

Nuclear reactions for Standard Solar Models

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Schools on Nuclear Astrophysics Questions
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Institute of
Space Sciences



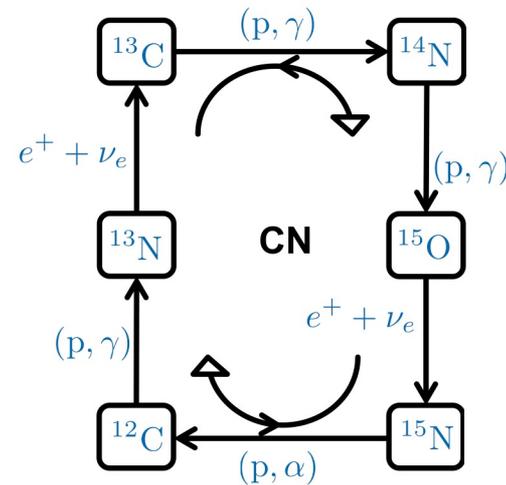


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CN-cycle initially proposed
in the 1930s

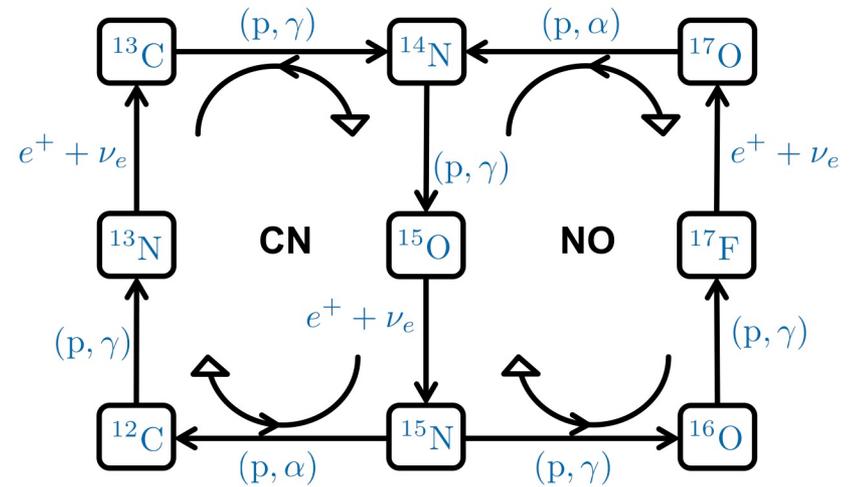




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Part of the larger CNO-cycle

CNO-(bi)cycle



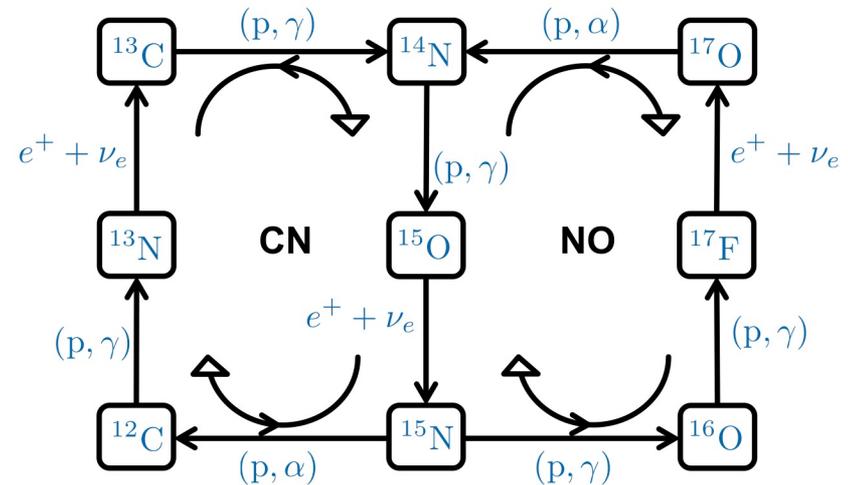


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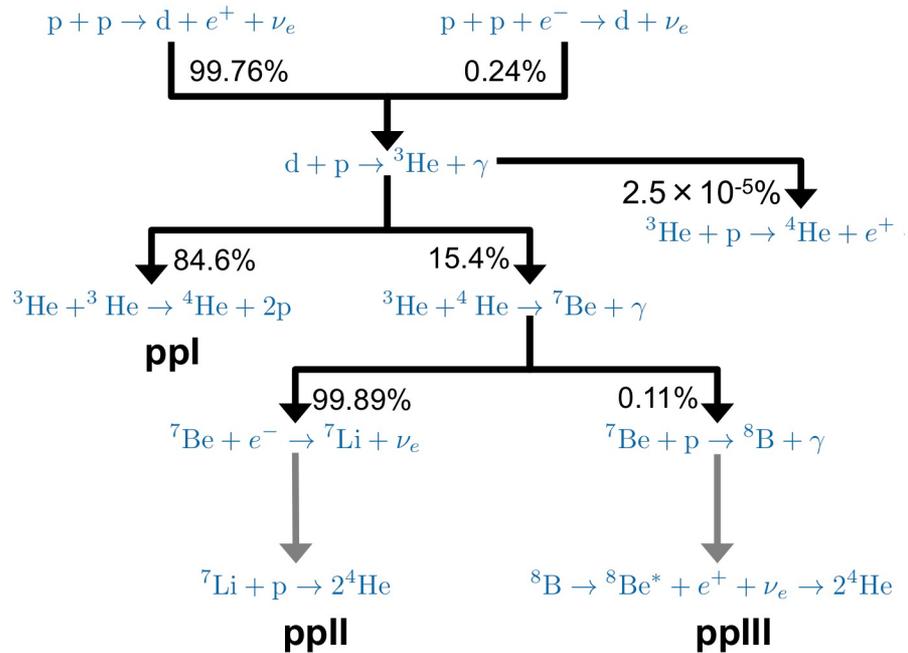
CN-cycle \sim 50 NO-cycle (Sun)

CNO-(bi)cycle





$$\gamma \leq 26.7\text{MeV} = c^2(4m_p - m_{{}^4\text{He}})$$

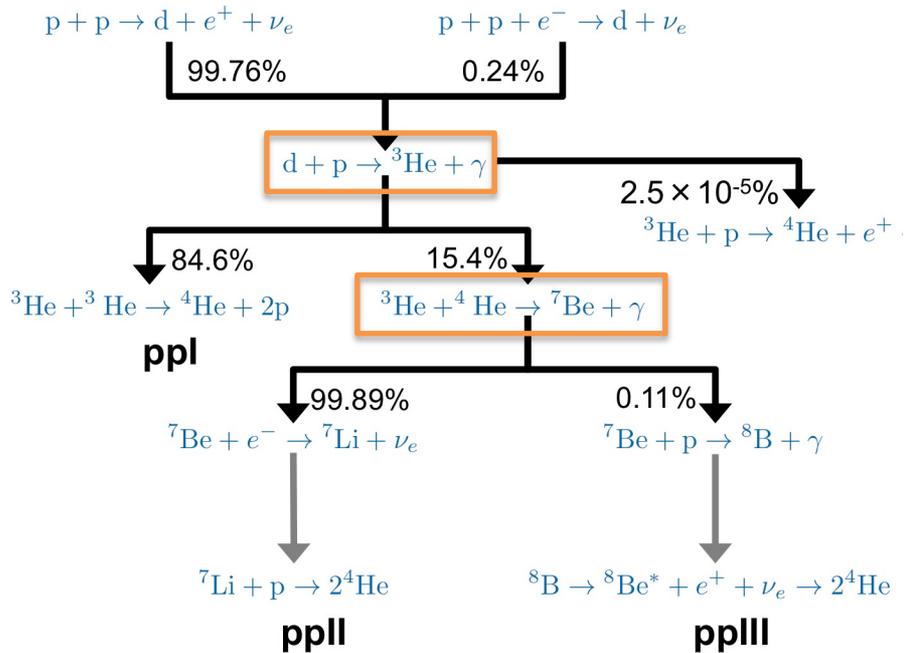


proton-proton chains



$$\gamma \leq 26.7\text{MeV} = c^2(4m_p - m_{{}^4\text{He}})$$

pp-chains



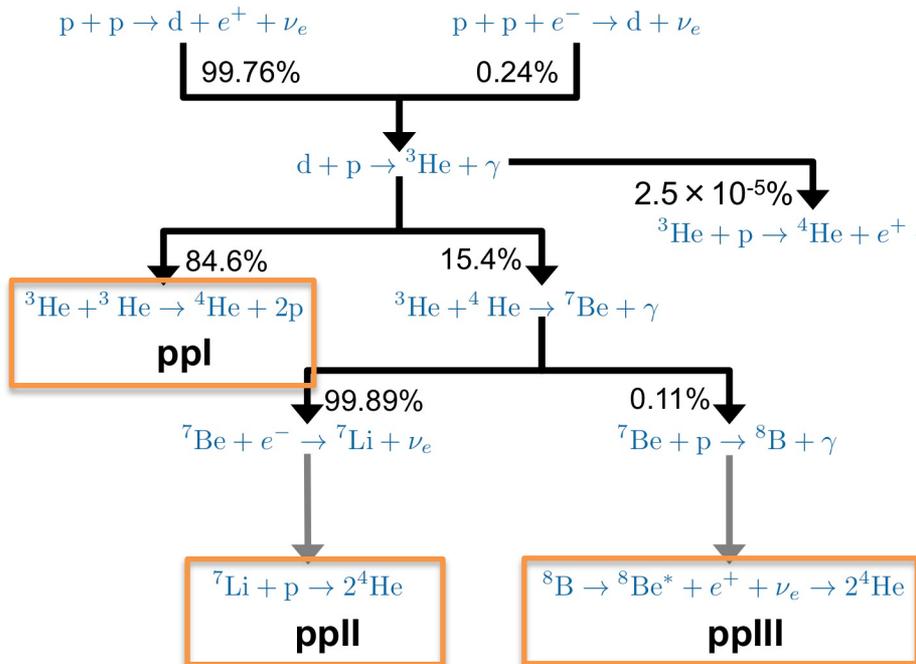
proton-proton chains

two main branching points



$$\gamma \leq 26.7\text{MeV} = c^2(4m_p - m_{{}^4\text{He}})$$

pp-chains



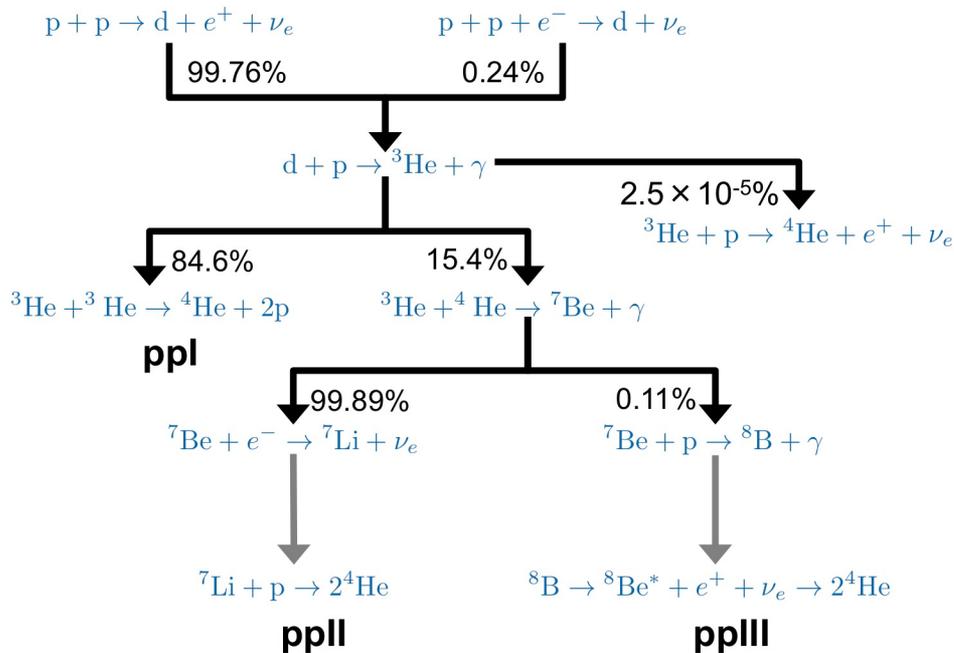
proton-proton chains

ppI:ppII:ppIII (& ${}^3\text{He}+p$)
85:15:1e-4

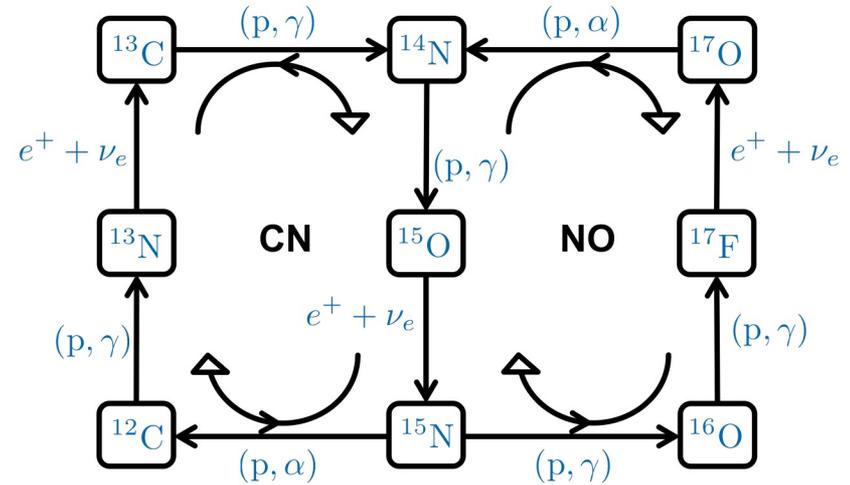


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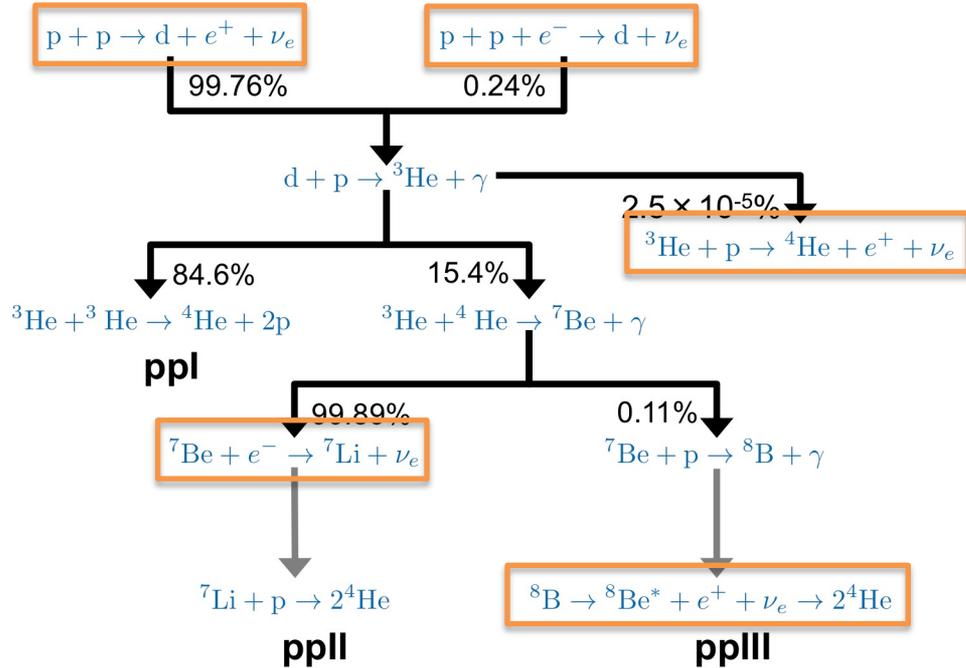


CNO-(bi)cycle



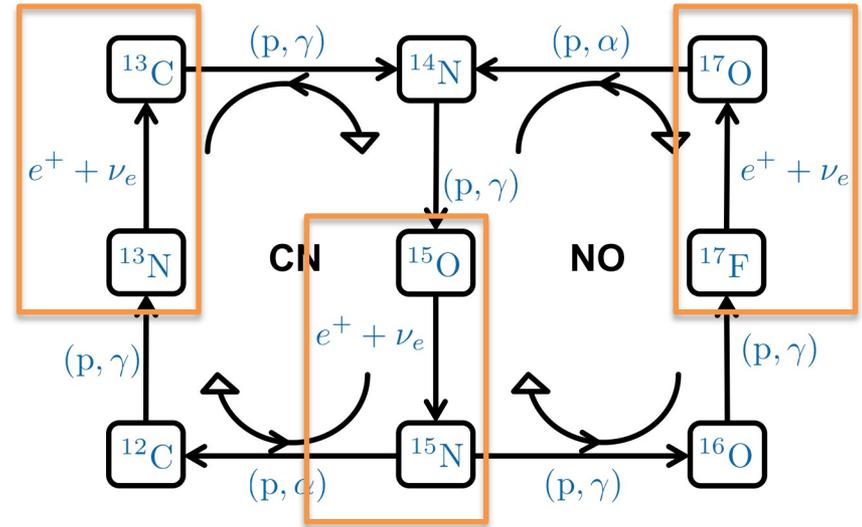
Eight fundamental neutrino sources can probe solar interior/physics

pp-chains



pp – pep – ${}^7\text{Be}$ – ${}^8\text{B}$ - hep

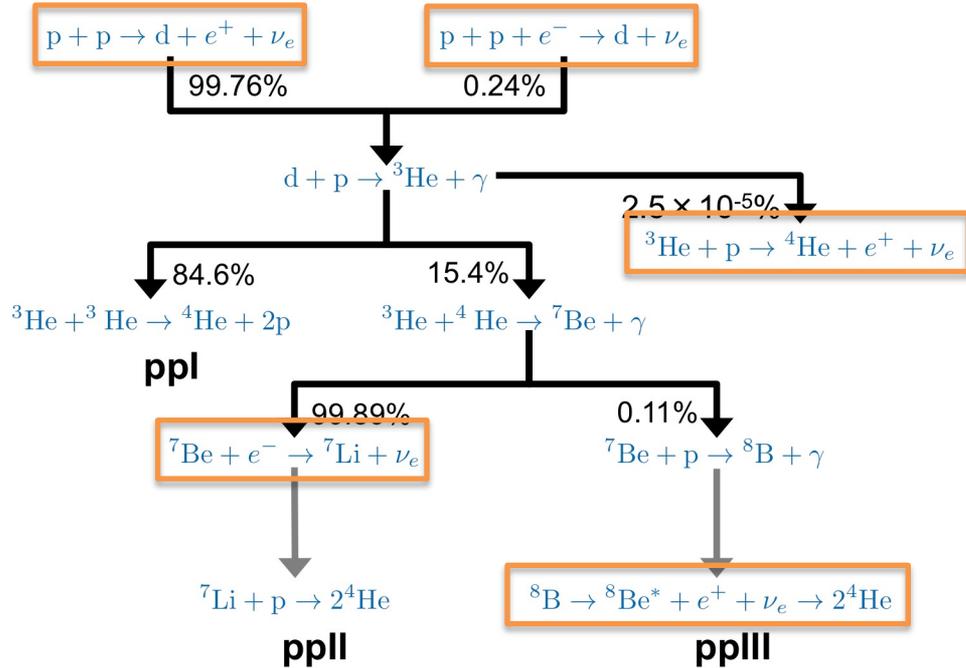
CNO-(bi)cycle



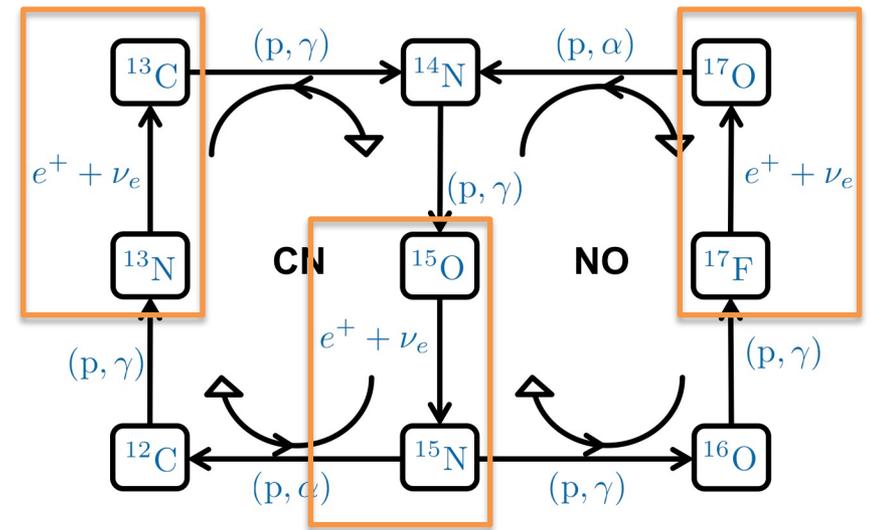
${}^{13}\text{N}$ – ${}^{15}\text{O}$ – ${}^{17}\text{F}$

Eight fundamental neutrino sources can probe solar interior/physics

pp-chains



CNO-(bi)cycle



A framework (physical model) for quantitative predictions is needed
(standard) solar models

stellar evolution - physical processes (equations)
constitutive physics (nuclear rates, EOS, opacity, etc.)
initial composition of the star

Reaction	Uncertainty	Ref.
$p + p \rightarrow d + \nu_e + e^+$	1%	A16
${}^3\text{He} + {}^3\text{He} \rightarrow 2p + {}^4\text{He}$	5.2%	A11
$p + {}^3\text{He} \rightarrow {}^4\text{He} + \nu_e + e^+$	30%	A11
${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$	5.2%	A11
$p + {}^7\text{Be} \rightarrow {}^8\text{B}$	4.7%	Z15
$e^- + {}^7\text{Be} \rightarrow {}^7\text{Li}$	2%	A11
$p + {}^{14}\text{N} \rightarrow {}^{15}\text{O}$	7.5%	M11
$p + {}^{16}\text{O} \rightarrow {}^{17}\text{F}$	7.6%	A11

Adelberger et al. 2011, Marta et al. 2011, Zhang et al. 2015, Acharya et al. 2016
 Other reactions have small influence on outcome of solar models

stellar evolution - physical processes (equations)
constitutive physics (nuclear rates, EOS, opacity, etc.)
initial composition of the star

3 observational constraints imposed at present age $\tau_{\odot}=4.57$ Gyr

- * photospheric $(Z/X)_{\odot}$ + solar mixture (fractional abundance of all metals)
- * solar radius R_{\odot}
- * solar luminosity $L_{\odot}= 3.8418 \times 10^{33}$ erg s⁻¹

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3 free parameters for the initial (pre-main sequence) model

- * Initial helium Y_{ini} (L_{\odot})
- * Initial metallicity Z_{ini} ($X+Y+Z=1$), so all composition determined ($Z/X)_{\odot}$
- * Efficiency of convection in mixing length α_{MLT} (R_{\odot})

Evolve until solar age τ_{\odot} and adjust free parameters until observational constraints are satisfied (iterative process)

A standard solar model imposes a strong constraint on solar energetics

$$L_{\odot} = \int_0^M (\varepsilon_{\text{nuc}} - \varepsilon_{\nu} + \varepsilon_{\text{eg}}) dm$$



gravothermal energy
(contraction/expansion/internal energy)
Negligible in main sequence star

A standard solar model imposes a strong constraint on solar energetics

$$L_{\odot} = \int_0^M (\underbrace{\varepsilon_{\text{nuc}} - \varepsilon_{\nu}}_{\substack{\text{nuclear energy production} \\ \text{and associated neutrino losses}}} + \varepsilon_{\text{eg}}) dm$$

nuclear energy production
and associated neutrino losses

$$\varepsilon_{\text{nuc}} = \sum_{\text{H-burn}} \varepsilon_i(T, \rho, \vec{X})$$

$$\varepsilon_{\nu} = \sum_{i=1,8} \langle \varepsilon_{\nu} \rangle_i$$

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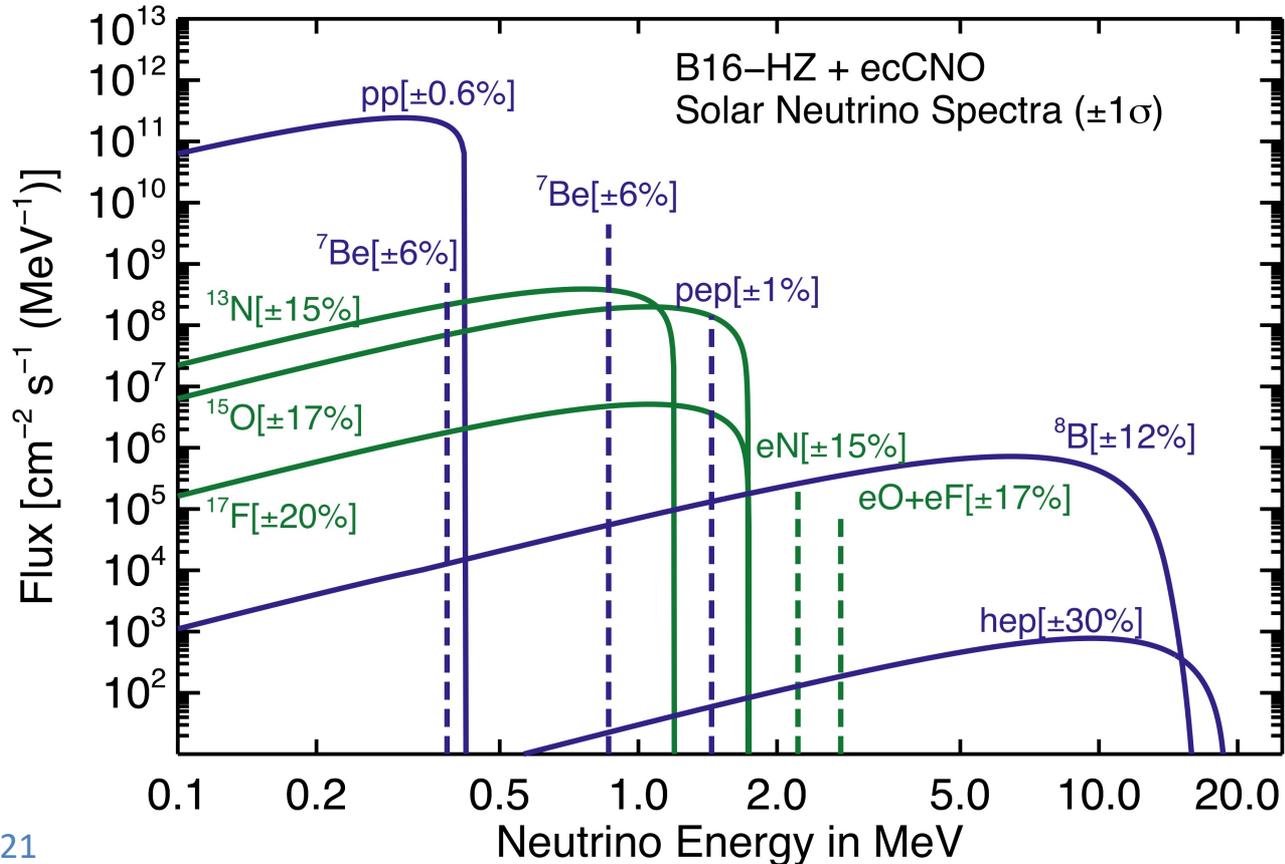
A linear relation between neutrino fluxes and solar luminosity can be obtained known as "Luminosity constraint" ([Bahcall 2002](#), [Vescovi et al. 2021](#))

$$L_{\odot} = \sum_{i=1,8} \alpha_i \Phi_i$$

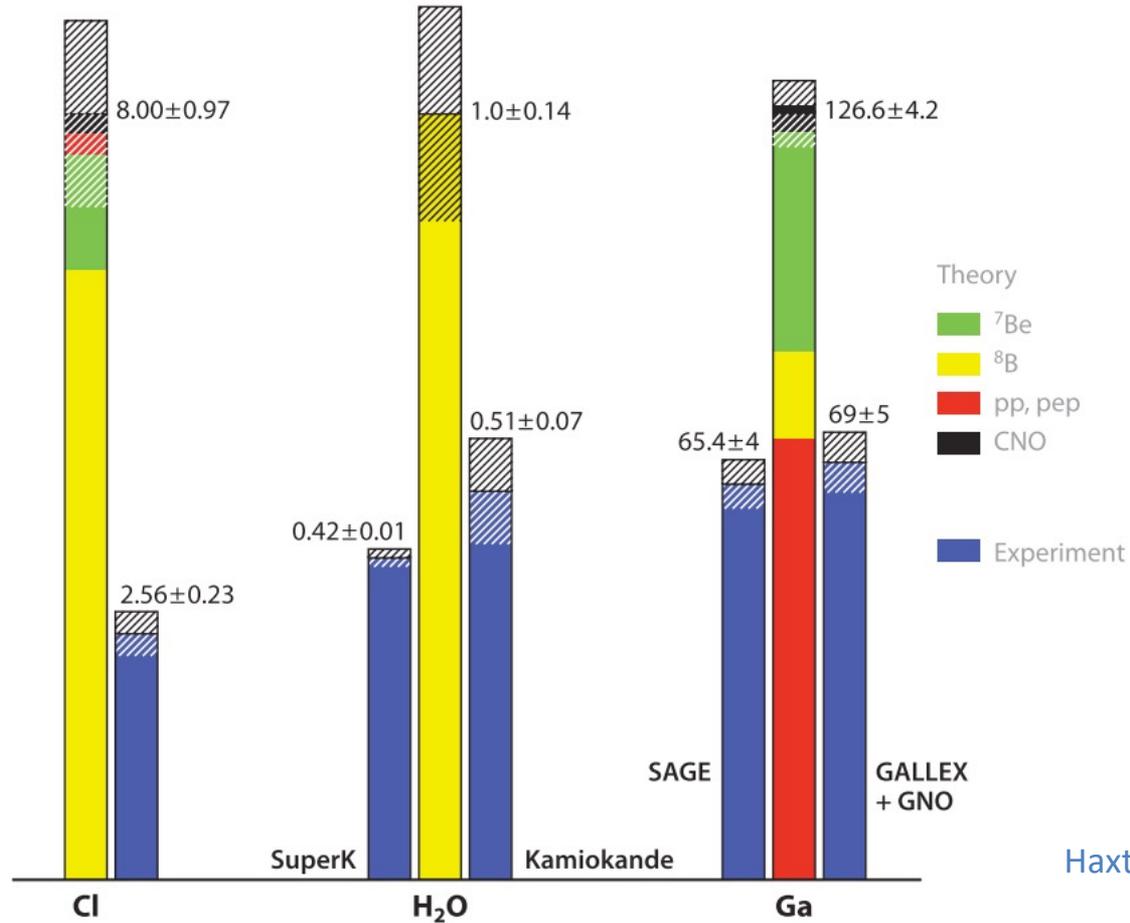
where α_i are given by **nuclear physics** and fluxes by the structure of the Sun (or model)

Spectral shape of neutrino fluxes well known (weak interactions)

Absolute fluxes depend on model



For ~30 years – ν_e only experiments showed a solar ν deficit



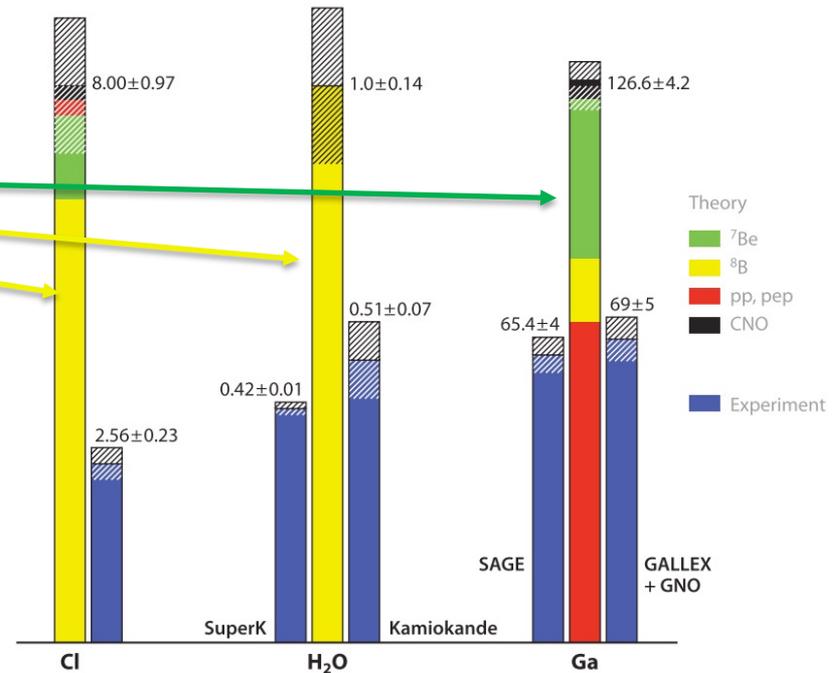
Two classes of astrophysical solution

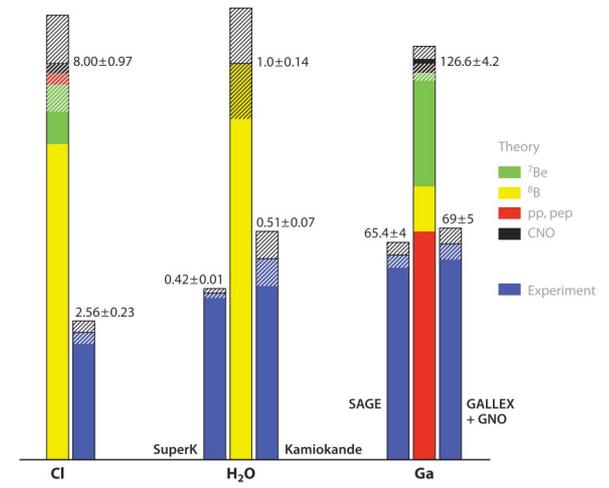
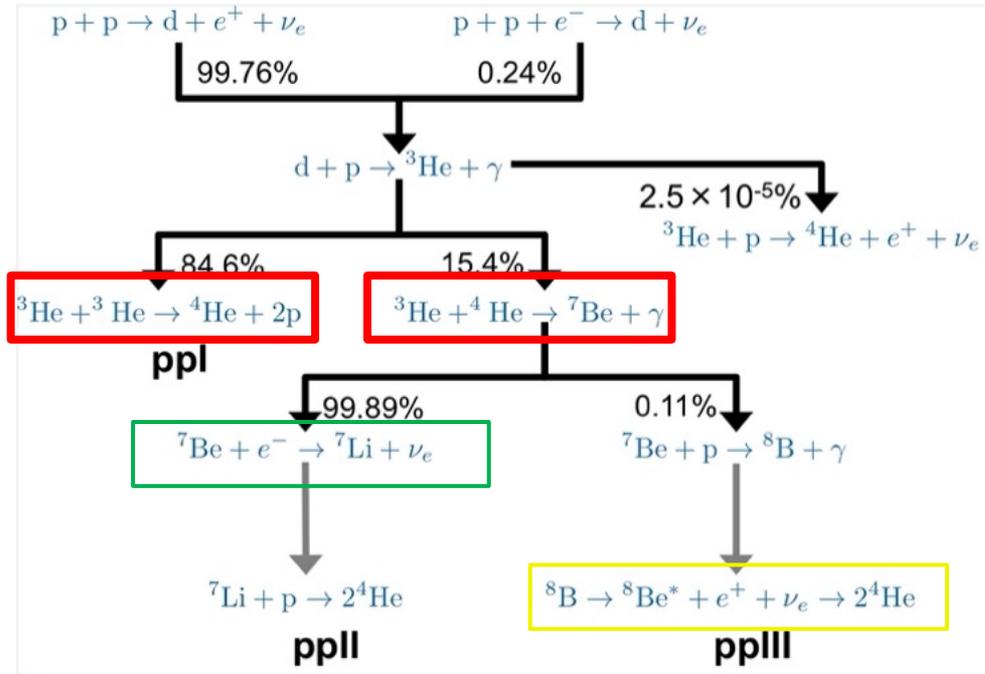
Modification of SSM:
solar dynamics, dark matter, etc, etc
(to lower ^8B and ^7Be)

$$L_{\odot} = \int_0^M (\epsilon_{\text{nuc}} - \epsilon_{\nu} + \epsilon_X) dm$$

Different ν -fluxes

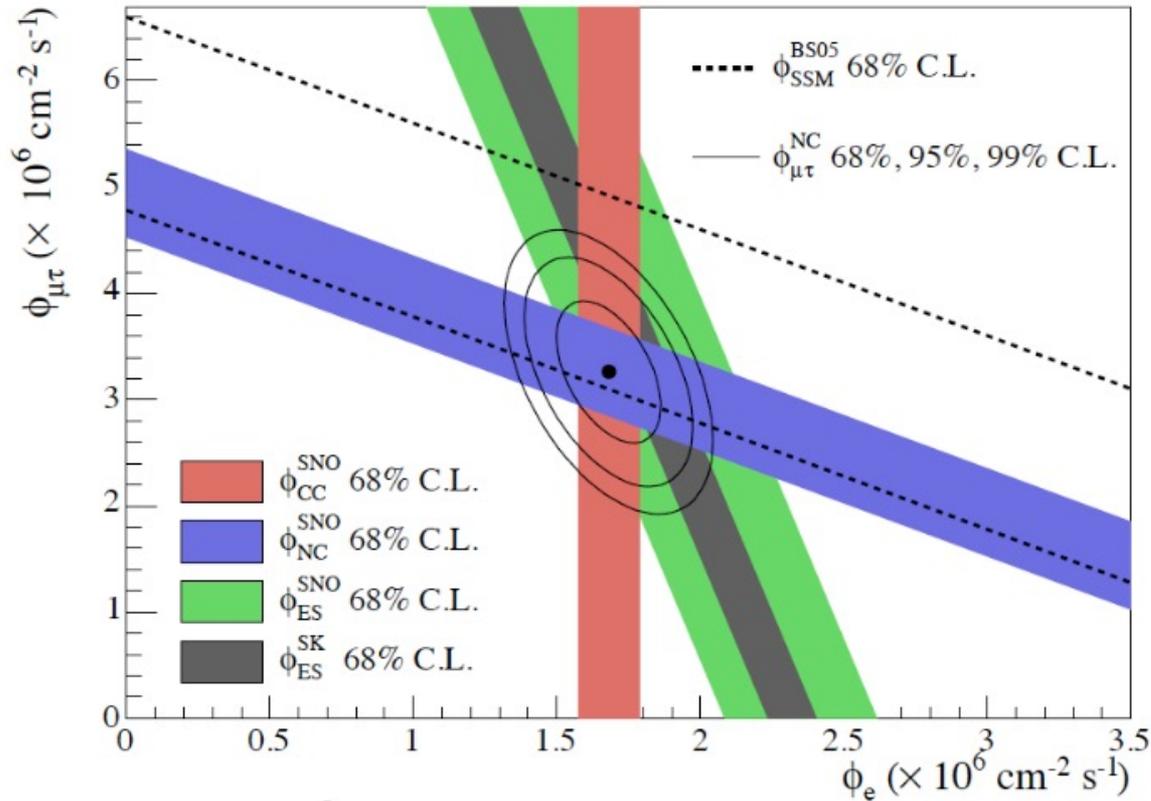
$$L_{\odot} = \sum_{i=1,8} \alpha_i \Phi_i$$





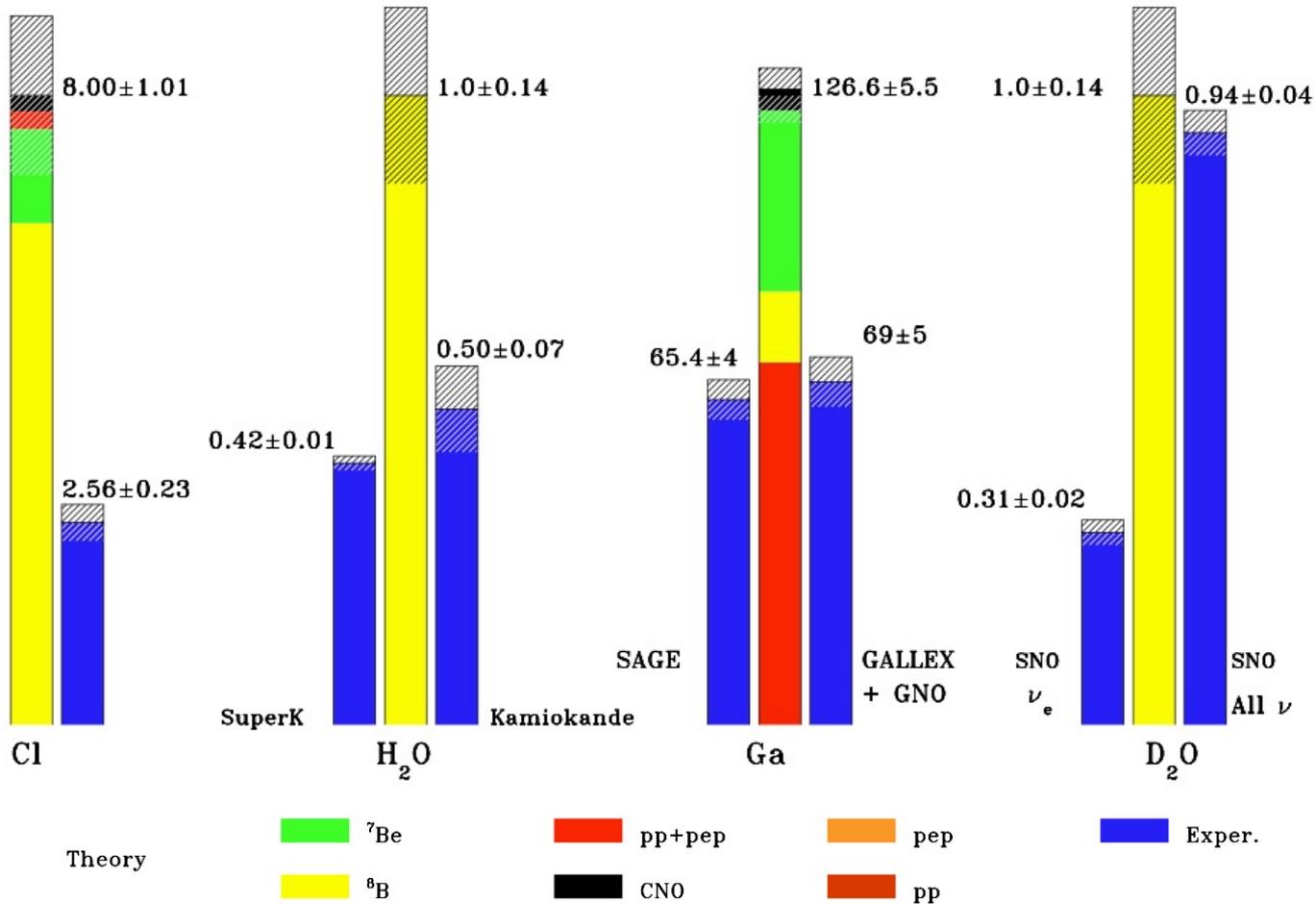
Accurate and precise ${}^3\text{He}+{}^4\text{He}$, ${}^3\text{He}+{}^3\text{He}$, $p+{}^7\text{Be}$ rates became crucial
lengthy coverage of literature – [Adelberger et al. 2011](#)

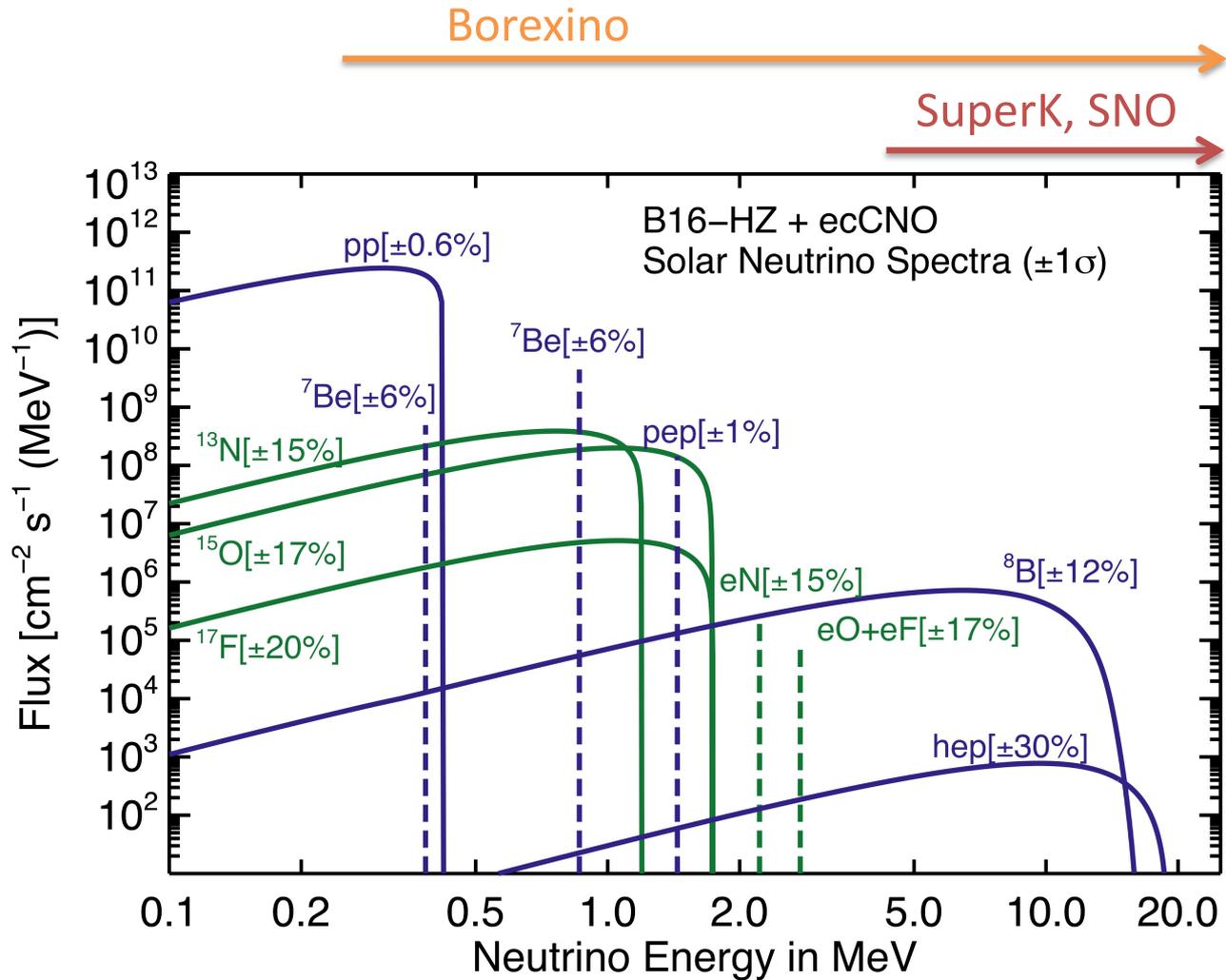
SNO (and SuperK) discovery of neutrino oscillations



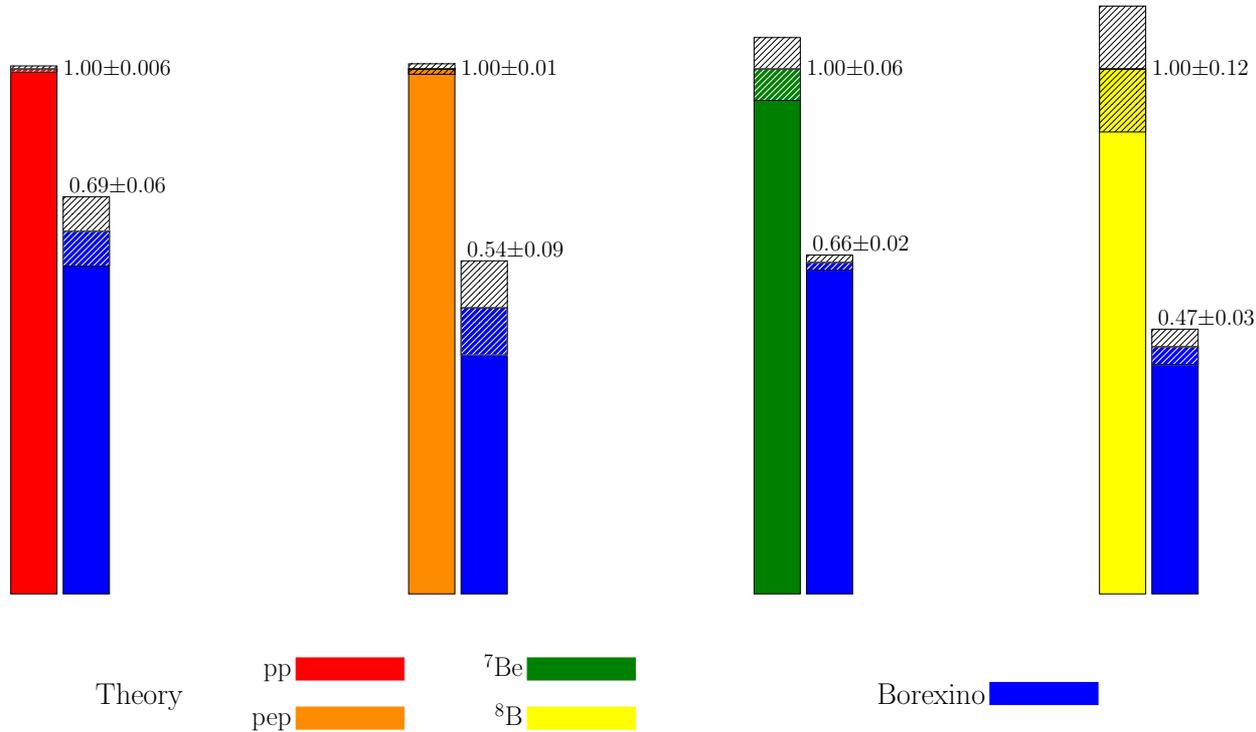
SNO coll.

SNO (and SuperK) discovery of neutrino oscillations





ν fluxes: Solar models vs. Borexino

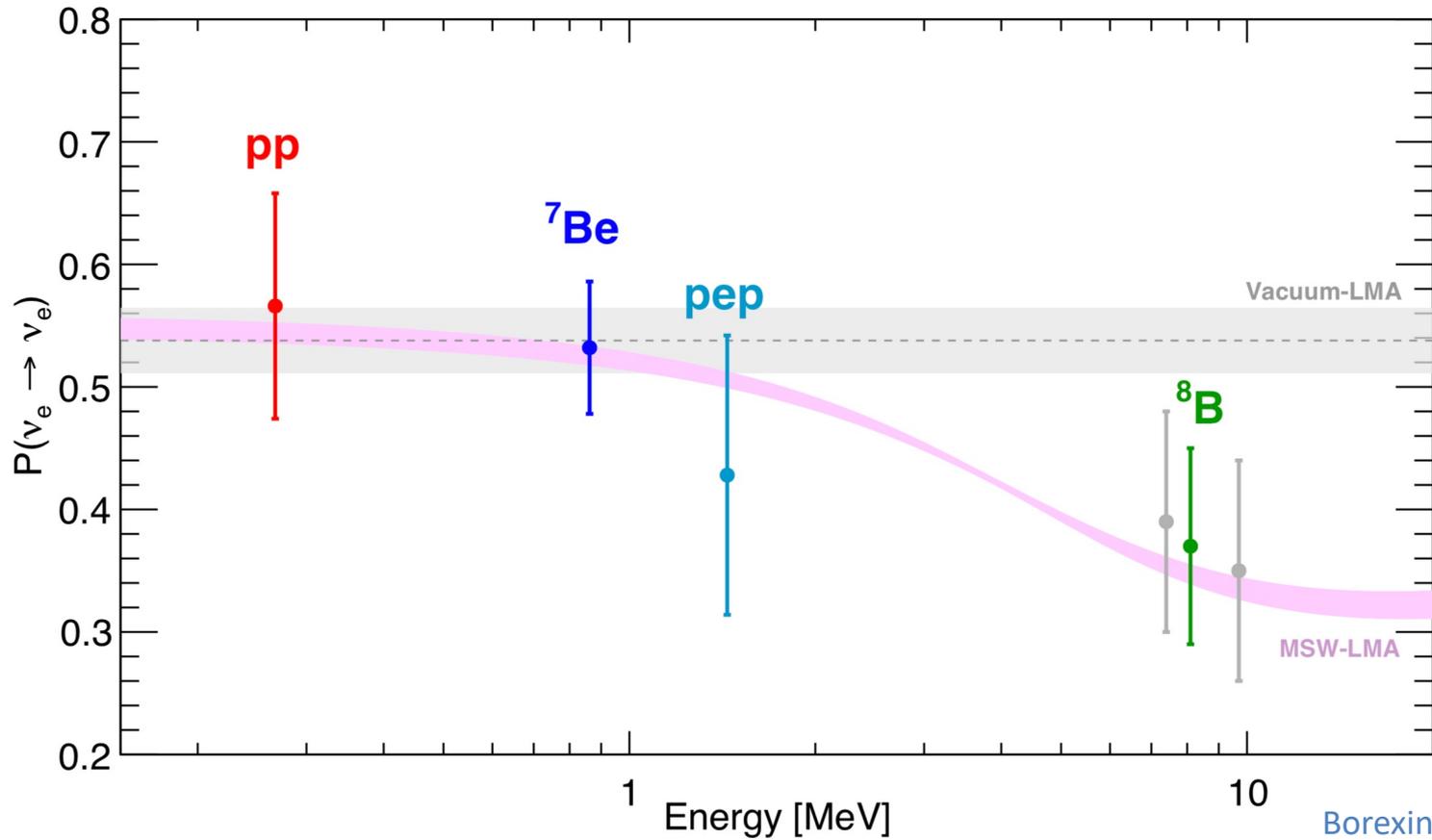


Once ν -oscillations are accounted for \rightarrow **good agreement with SSM** (Borexino coll. 2018)

$$L = 3.89^{+0.35}_{-0.42} \times 10^{33} \text{ erg/s}$$

$$L_{\odot} = 3.842 \times 10^{33} \text{ erg/s}$$

ν -physics and solar models remain associated
survival probability computed using SSMs as reference fluxes



Detailed tests of solar physics

$$L = 3.89_{-0.42}^{+0.35} \times 10^{33} \text{ erg/s}$$

$$L_{\odot} = 3.842 \times 10^{33} \text{ erg/s}$$

In agreement, but 10% uncertainty

$$L_{\odot} = \int_0^M (\varepsilon_{\text{nuc}} - \varepsilon_{\nu} + \varepsilon_{\text{eg}}) dm$$

still room for non-standard energy sources

Individual fluxes

Flux	Solar (Global)		SSM - B16	
	(no LC)		high-Z	
$\Phi(\text{pp})$	6.21±0.50		5.98(0.6%)	
$\Phi(\text{pep})$	1.51±0.12		1.44(1%)	
$\Phi(\text{hep})$	1.9 ⁺¹² ₋₉		7.98(30%)	
$\Phi(^7\text{Be})$	4.85±0.19	3%	4.93(6%)	
$\Phi(^8\text{B})$	5.16 ^{+0.13} _{-0.09}	2%	5.46(12%)	

Experiments Solar model

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	Experiments		Solar model	

Experiment better than models

Nuclear reactions a limiting factor

$S_{34} - 5\%$

$S_{17} - 5\%$

Individual fluxes

Flux	Solar (Global)		SSM - B16	
	(no LC)		high-Z	low-Z
$\Phi(\text{pp})$	6.21 ± 0.50		5.98(0.6%)	6.03(0.5%)
$\Phi(\text{pep})$	1.51 ± 0.12		1.44(1%)	1.46(1%)
$\Phi(\text{hep})$	19^{+12}_{-9}		7.98(30%)	8.25(30%)
$\Phi(^7\text{Be})$	4.85 ± 0.19	3%	4.93(6%)	4.50(6%)
$\Phi(^8\text{B})$	$5.16^{+0.13}_{-0.09}$	2%	5.46(12%)	4.50(12%)



Also solar studies

Two solar models – different solar composition

^7Be and ^8B different by 10% and 20%

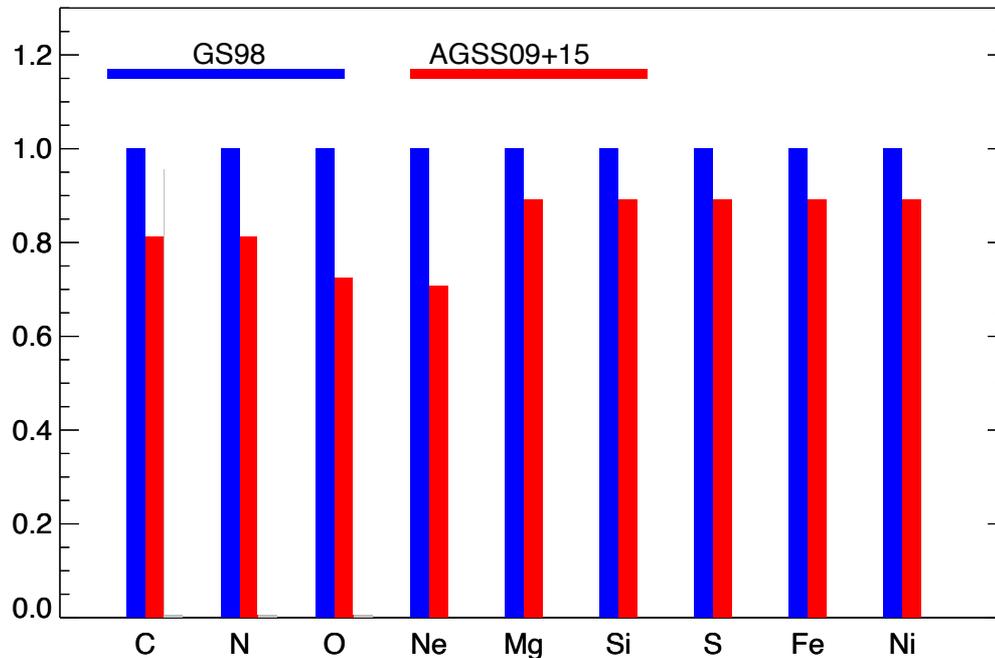
Discrimination between them depends crucially
on reducing model uncertainties

3 observational constraints imposed at present age $\tau_{\odot}=4.57$ Gyr

* **photospheric $(Z/X)_{\odot}$ + solar mixture (fractional abundance of all metals)**

* solar radius R_{\odot}

* solar luminosity $L_{\odot}= 3.8418 \times 10^{33}$ erg s^{-1}



Spectroscopic analysis

new (AGSS09) vs old (GS98)

3D vs 1D model atmospheres
NLTE vs LTE line formation
identification of line blends

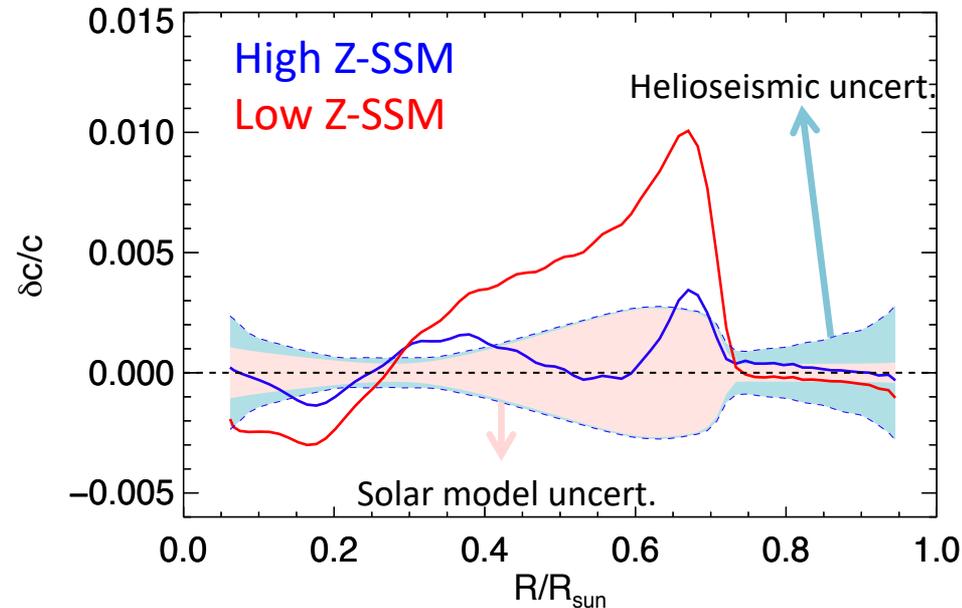
CNO(Ne) < 30-40%

“Modern” solar composition: low in metals

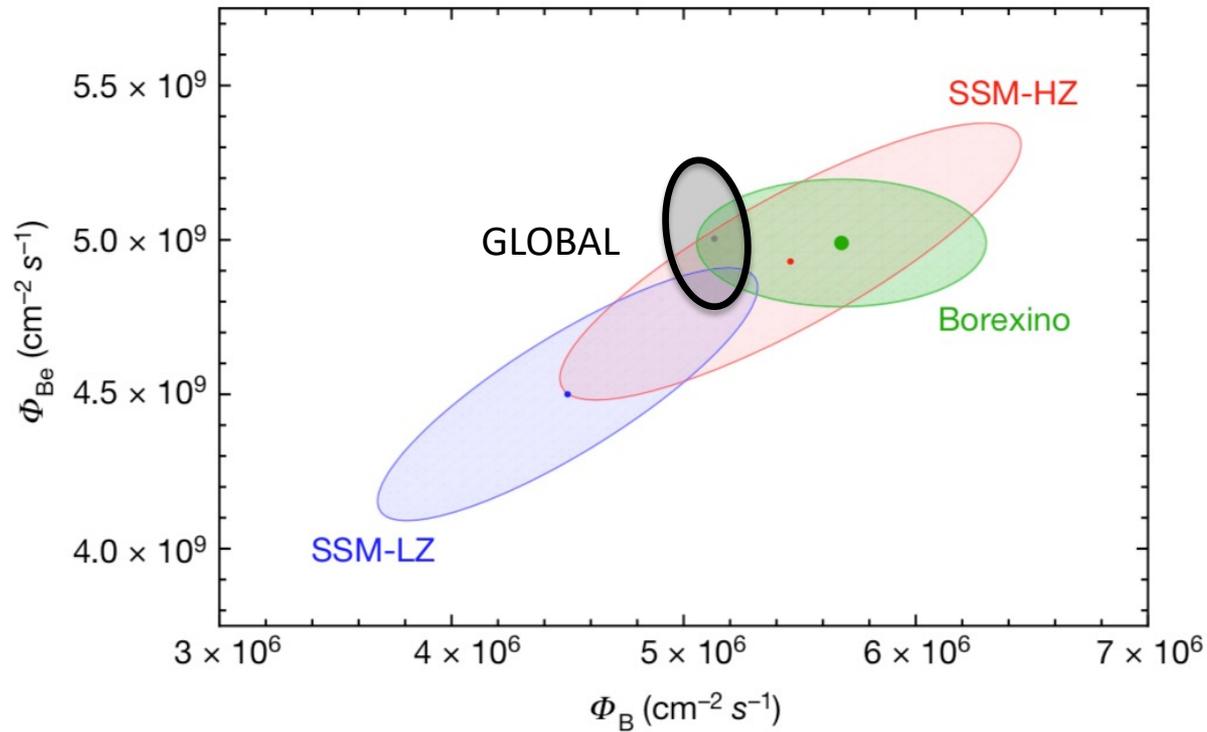
- smaller radiative opacity in solar interior
- impact on radiative energy transport (degeneracy with uncertain opacities)
- changes internal solar structure

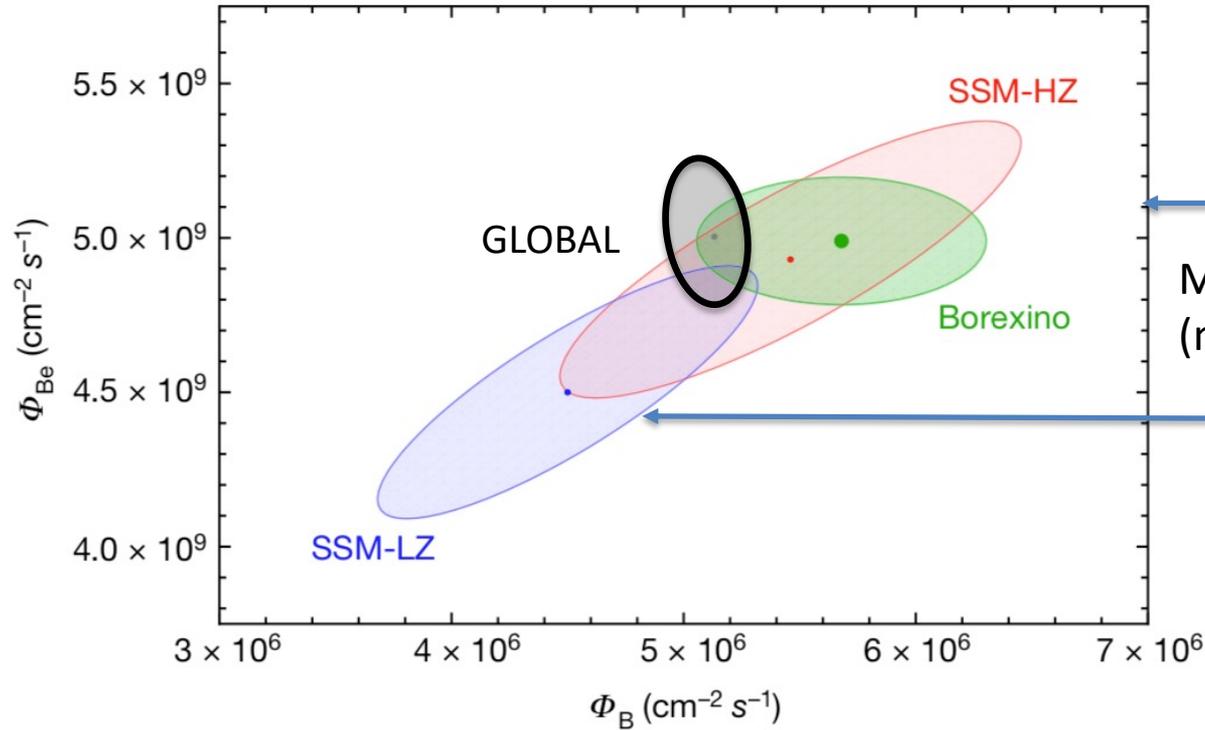
Most clear evidence – solar sound speed profile

(measurable by studying solar oscillations – global sound waves – helioseismology)



Solar abundance problem – no solution so far: limit in modeling Sun/stars?





Model uncertainties
(nuclear) should be reduced

Fluxes on Earth $\text{cm}^{-2}\text{s}^{-1}$

Flux	B16-GS98	B16-AGSS09met
$\Phi(\text{pp})$	$5.98(1 \pm 0.006)$	$6.03(1 \pm 0.005) \times 10^{10}$
$\Phi(\text{pep})$	$1.44(1 \pm 0.01)$	$1.46(1 \pm 0.009) \times 10^8$
$\Phi(\text{hep})$	$7.98(1 \pm 0.30)$	$8.25(1 \pm 0.30) \times 10^3$
$\Phi(^7\text{Be})$	$4.93(1 \pm 0.06)$	$4.50(1 \pm 0.06) \times 10^9$
$\Phi(^8\text{B})$	$5.46(1 \pm 0.12)$	$4.50(1 \pm 0.12) \times 10^6$
$\Phi(^{13}\text{N})$	$2.78(1 \pm 0.15)$	$2.04(1 \pm 0.14) \times 10^8$
$\Phi(^{15}\text{O})$	$2.05(1 \pm 0.17)$	$1.44(1 \pm 0.16) \times 10^8$
$\Phi(^{17}\text{F})$	$5.29(1 \pm 0.20)$	$3.26(1 \pm 0.18) \times 10^6$

} CNO fluxes reflect linearly the CNO abundances
 30-40% differences
 15-20% uncertainties

Can we exploit ν -fluxes to learn about solar core composition and point towards a solution of solar abundance problem?

Uncertainties in model ν -fluxes separated qualitatively in:

- environmental (thermal): factors contributing to temperature in solar core (T_c)
radiative opacities, solar luminosity, abundance of metals (but CNO)
- nuclear rates: affect individual or a few fluxes (e.g. ${}^3\text{He}+{}^4\text{He}$)
- CN(O) abundances: affect directly CN(O) fluxes

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Flux dependences

$$\frac{\Phi_i}{\Phi_{i,0}} = \prod_{j=\text{env}} \left(\frac{p_j}{p_{j,0}} \right)^{\beta_{ij}} \prod_{j=\text{nuc}} \left(\frac{p_j}{p_{j,0}} \right)^{\beta_{ij}} \prod_{j=\text{CNO}} \left(\frac{p_j}{p_{j,0}} \right)^{\beta_{ij}}$$

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$$\delta\Phi_i = \sum_{j=\text{env}} \beta_{ij} \delta p_j + \sum_{j=\text{nuc}} \beta_{ij} \delta p_j + \sum_{j=\text{CNO}} \beta_{ij} \delta p_j \quad \text{1st order approximation}$$

β_{ij} obtained from solar models ([Serenelli et al. 2013](#)) or linear perturbation ([Villante & AS 2021](#))

Environmental uncertainties are fully correlated among fluxes

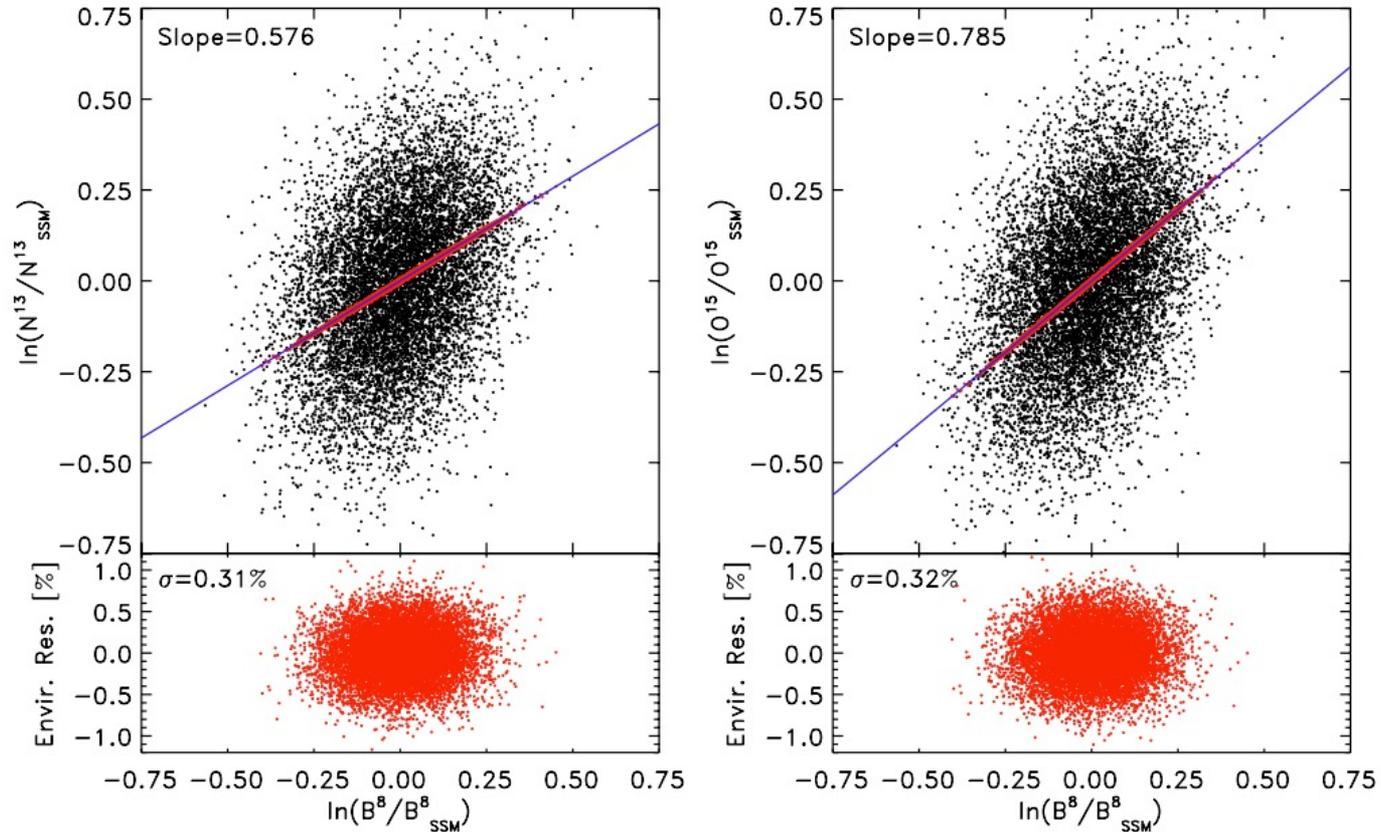
In linear regime, it is reasonable that for any two fluxes (i,k), a coefficient c_{ik}

$$\delta\Phi_i - c_{ik}\delta\Phi_k = \sum_{j=\text{env}} (\beta_{ij} - c_{ik}\beta_{kj})\delta p_j + \sum_{j=\text{nuc}} (\beta_{ij} - c_{ik}\beta_{kj})\delta p_j + \sum_{j=\text{CNO}} (\beta_{ij} - c_{ik}\beta_{kj})\delta p_j$$

That will give $\sum_{j=\text{env}} (\beta_{ij} - c_{ik}\beta_{kj})\delta p_j \approx 0$

i.e. build a combination of ν -fluxes that minimizes environmental uncertainties

Environmental uncertainties are fully correlated among fluxes



Environmental uncertainties are fully correlated among fluxes

$$\begin{aligned} & \frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.576} \\ &= x_{\text{C}}^{0.840} x_{\text{N}}^{0.161} D^{0.183} [L_{\odot}^{0.553} O^{-0.017} A^{0.157}] \\ & \times [S_{11}^{-0.639} S_{33}^{0.264} S_{34}^{-0.526} S_{17}^{-0.576} S_{e7}^{0.576} S_{114}^{0.743}] \\ & \times [x_{\text{O}}^{0.002} x_{\text{Ne}}^{-0.005} x_{\text{Mg}}^{-0.004} x_{\text{Si}}^{0.0} x_{\text{S}}^{0.0} x_{\text{Ar}}^{0.001} x_{\text{Fe}}^{0.005}] \end{aligned}$$

$$\begin{aligned} & \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.785} \\ &= x_{\text{C}}^{0.794} x_{\text{N}}^{0.212} D^{0.172} [L_{\odot}^{0.515} O^{-0.016} A^{0.308}] \\ & \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ & \times [x_{\text{O}}^{0.003} x_{\text{Ne}}^{-0.005} x_{\text{Mg}}^{-0.003} x_{\text{Si}}^{-0.001} x_{\text{S}}^{-0.001} x_{\text{Ar}}^{0.001} x_{\text{Fe}}^{0.003}] \end{aligned}$$

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Environmental uncertainties are fully correlated among fluxes

$$\begin{aligned} & \frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.576} \\ &= x_C^{0.840} x_N^{0.161} D^{0.183} \left[\frac{L_\odot}{L_\odot} \frac{\mathcal{O}}{\mathcal{O}}^{-0.017} \frac{A}{A}^{0.157} \right] \\ & \times [S_{11}^{-0.639} S_{33}^{0.264} S_{34}^{-0.526} S_{17}^{-0.576} S_{e7}^{0.576} S_{114}^{0.743}] \\ & \times \left[\frac{x_{\text{O}}}{x_{\text{O}}}^{-0.002} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.004} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{0.0} \frac{x_{\text{S}}}{x_{\text{S}}}^{0.0} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.005} \right] \end{aligned}$$

$$\begin{aligned} & \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.785} \\ &= x_C^{0.794} x_N^{0.212} D^{0.172} \left[\frac{L_\odot}{L_\odot} \frac{\mathcal{O}}{\mathcal{O}}^{-0.016} \frac{A}{A}^{0.308} \right] \\ & \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ & \times \left[\frac{x_{\text{O}}}{x_{\text{O}}}^{-0.003} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.003} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{-0.001} \frac{x_{\text{S}}}{x_{\text{S}}}^{-0.001} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.003} \right] \end{aligned}$$

$$\frac{\Phi(^{13}\text{N})}{\Phi(^{13}\text{N})^{\text{SSM}}} = \left(\frac{\Phi(^8\text{B})}{\Phi(^8\text{B})^{\text{SSM}}} \right)^{0.576} \approx X_{\text{C+N}} [0.5\%(\text{env}) + 9\%(\text{nuc}) + 2\%(\text{diff})]$$

Environmental uncertainties are fully correlated among fluxes

$$\begin{aligned} & \frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.576} \\ &= x_C^{0.840} x_N^{0.161} D^{0.183} \left[\frac{L_\odot}{L_\odot} \frac{\mathcal{O}}{\mathcal{O}}^{-0.017} \frac{A}{A}^{0.157} \right] \\ & \times [S_{11}^{-0.639} S_{33}^{0.264} S_{34}^{-0.526} S_{17}^{-0.576} S_{e7}^{0.576} S_{114}^{0.743}] \\ & \times \left[\frac{x_{\text{O}}}{x_{\text{O}}}^{-0.002} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.004} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{0.0} \frac{x_{\text{S}}}{x_{\text{S}}}^{0.0} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.005} \right] \end{aligned}$$

$$\begin{aligned} & \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.785} \\ &= x_C^{0.794} x_N^{0.212} D^{0.172} \left[\frac{L_\odot}{L_\odot} \frac{\mathcal{O}}{\mathcal{O}}^{-0.016} \frac{A}{A}^{0.308} \right] \\ & \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ & \times \left[\frac{x_{\text{O}}}{x_{\text{O}}}^{-0.003} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.003} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{-0.001} \frac{x_{\text{S}}}{x_{\text{S}}}^{-0.001} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.003} \right] \end{aligned}$$

$$\frac{\Phi(^{13}\text{N})}{\Phi(^{13}\text{N})^{\text{SSM}}} = \left(\frac{\Phi(^8\text{B})}{\Phi(^8\text{B})^{\text{SSM}}} \right)^{0.576} \approx X_{\text{C+N}} [0.5\%(\text{env}) + 9\%(\text{nuc}) + 2\%(\text{diff})]$$

$\Phi(^8\text{B})$ → Fixed by experiments (SNO, SuperK) to 2%

Environmental uncertainties are fully correlated among fluxes

$$\begin{aligned} & \frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.576} \\ &= x_C^{0.840} x_N^{0.161} D^{0.183} \left[L_\odot^{0.553} \mathcal{O}^{-0.017} A^{0.157} \right] \\ & \times [S_{11}^{-0.639} S_{33}^{0.264} S_{34}^{-0.526} S_{17}^{-0.576} S_{e7}^{0.576} S_{114}^{0.743}] \\ & \times \left[\lambda_{\text{O}}^{-0.002} \lambda_{\text{Ne}}^{-0.005} \lambda_{\text{Mg}}^{-0.004} \lambda_{\text{Si}}^{0.0} \lambda_{\text{S}}^{0.0} \lambda_{\text{Ar}}^{0.001} \lambda_{\text{Fe}}^{0.005} \right] \end{aligned}$$

$$\begin{aligned} & \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.785} \\ &= x_C^{0.794} x_N^{0.212} D^{0.172} \left[L_\odot^{0.515} \mathcal{O}^{-0.016} A^{0.308} \right] \\ & \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ & \times \left[\lambda_{\text{O}}^{-0.003} \lambda_{\text{Ne}}^{-0.005} \lambda_{\text{Mg}}^{-0.003} \lambda_{\text{Si}}^{-0.001} \lambda_{\text{S}}^{-0.001} \lambda_{\text{Ar}}^{0.001} \lambda_{\text{Fe}}^{0.003} \right] \end{aligned}$$

$$\frac{\Phi(^{13}\text{N})}{\Phi(^{13}\text{N})^{\text{SSM}}} = \left(\frac{\Phi(^8\text{B})}{\Phi(^8\text{B})^{\text{SSM}}} \right)^{0.576} \approx X_{\text{C+N}} [0.5\%(\text{env}) + \boxed{9\%(\text{nuc})} + 2\%(\text{diff})]$$

Dominant uncertainty – nuclear cross sections from several reactions (S_{114} , S_{34} , S_{17})

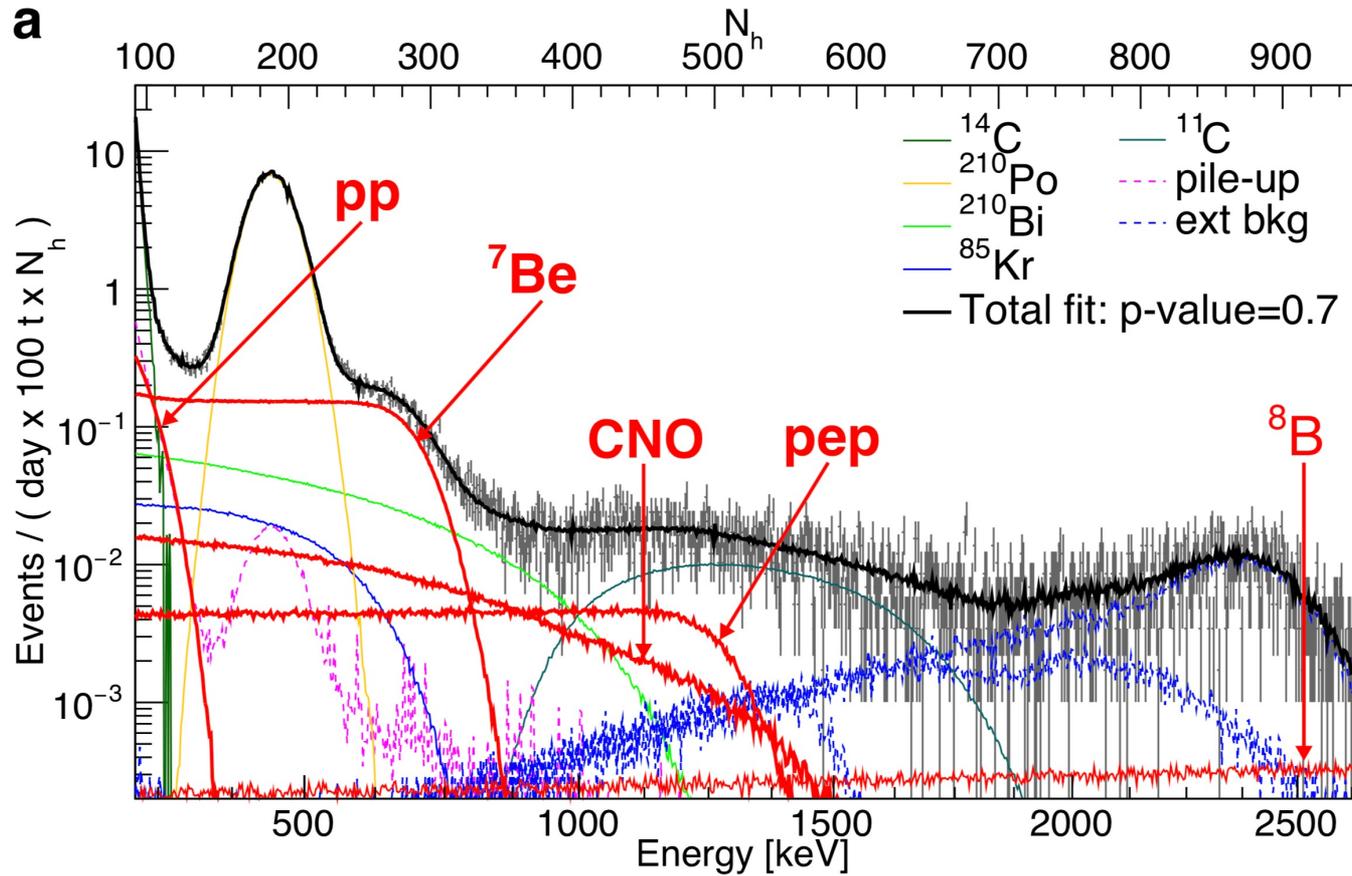
Environmental uncertainties are fully correlated among fluxes

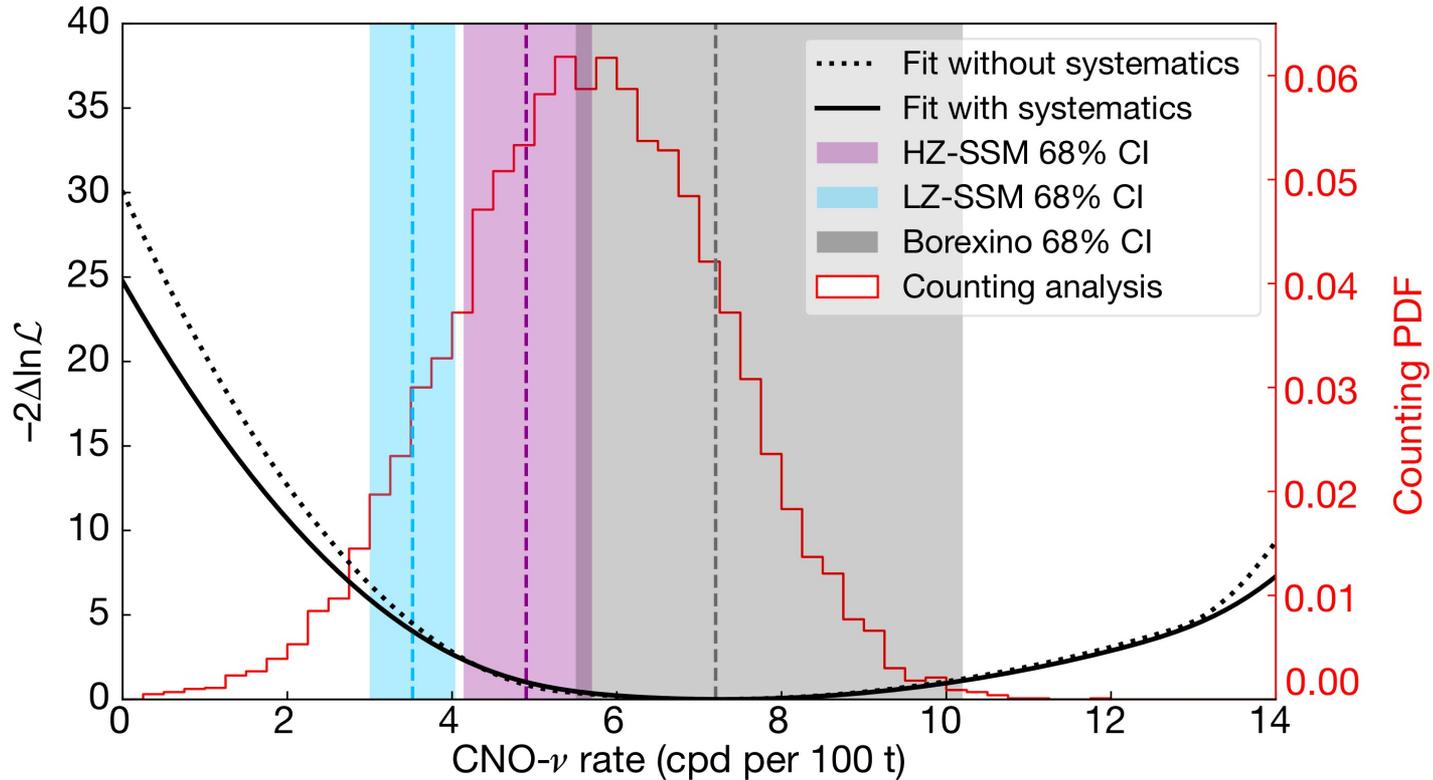
$$\begin{aligned} & \frac{\phi(^{13}\text{N})}{\phi(^{13}\text{N})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.576} \\ &= x_C^{0.840} x_N^{0.161} D^{0.183} \left[\frac{L_\odot}{L_\odot} \frac{O}{O}^{-0.017} \frac{A}{A}^{0.157} \right] \\ &\quad \times [S_{11}^{-0.639} S_{33}^{0.264} S_{34}^{-0.526} S_{17}^{-0.576} S_{e7}^{0.576} S_{114}^{0.743}] \\ &\quad \times \left[\frac{x_O}{x_O}^{-0.002} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.004} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{0.0} \frac{x_{\text{S}}}{x_{\text{S}}}^{0.0} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.005} \right] \end{aligned}$$

$$\begin{aligned} & \frac{\phi(^{15}\text{O})}{\phi(^{15}\text{O})^{\text{SSM}}} \bigg/ \left[\frac{\phi(^8\text{B})}{\phi^{\text{SSM}}(^8\text{B})} \right]^{0.785} \\ &= x_C^{0.794} x_N^{0.212} D^{0.172} \left[\frac{L_\odot}{L_\odot} \frac{O}{O}^{-0.016} \frac{A}{A}^{0.308} \right] \\ &\quad \times [S_{11}^{-0.831} S_{33}^{0.342} S_{34}^{-0.685} S_{17}^{-0.785} S_{e7}^{0.785} S_{114}^{0.995}] \\ &\quad \times \left[\frac{x_O}{x_O}^{-0.003} \frac{x_{\text{Ne}}}{x_{\text{Ne}}}^{-0.005} \frac{x_{\text{Mg}}}{x_{\text{Mg}}}^{-0.003} \frac{x_{\text{Si}}}{x_{\text{Si}}}^{-0.001} \frac{x_{\text{S}}}{x_{\text{S}}}^{-0.001} \frac{x_{\text{Ar}}}{x_{\text{Ar}}}^{0.001} \frac{x_{\text{Fe}}}{x_{\text{Fe}}}^{0.003} \right] \end{aligned}$$

$$\frac{\Phi(^{13}\text{N})}{\Phi(^{13}\text{N})^{\text{SSM}}} = \left(\frac{\Phi(^8\text{B})}{\Phi(^8\text{B})^{\text{SSM}}} \right)^{0.576} \approx X_{\text{C+N}} [0.5\%(\text{env}) + 9\%(\text{nuc}) + 2\%(\text{diff})]$$

Provided a ^{13}N (or ^{15}O , or a combination of them) is available we obtain $X_{\text{C+N}}$ in the solar core!





Borexino coll. 2020

Still large experimental uncertainty – further improvements in data analysis
 Pioneering study for future experiments
 In solar models, reducing nuclear uncertainties is required

Solar models:

reference neutrino fluxes for neutrino physics – survival probability

vacuum limit

potentially in transition region (CNO fluxes)

neutrino inferred luminosity tests solar energy source and non-standard mechanisms

learning about solar properties

CN composition in the core – solar abundance problem

breaking degeneracy with radiative opacities

surface vs core composition → tests of chemical mixing in stars
non-standard events (e.g. accretion in early
solar system)

Underlying these topics is the requirement of accurate and precise nuclear reaction rates

Fundamental physics (and astrophysics) requires excellent nuclear physics

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