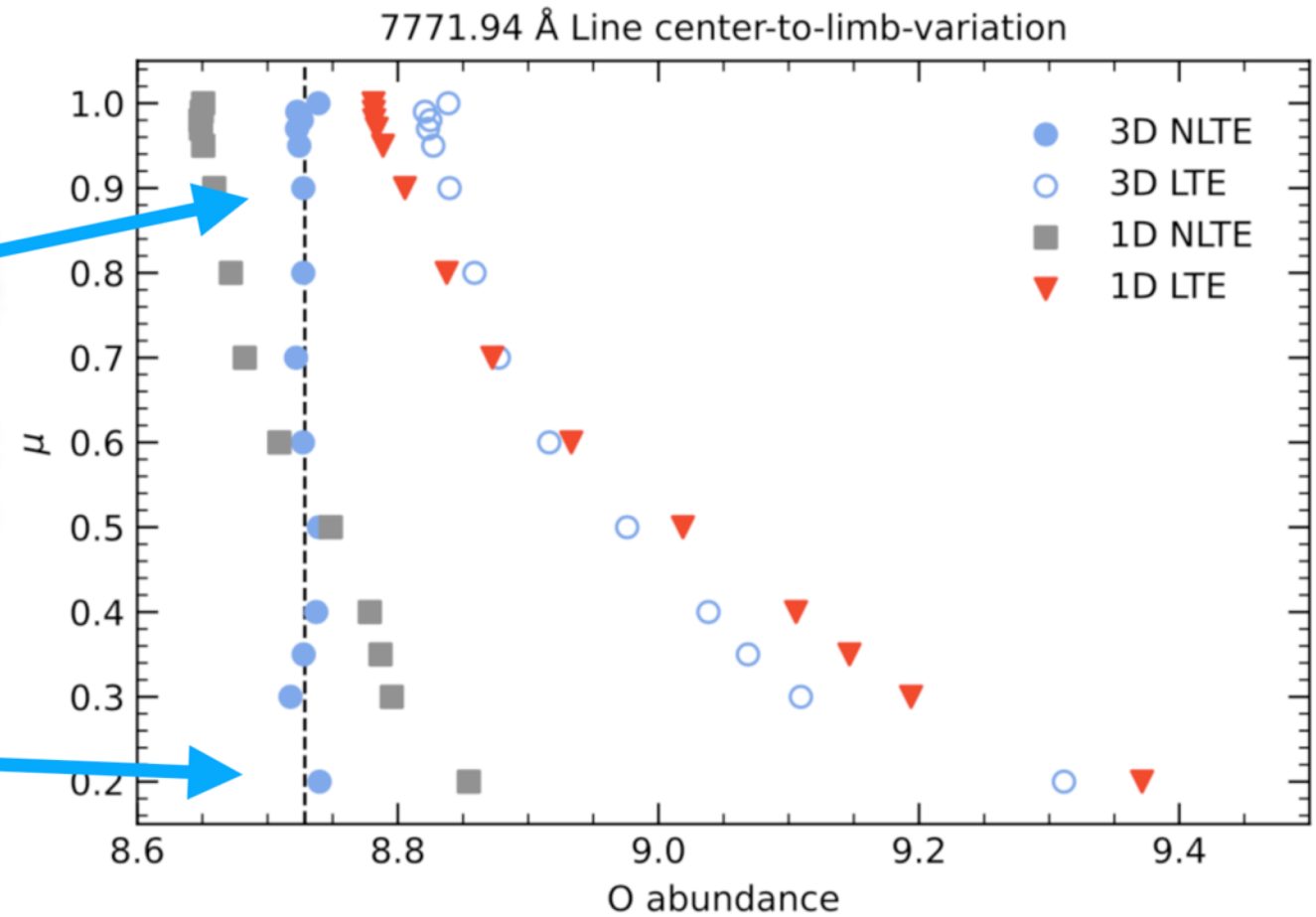
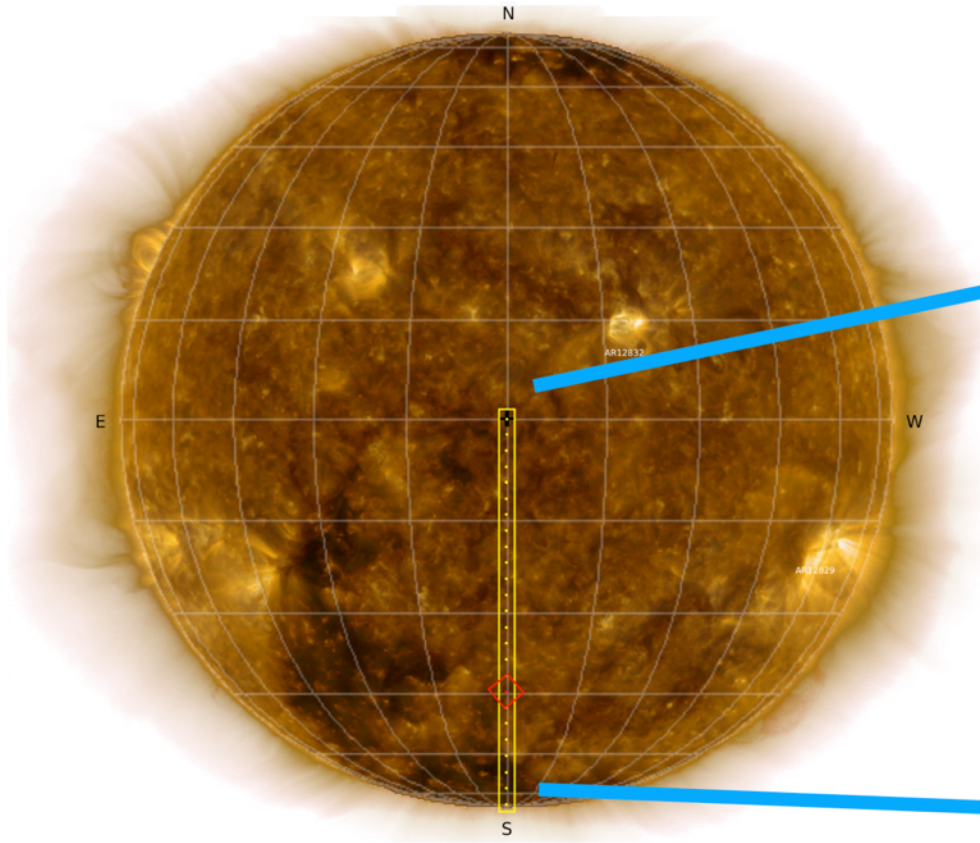


Solar envelope is convective
→ hydrodynamic models
→ 3D atmosphere model

Model atmosphere
→ detailed radiative transfer
→ synthetic spectrum to compare with observed one
→ determination of abundances

Foundation science: Solar spectrum & abundances

Only star that allows detailed tests, e.g. center-to-limb variations

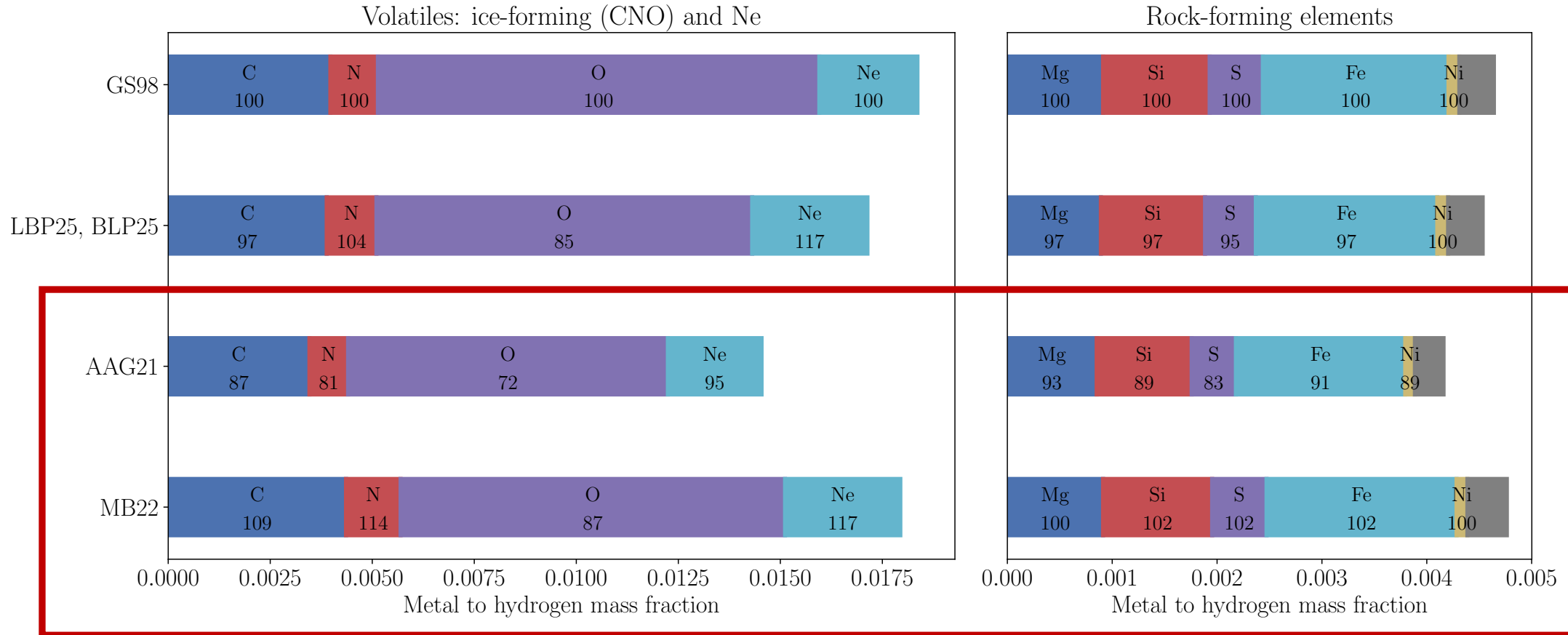


Pietrow, Hoppe, Bergemann et al. 2023

Bergemann, Hoppe, et al. 2021

(plots from M. Bergemann)

Which solar composition?



GS98: Grevesse & Sauval 1998

LBP25/BLP25: Lodders, Bergemann, Palme 2025

AAG21: Asplund et al. 2021,

MB22: Magg et al. 2022

Chemical abundances are a constraint, not a prediction, of (non-) standard solar models

Building a standard solar model

Boundary conditions

Solar mass – $M_{\odot}$ – determined from $GM_{\odot} \rightarrow$ limited by knowledge of G (~one part in 10^5)

Solar radius – $R_{\odot}$ – several methods: radio occultations, solar oscillations, Venus transit, (< one part in 10^3)
more loosely defined concept

Solar luminosity – $L_{\odot}$ – bolometric measurements (< one part in 10^3)

Solar (photospheric) composition (?) – solar spectrum, meteorites, (corona & wind)
AAG21, MB22

Solar age – $\tau_{\odot}$ – radioactive dating of meteorites (~one part in 10^3)

Input to standard solar models

solar mixture (relative abundances, no normalization)

radiative opacities, equation of state

nuclear reaction rates

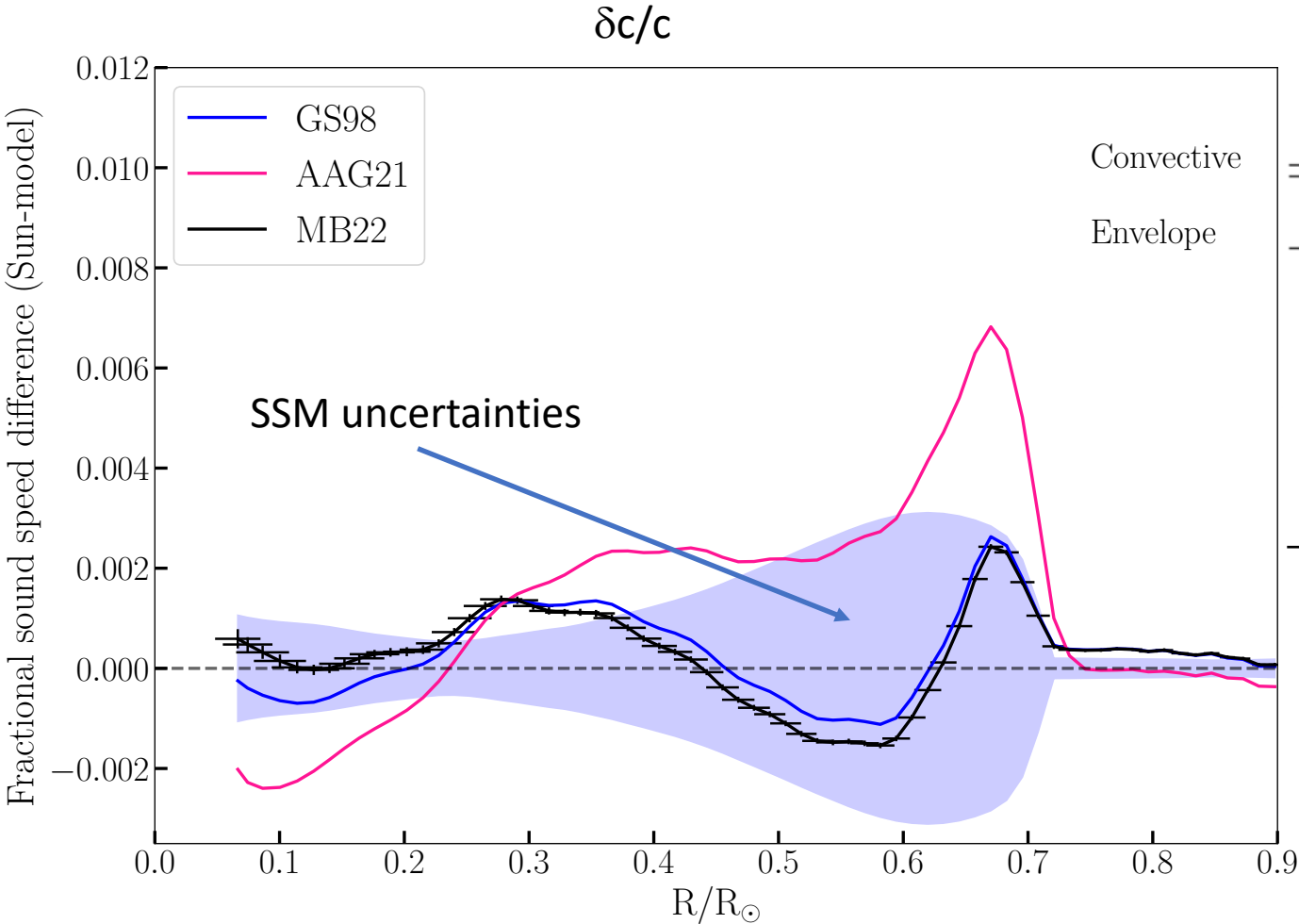
mixing processes: convection, microscopic diffusion

Find the 3 free parameters: mixing length (convection), initial helium, initial metallicity that match observables at $\tau_{\odot}$

SSM framework IS NOT INTENDED to be a full description of the Sun (rotation, extramixing, magn. fields)



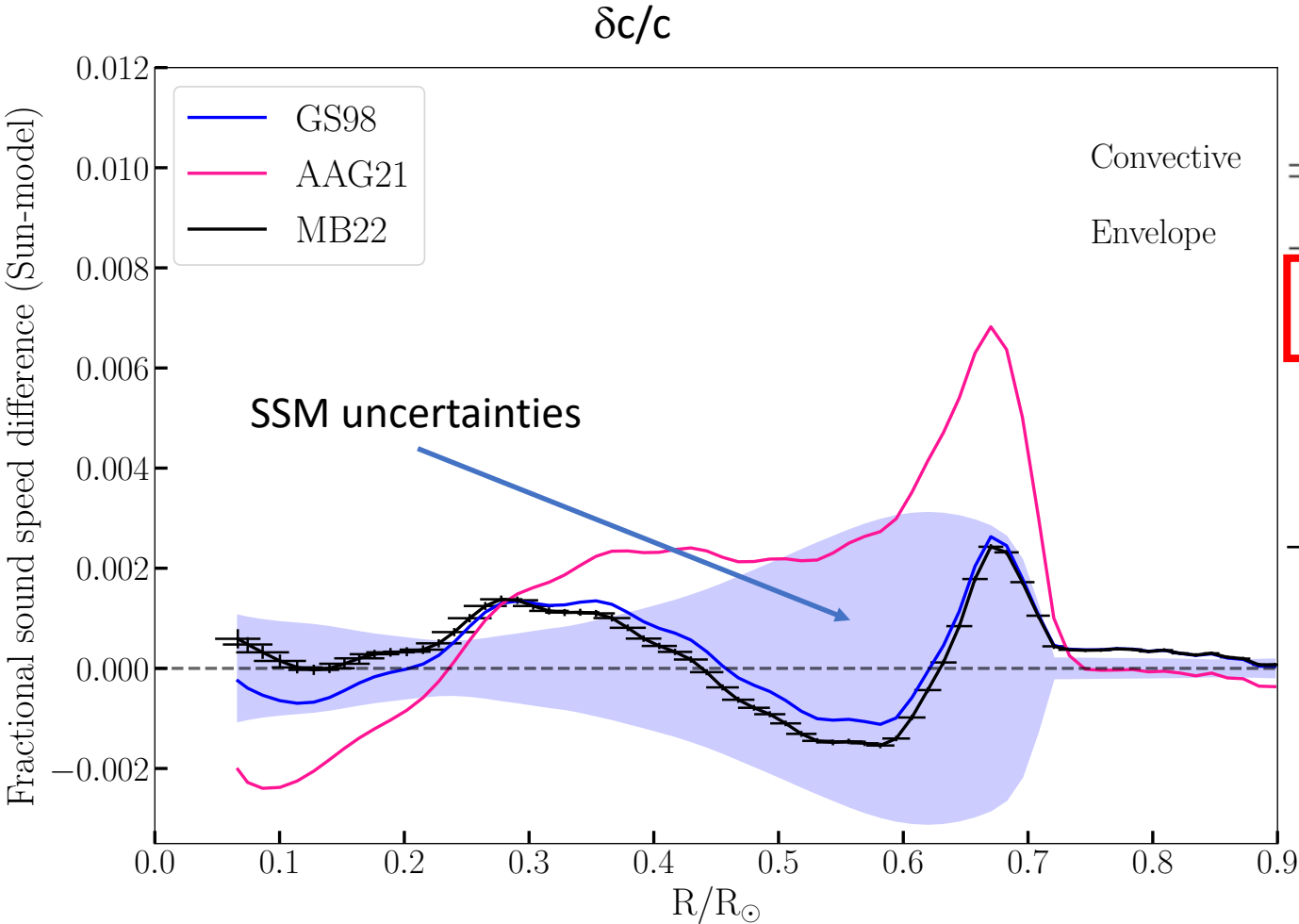
What helioseismology tells us



Model	$R_{CZ}/R_{\odot}$	Y_S	$\langle \delta c/c \rangle$	Y_{ini}	Z_{ini}
MB22	0.7123	0.2439	0.0010	0.2734	0.0176
AAG21	0.7197	0.2343	0.0027	0.2638	0.0155
GS98	0.7122	0.2425	0.0010	0.2718	0.0187
Solar	0.713 ± 0.001	0.2485 ± 0.0035			



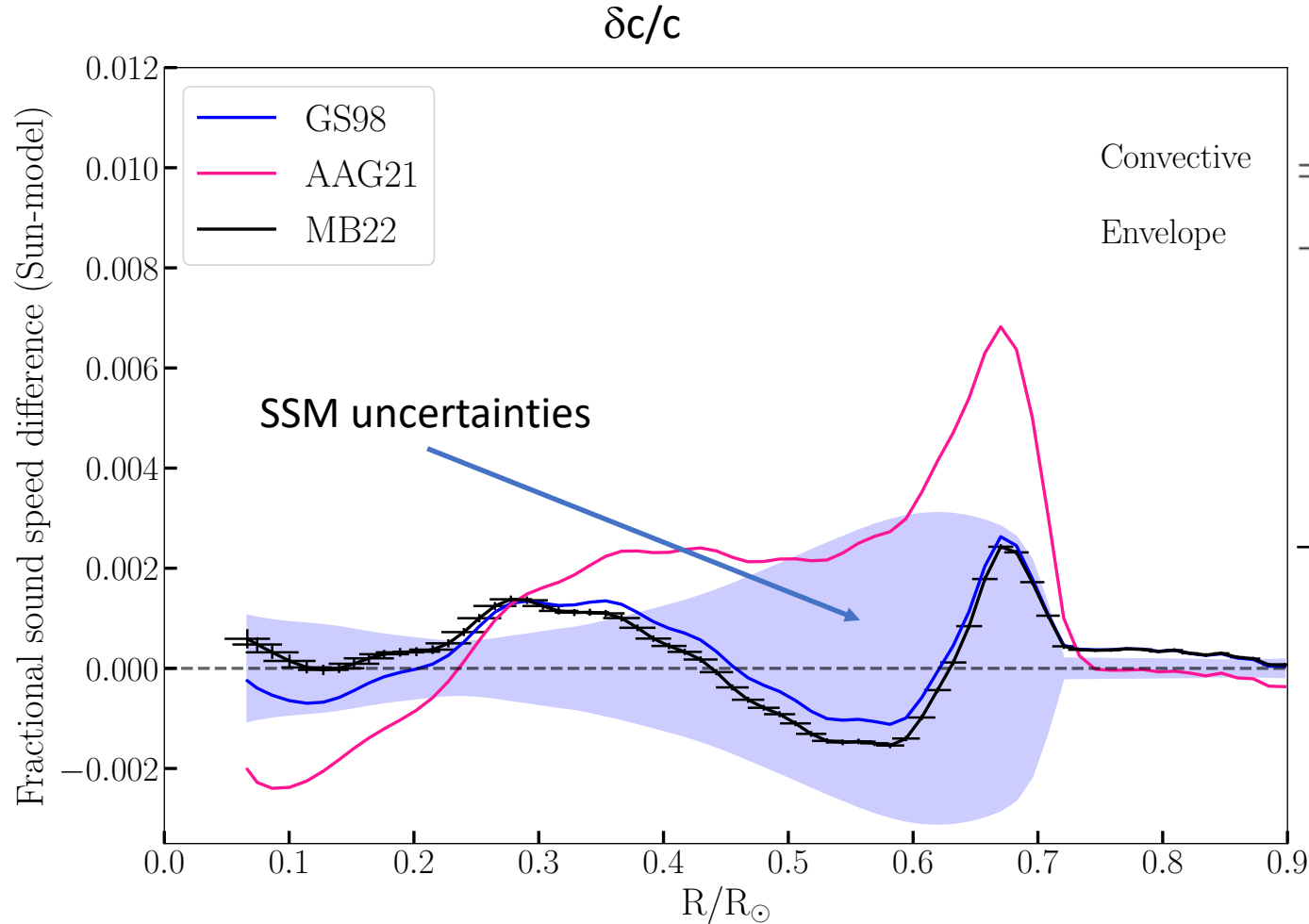
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**Results sensitive to thermal structure
because sound speed scales with $T^{1/2}$**

$$\nabla T^4 \propto \kappa \quad c^2 \propto \frac{T}{\mu}$$

→ (composition + radiative opacities)

What helioseismology tells us

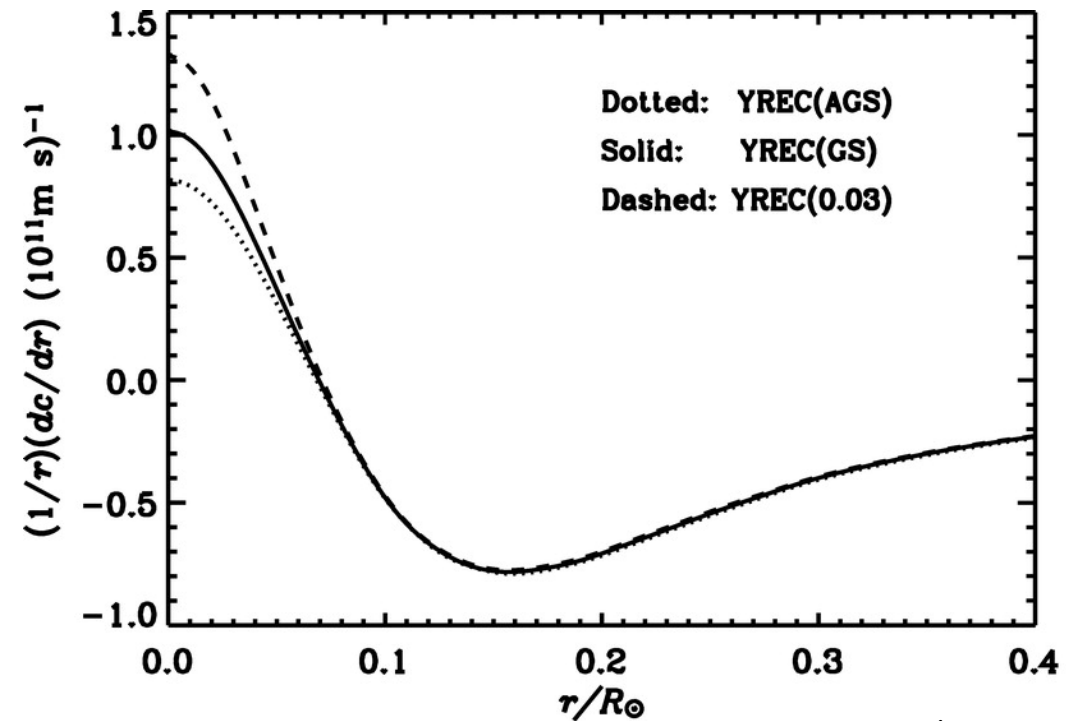
$$\nu_{n\ell} \simeq \left(n + \frac{\ell}{2}\right) \Delta\nu - A\ell(\ell+1) \frac{\Delta\nu^2}{\nu_{n\ell}}$$

$$A = -\frac{1}{4\pi^2 \Delta\nu} \left[\int_0^R \frac{dc}{dr} \frac{dr}{r} \right]$$

Ratios of frequency separation
Small frequency/large frequency

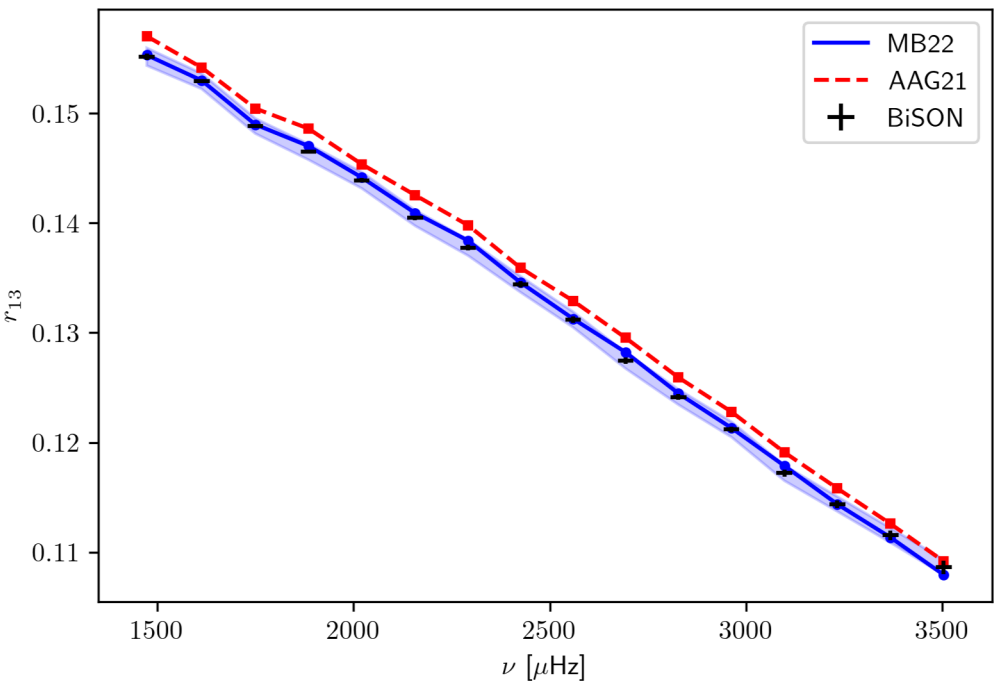
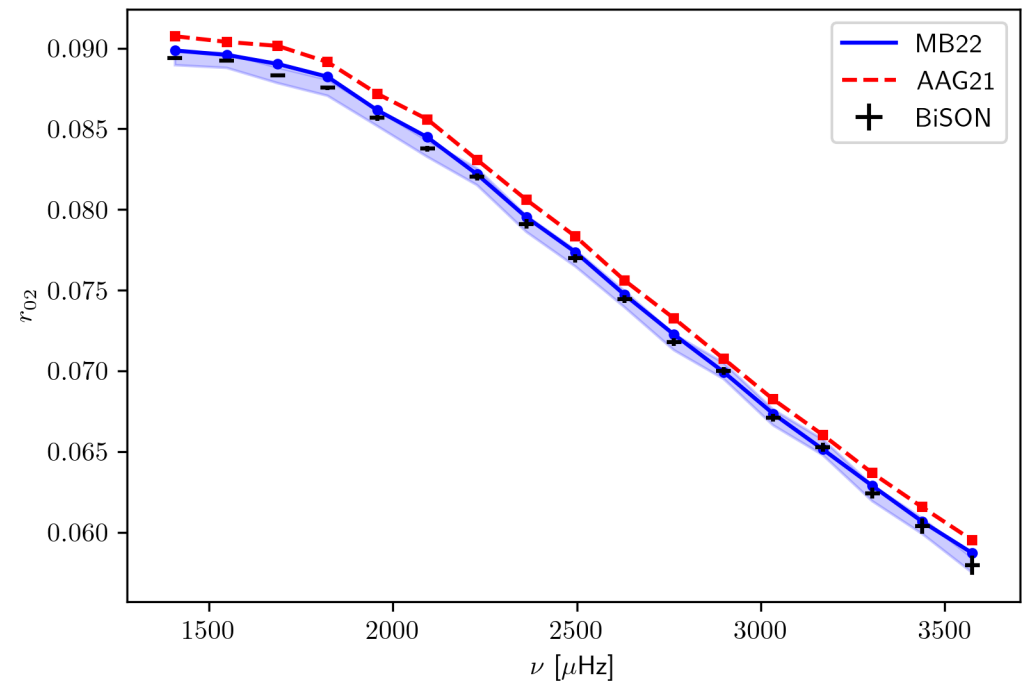
$$\left. \begin{aligned} r_{02}(n) &= \frac{V_{n,0} - V_{n-1,2}}{V_{n,1} - V_{n-1,1}} \\ r_{13}(n) &= \frac{V_{n,1} - V_{n-1,3}}{V_{n+1,0} - V_{n,0}} \end{aligned} \right\} \propto \int_0^R \frac{dc}{dr} \frac{dr}{r}$$

Roxburgh & Vorontsov 2003



Basu et al. 2007

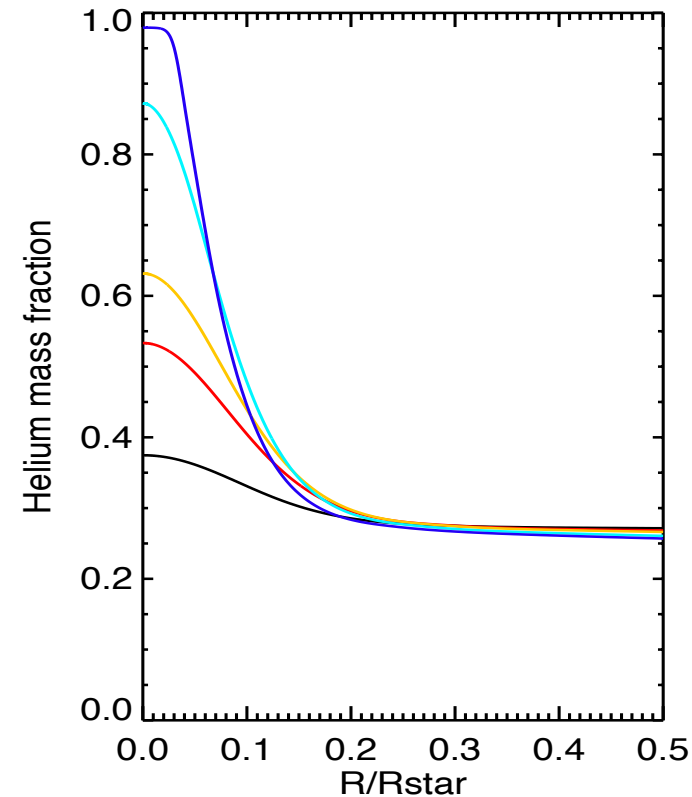
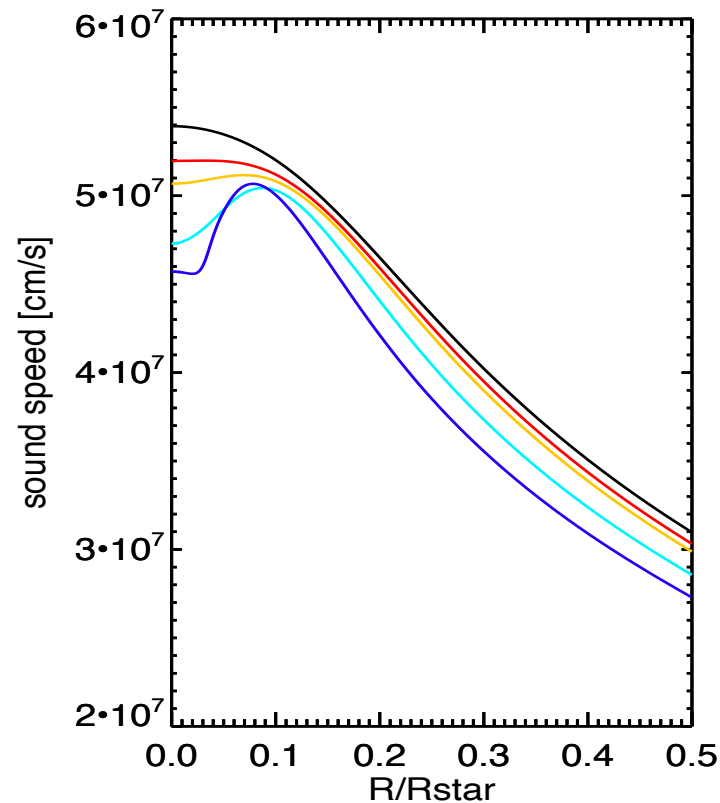
Low-Z & High-Z models vs BiSON data – OP opacities



Dating the Sun “as a star”

Cancellation effects limit modes to $l=0, 1, 2, (3)$ for other stars (e.g. Kepler, TESS, PLATO)

$$\nu_{n,\ell} - \nu_{n-1,\ell+2} \propto \frac{1}{4\pi\nu_{n,\ell}} \int_0^R \boxed{\frac{dc}{dr}} \frac{dr}{r} \longrightarrow \text{age diagnostics}$$

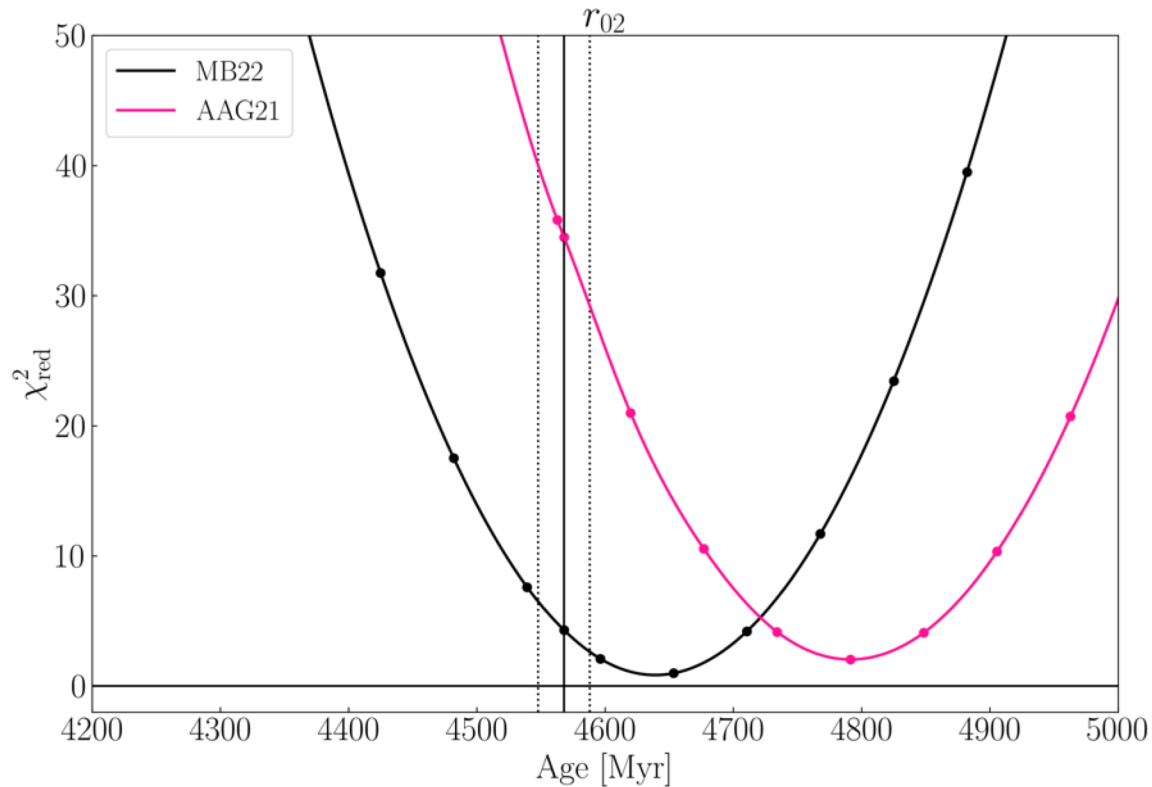


$$c^2 \propto \frac{T}{\mu}$$

The Sun from afar

No independent age for other stars

$$\nu_{n,\ell} - \nu_{n-1,\ell+2} \propto \frac{1}{4\pi\nu_{n,\ell}} \int_0^R \frac{dc}{dr} \frac{dr}{r}$$



	Solar age (Gyr)	χ^2 (33 dofs)
Sun	4.568 ± 0.020	---
AAG21	4.755 ± 0.034	76.6
MB22	4.611 ± 0.032	38.4

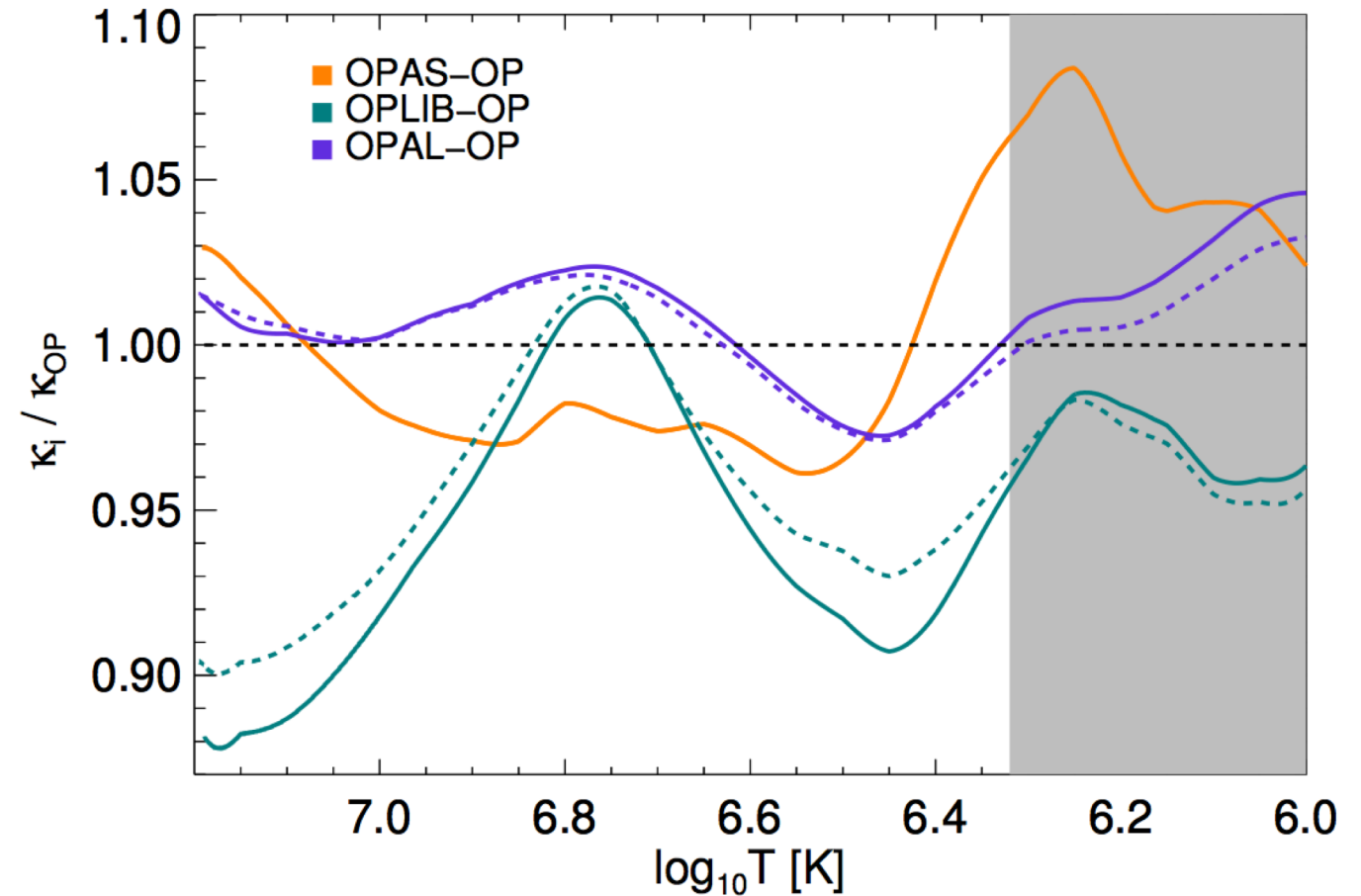
Composition introduces a systematic effect on age determination of about to 250Myr (5%)



Impact of opacity (tables)

Status of solar (stellar) opacities

- OPAL (1996)
- Opacity Project (OP; 2005)
- OPAS (2012, 2015 – Blancard et al., Mundet et al.)
- Los Alamos/OPLIB (2016 – Colgan et al.)



Impact of opacity (tables)

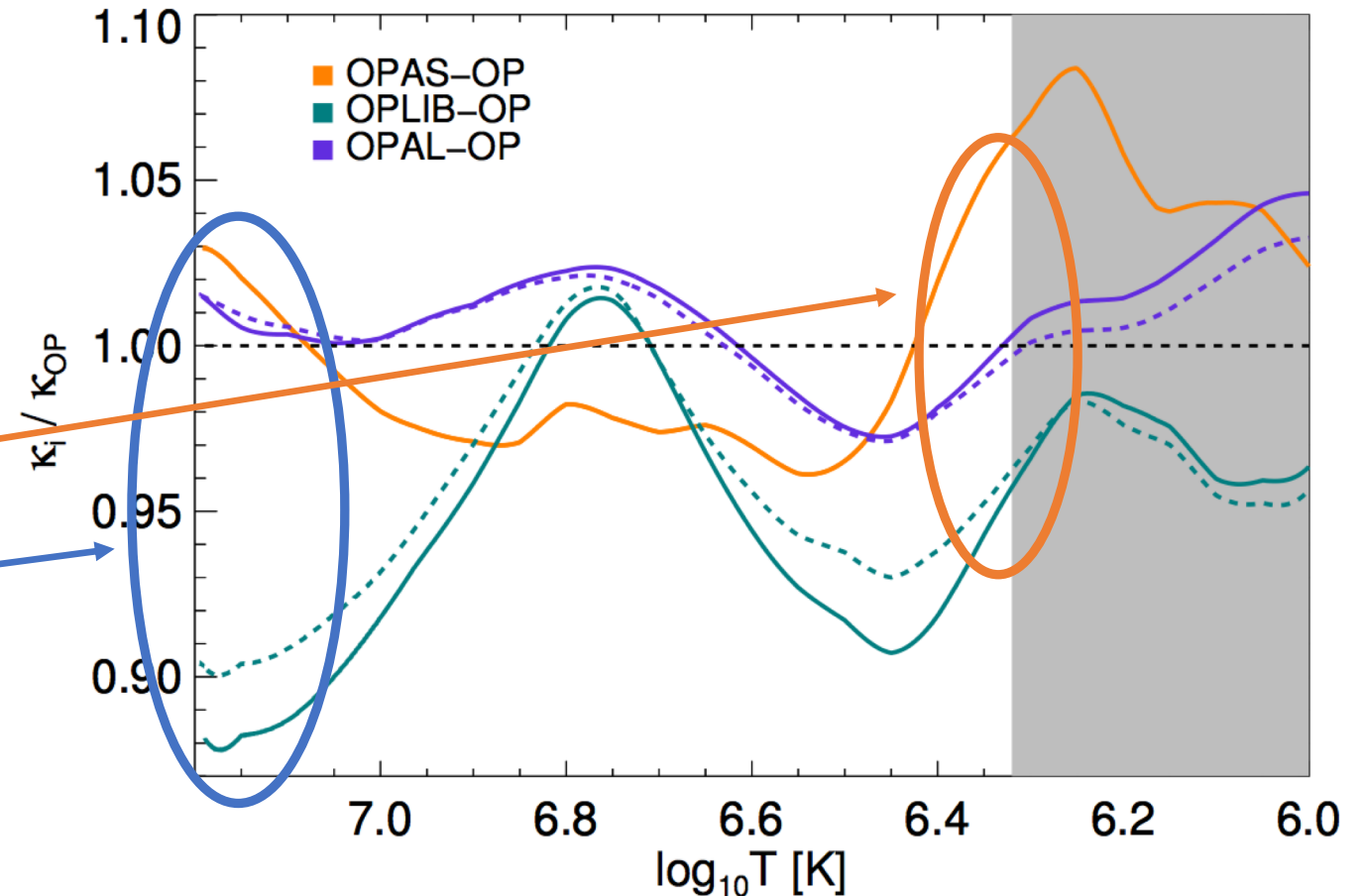
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Fractional opacity differences wrt OP

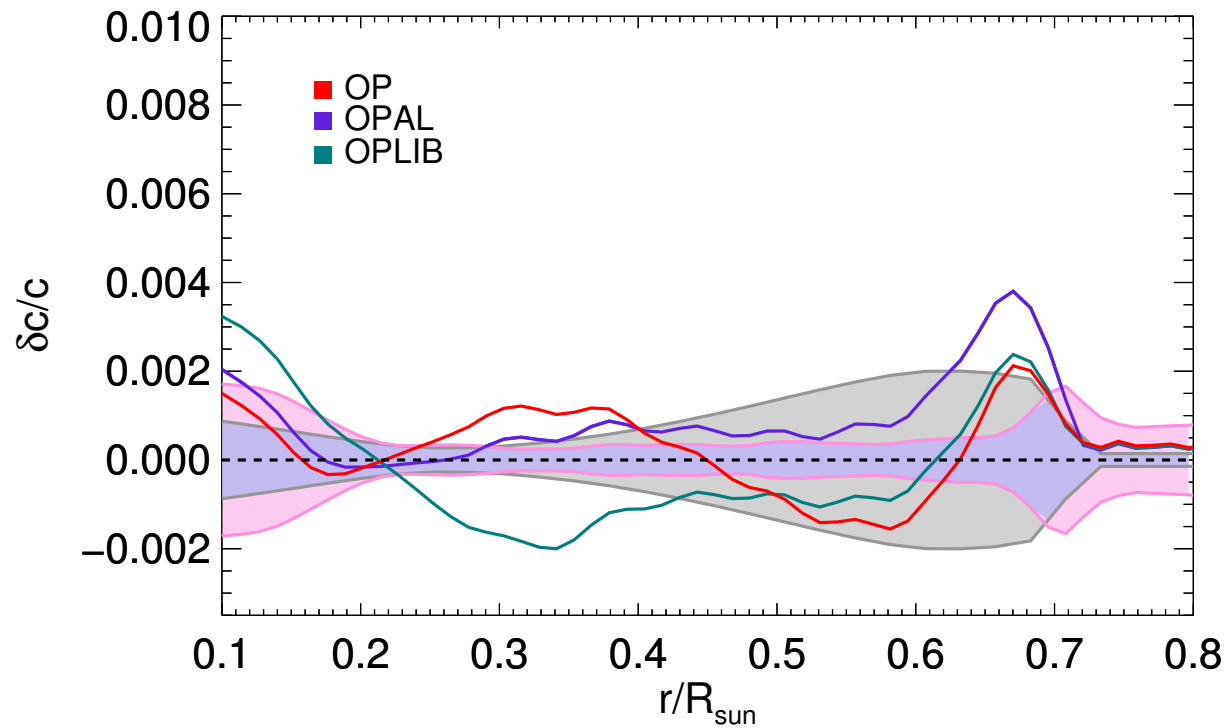
±5% at base of convective envelope

OPAS-OP-OPAL similar in the core
OPLIB (Los Alamos) up to 15% lower

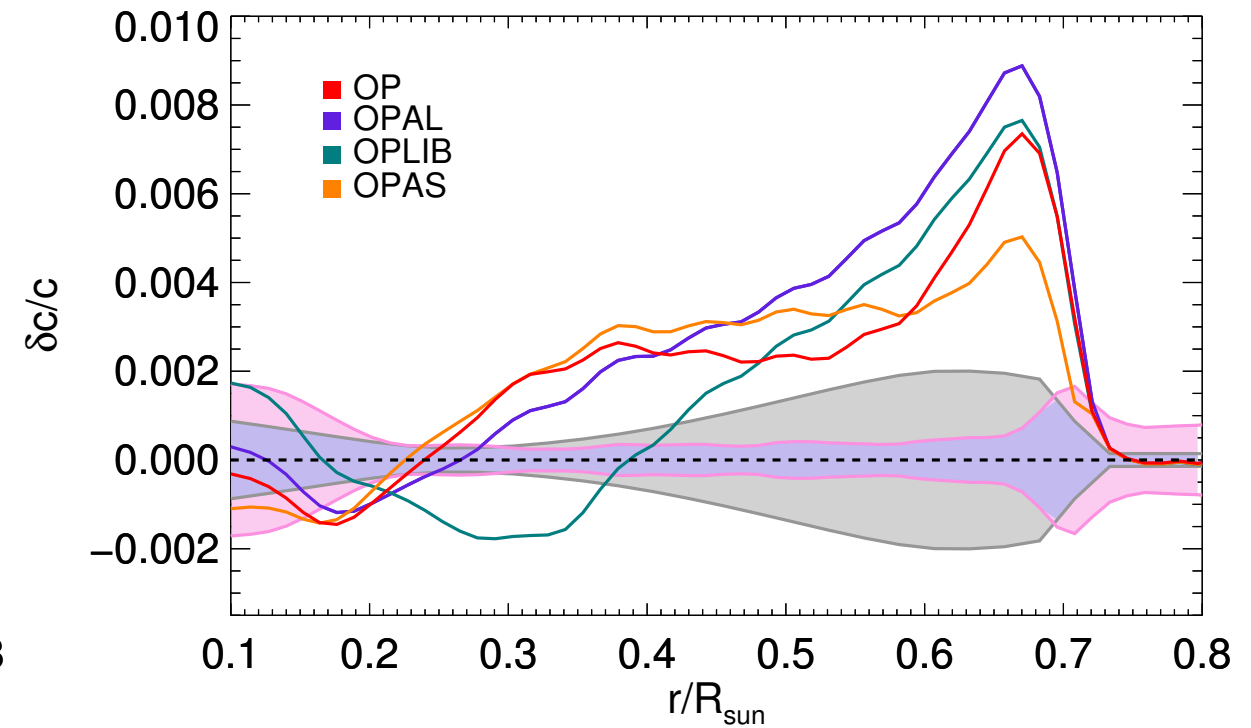


Impact on sound speed

GS98 – MB22 – proxy for high-Z

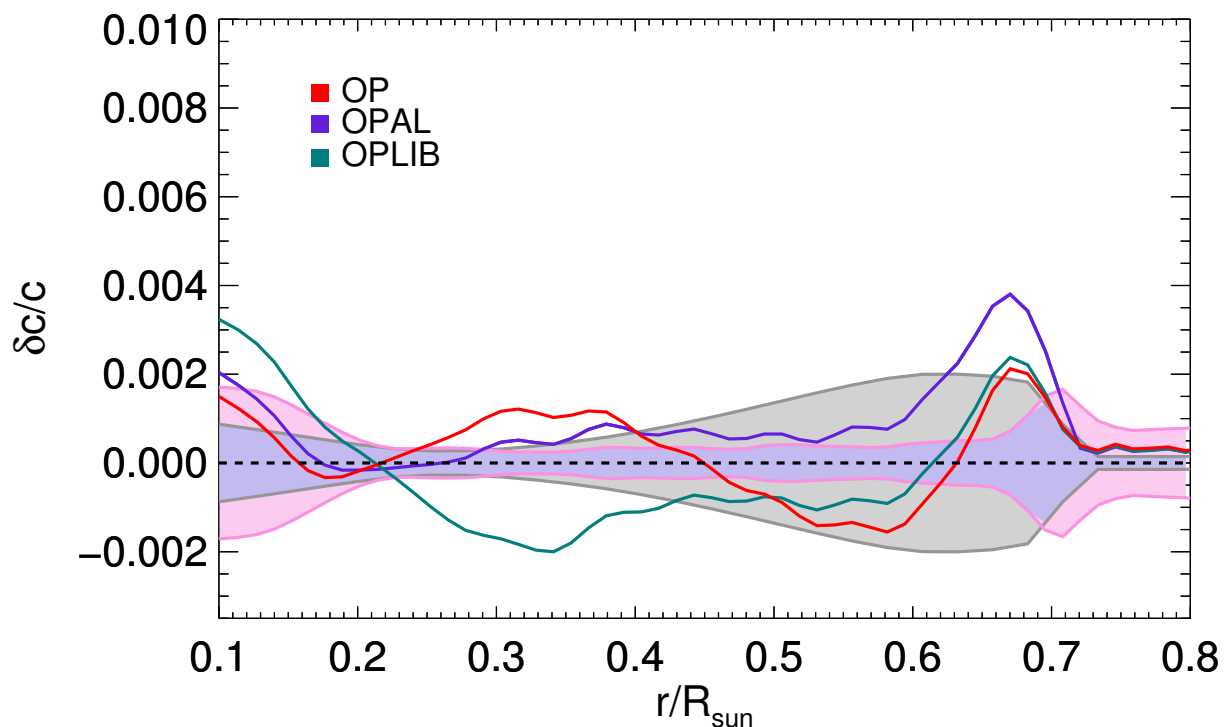


AGSS09 - proxy for low-Z

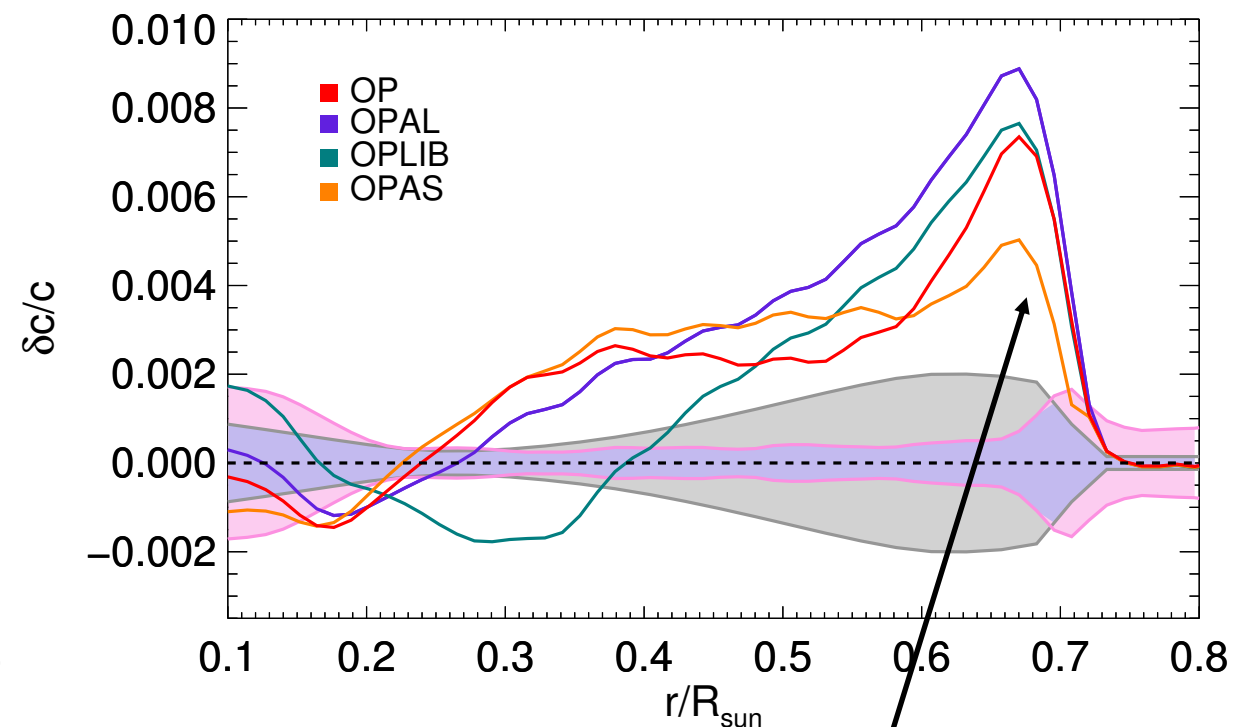


Impact on sound speed

GS98 – MB22 – proxy for high-Z



AGSS09 - proxy for low-Z

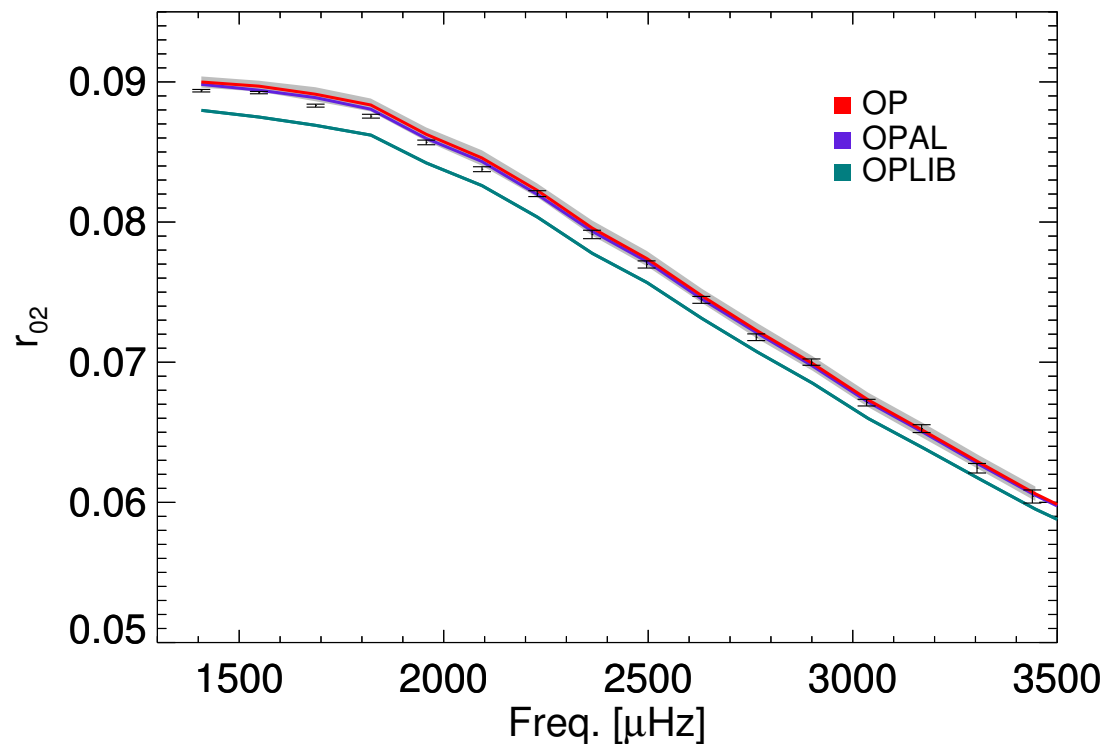


No dramatic differences, most significant might be the OPAS behavior at R_{CZ}

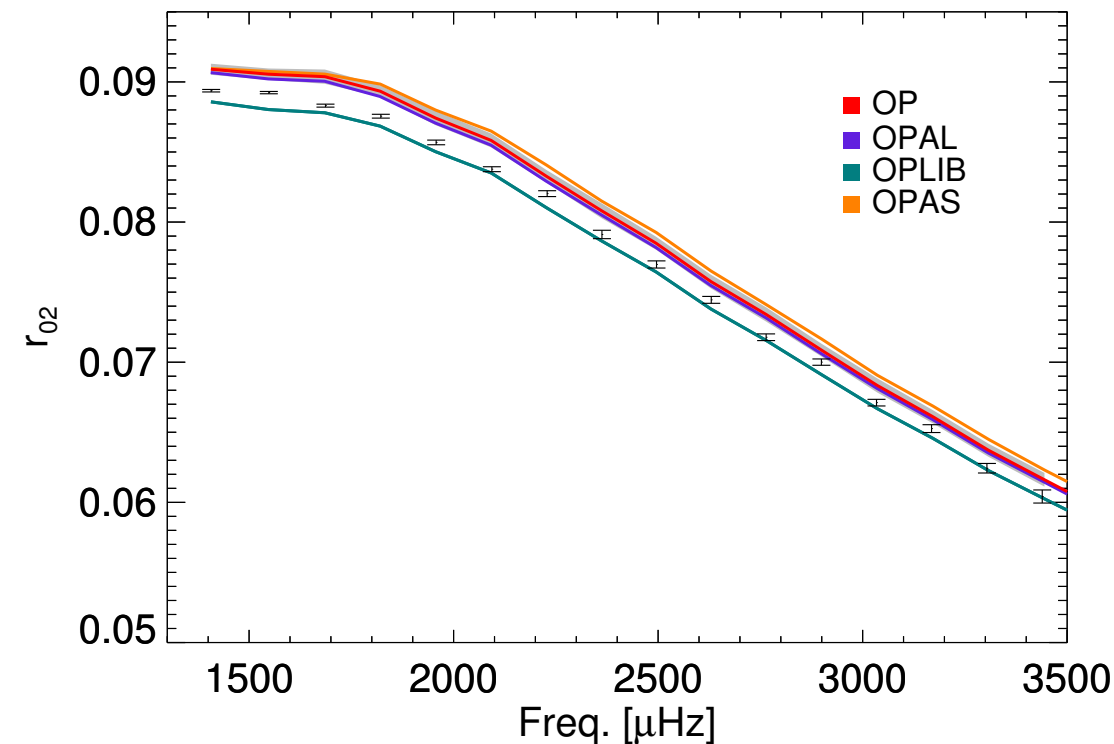


Frequency ratios

GS98 – MB22 – proxy for high-Z



AGSS09 - proxy for low-Z



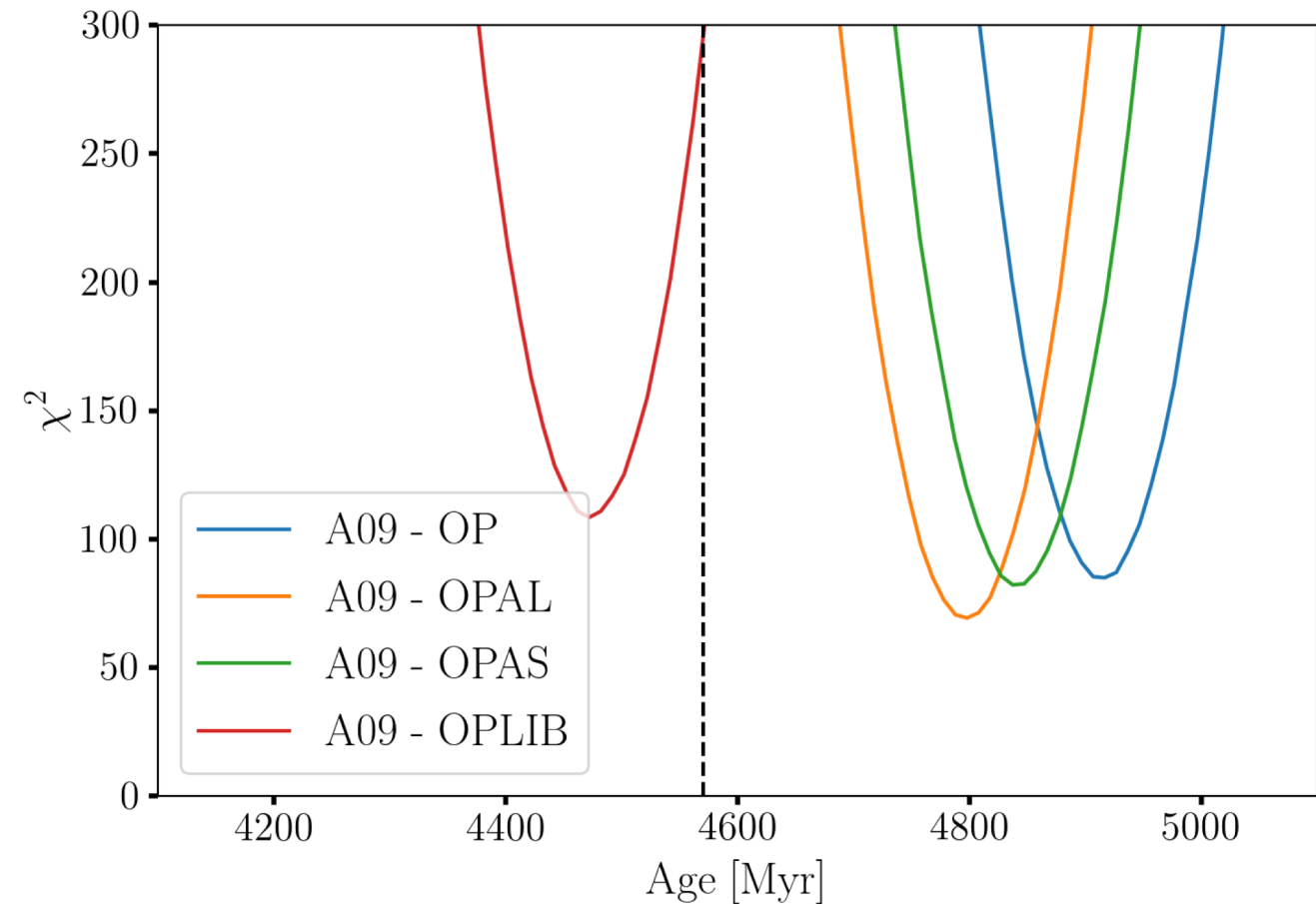
OPLIB → helioseismic probes do not provide a coherent picture (see also Buldgen et al. 2017)

OPLIB – high-Z: good c_s , R_{CZ} , Y_s , bad frequency ratios (core)

OPLIB – low-Z: bad c_s , R_{CZ} , Y_s , good frequency ratios (core)

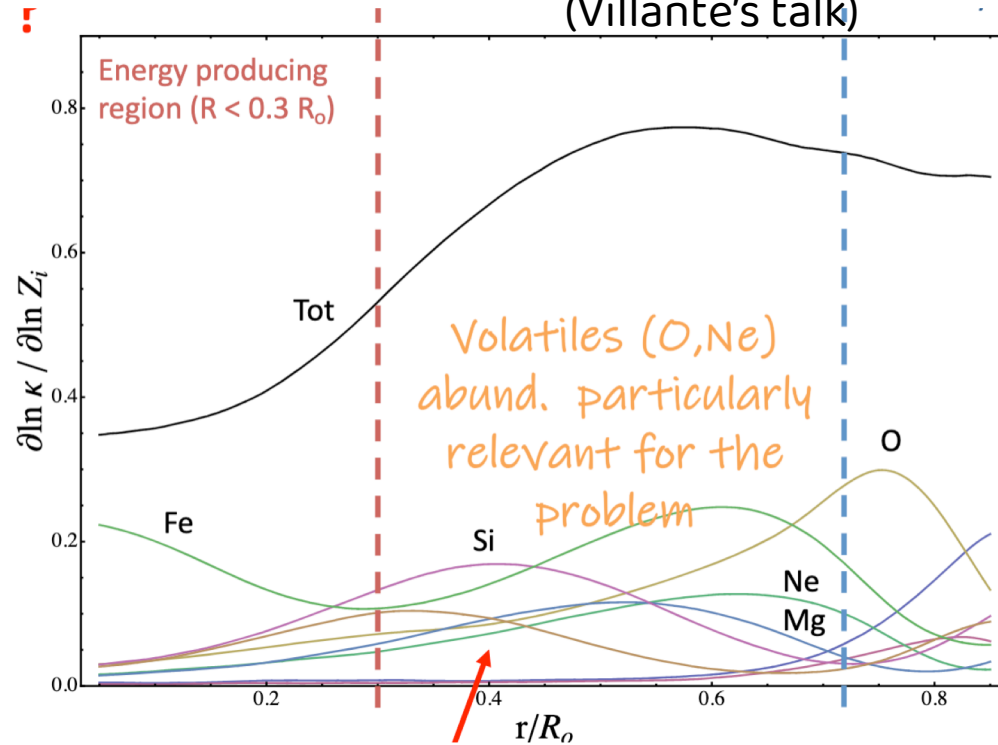


**Age determination using frequency ratios
→ more than 10% spread just on opacity
tables**

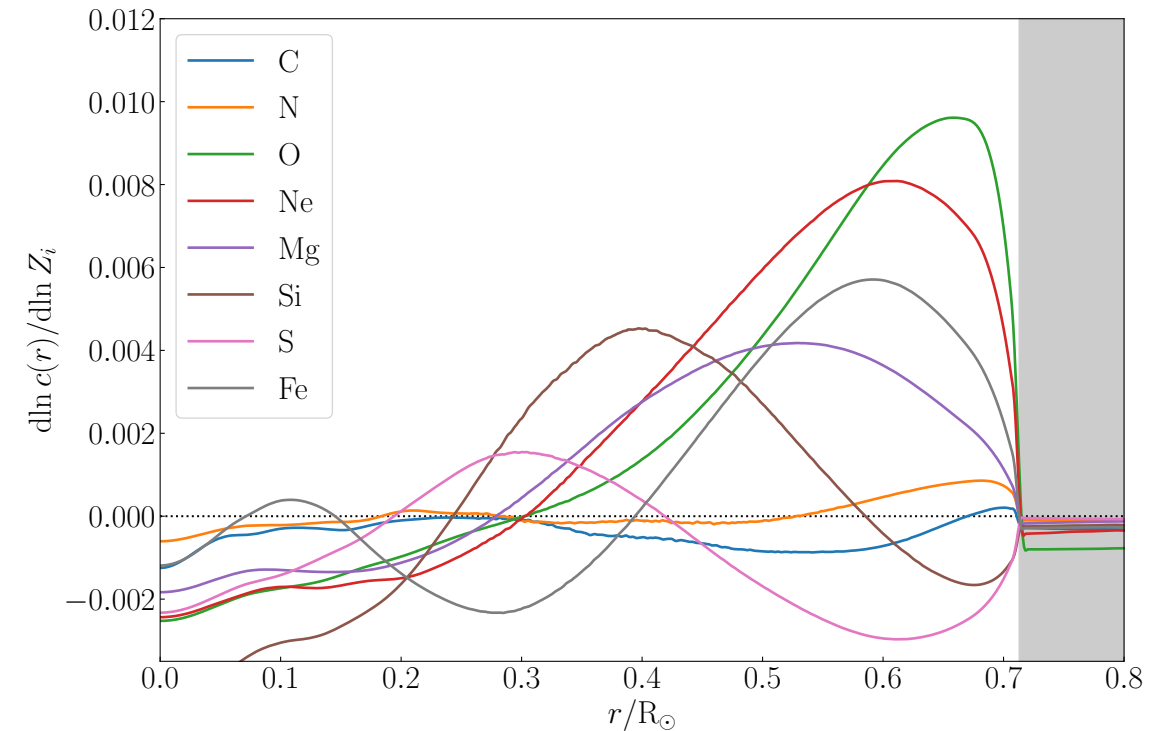


Before going away from opacities – a word on neon

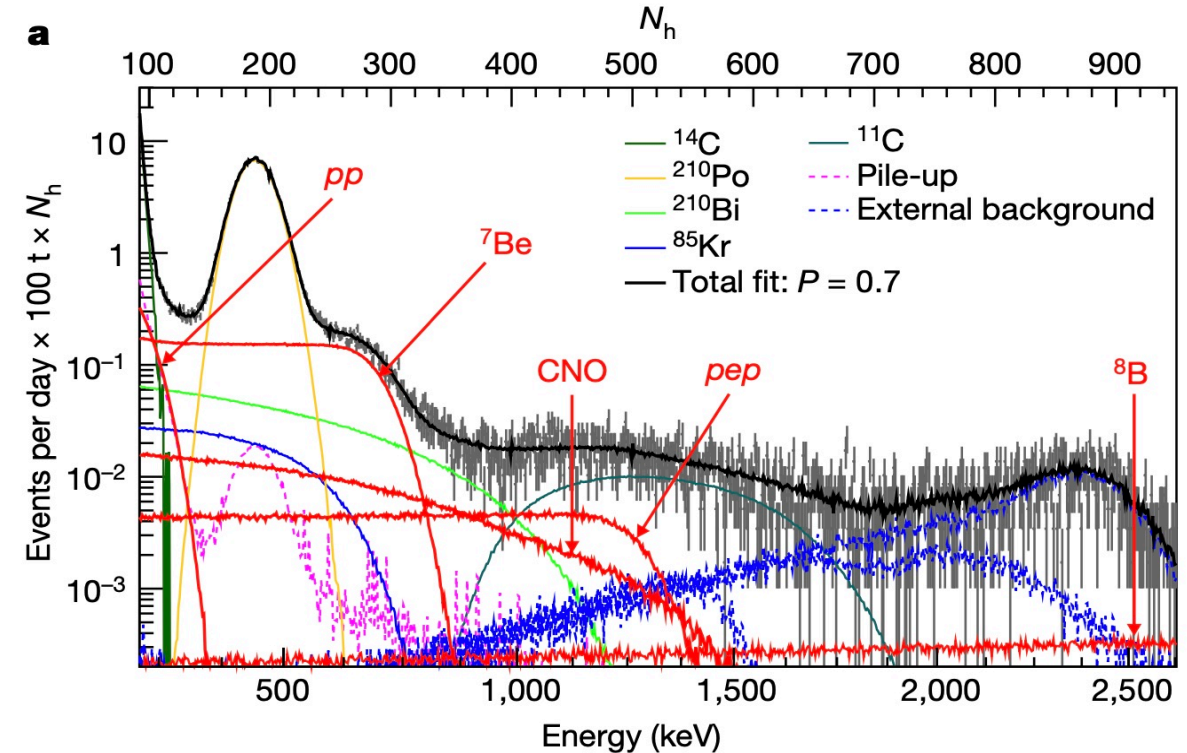
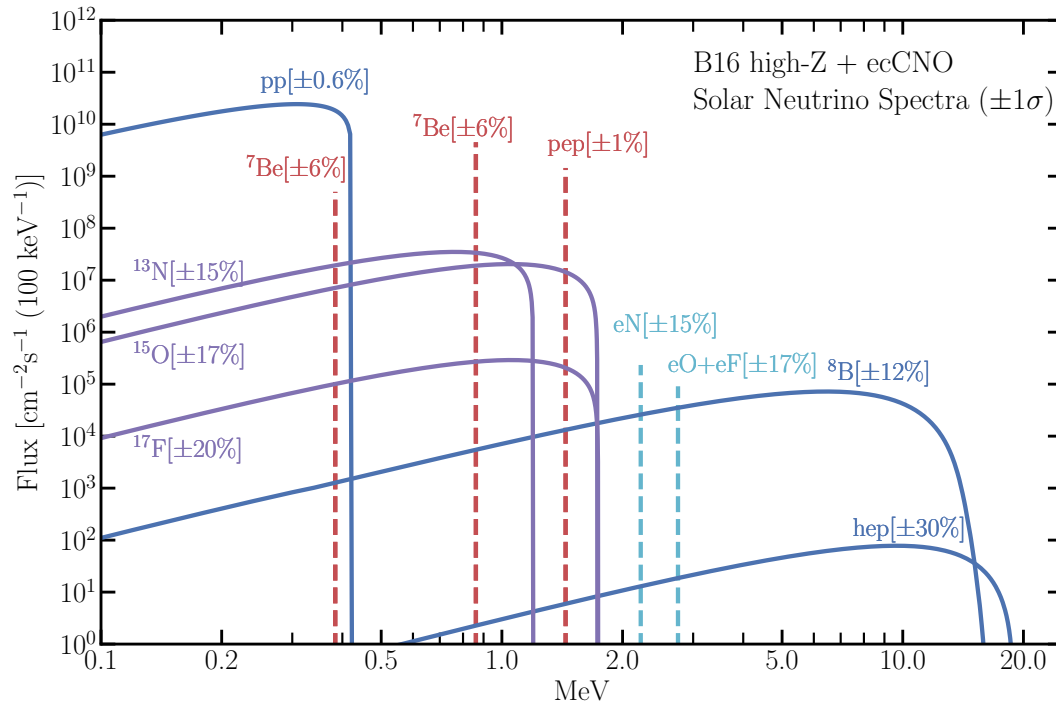
Opacities: $O > Fe > Ne$
(Villante's talk)



Sound speed dependence: $O > Ne > Fe$

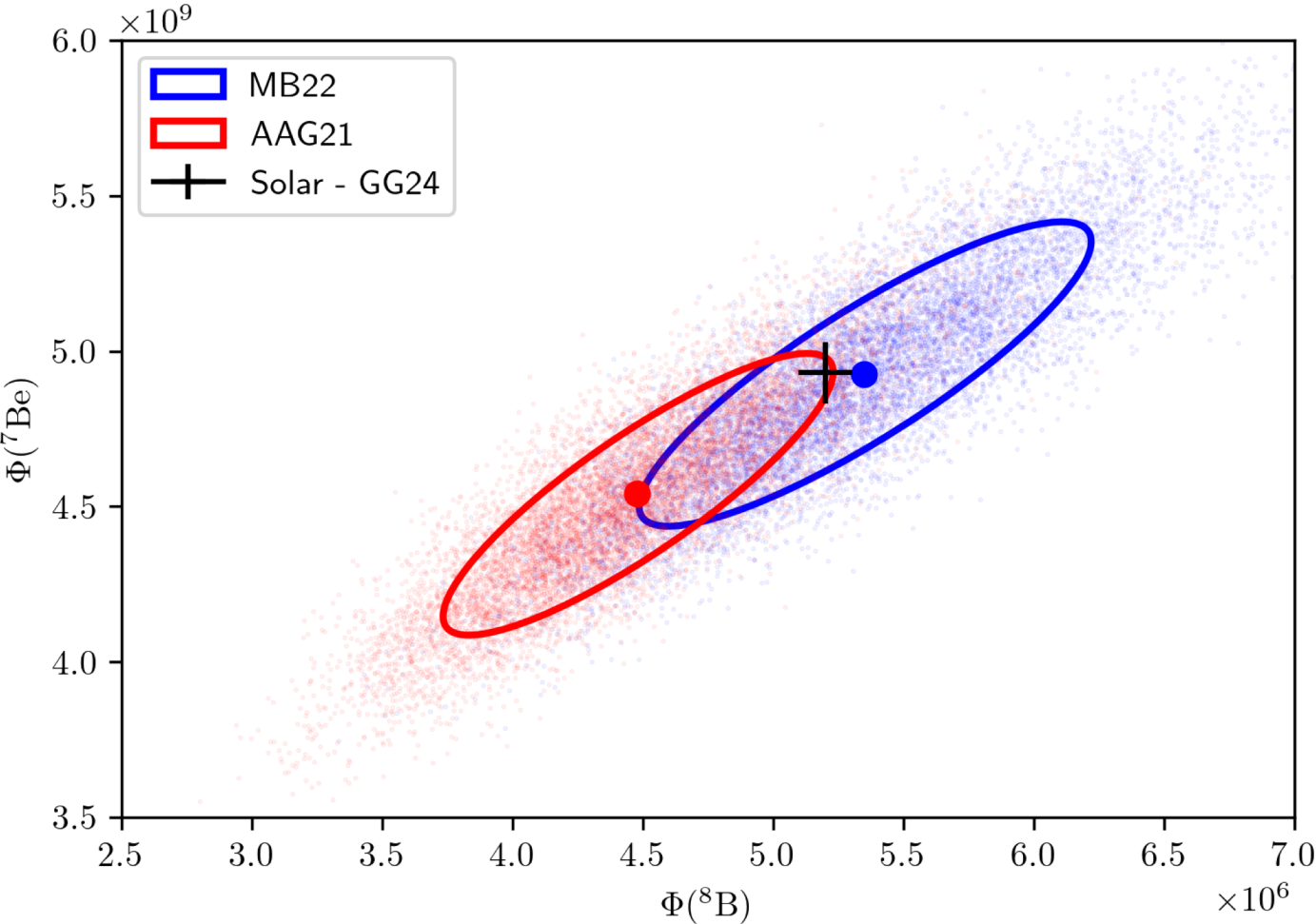


- Ne: coronal emission lines – SHO observations – Ne/Mg to control FIP – Ne/O to bring it to photosphere
- All hangs on Young 2018 paper, 40% increase wrt previous results (also Young) based on revision of ionization/recombination rates
- Uncertainties quote in compilations of solar abundances: 12% (AAG21), 23% (MB22)



Borexino coll 2022, (Basilico) 2023

Bringing pp-chain neutrinos into the game



Solar Fusion III rates

cm ⁻² s ⁻¹	AAG21	MB22	Sun
pp (10 ¹⁰)	6.00 (0.6%)	5.95 (0.6%)	5.94 (0.4%)
pep (10 ⁸)	1.45 (1.1%)	1.42 (1.1%)	1.42 (1.6%)
hep (10 ³)	8.16 (30%)	7.92 (30%)	30 (33%)
⁷ Be (10 ⁹)	4.55 (7.4%)	4.92 (7.6%)	4.93 (2%)
⁸ B (10 ⁶)	4.48 (12.6%)	5.35 (13.1%)	5.20 (1.9%)

“Sun”: experimental results from GG-2024

Model uncertainties >> experimental ones

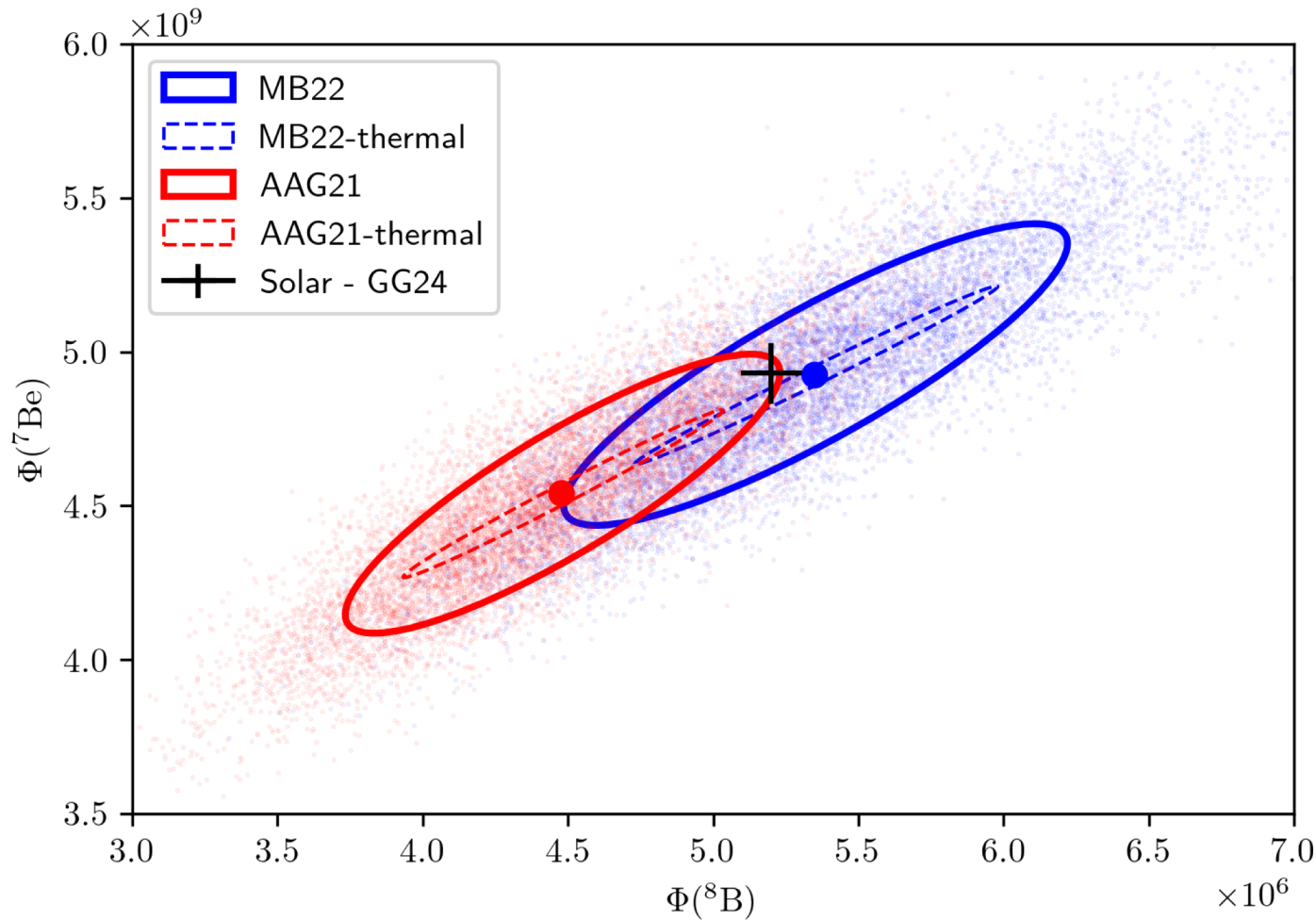
x-sections (S₁₇, S₃₄, S₁₁)

6% for ⁷Be

8% for ⁸B

radiative opacity

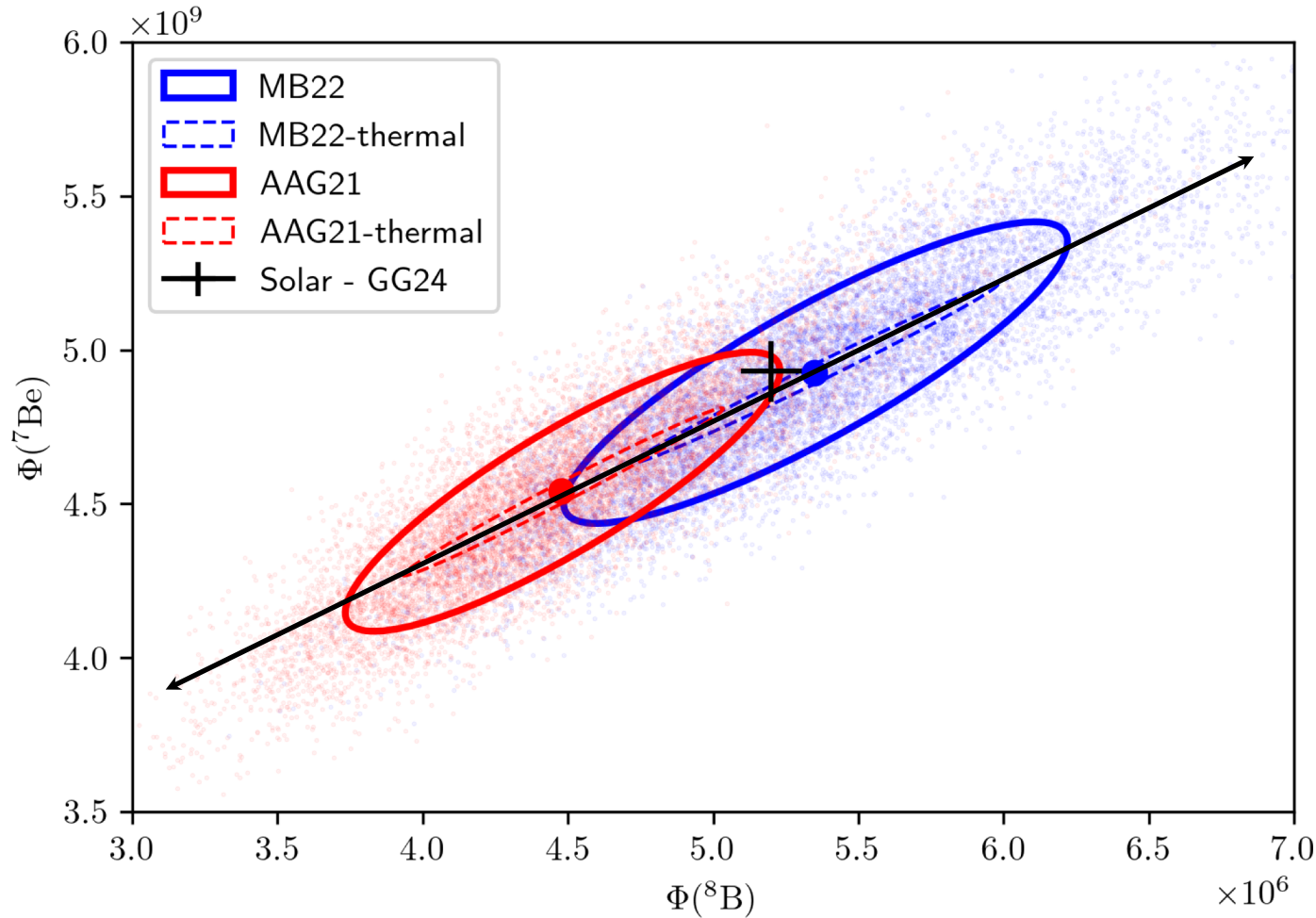
Bringing pp-chain neutrinos into the game



Physics affecting core temperature T_c produce fully correlated changes

- $L_\odot$, age
- opacity & metals
- grav. settling
- p+p rate
- any non-standard process

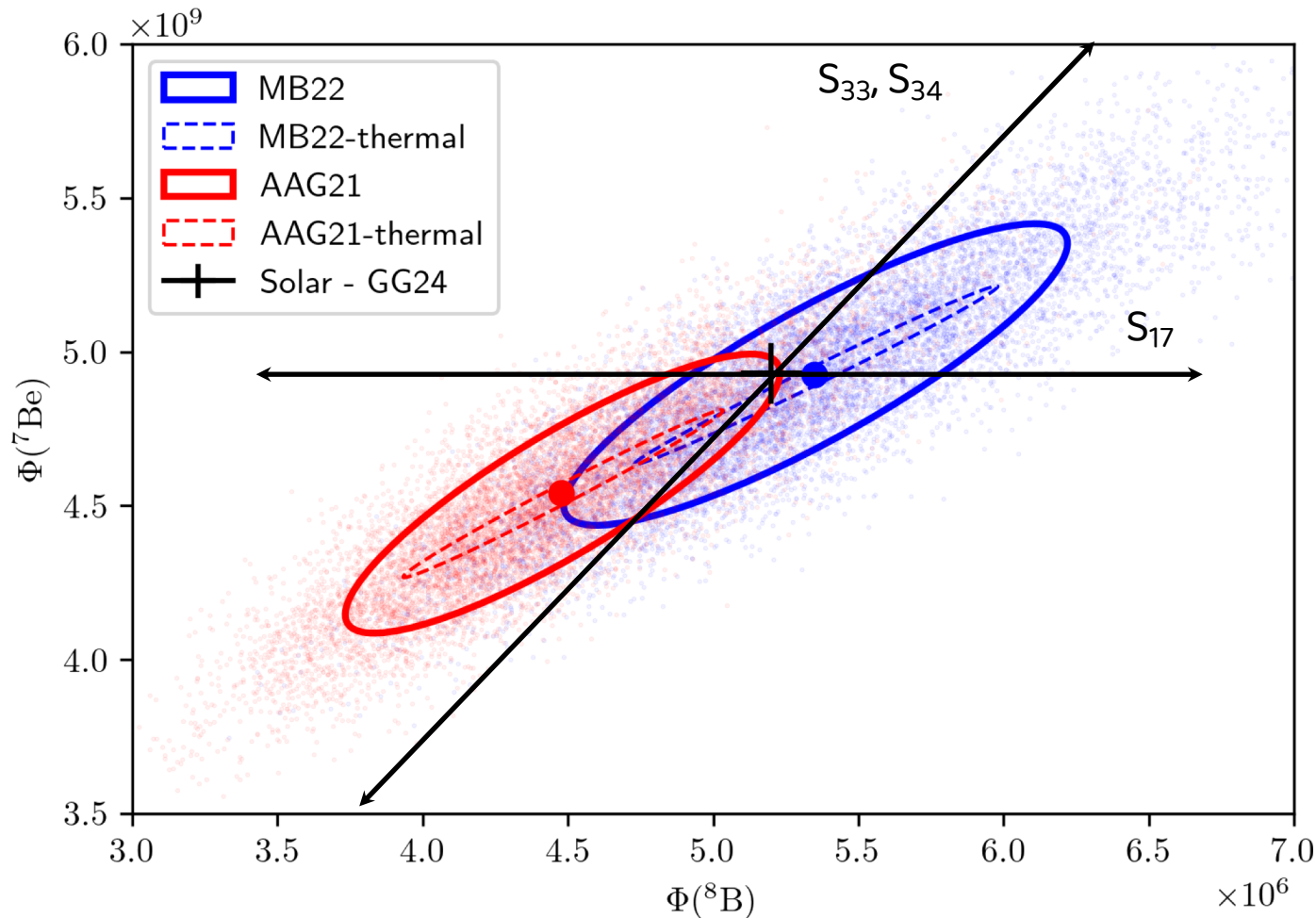
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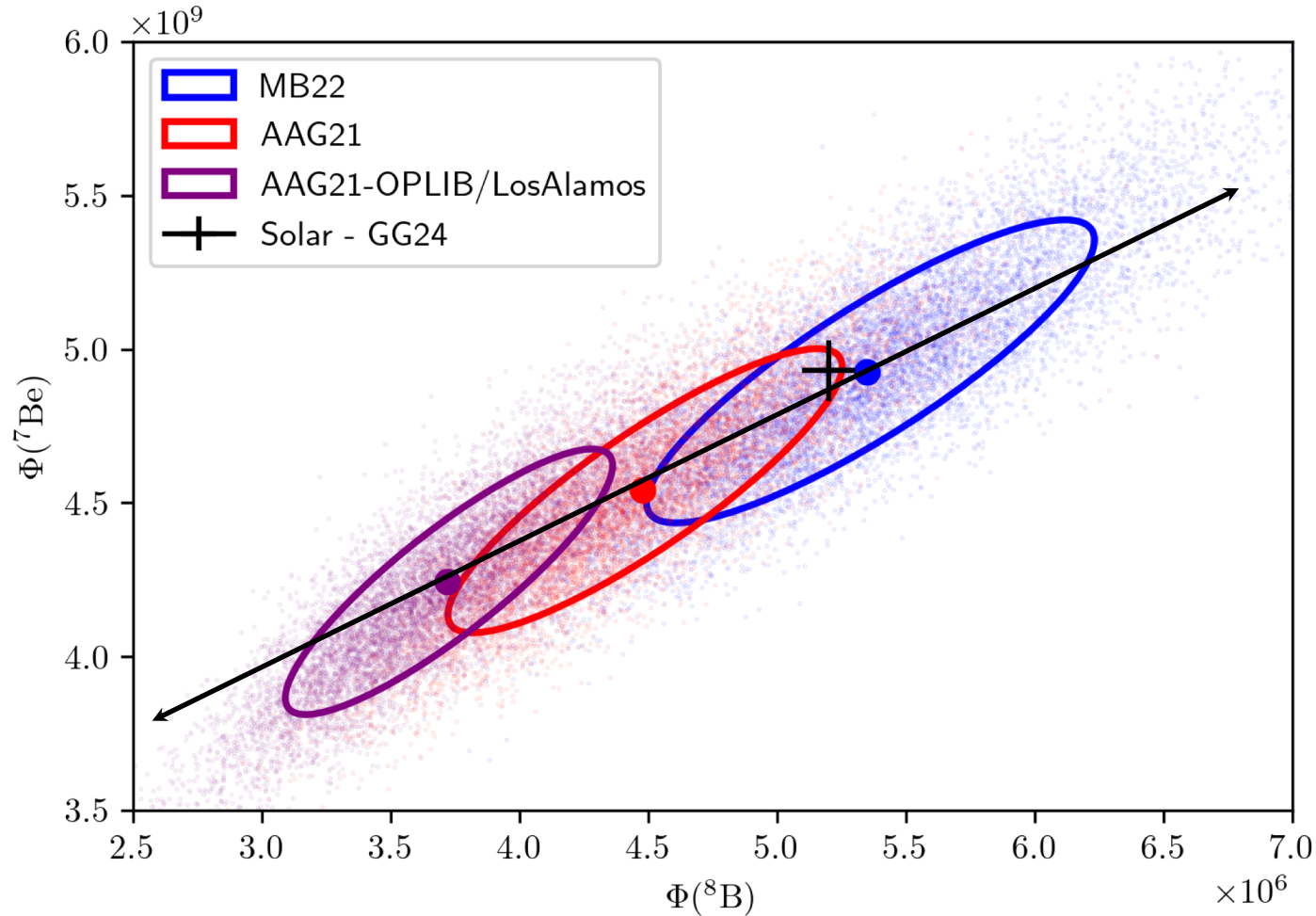


Physics affecting core temperature T_c produce fully correlated changes

- $L_\odot$, age
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- any non-standard process

Experimental and model data lie along the T_c sequence $\rightarrow$ central values for nuclear cross ($S_{33}, S_{34}, S_{e7}, S_{17}$) sections are robust

Bringing pp-chain neutrinos into the game



${}^7\text{Be}$ & ${}^8\text{B}$ fix the solar core temperature

Solar neutrinos fix the opacity scale at the core
(not reachable through helioseism., see Villante's talk)

At low-Z OPLIB/LA opacities $\rightarrow T_c$ too low

Non-standard processes $\rightarrow$ lower T_c

Breaking the degeneracy between opacity & composition



- SSMs (and non-SSMs that are not too contrived) show lack of effective opacity if low-Z is assumed
- Determinations of solar opacity profile through inversions (Buldgen et al. 2025), helios & solar neutrinos (Song et al. 2018, Villante et al. 2014, others) cannot separate opacity from composition



Breaking the degeneracy between opacity & composition

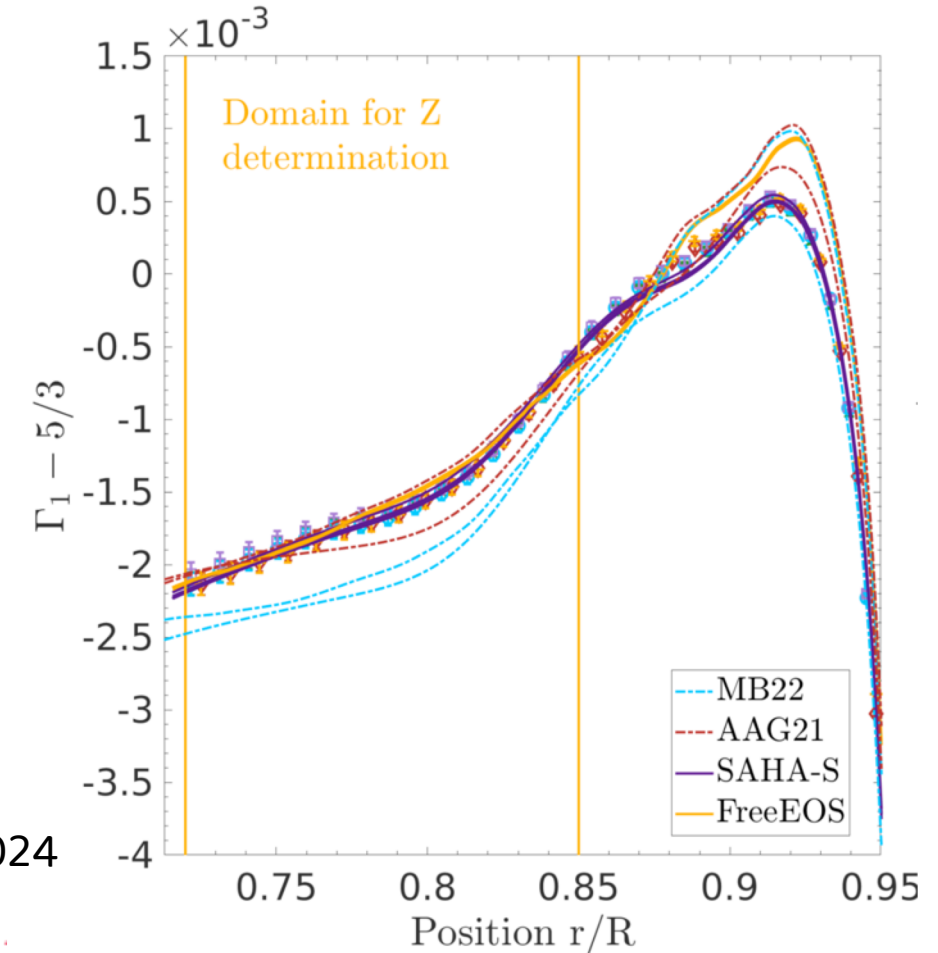
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$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_{\text{ad}} = 5/3 \text{ (for fully ionized gas)}$$

- It can be determined through inversion of solar oscillations and compared to solar models.
- Γ_1 determination depends on equation of state

Results indicate agreement with AAG21 (low-Z) rather than MB22

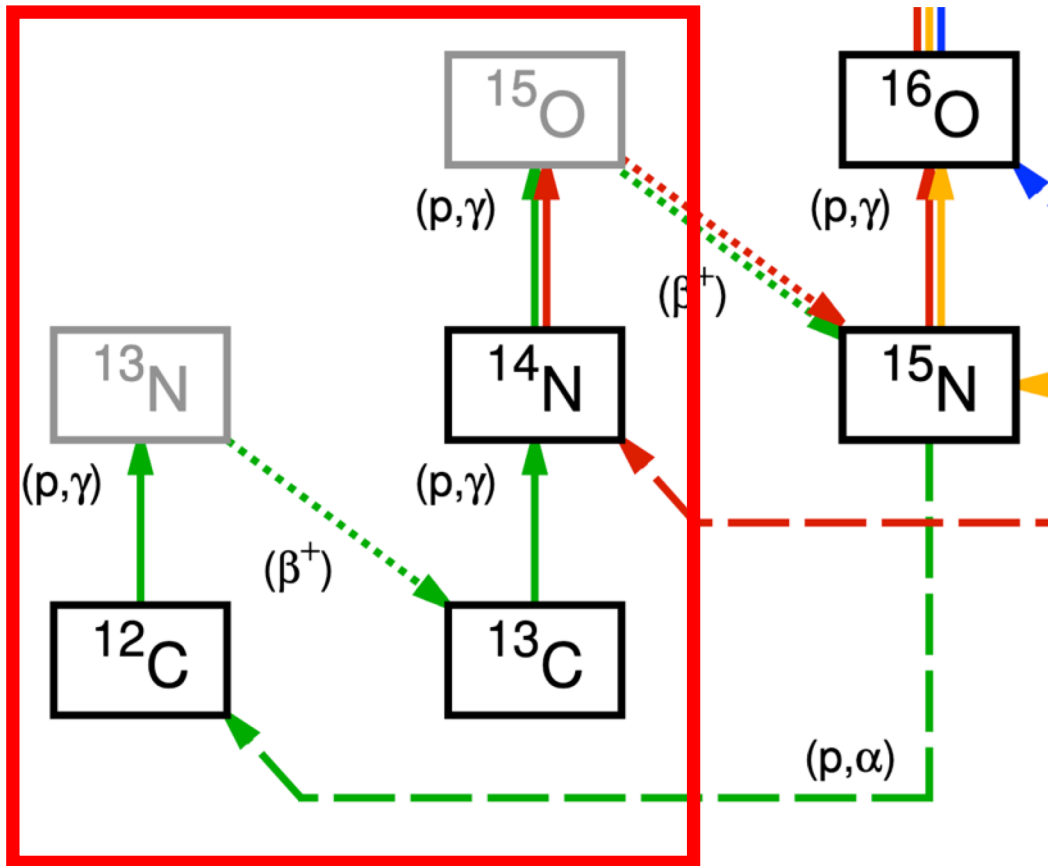
Buldgen et al. 2024



.....

CN-cycle is a marginal contributor to solar structure

CN operates against a “fixed” structure determined by pp-chains

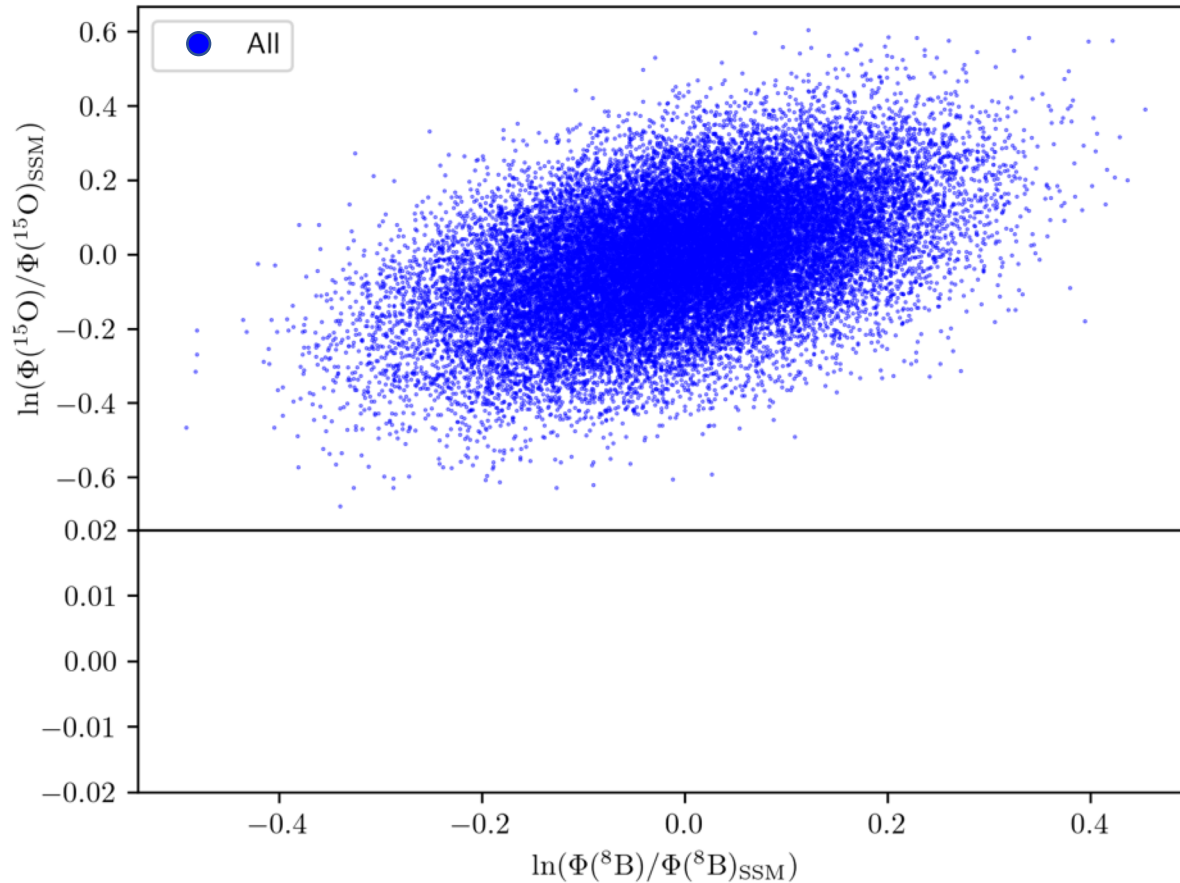


Changes in physics affecting CN do not change structure, **i.e. core temperature**,
→ retain explicit dependences:

- e.g. linear response to bottleneck nuclear reaction
 $^{14}\text{N}(p, \gamma)^{15}\text{O}$
- **linear dependence on abundance of catalyzers in solar core: C+N**
- **one-to-one relation between neutrino fluxes and CN abundance**

What CN solar neutrinos tell us

^8B as a thermometer



Neutrino fluxes depend on:

➤ **solar core temperature – environmental quantities**

opacity
heavy elements (Si, Mg, Fe)
luminosity, age

uncertainties in these quantities affect ν -fluxes in a fully correlated way

➤ **nuclear reaction rates**

specific dependence for specific fluxes
(e.g. $^{14}\text{N}(p,g)^{15}\text{O}$ does not affect pp-chain)

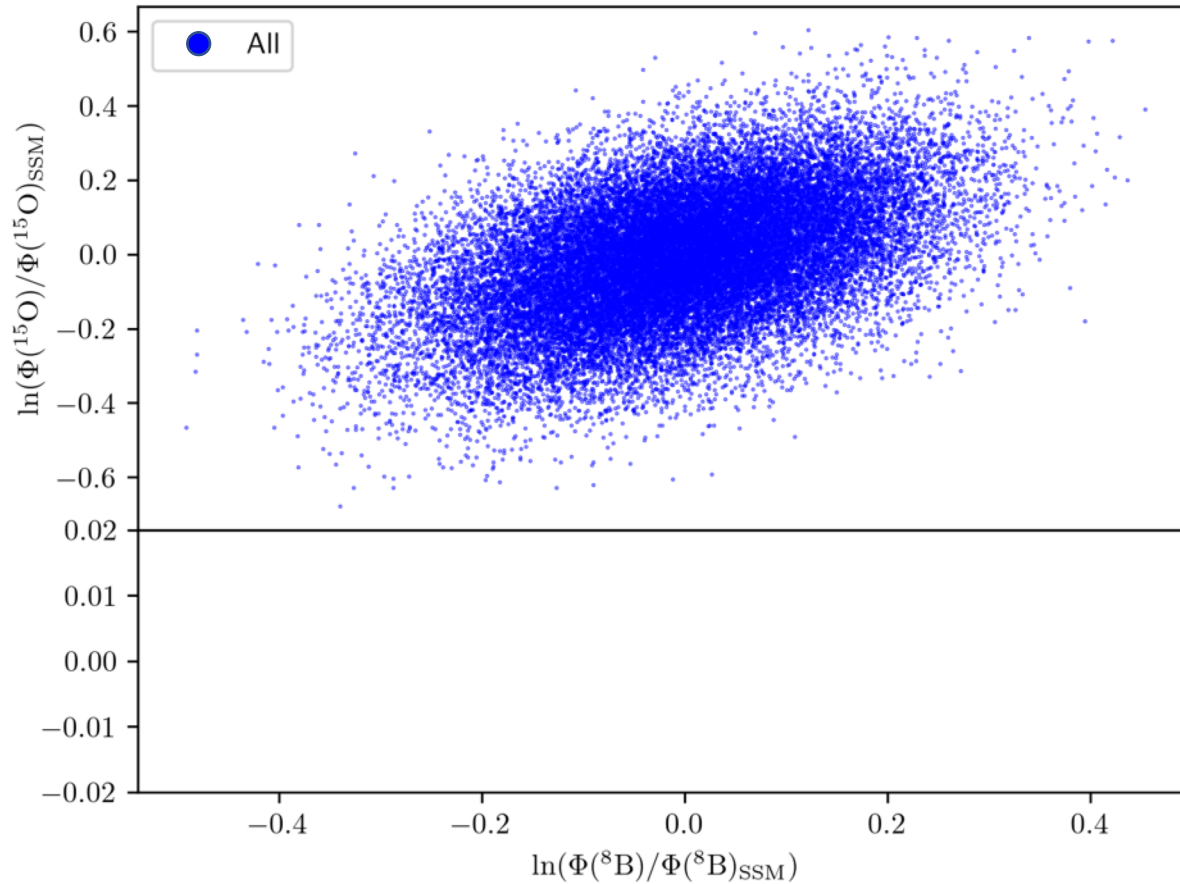
➤ **catalyzing effect of abundances**

C & N abundance in the solar core → CN-cycle



What CN solar neutrinos tell us

^8B as a thermometer



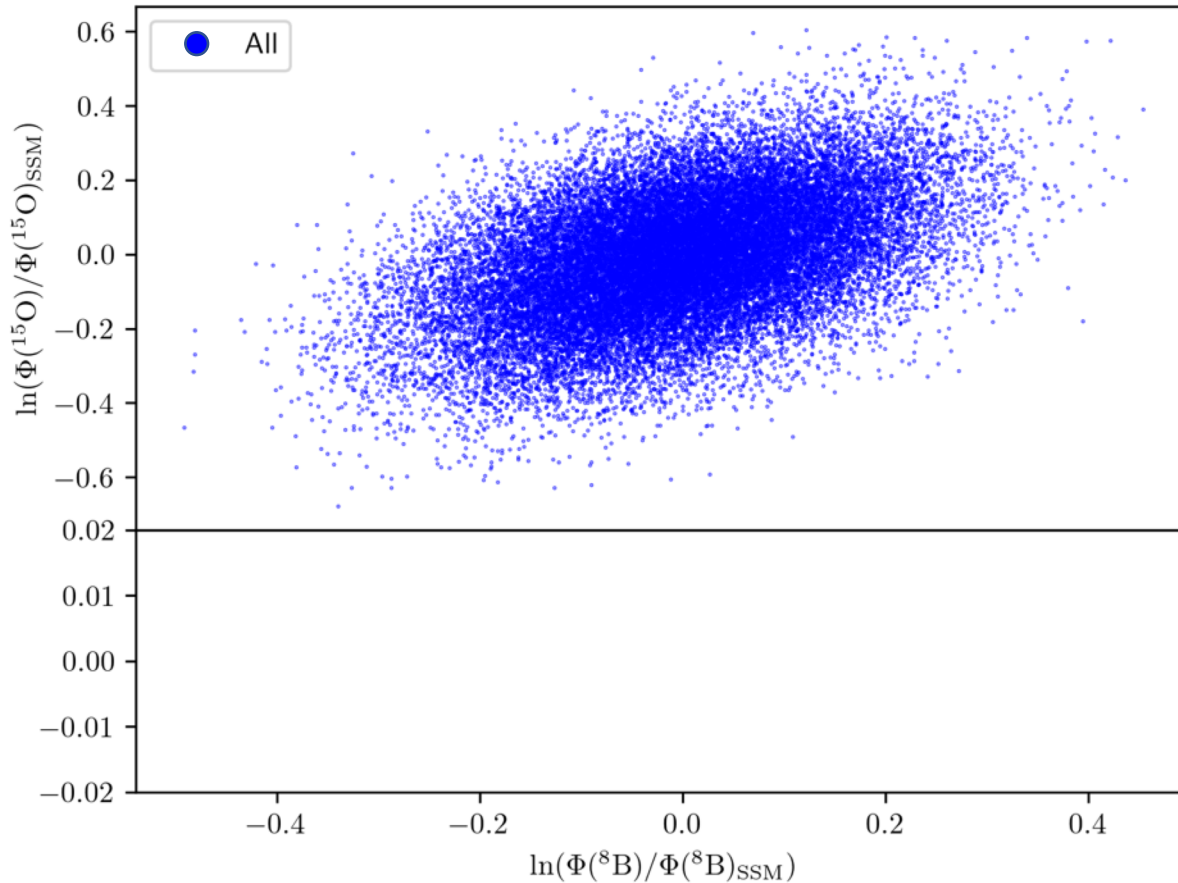
Neutrino fluxes as power-laws:

$$\frac{\partial \log \Phi_i}{\partial \log q_j} = \alpha_{ij} \longrightarrow \frac{\Phi_i}{\Phi_{i,\text{ref}}} = \left(\frac{q_j}{q_{j,\text{ref}}} \right)^{\alpha_{ij}}$$



What CN solar neutrinos tell us

^8B as a thermometer



Neutrino fluxes as power-laws:

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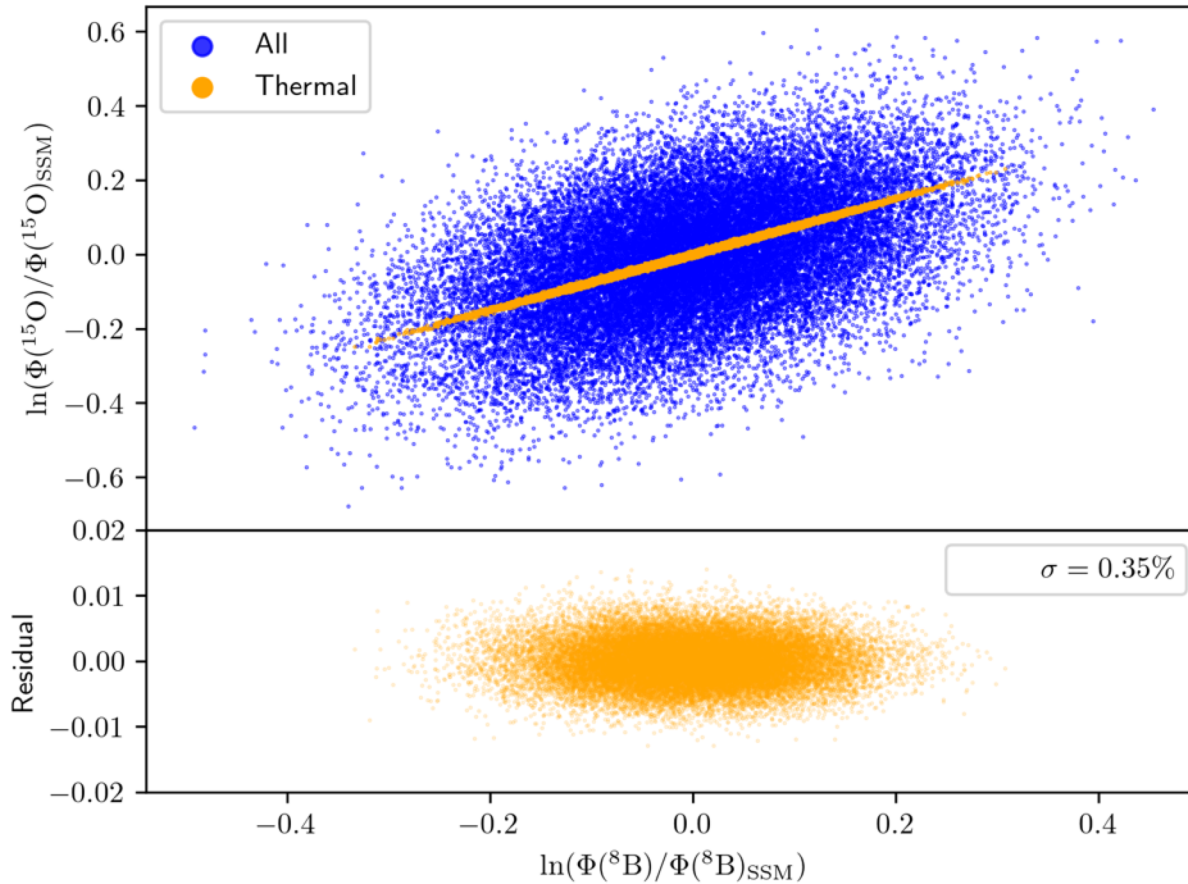
$$\begin{aligned} \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} &= [L_{\odot}^{5.942} O^{2.034} A^{1.364} D^{0.382}] \\ &\times [S_{11}^{-2.912} S_{33}^{0.024} S_{34}^{-0.052} S_{17}^{0.0} S_{e7}^{0.0} S_{114}^{1.00}] \\ &\times [x_C^{0.815} x_N^{0.217} x_O^{0.112} x_{\text{Ne}}^{0.081} x_{\text{Mg}}^{0.069} x_{\text{Si}}^{0.150} x_S^{0.109} x_{\text{Ar}}^{0.028} x_{\text{Fe}}^{0.397}] \end{aligned}$$

$$\begin{aligned} \frac{\phi(^8\text{B})}{\phi(^8\text{B})_{\text{SSM}}} &= [L_{\odot}^{6.966} O^{2.734} A^{1.319} D^{0.278}] \\ &\times [S_{11}^{-2.665} S_{33}^{-0.419} S_{34}^{0.831} S_{17}^{1.028} S_{e7}^{-1} S_{114}^{0.00}] \\ &\times [x_C^{0.022} x_N^{0.007} x_O^{0.128} x_{\text{Ne}}^{0.102} x_{\text{Mg}}^{0.092} x_{\text{Si}}^{0.198} x_S^{0.138} x_{\text{Ar}}^{0.034} x_{\text{Fe}}^{0.498}] \end{aligned}$$



What CN solar neutrinos tell us

^8B as a thermometer

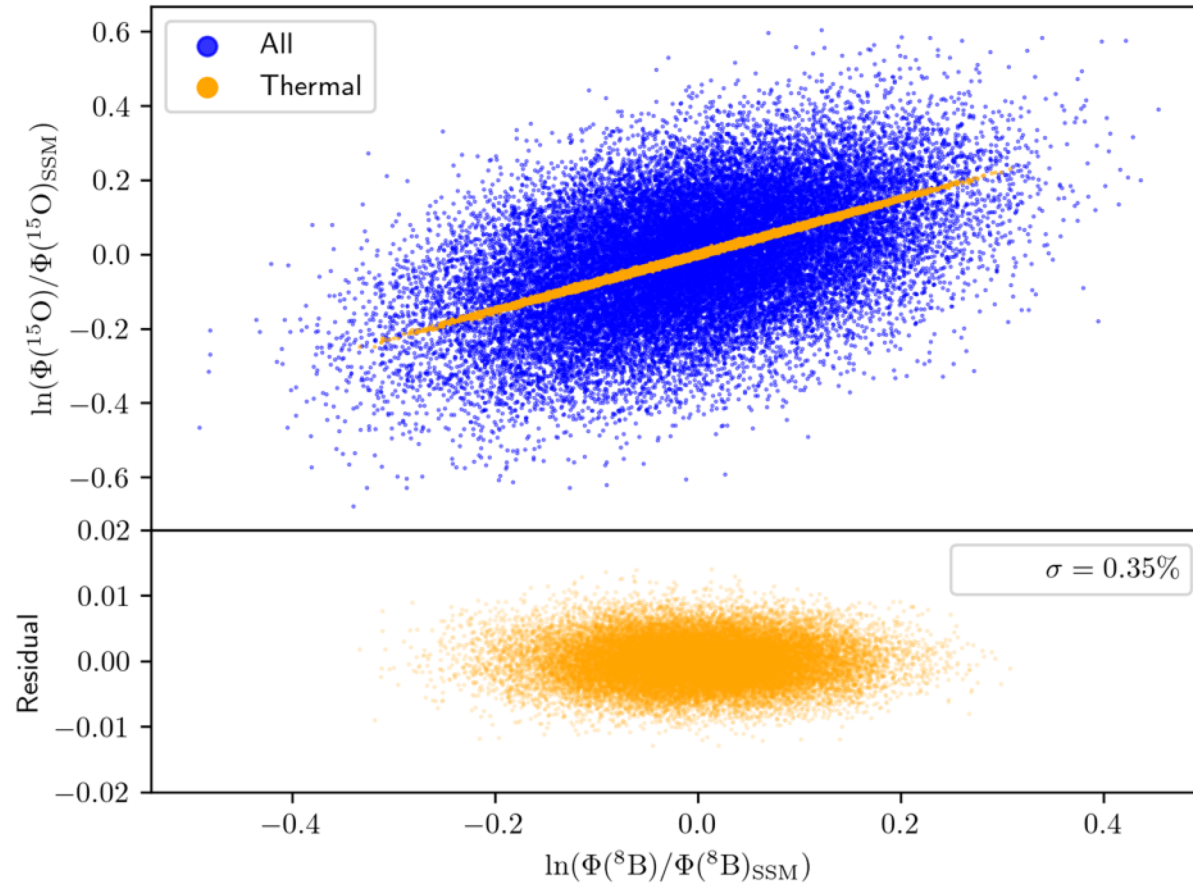


Thermal uncertainties are cancelled out, absorbed by a ^8B experimental measurement, down to 0.3%

$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} / \left[\frac{\phi(^8\text{B})}{\phi_{\text{SSM}}(^8\text{B})} \right]^{0.785} = x_C^{0.794} x_N^{0.212} D^{0.172} \\ \times [L_{\odot}^{0.515} O^{-0.016} A^{0.308}] \\ \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ \times [x_O^{0.003} x_{\text{Ne}}^{-0.005} x_{\text{Mg}}^{-0.003} x_{\text{Si}}^{-0.001} x_S^{-0.001} x_{\text{Ar}}^{0.001} x_{\text{Fe}}^{0.003}]$$

What CN solar neutrinos tell us

^8B as a thermometer

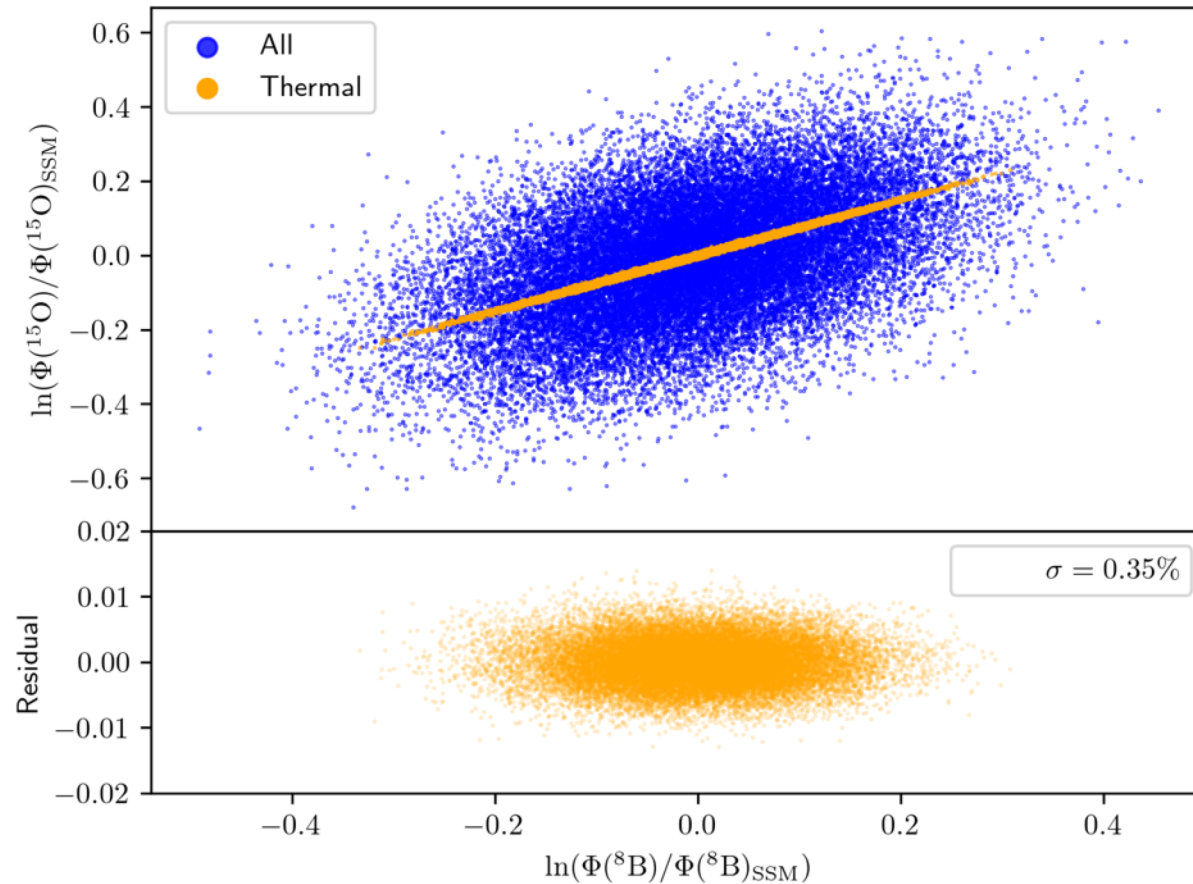


$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} / \left[\frac{\phi(^8\text{B})}{\phi_{\text{SSM}}(^8\text{B})} \right]^{0.785} = x_C^{0.794} x_N^{0.212} D^{0.172} \times [L_{\odot}^{0.515} O^{-0.016} A^{0.308}] \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \times [x_O^{0.003} x_{\text{Ne}}^{-0.001} x_{\text{Fe}}^{0.003}]$$

Nuclear reaction rates

What CN solar neutrinos tell us

^8B as a thermometer



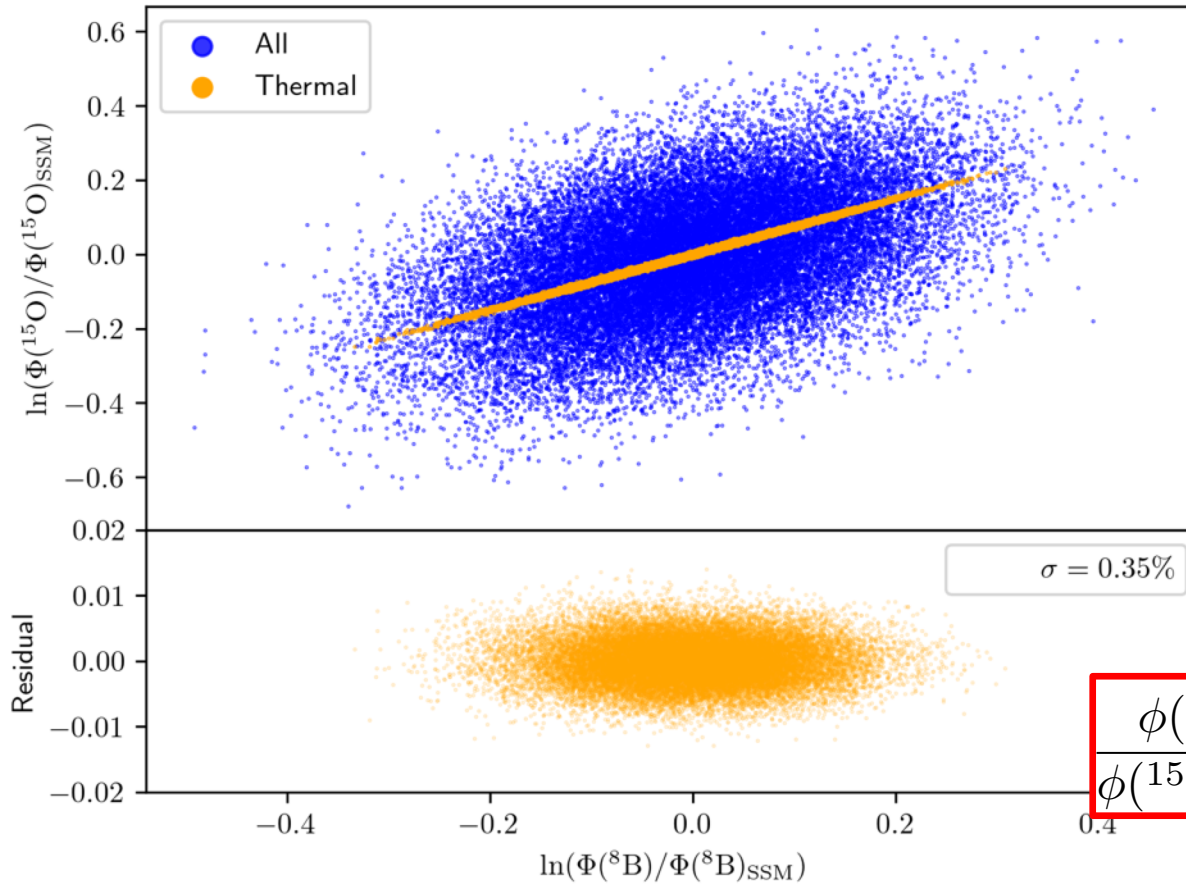
**Linear dependence
on C+N**

$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} / \left[\frac{\phi(^8\text{B})}{\phi_{\text{SSM}}(^8\text{B})} \right]^{0.785} = x_C^{0.794} x_N^{0.212} O^{0.172} \times [L_{\odot}^{0.515} O^{-0.016} A^{0.308}] \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \times [x_O^{0.003} x_{\text{Ne}}^{-0.005} x_{\text{Mg}}^{-0.003} x$$

**Linear dependence on
 $^{14}\text{N}+\text{p}$ rate**

What CN solar neutrinos tell us

^8B as a thermometer



**Linear dependence
on C+N**

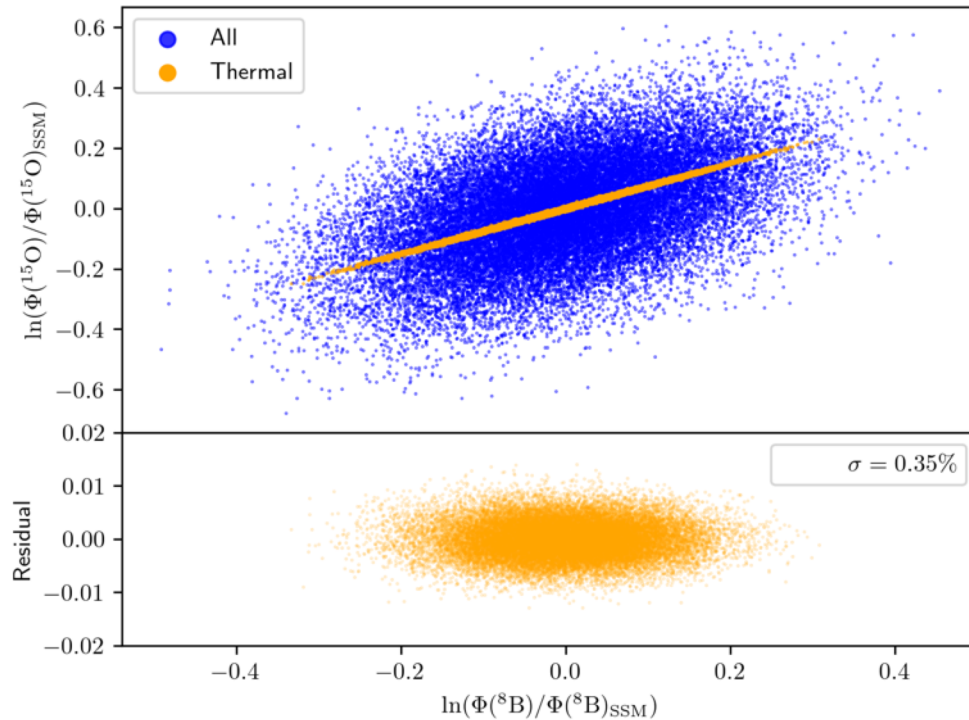
$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} / \left[\frac{\phi(^8\text{B})}{\phi_{\text{SSM}}(^8\text{B})} \right]^{0.785} = x_C^{0.794} x_N^{0.212} D^{0.172} \times [L_\odot^{0.515} O^{-0.016} A^{0.308}] \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \times [x_O^{0.003} x_{\text{Ne}}^{-0.001} x_{\text{Fe}}^{0.003}]$$

Nuclear reaction rates

$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} \simeq \left(\frac{\phi(^8\text{B})}{\phi(^8\text{B})_{\text{SSM}}} \right)^{0.785} (x_{\text{C+N}}) [1 \pm 0.10(\text{nuc}) + 0.03(\text{D})]$$

What CN solar neutrinos tell us

^8B as a thermometer



$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} \simeq \left(\frac{\phi(^8\text{B})}{\phi(^8\text{B})_{\text{SSM}}} \right)^{0.785} (x_{\text{C+N}}) [1 \pm 0.10(\text{nuc}) + 0.03(\text{D})]$$

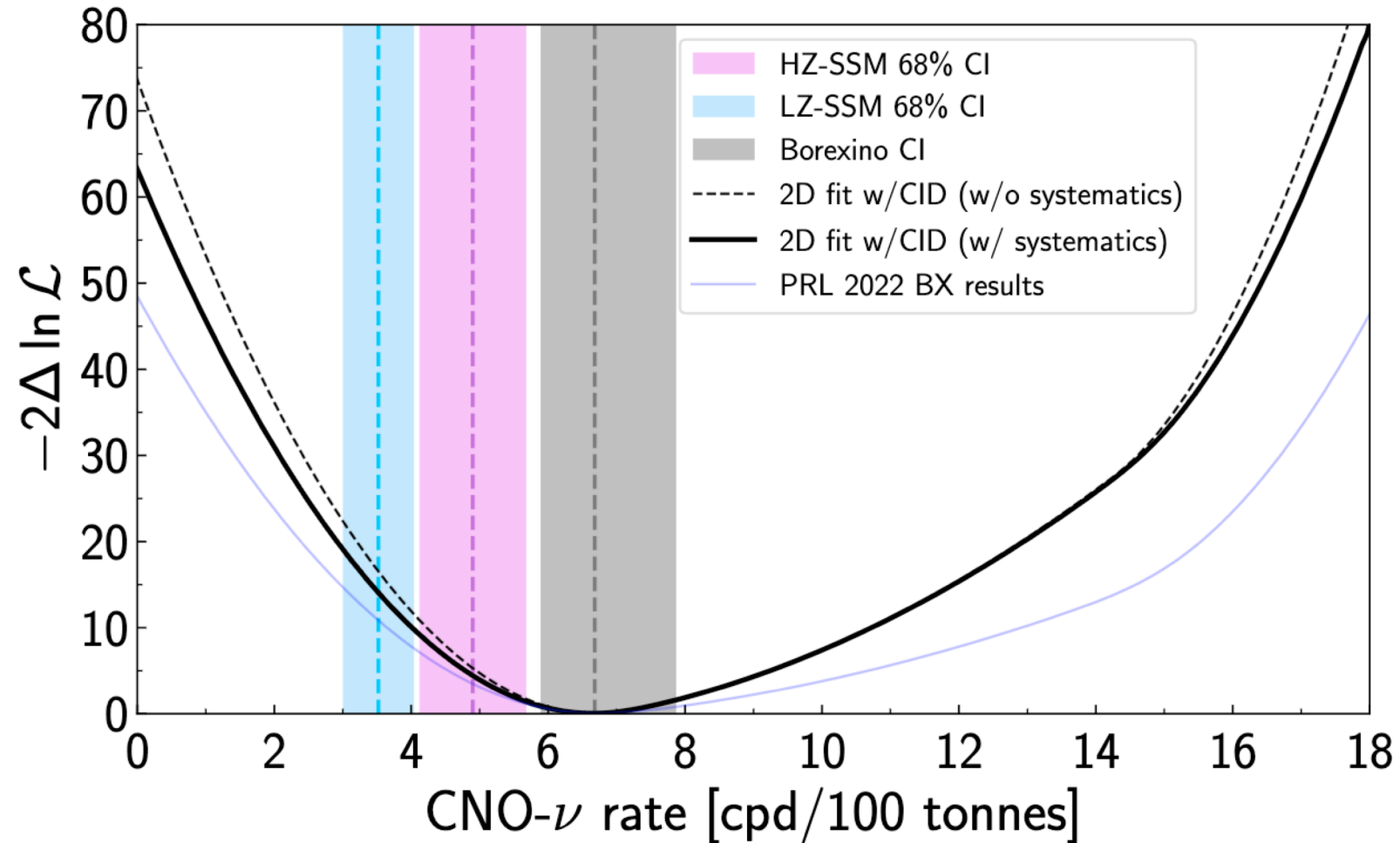
C+N abundance if ^{15}O flux is measured
(or any combination of ^{13}N & ^{15}O fluxes)

10% uncertainty (nuclear rates)

Using SFIII results, experimental uncertainty dominated by

- $^{14}\text{N}(\text{p},\gamma)^{15}\text{O}$ (8.5%)
- $^3\text{He}(^4\text{He},\gamma)^7\text{Be}$ (5%)

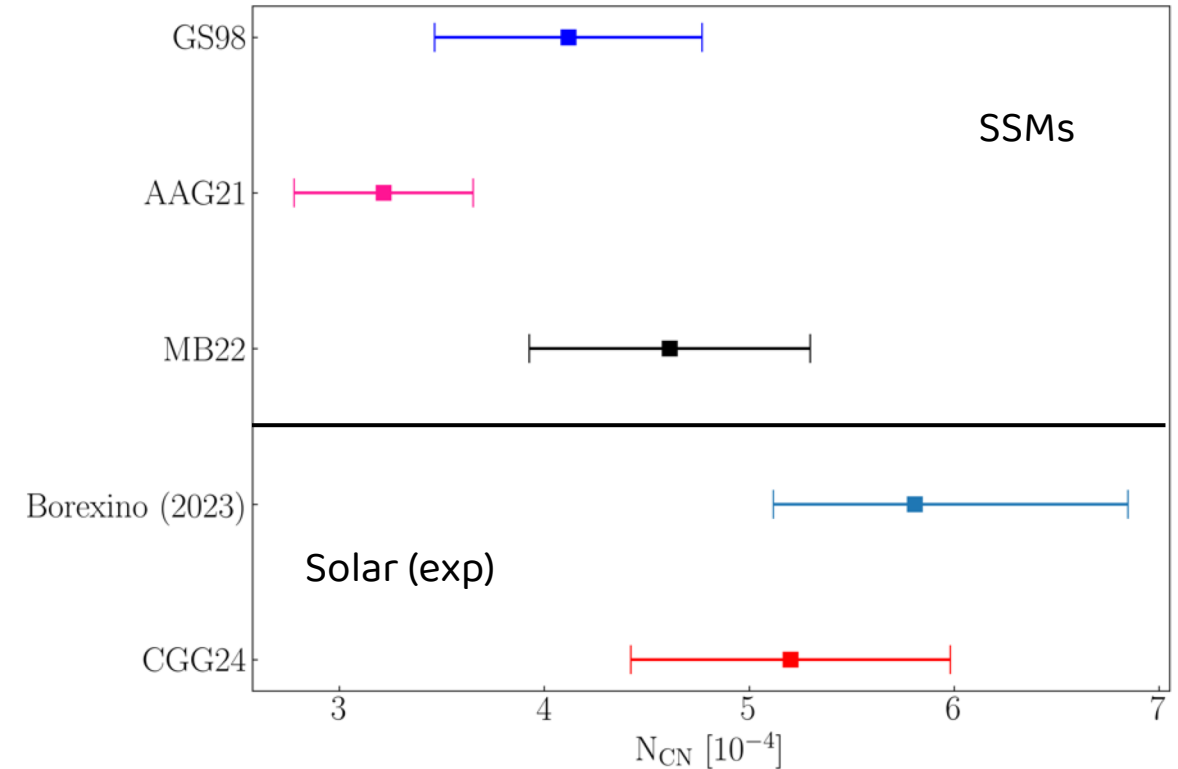
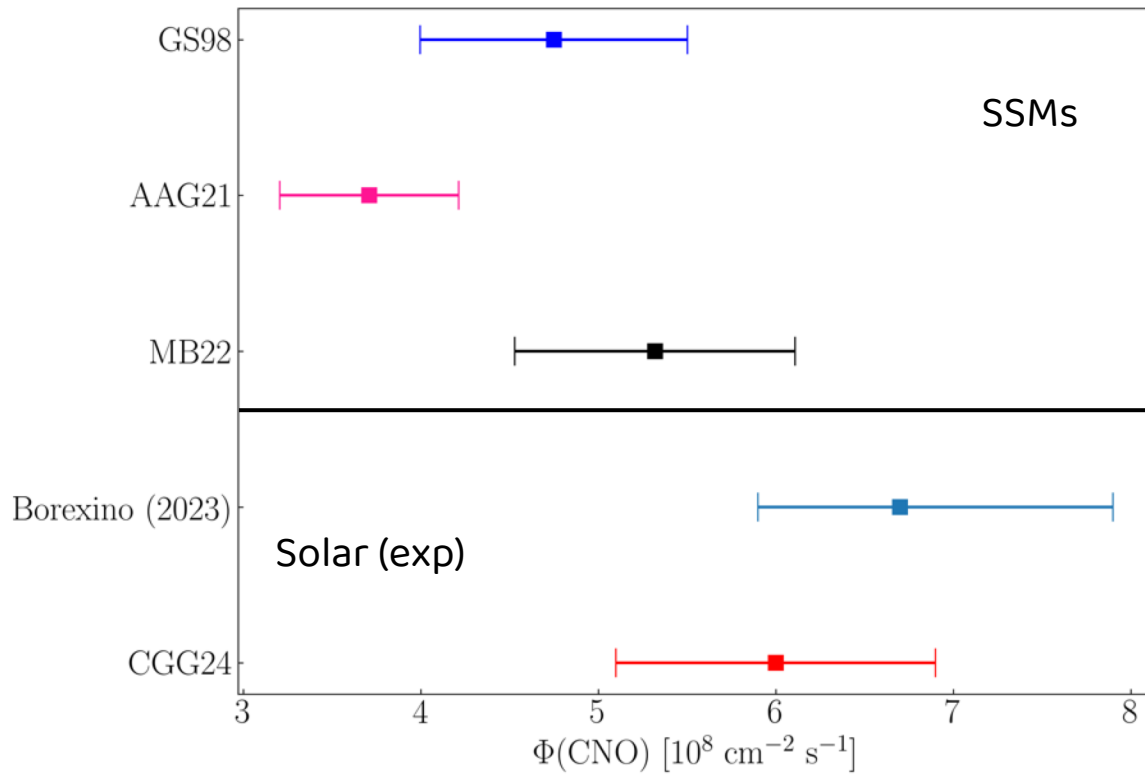
Borexino measurement of $^{13}\text{N}+^{15}\text{O}$ fluxes (Borexino coll. 2022, 2023)



What CN solar neutrinos tell us

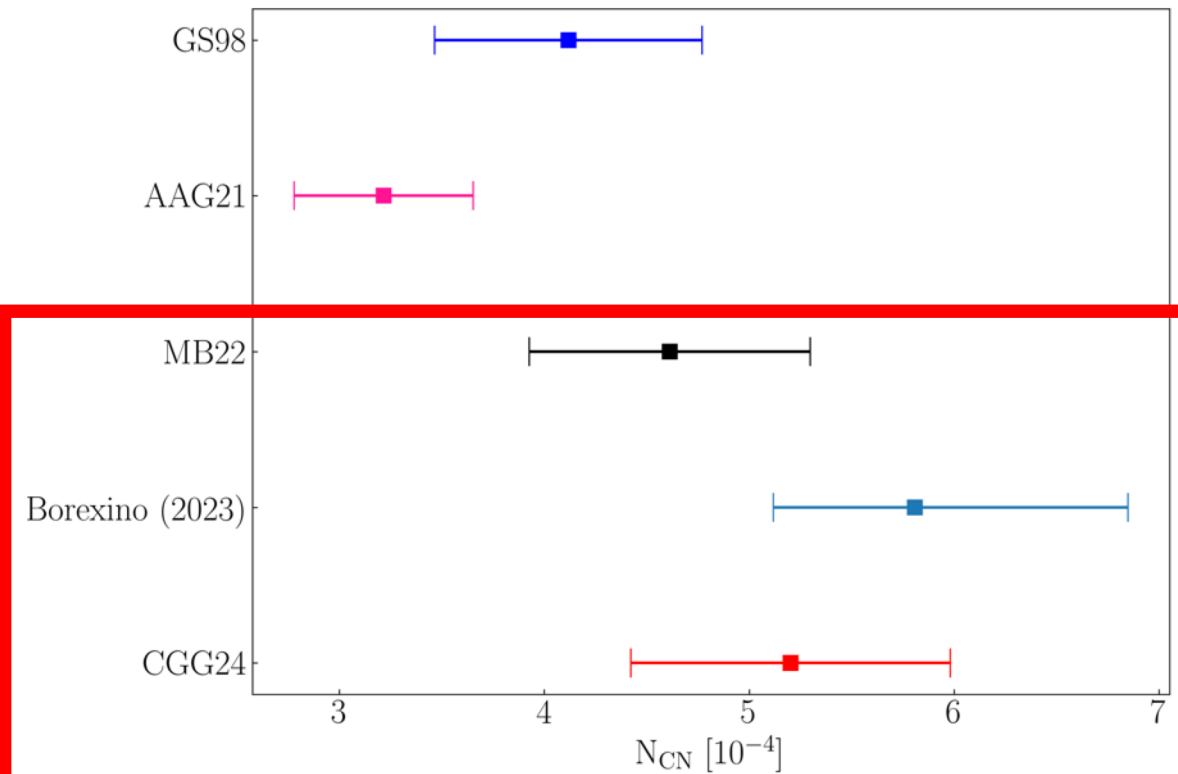
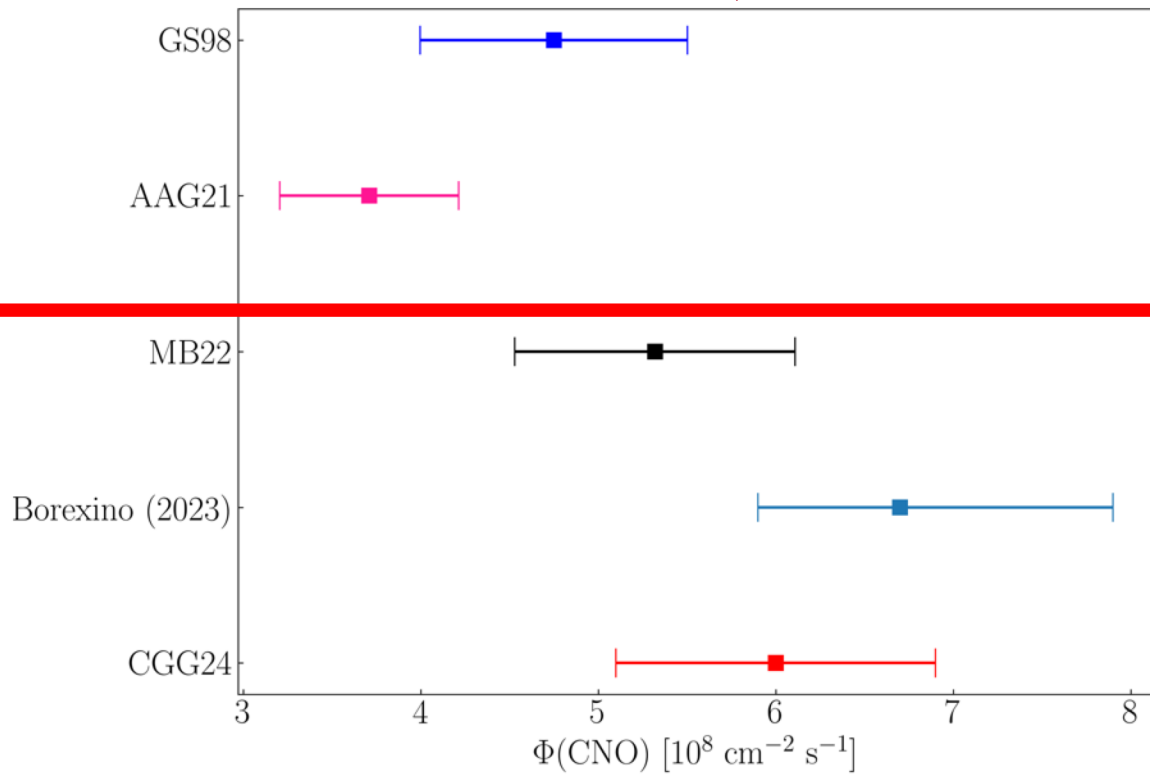
SuperK & SNO (2%)

$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} \simeq \left(\frac{\phi(^8\text{B})}{\phi(^8\text{B})_{\text{SSM}}} \right)^{0.785} (x_{\text{C+N}})[1 \pm 0.10(\text{nuc}) + 0.03(\text{D})]$$



What CN solar neutrinos tell us

$$\frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})_{\text{SSM}}} \simeq \left(\frac{\phi(^8\text{B})}{\phi(^8\text{B})_{\text{SSM}}} \right)^{0.785} (x_{\text{C+N}})[1 \pm 0.10(\text{nuc}) + 0.03(\text{D})]$$



CN neutrinos break the degeneracy between composition and opacity

Favor high CN abundance

Nuclear rates largest source of uncertainty, but one we can control

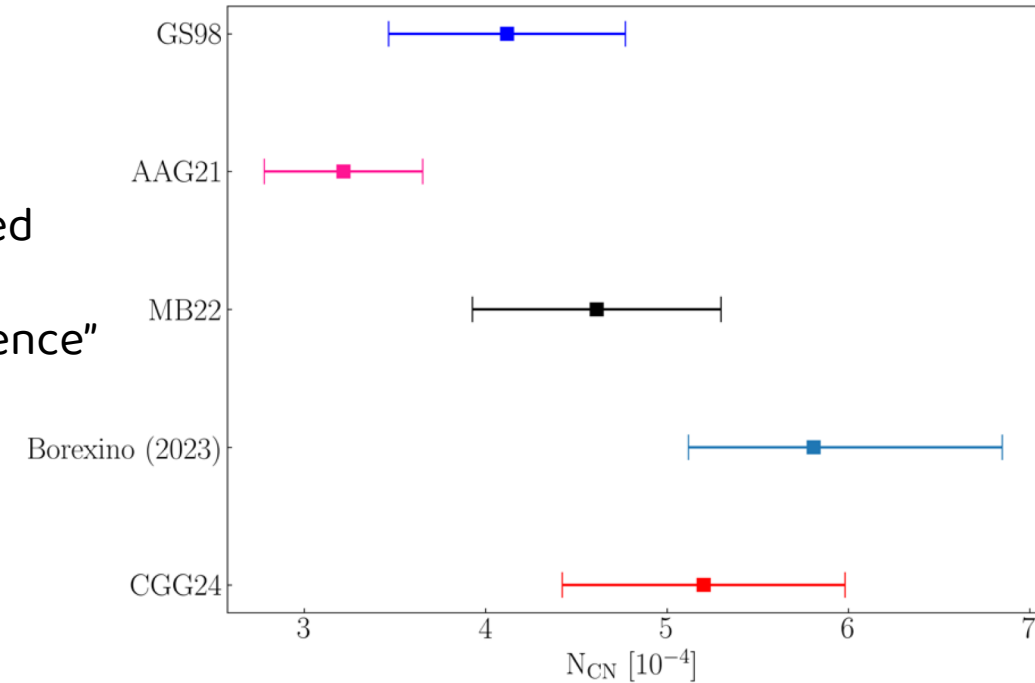
What CN solar neutrinos tell us

What can affect core CN measurement?

- $^{14}\text{N}(p,\gamma)$ central value $\rightarrow$ accurate and precise measurement needed
- $^7\text{Be}(p,\gamma)$ or $^3\text{He}(^4\text{He},g)$ central values $\rightarrow$ but $^7\text{Be} - ^8\text{B}$ "thermal sequence" make it unlikely, but better uncertainties needed
- CN- ν measurement uncertainties (still) large

What can NOT affect core CN measurement?

- model dependence down to a bare minimum
- uncertainties in opacities, macroscopic mixing processes



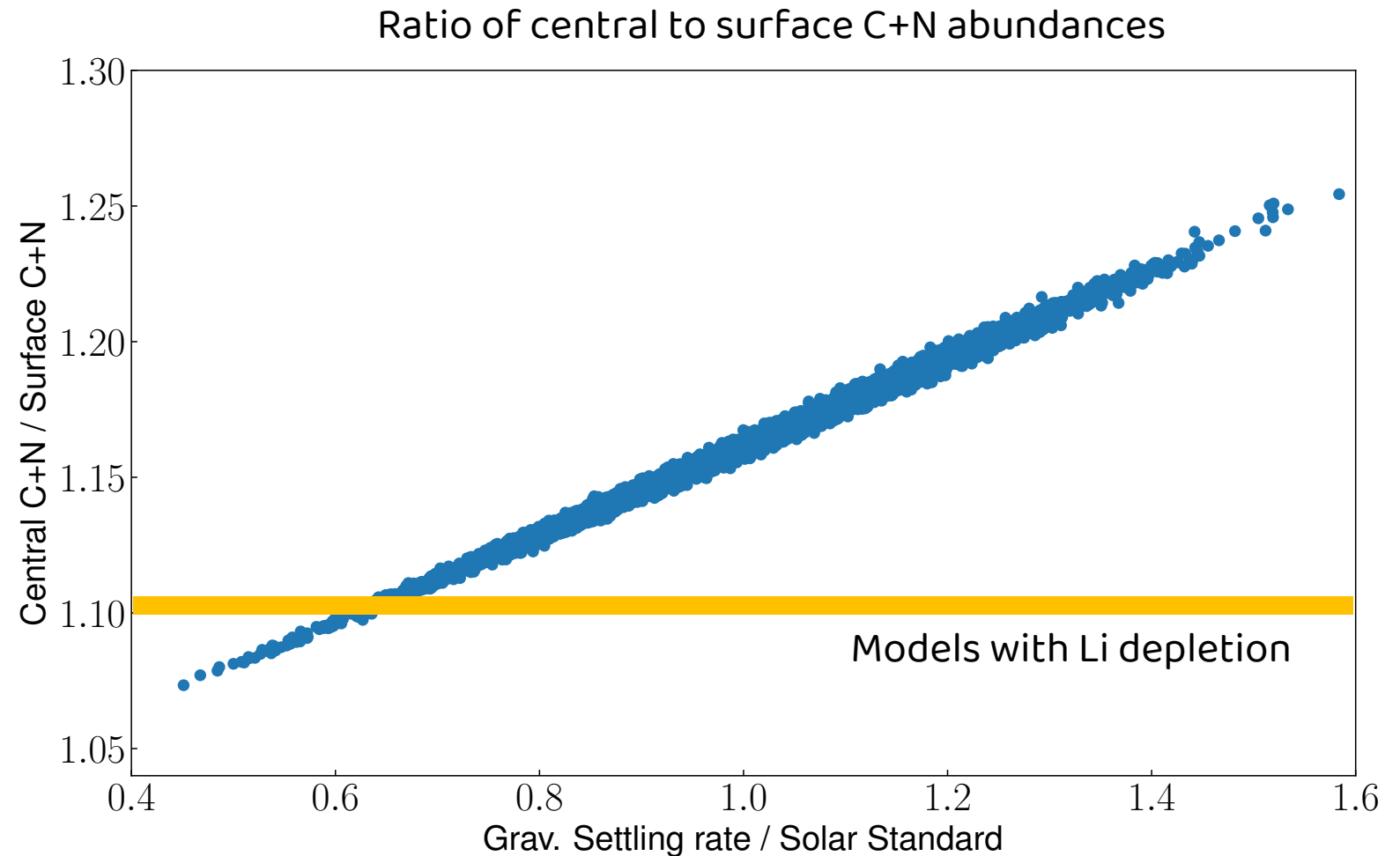
Future: measurement of mixing processes in the Sun?

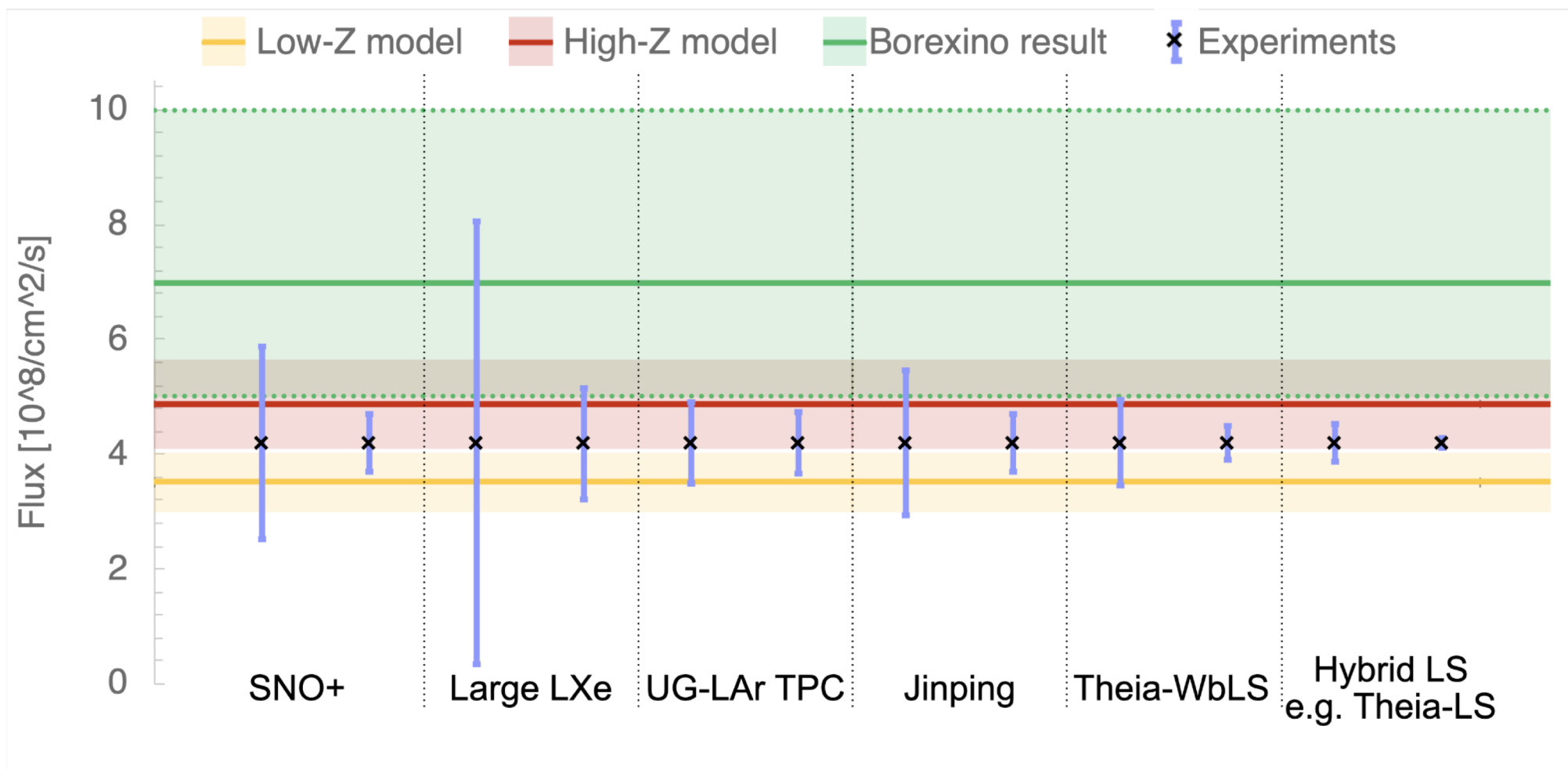
Current errors too large

- 10-15% spectroscopy
- 20% neutrinos

Errors of a few % would
be needed

Possible in the (distant?)
future





Orebi-Gann et al. 2021

Traditional helioseismic probes – sound speed/density, depth of convective zone, surface helium

favor combination of current opacity tables + high-Z composition

frequency ratios as well

however quantitative result might be affected by non-SSM models (see Buldgen's talk)

regardless, combination of low-Z and current opacities are not satisfactory

pp-chain neutrinos

similar conclusions regarding metallicity and opacities

useful to fix core scale for opacities – see example with OPLIB/Los Alamos

Breaking degeneracy

Γ_1 adiabatic index in solar envelope favors low-Z - modulo equation of state

CN neutrinos favor high-Z (actually, C+N)

General conclusions

state of the art for opacities is highly unsatisfactory: discrepant with experiment, large differences among calculations

key nuclear reactions require confirmation of accuracy and improvement in precision (e.g. ideally $^{14}\text{N}+\text{p} < 3\%$)

future neutrino experiments – not directly solar experiments – might provide CN- ν better measurements

