The $^{12}C(\alpha,\gamma)^{16}O$ reaction at astrophysical energies – status and missing inputs for R-matrix extrapolations

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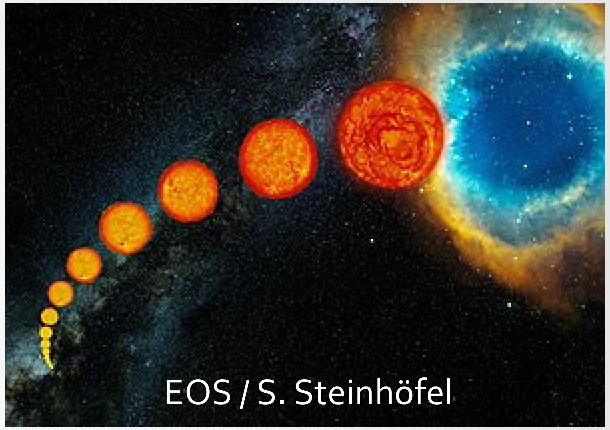
Motivation



- Together with the 3α process, the $^{12}C(\alpha,\gamma)^{16}O$ reaction determines the $^{12}C/^{16}O$ ratio in the universe.
- For stellar evolution, the $^{12}C/^{16}O$ ratio determines the evolution of massive stars, which in turn effects all later stages of nucleosynthesis.

TABLE I. Astrophysical environments and burning stages where the $^{12}C(\alpha,\gamma)^{16}O$ reaction plays an important role. The temperatures of these environments dictate the energy ranges where the $^{12}C(\alpha,\gamma)^{16}O$ cross section must be well known for an accurate calculation of the reaction rate.

Burning stages	Astrophysical sites	Temperature range (GK)	Gamow energy range (MeV)
Core helium burning	AGB stars and massive stars	0.1–0.4	0.15–0.65
Core carbon and oxygen burning	Massive stars	0.6–2.7	0.44–2.5
Core silicon burning	Massive stars	2.8–4.1	1.1–3.4
Explosive helium burning	Supernovae and x-ray bursts	≈1	0.6–1.25
Explosive oxygen and silicon burning	Supernovae	> 5	> 1.45

Motivation Highlight: Black Hole Mass Gap
Link to LIGO

- **Farmer *et al.* (2020), Mehta *et al.* (2022)**

THE ASTROPHYSICAL JOURNAL, 924:39 (21pp), 2022 January 1

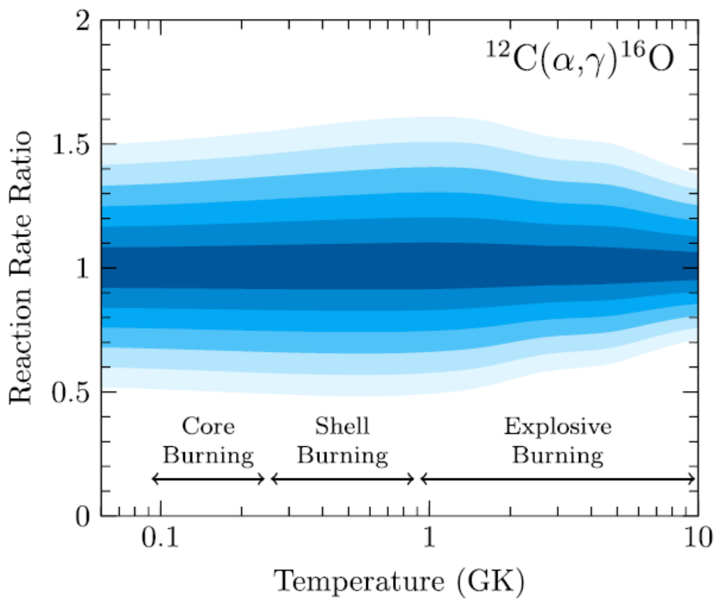
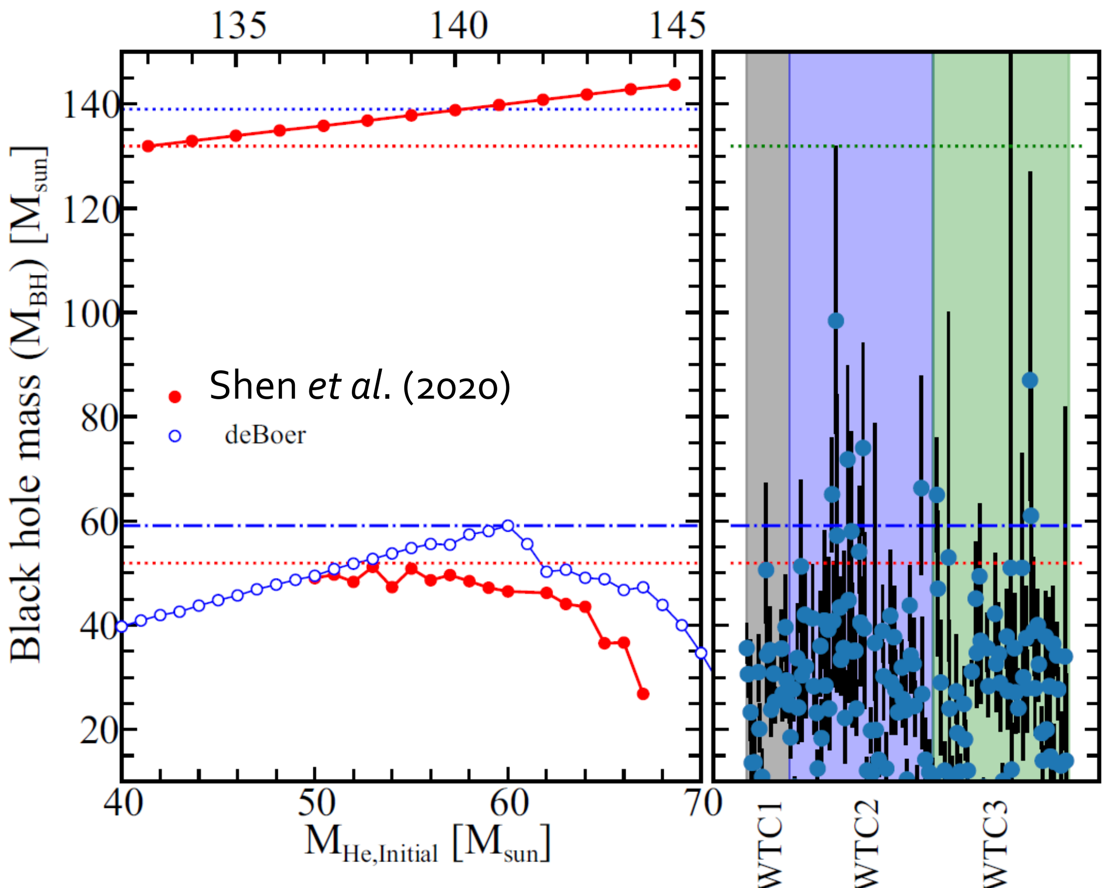


Figure 9. Relative uncertainties in the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate of this work, expanded from those presented in deBoer et al. (2017). The uncertainties are normalized to the central value for clearer presentation. The regions of fading blue color represent 0.5σ steps in the Gaussian uncertainty distribution.

Following Gialanella *et al.* (2001)



deBoer *et al.* (2025)

Motivation Highlight: White Dwarf Seismology

- **Chidester *et al*. (2022,2023)**

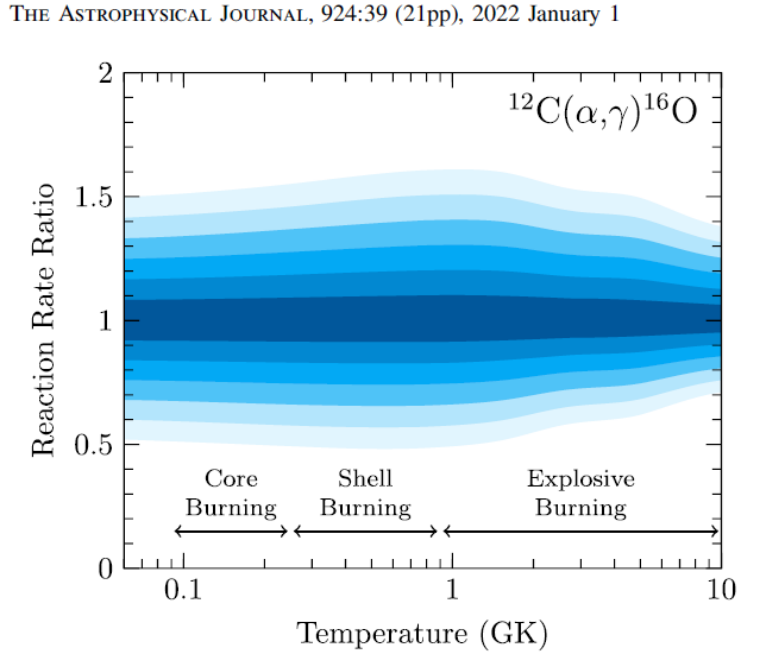
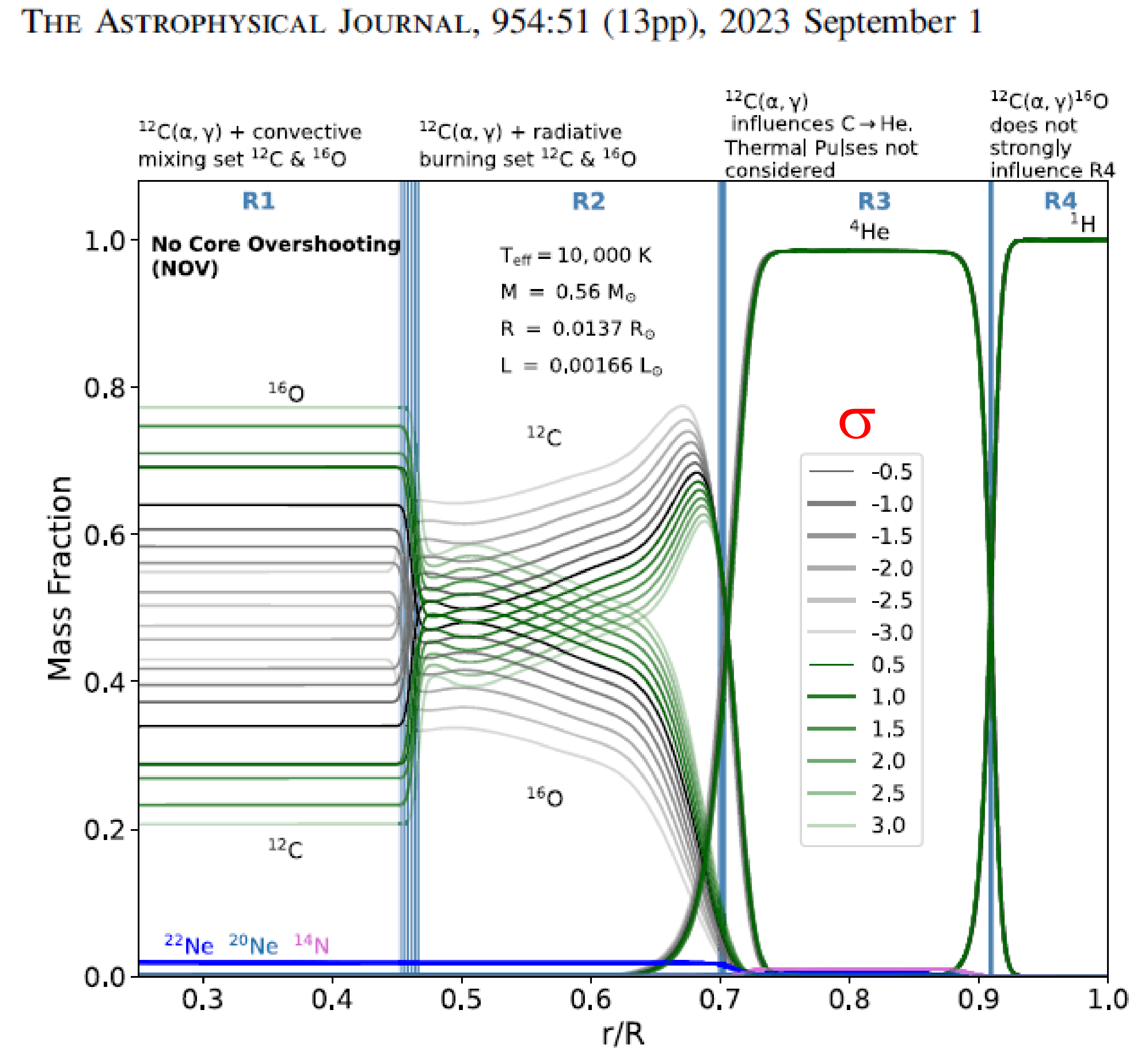
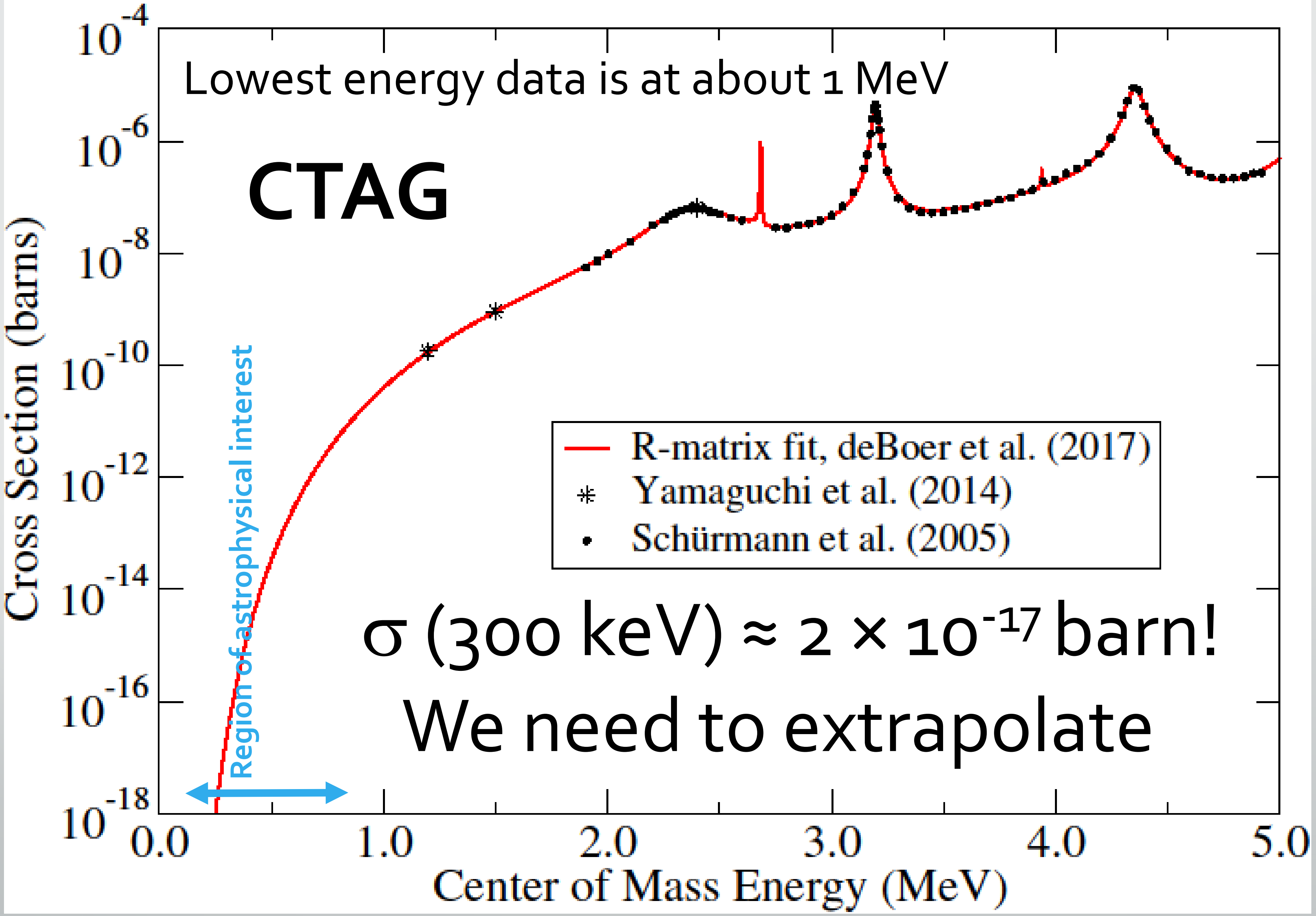


Figure 9. Relative uncertainties in the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate of this work, expanded from those presented in deBoer et al. (2017). The uncertainties are normalized to the central value for clearer presentation. The regions of fading blue color represent 0.5σ steps in the Gaussian uncertainty distribution.



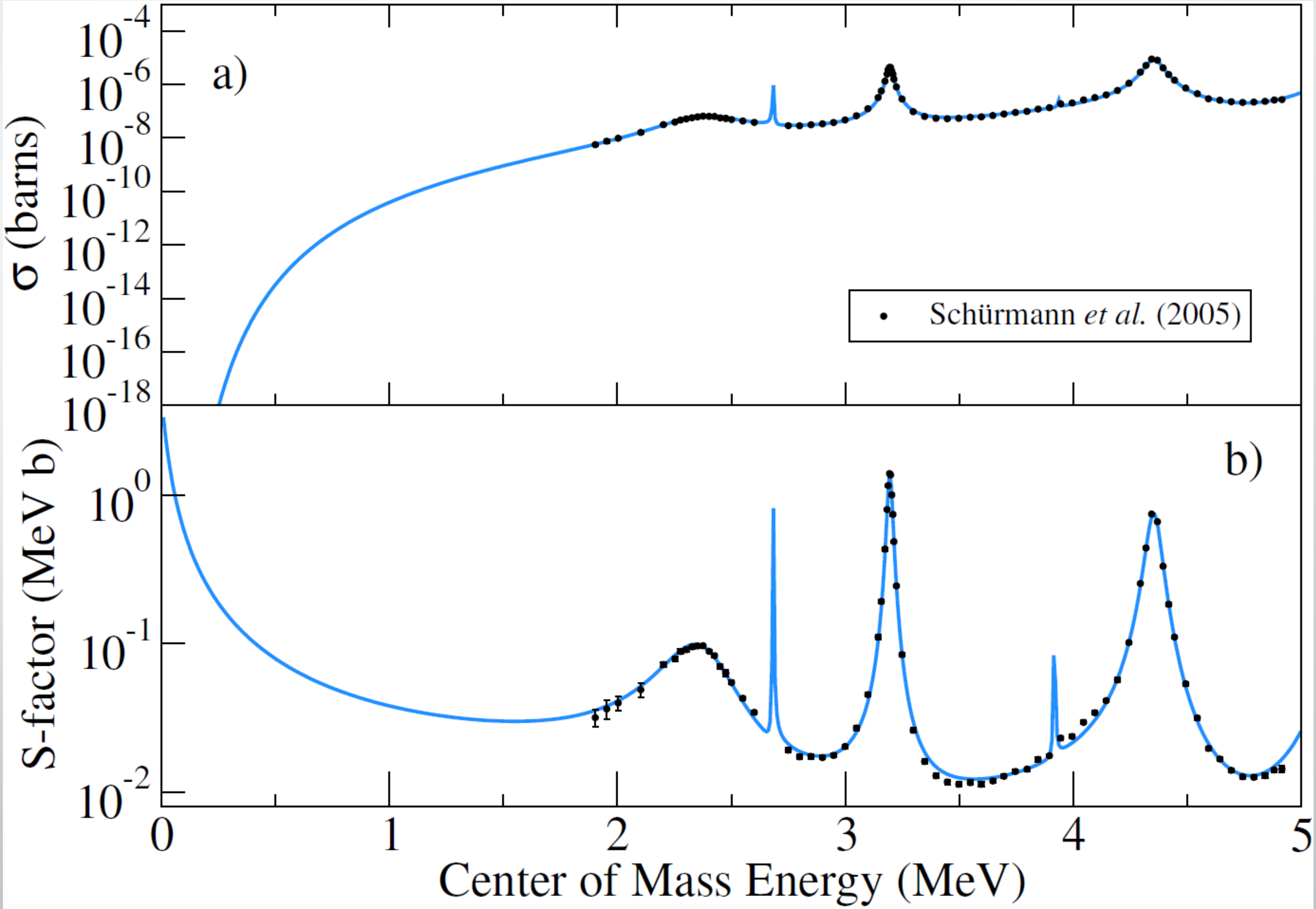

Lowest energy data is at about 1 MeV
CTAG
Cross Section (barns)
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
10^-18
Region of astrophysical interest
R-matrix fit, deBoer et al. (2017)
Yamaguchi et al. (2014)
Schürmann et al. (2005)
σ (300 keV) ≈ 2 × 10^-17 barn!
We need to extrapolate
0.0
1.0
2.0
3.0
4.0
5.0
Center of Mass Energy (MeV)

S-factor

Approximately divide out the Coulomb penetrability

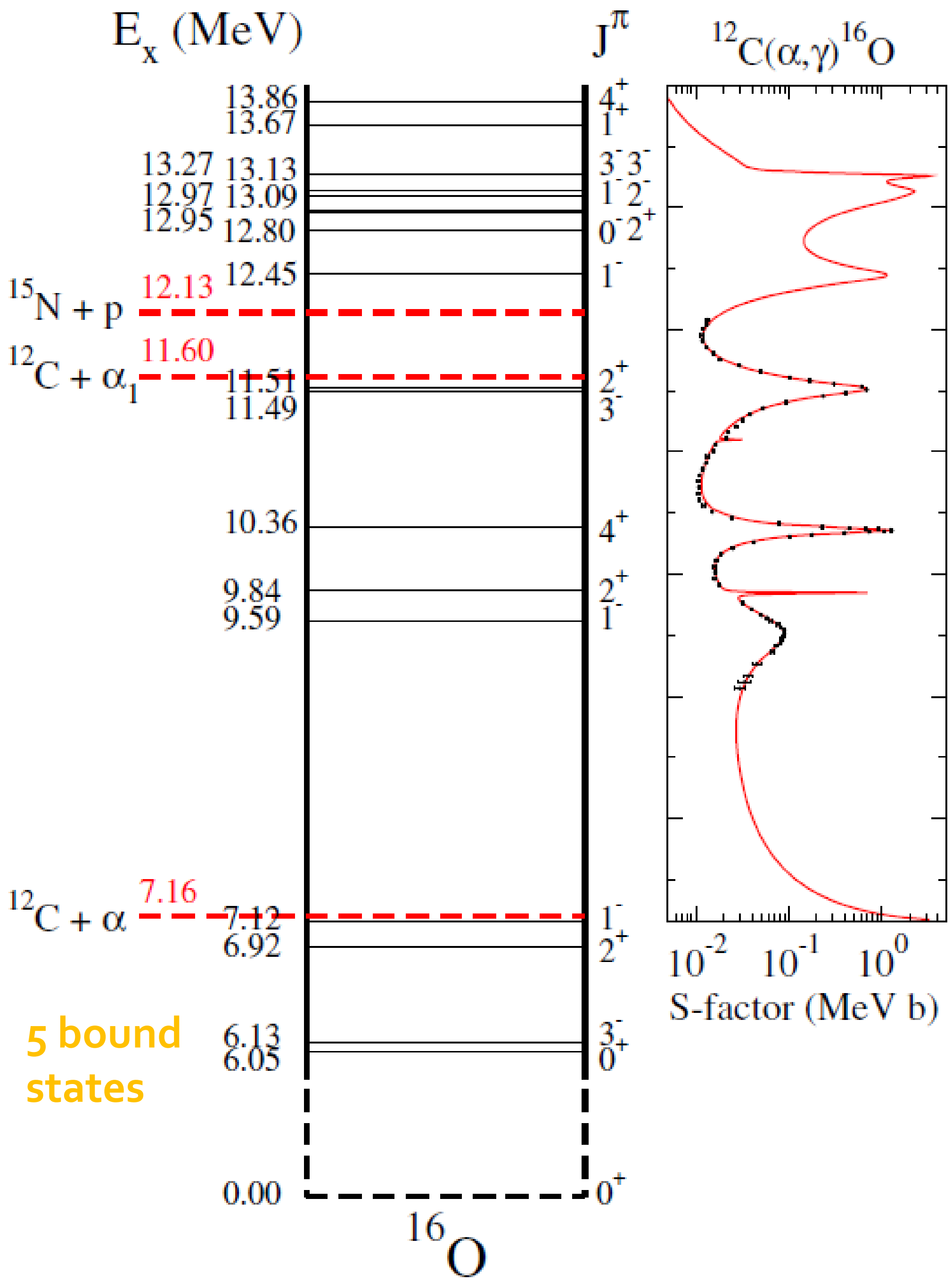
$$S(E) = \sigma(E) E e^{2\pi\eta}$$

$$\eta = \sqrt{\mu/2E} Z_1 Z_2 e^2 / \hbar^2$$

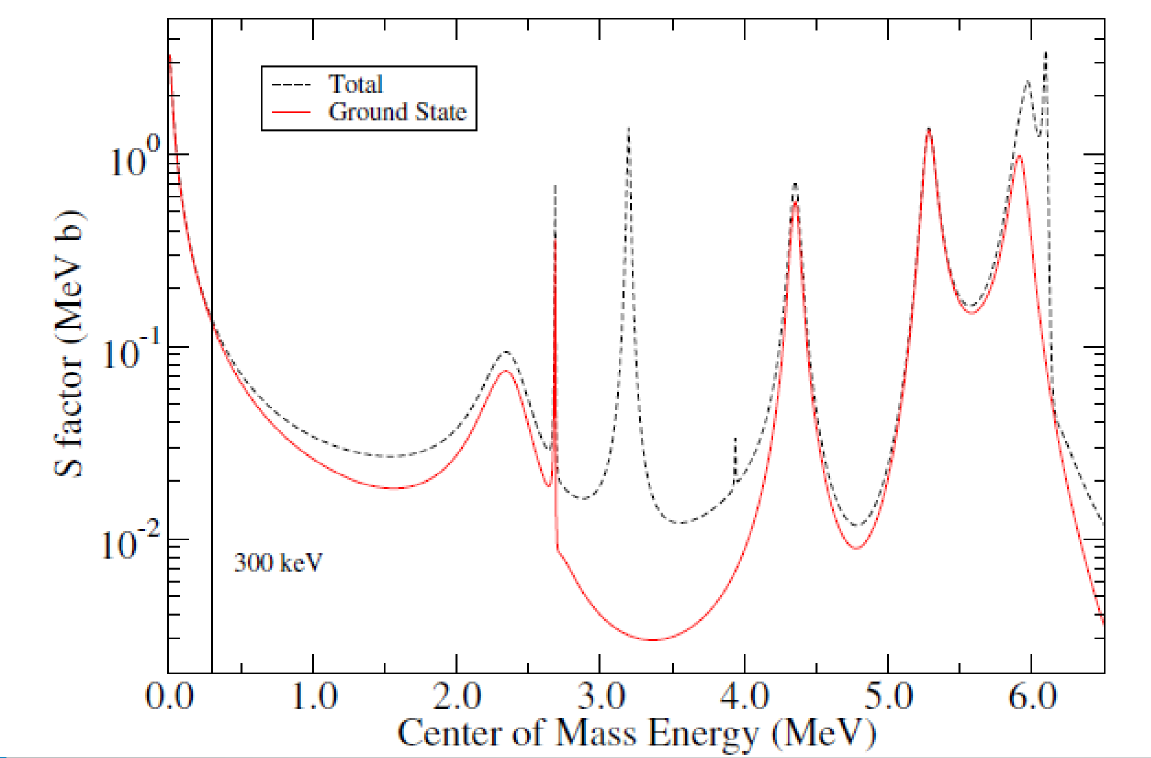


Level structure

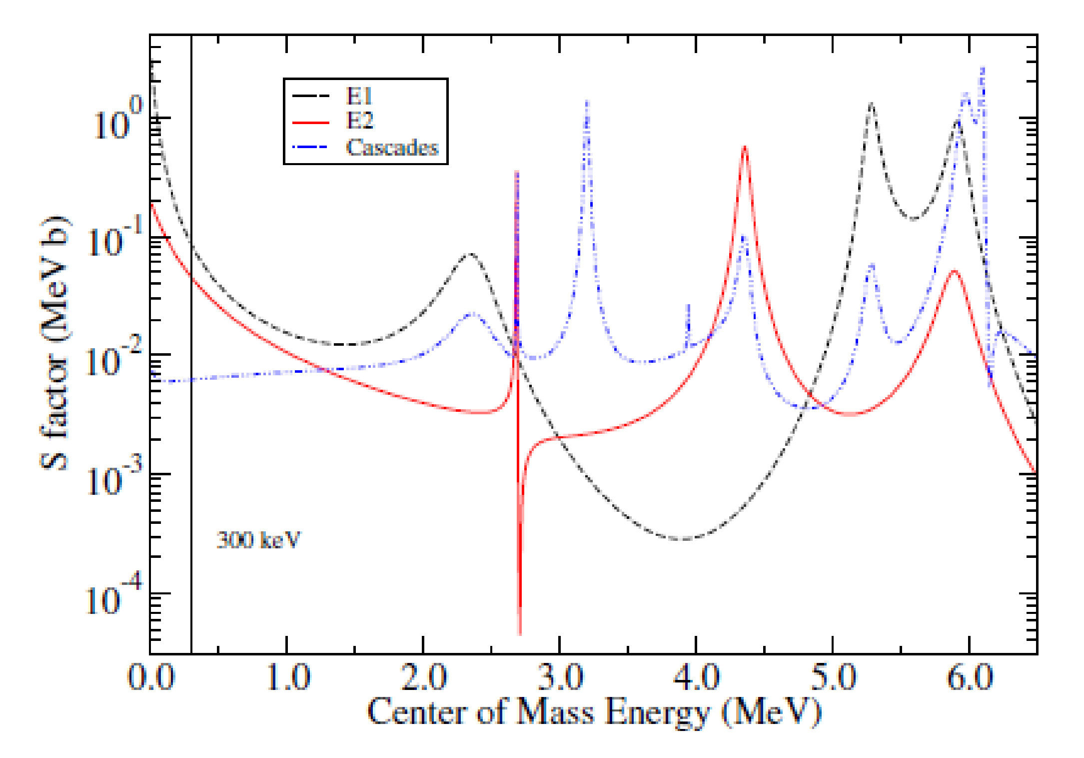
- Well known level structure
- α-separation energy is the lowest
- Five bound states
- Bound states decay with nearly 100% probability directly to the ground state



Ground state transition dominates at low energy



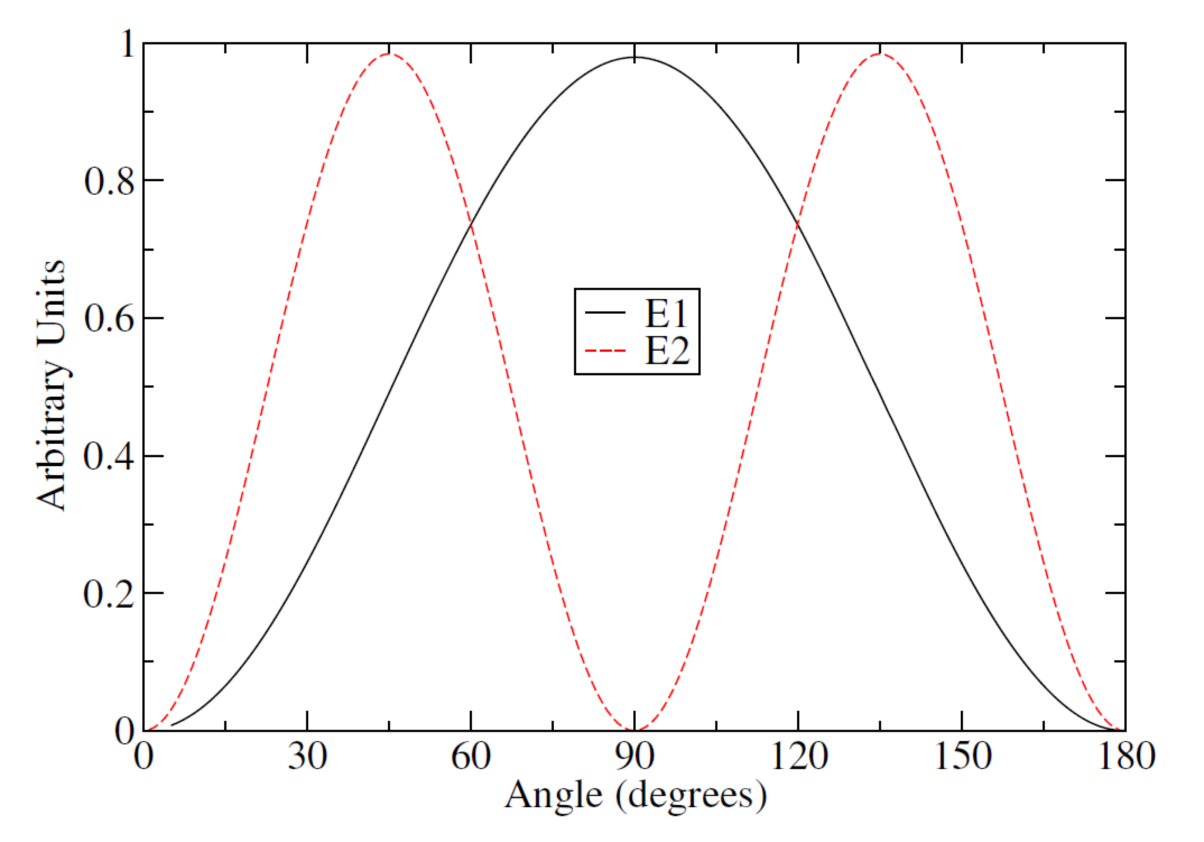
Only electric γ-ray transitions are possible because both ^{4}He and ^{12}C have intrinsic spin 0^+



Only E1 and E2 have been observed, and they have similar strength at very low energy

Ground State E1 and E2 transitions

- E1 transition can be measured by observing the cross section at 90 degrees
  - E2 contamination possible for real experiments
- E2 component must be determined through angular distribution measurements



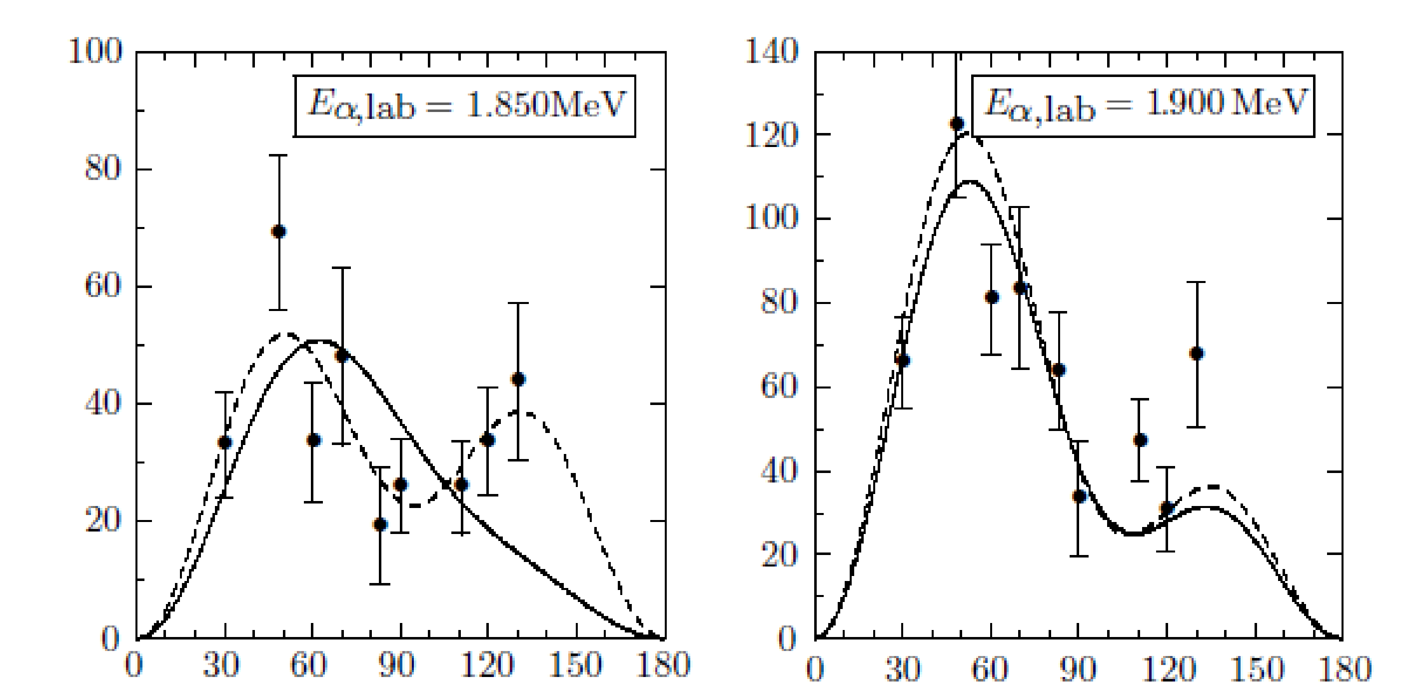
$$\sigma_{E1} = 4\pi\left(\frac{2}{3}\right)\left(\frac{d\sigma}{d\Omega}\right)_{90^\circ}$$

Extraction of E2 ground state cross section has a lot of uncertainty

- Ambiguity in the fitting
- ϕ_{12} should be consistent with scattering data

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{tot}}}{4\pi}\Big[(3|A_{E1}|^2 + 5|A_{E2}|^2)\,Q_0\,P_0(\cos\vartheta) + \left(\frac{25}{7}|A_{E2}|^2 - 3|A_{E1}|^2\right)Q_2\,P_2(\cos\vartheta) - \frac{60}{7}|A_{E2}|^2\,Q_4\,P_4(\cos\vartheta) + 6\sqrt{3}\,|A_{E1}|\,|A_{E2}| \times \cos\phi_{12}(Q_1\,P_1(\cos\vartheta) - Q_3\,P_3(\cos\vartheta))\Big], \quad (4.1)$$

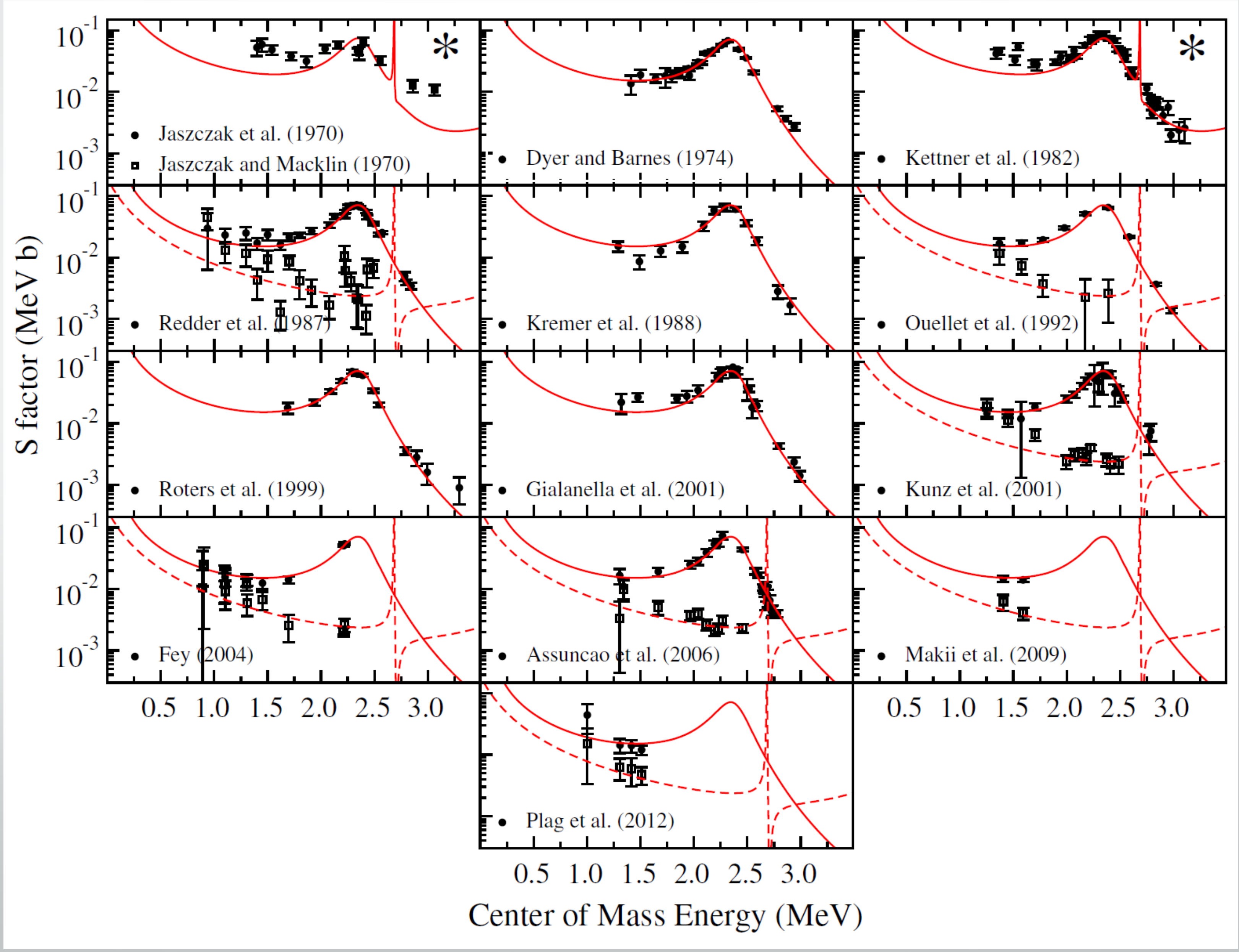
$$\phi_{12} = \delta_2 - \delta_1 + \arctan(\eta/2),$$



Assunção, et al. (2006)

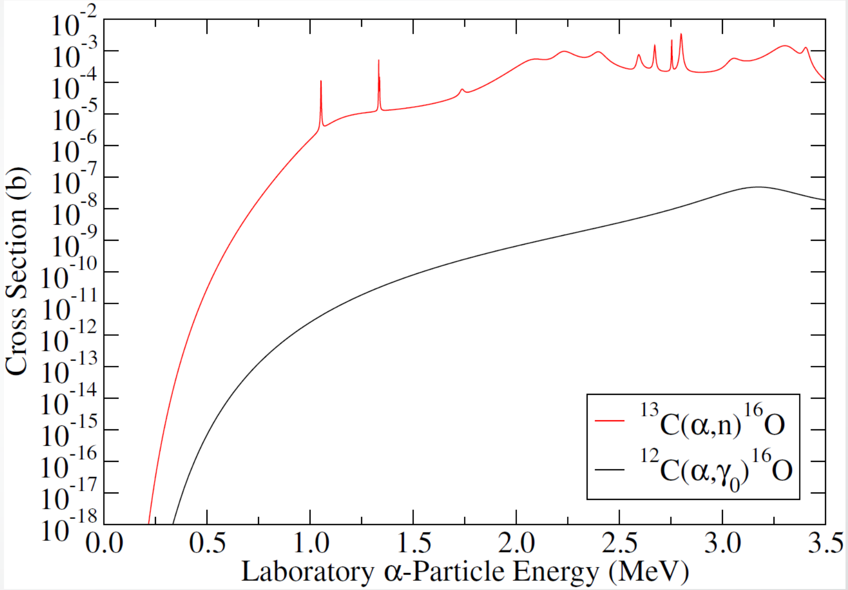
"World Data Set" for the ground state transition

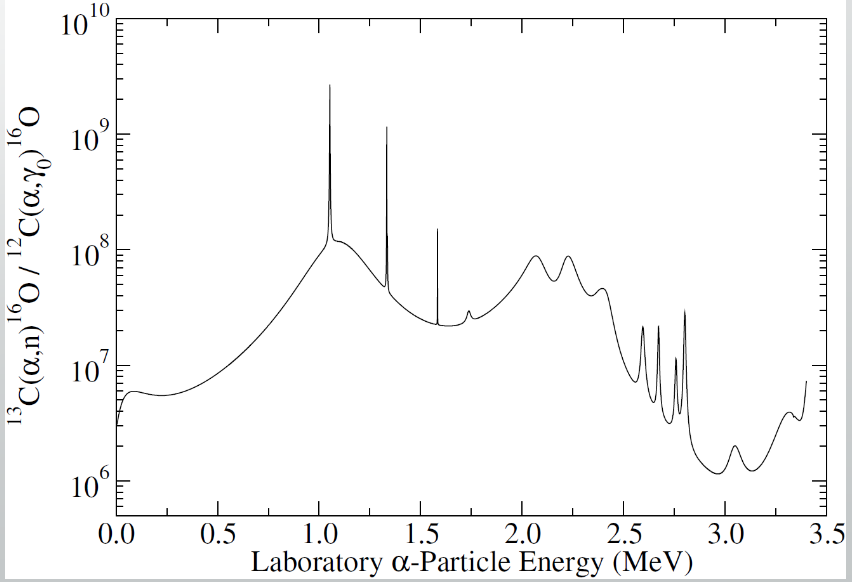
- In hindsight, it was not the best idea to only report "E1 and E2" cross sections, since this makes it harder to understand discrepancies between the different measurements



Major experimental challenges for direct measurements

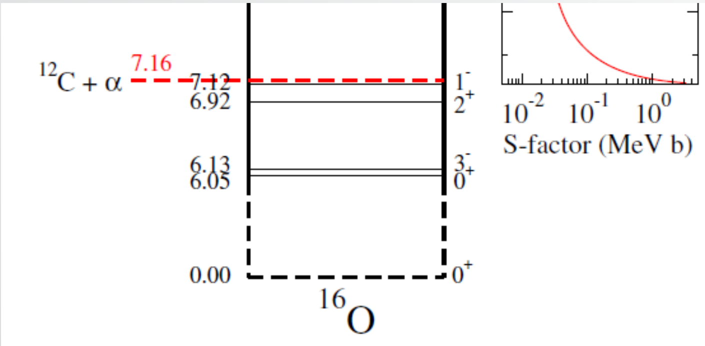
- Very low cross section
- Background reactions
  - $^{13}C(\alpha,n)^{16}O$ *et al.*
  - High Q-value (n,γ) on nearby material
- Inverse kinematics has issues
- Solutions
  - Very clean target / beam lines
  - Time-of-flight method to separate prompt γ-ray signals from (n,γ) delayed ones
  - Recoil separators
  - More exotic solutions





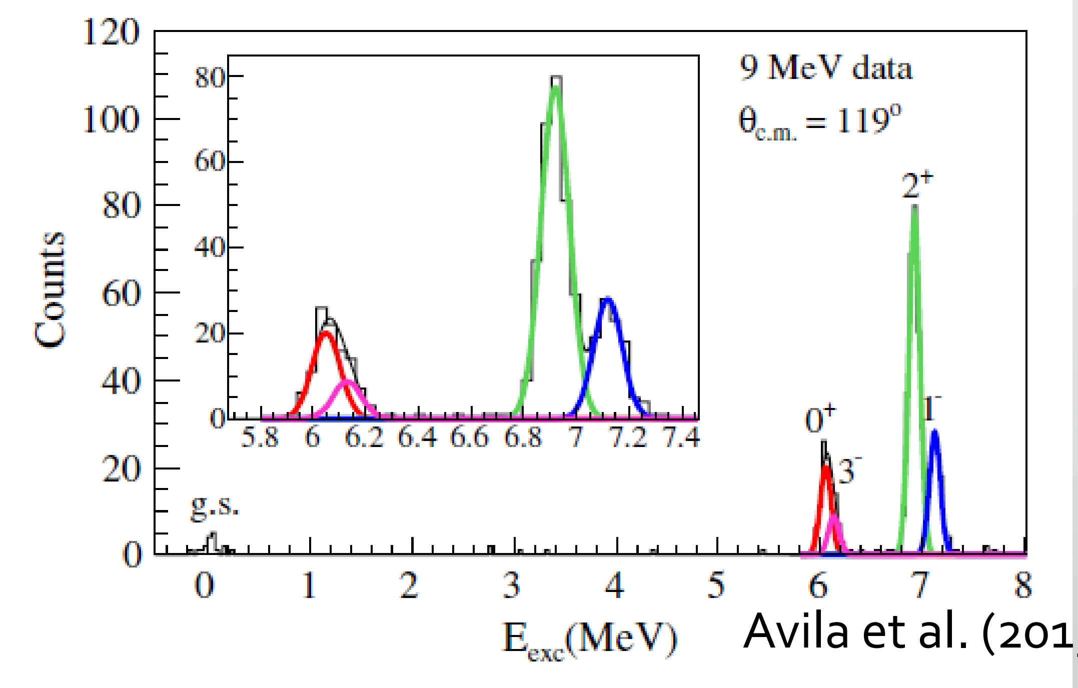
Transfer reaction studies

- SubCoulomb measurements of Asymptotic Normalization Coefficients ($C_{\lambda c}$)



$$C_{\lambda c} = \frac{(2\mu_\alpha a_c)^{1/2}}{\hbar W_c(a_c)} \frac{\tilde{\gamma}_{\lambda c}}{[1 + \sum_{c'} \tilde{\gamma}^2_{\lambda c'} (dS_{c'}/dE)(\tilde{E}_\lambda)]^{1/2}},$$

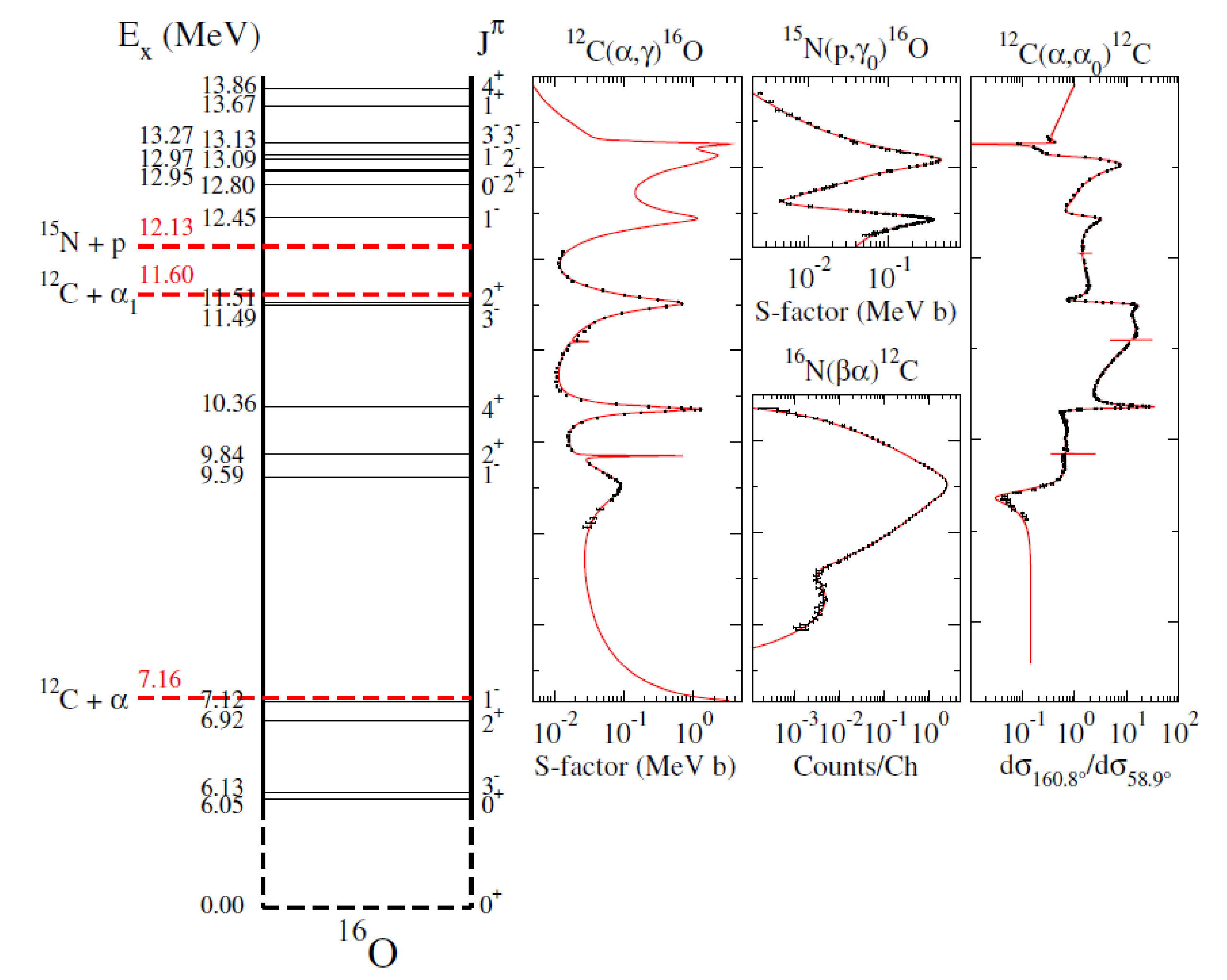
^{6}Li(^{12}C,d)^{16}O or ^{7}Li(^{12}C,t)^{16}O



Avila et al. (2015)

Reference	ANC$_\alpha$ (fm$^{-1/2}$) 6.92 MeV, 2$^+$	7.12 MeV, 1$^-$
Transfer		
Brune *et al.* (1999)	$1.14(10) \times 10^5$	$2.08(20) \times 10^{14}$
Belhout *et al.* (2007)	$1.40(50) \times 10^{5c}$	$1.87(54) \times 10^{14}$
Oulebsir *et al.* (2012)	$1.44(28) \times 10^5$	$2.00(35) \times 10^{14}$
Avila *et al.* (2015)	$1.22(7) \times 10^5$	$2.09(14) \times 10^{14}$

Other compound nucleus reactions

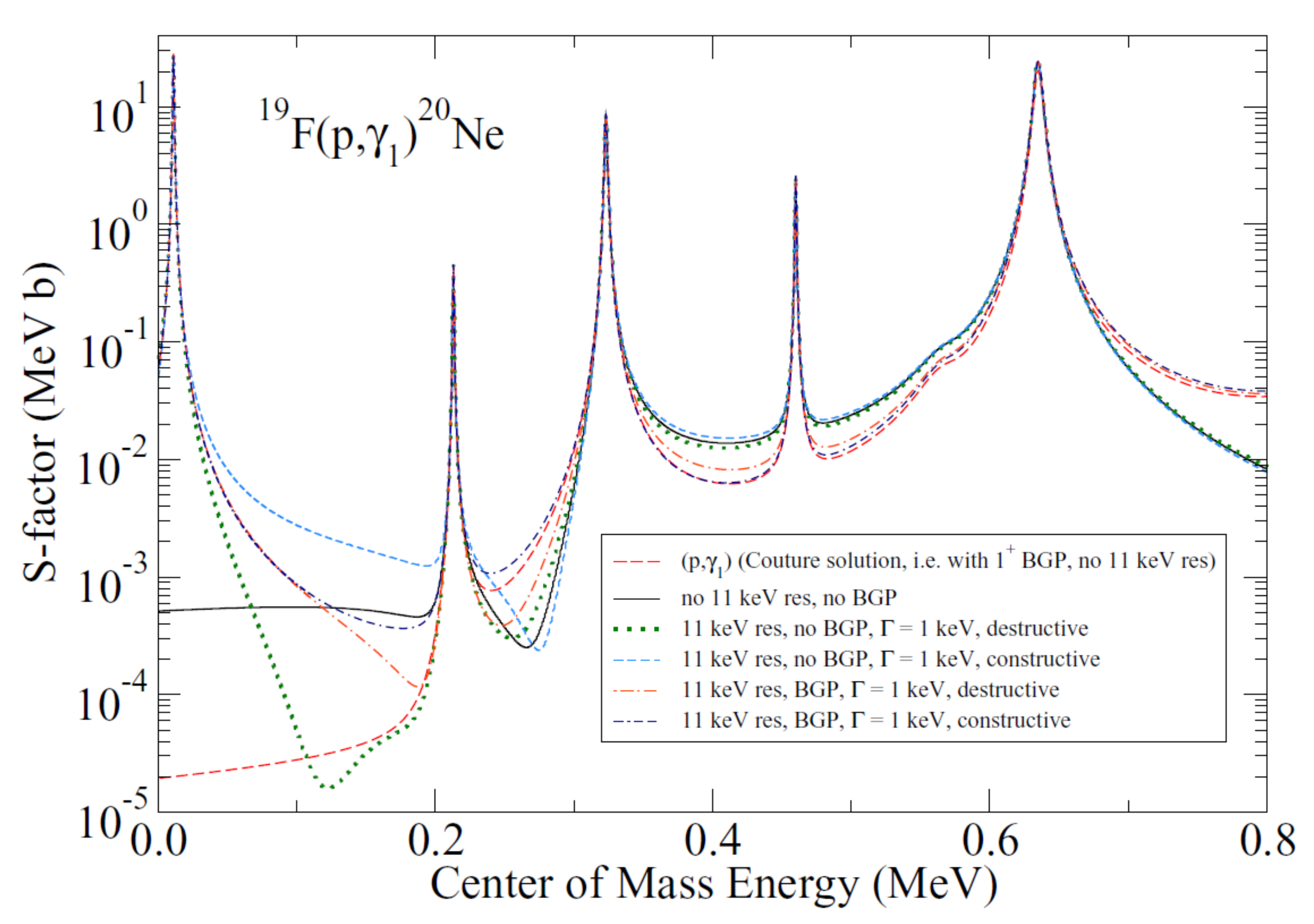


Phenomenological R-matrix

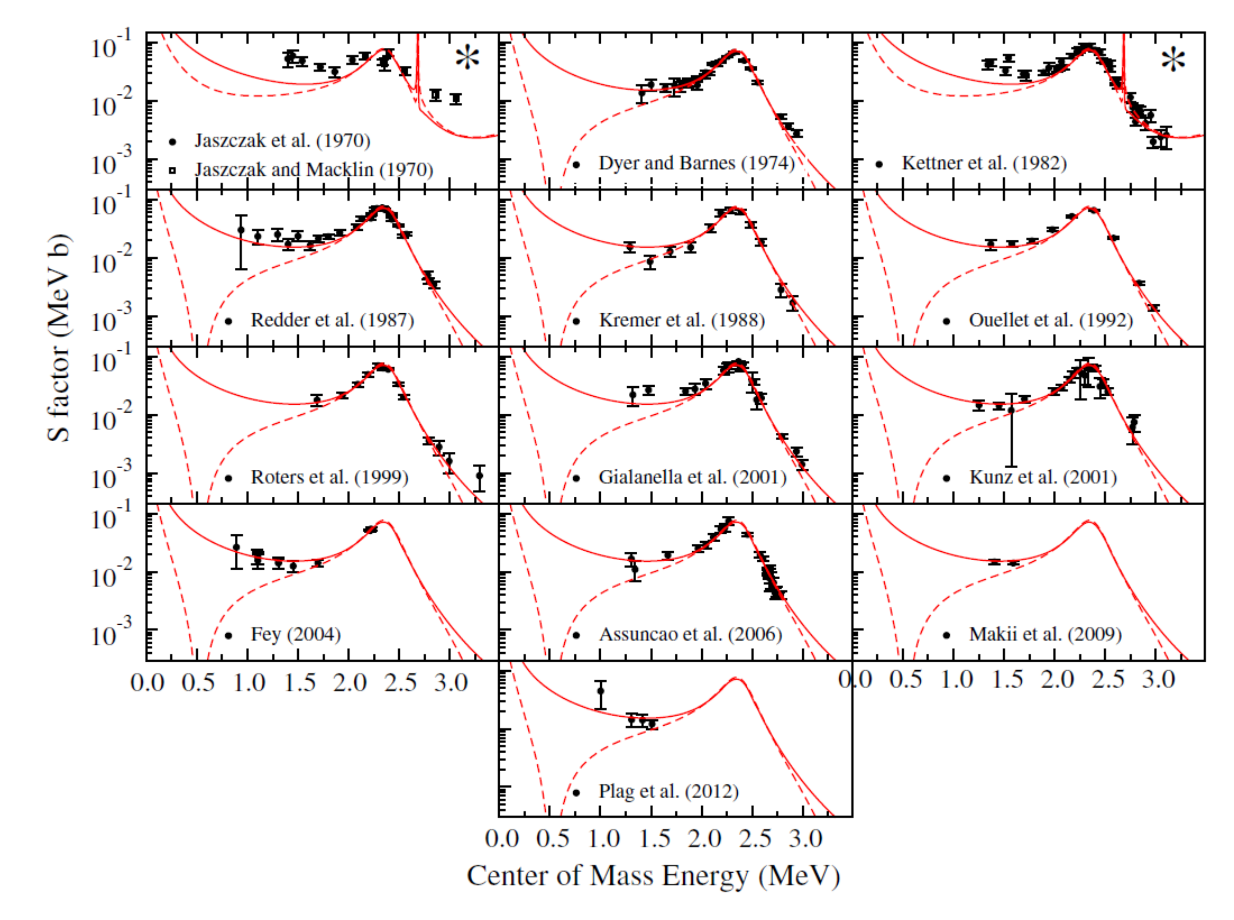
- Reaction framework that parameterizes the cross section in terms of the energy levels of the compound system (mostly)
  - J^{π}, E_{level}, $\Gamma_{level,channel}$
  - Can calculate the interference between resonances

$$R_{cc'} = \sum_{\lambda=1}^{\infty} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_{\lambda} - E}$$

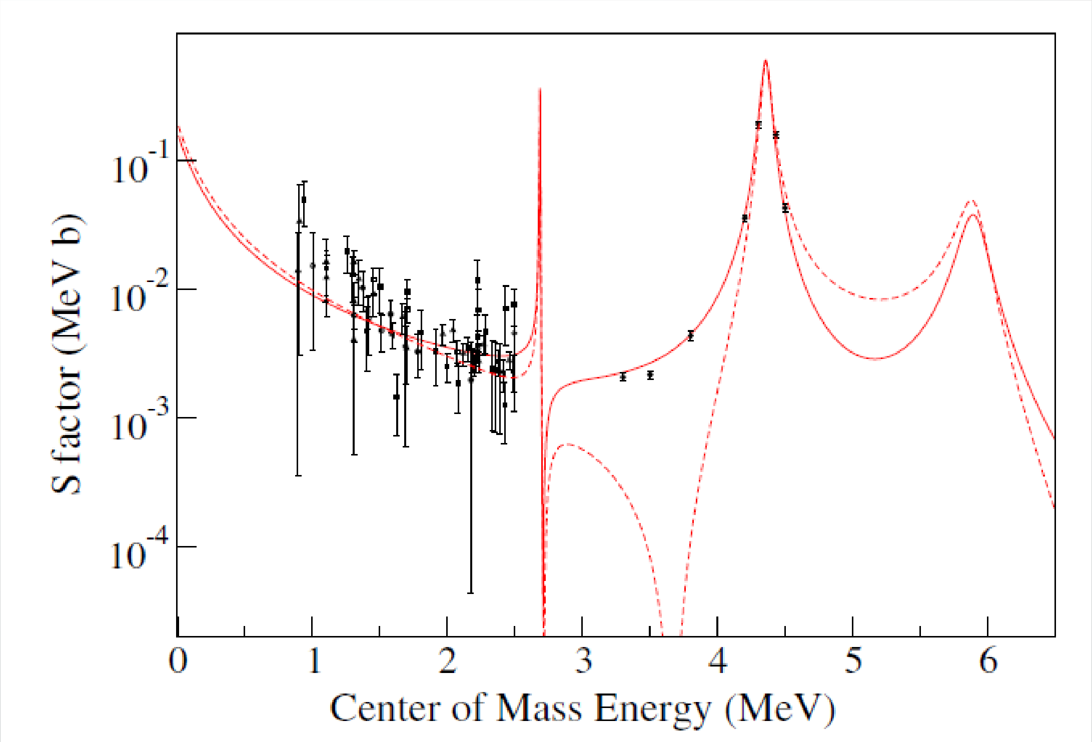
$$\sigma \propto |R_{cc'}|^2$$



One example of interference solutions for $^{12}C(\alpha,\gamma)^{16}O$

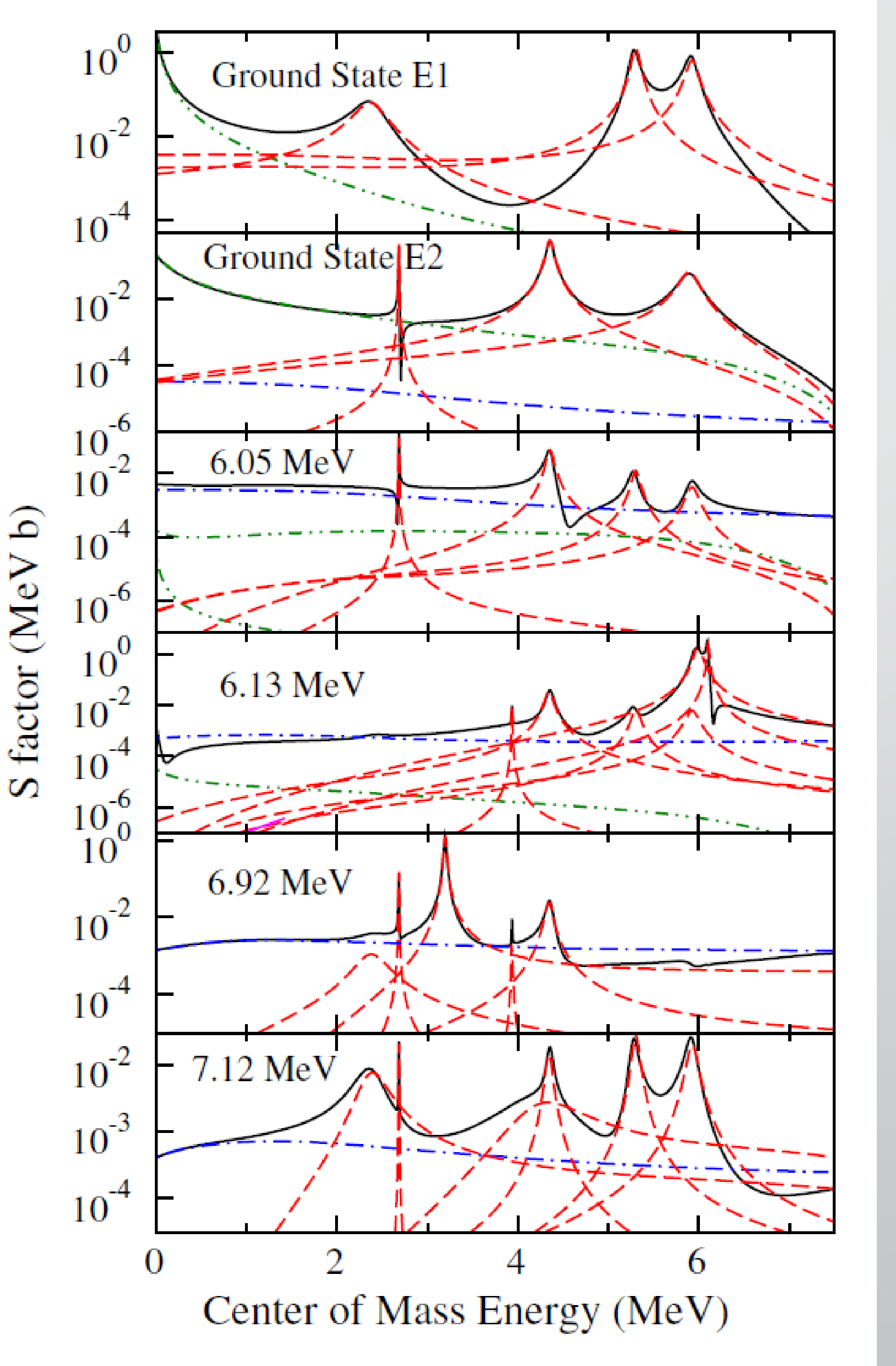


One reason why higher energy data can also be very important for an *R*-matrix fit

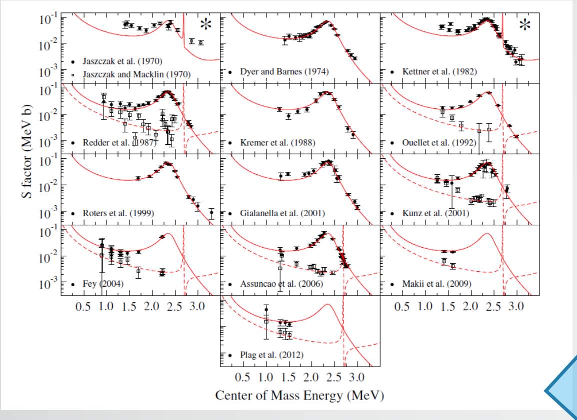


Breaking the S-factor down into R-matrix components

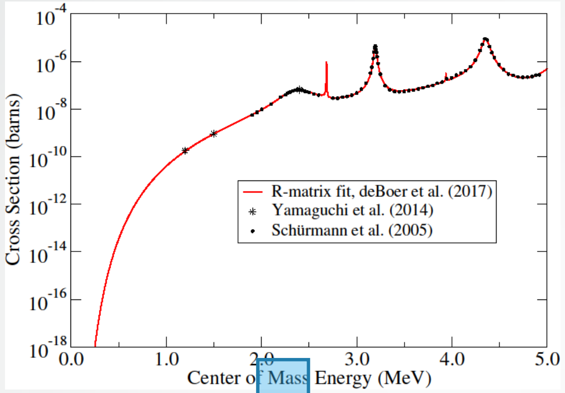
- Green = Subthreshold state
  - Strong ones for the ground state (1^- and 2^+)
- Blue = Direct capture
  - Modeled using external capture in these calculations
- Red = Resonances



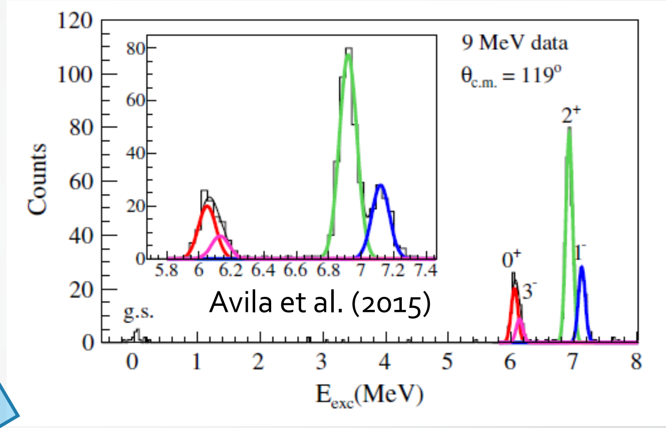
Phenomenological R-matrix



Low energy $^{12}C(\alpha,\gamma)^{16}O$ data



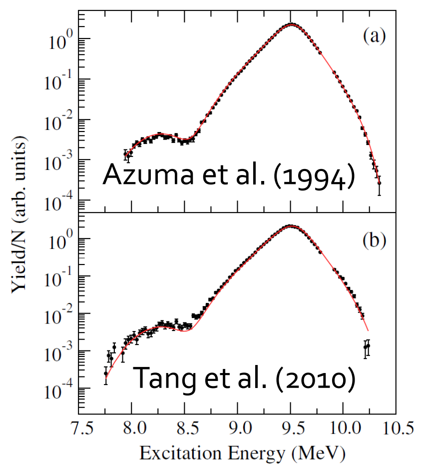
Total capture cross sections (recoils)



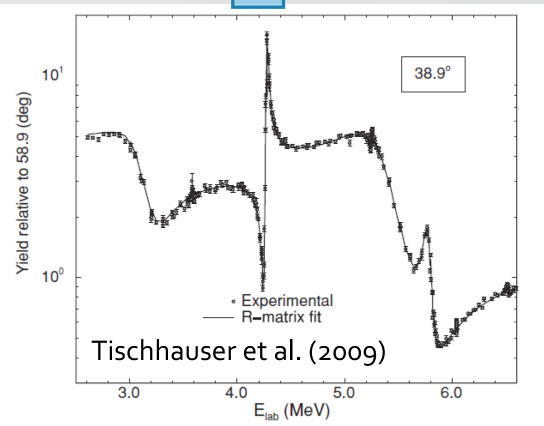
Bound state ANCs

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E},$$

$^{16}N(\beta\alpha)^{12}C$ spectrum



$^{12}C(\alpha,\alpha)^{12}C$ data



Multichannel R-matrix

$$\mathbf{U} = \rho^{\frac{1}{2}}\mathbf{O}^{-1}(\mathbf{1}-\mathbf{RL}_0)^{-1}(\mathbf{1}-\mathbf{RL}_0^*)\mathbf{I}\rho^{-\frac{1}{2}}$$

$$T_{cc'} = e^{2i\omega_c}\delta_{cc'} - U_{cc'},$$

$$\sigma_{\alpha\alpha'} = \frac{\pi}{k_\alpha^2}\sum_{Jll'ss'} g_J \left|T_{cc'}^J\right|^2,$$

CALCULATION OF THE $^{12}C+\alpha$ CAPTURE CROSS SECTION AT STELLAR ENERGIES*

By F. C. Barker†

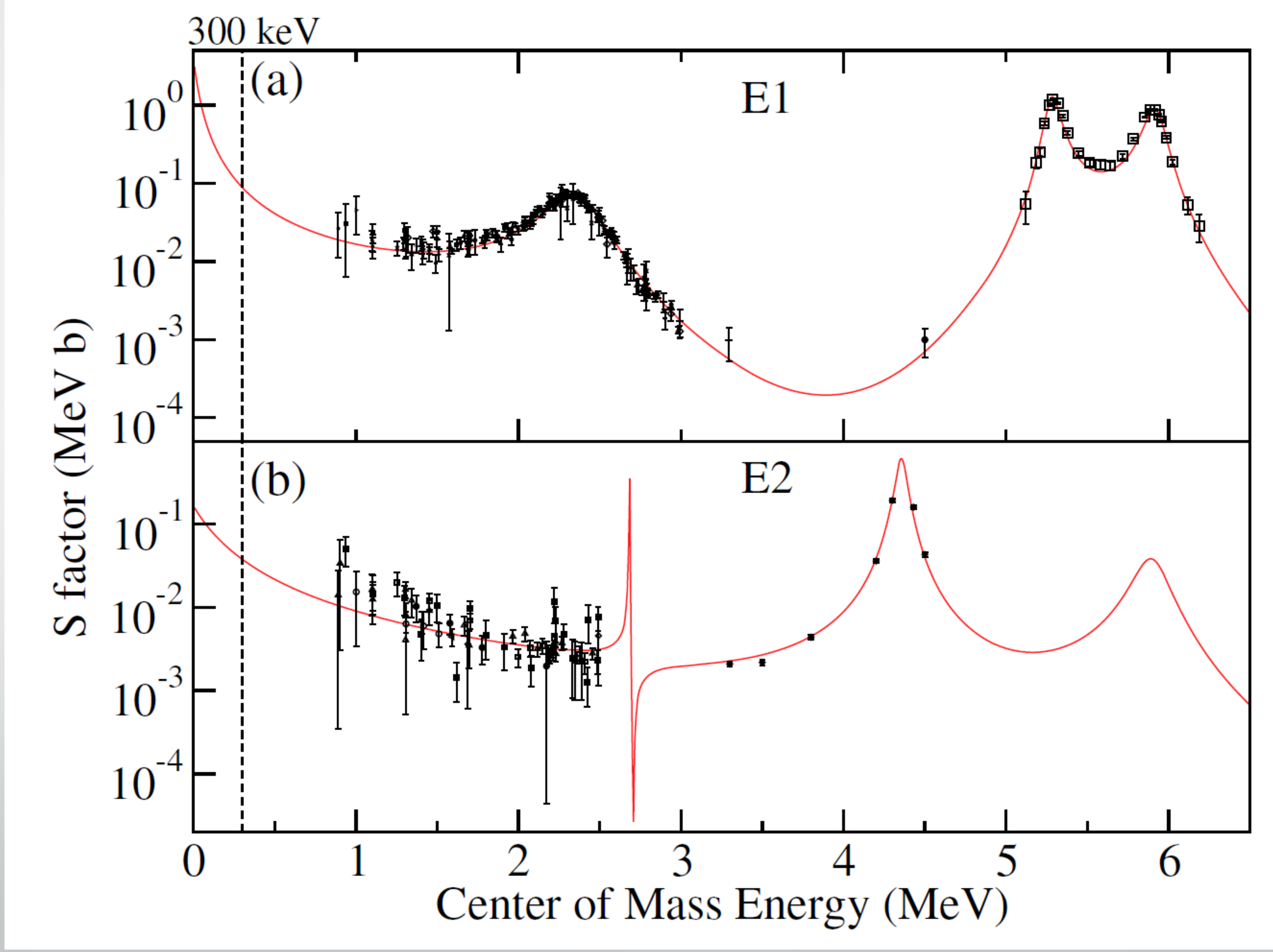
[Manuscript received 26 July 1971]

Abstract

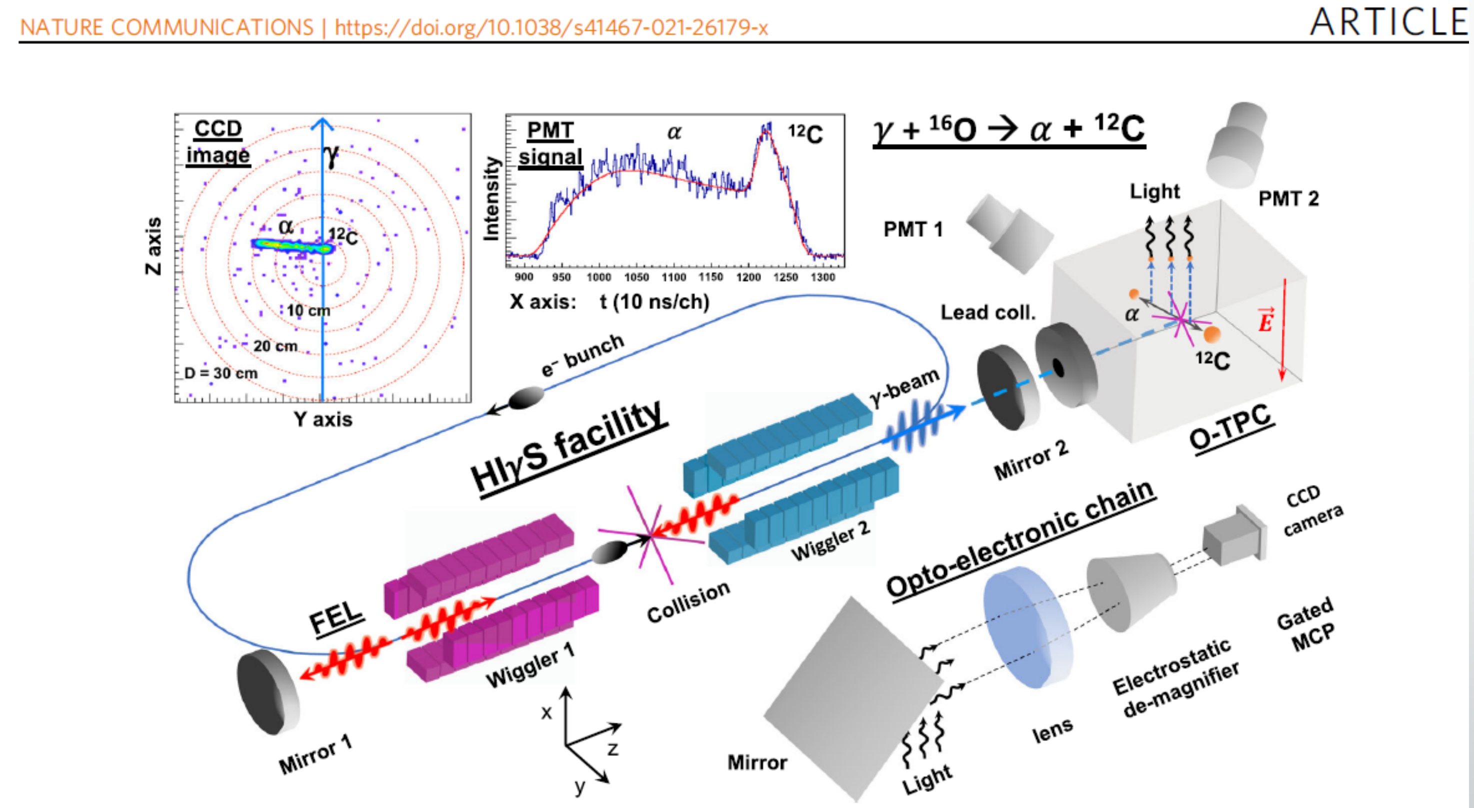
The $^{12}C(\alpha,\gamma)^{16}O$ cross section is calculated at stellar energies, using R-matrix parameters obtained by fitting consistently the $^{12}C+\alpha$ scattering phase shifts and the α-spectrum from ^{16}N β-decay. This limits the $^{12}C+\alpha$ channel radius to the range 5–7 fm. The S-factor at $E_\alpha = 400$ keV is calculated to lie in the range 0·05–0·33 MeV . b.

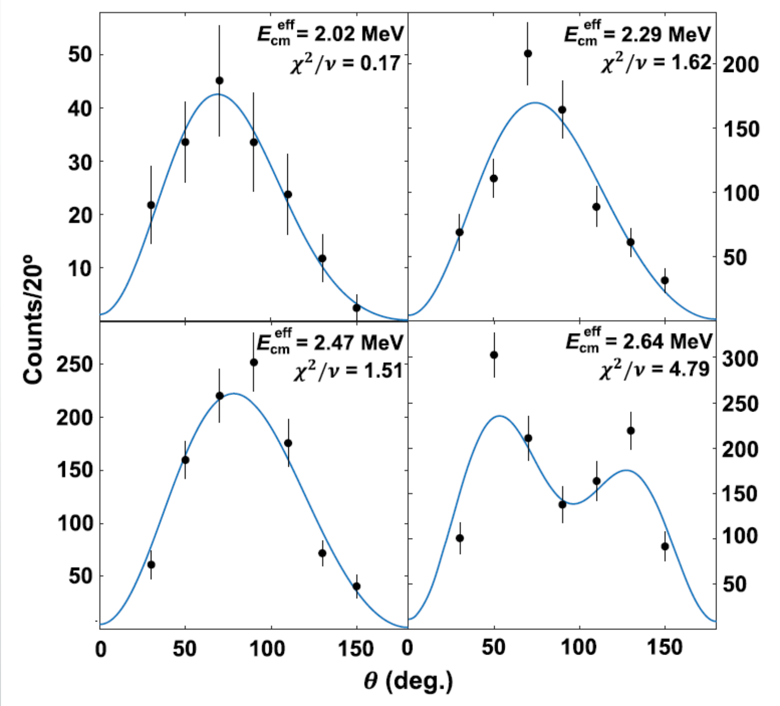
Data inconsistencies

- "World data sets"
- Normalized here, but still show a lot of scatter
- Larger discrepancies in ground state E2 data
- However, more modern measurements have, usually, produced more consistent data



What's new? $^{16}O(\gamma,\alpha)^{12}C$





Smith *et al*. (2021)

Long campaign (hundreds of hours) led by Moshe Gai

New results are on the way!

What's new? Alternative transfer reactions

- Shen et al. (2019, 2020)
- **$^{12}C(^{11}B,^{7}Li)^{16}O$**
- Probing the model and reaction sensitivity of ANC determination
- 2^{+} ANC is re-measured in (2019)
- GS ANC is measured in (2020)

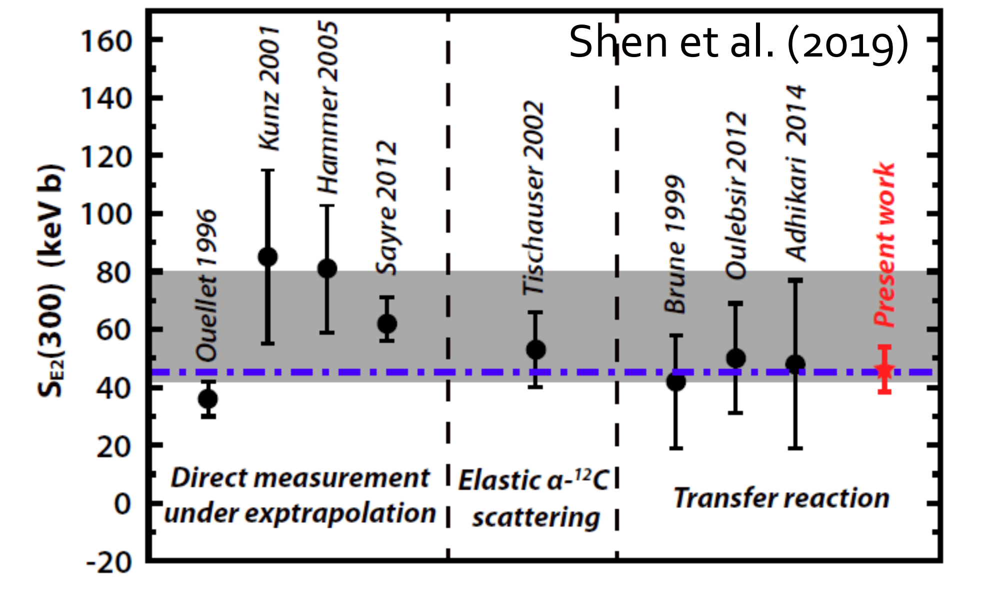


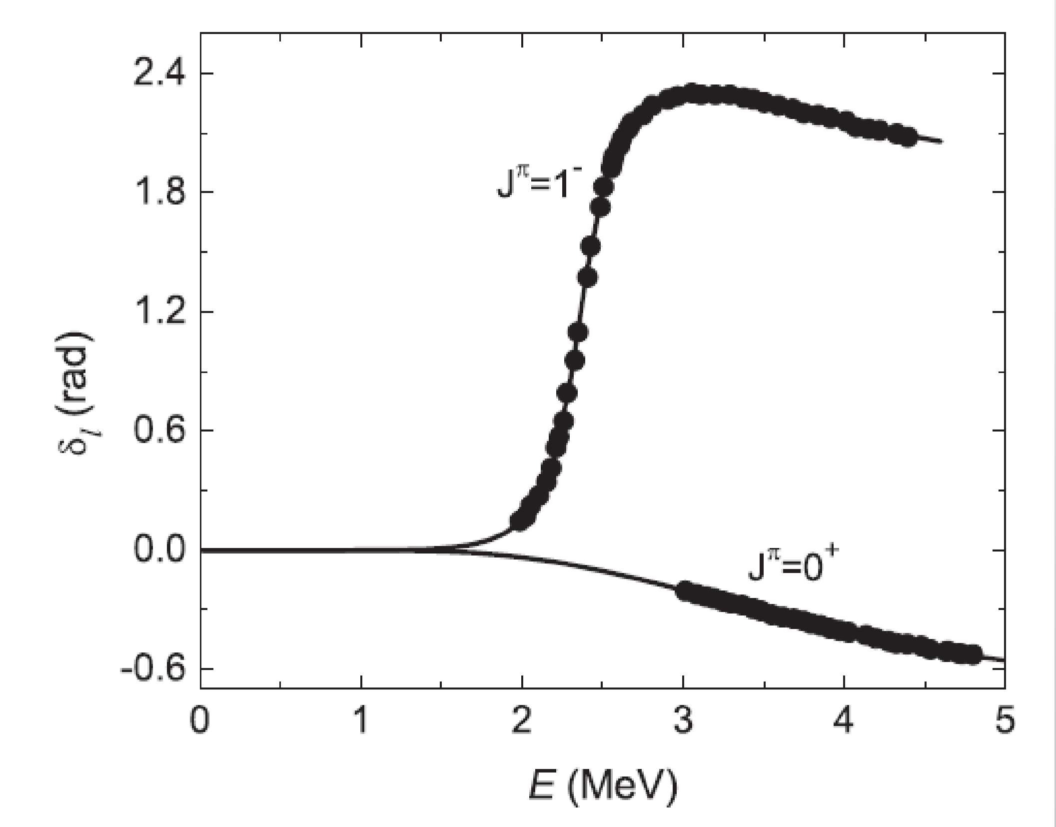
TABLE I. Present ANC of the ^{16}O GS and other available results in the literature. Shen et al. (2020)

Reference	ANC ($fm^{-1/2}$)	Method
Adhikari (2009) [14]	13.9 ± 2.4	$^{16}O + Pb$ breakup
Morais (2011) [16]	3390 (WS1)	$^{12}C(^{16}O, ^{12}C)^{16}O$
	1230 (WS2)	
	750 (FP)	
Sayre (2012) [11]	709	*R* matrix
Adhikari (2017) [15]	637 ± 86	$^{12}C(^{7}Li, t)^{16}O$
Present	337 ± 45	$^{12}C(^{11}B, ^{7}Li)^{16}O$

What's new? ANCs from scattering data revisited

- Orlov *et al.* (2017)
- Ramirez-Suarez and Sparenberg (2017)
- Blokhintsev *et al.* (2022,2023)
- Mukhamedzhanov *et al.* (2024)

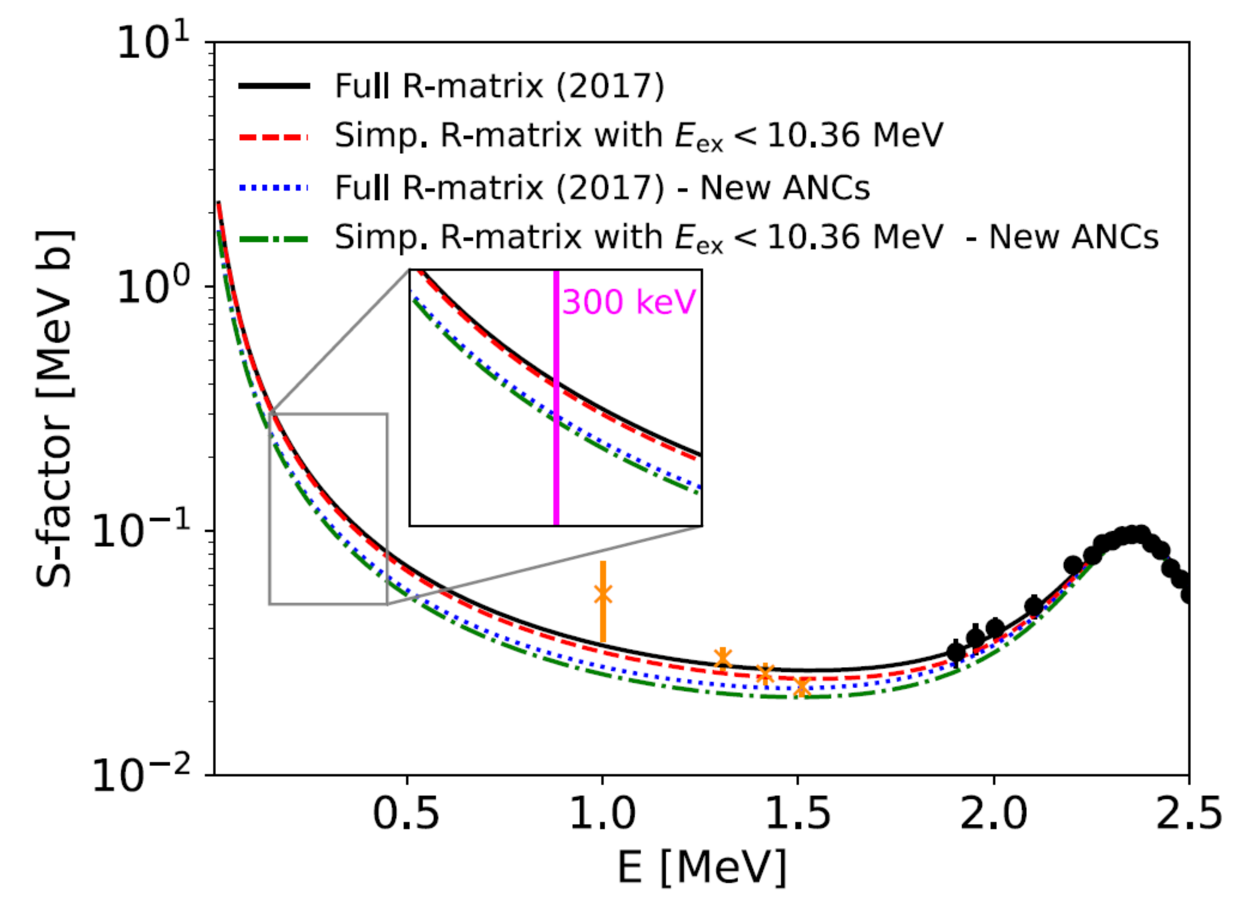
- Investigating different mathematical approaches for extracting ANCs from phase shifts



Phase shifts come from R-matrix fit to Tischhauser *et al.* elastic scattering data

(^{6}Li,d) transfer dependance on ^{6}Li ANC

- Hebborn et al. (2024)
- Calculation of ^{6}Li ANC from first principles leads to a 21% reduction in the S-factor at 300 keV



Recent sources of tension

- Shen et al. (2020)
  - Two solutions for GS and 2+ ANCs lead to similar reproduction of the experimental data.
  - But each solution requires an ANC that is inconsistent with previous measurements
  - One solution does produce a **15% increase** in the low energy S-factor compared to that of deBoer et al. (2017)
- Hebborn et al. (2024)
  - Calculation of ^{6}Li ANC from first principles results in a **21% reduction** in the S-factor compared to deBoer et al. (2017)
- Mukhamedzhanov et al. (2024)
  - ANCs extracted from scattering data indicate a **24% increase** in the S-factor compared to deBoer et al. (2017)

On the horizon: $^{16}O(e,e'\alpha)^{12}C$

- MIT group
  - See Friščić et al. (2019)

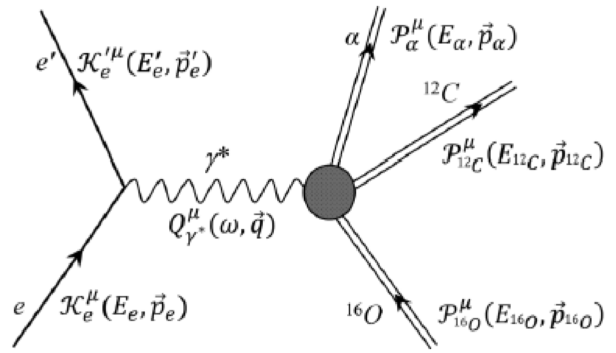
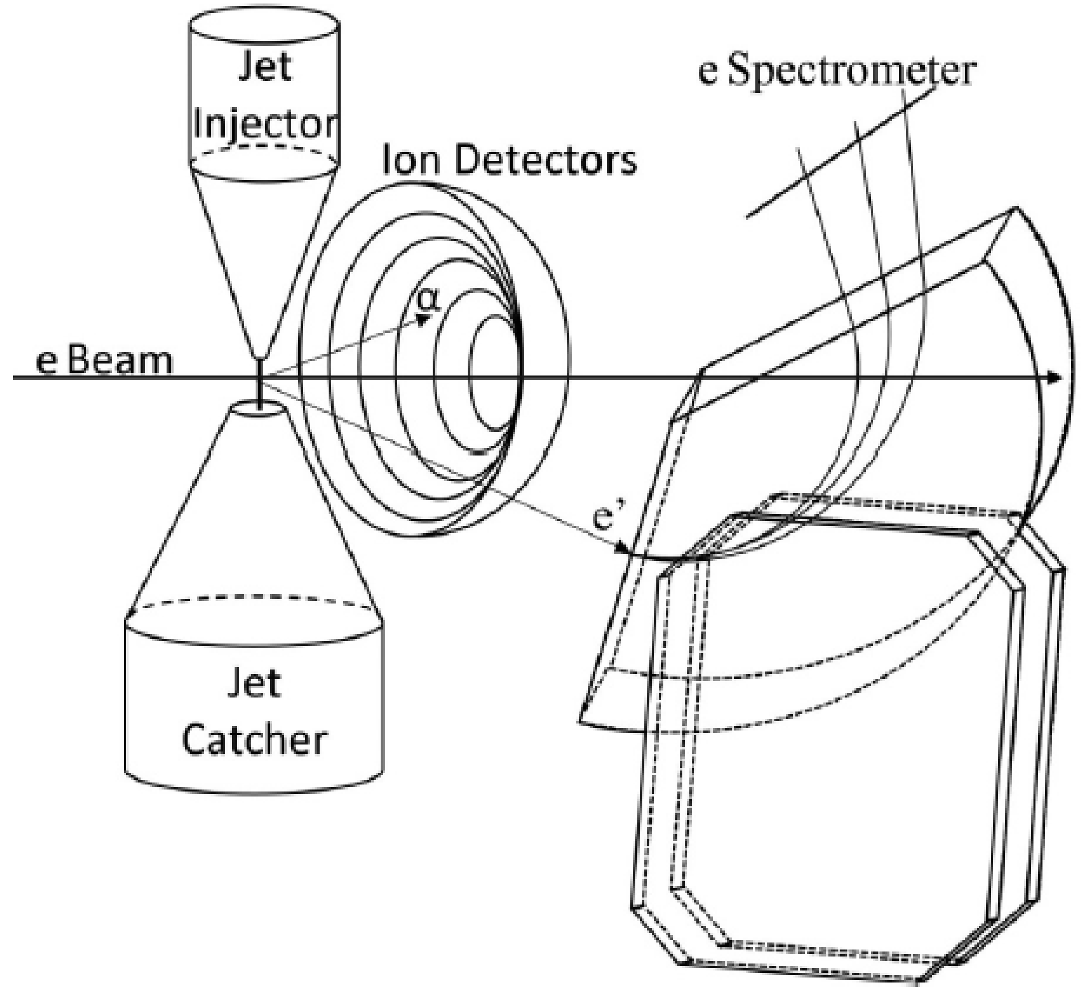
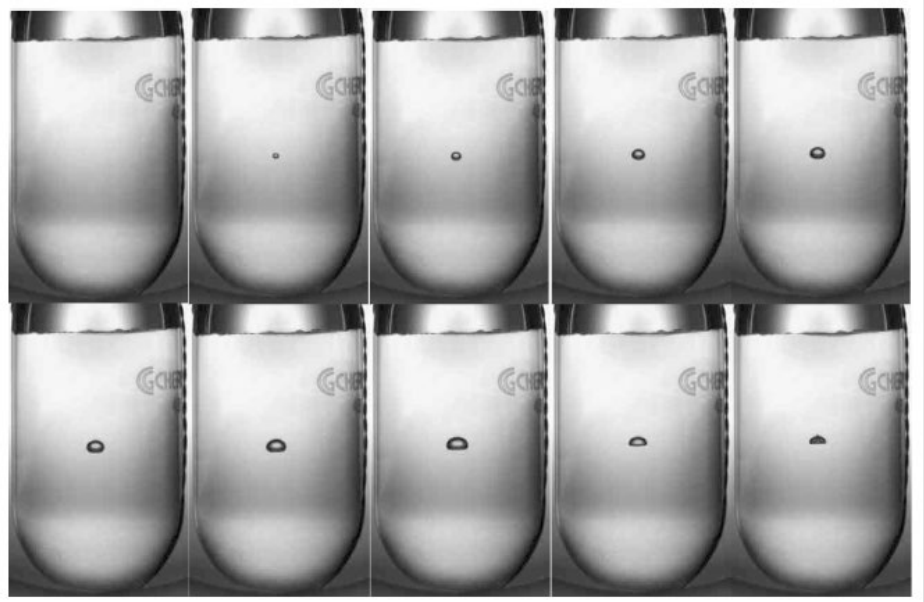


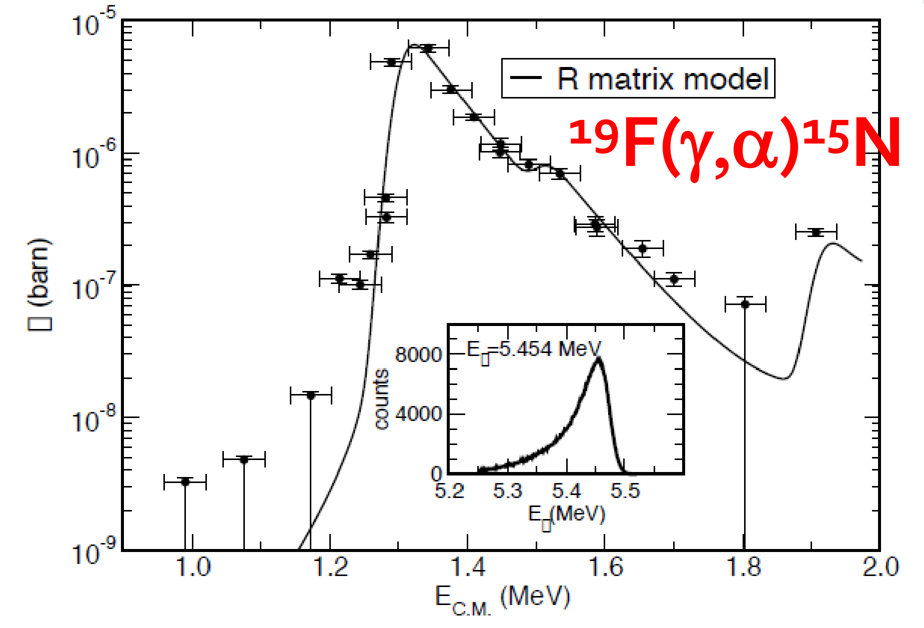
FIG. 3. First-order Feynman diagram for the electrodisintegration of ^{16}O involving one virtual photon γ^* exchange to be compared with Fig. 2. Again, the kinematic variables here will be discussed in more detail in Sec. III.



On the horizon: $^{16}O(\gamma,\alpha)^{12}C$

- Jefferson Laboratory
- Ernst Rehm and Claudio Ugalde
- Bubble Chamber + Bremsstrahlung beam
- Previous tests at HIγS
- Not sure on current status?





On the horizon: Coulomb dissociation of ^{16}O on lead

- FAIR at GSI
  - Aims to get 10% uncertainty at 1 MeV
  - Will cover a wide energy range
  - Some measurements made, but still under analysis

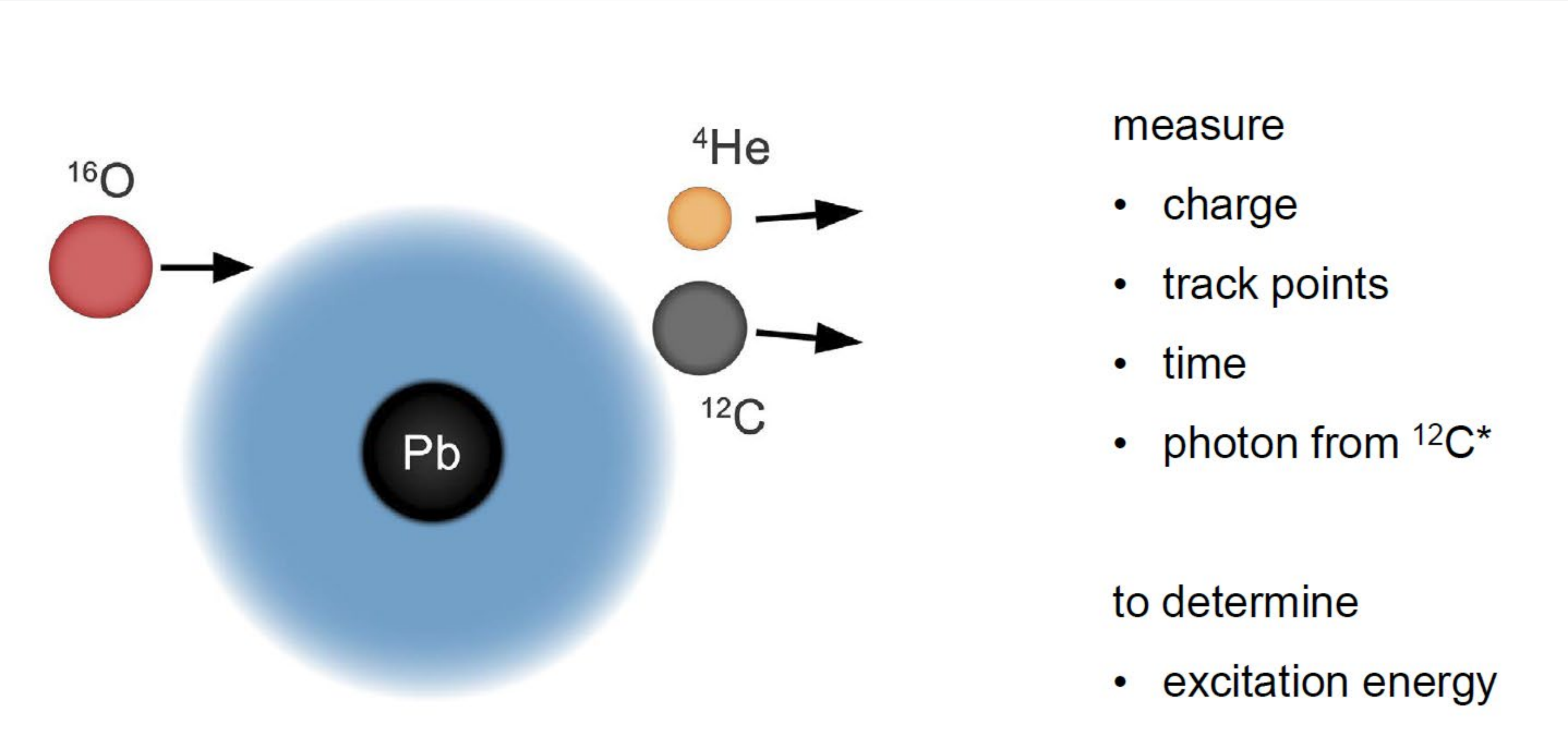


Figure courtesy of Rene Reifarth

Impact studies

New approach to determining radiative capture reaction rates at astrophysical energies

I. Friščić,[*] T. W. Donnelly, and R. G. Milner
Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Impact of $^{16}O(e, e'\alpha)^{12}C$ measurements on the $^{12}C(\alpha, \gamma)^{16}O$ astrophysical reaction rate

R. J. Holt[1,2,*] and B. W. Filippone[2,†]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA*

Impact of $^{16}O(\gamma, \alpha)^{12}C$ measurements on the $^{12}C(\alpha, \gamma)^{16}O$ astrophysical reaction rate

R. J. Holt,[1,2,*] B. W. Filippone,[2,†] and Steven C. Pieper[1,‡]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
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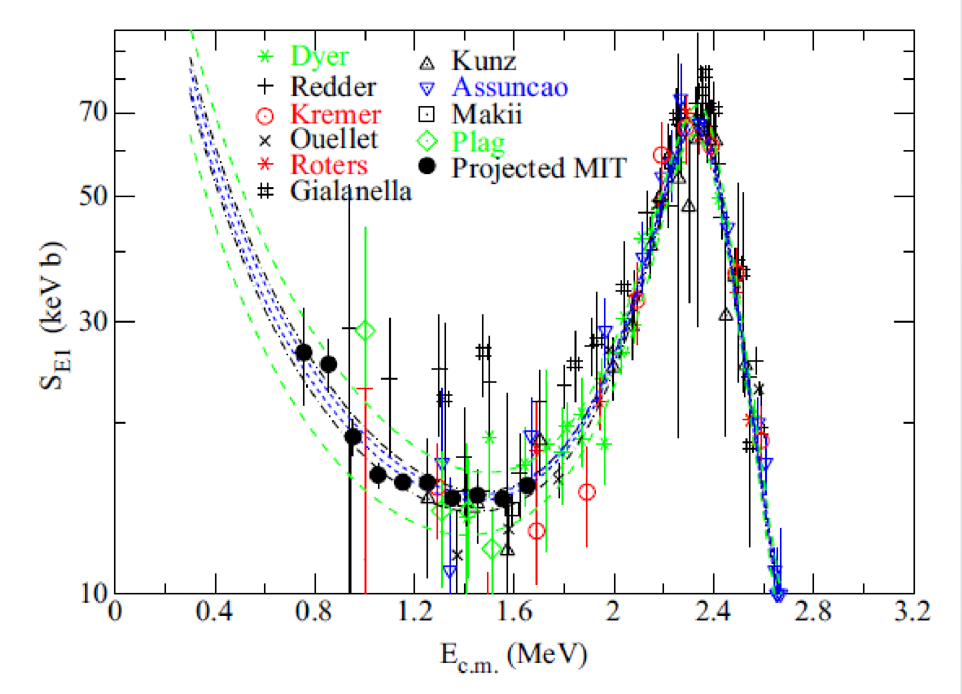
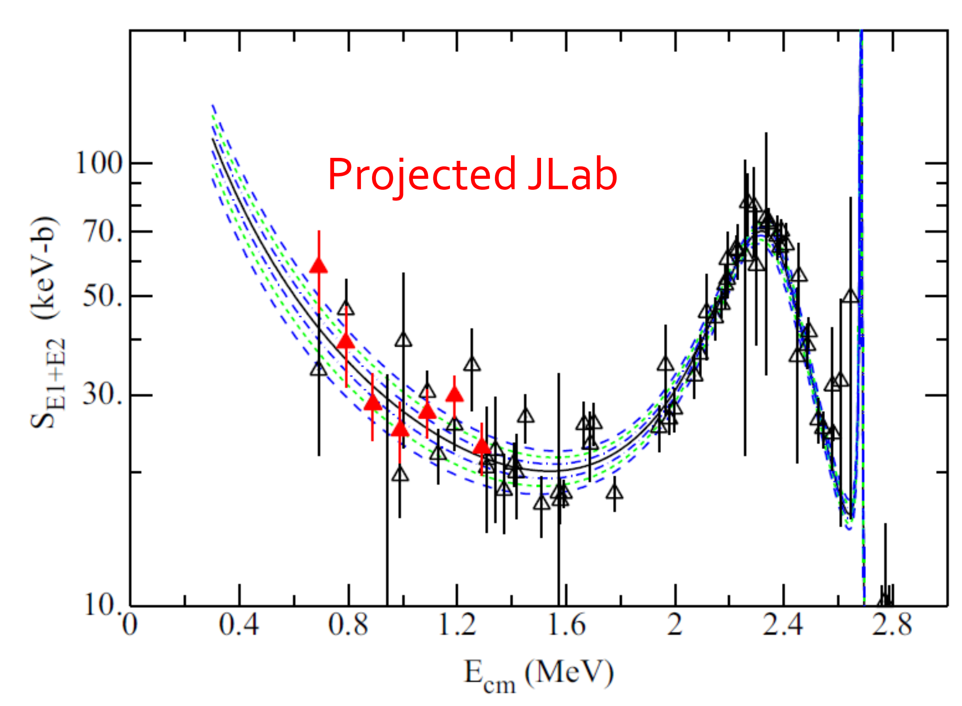
Projected uncertainty improvement

- These works argue that these new measurements can produce data with uncertainties that are competitive with direct measurements and can even extend to lower energies





$^{16}O(e, e'\alpha)^{12}C$ measurements and the $^{12}C(\alpha, \gamma)^{16}O$ astrophysical reaction rate

D. H. Potterveld,[1,*] B. W. Filippone,[2,†] R. J. Holt,[2,‡] and I. Friščić[3,§]
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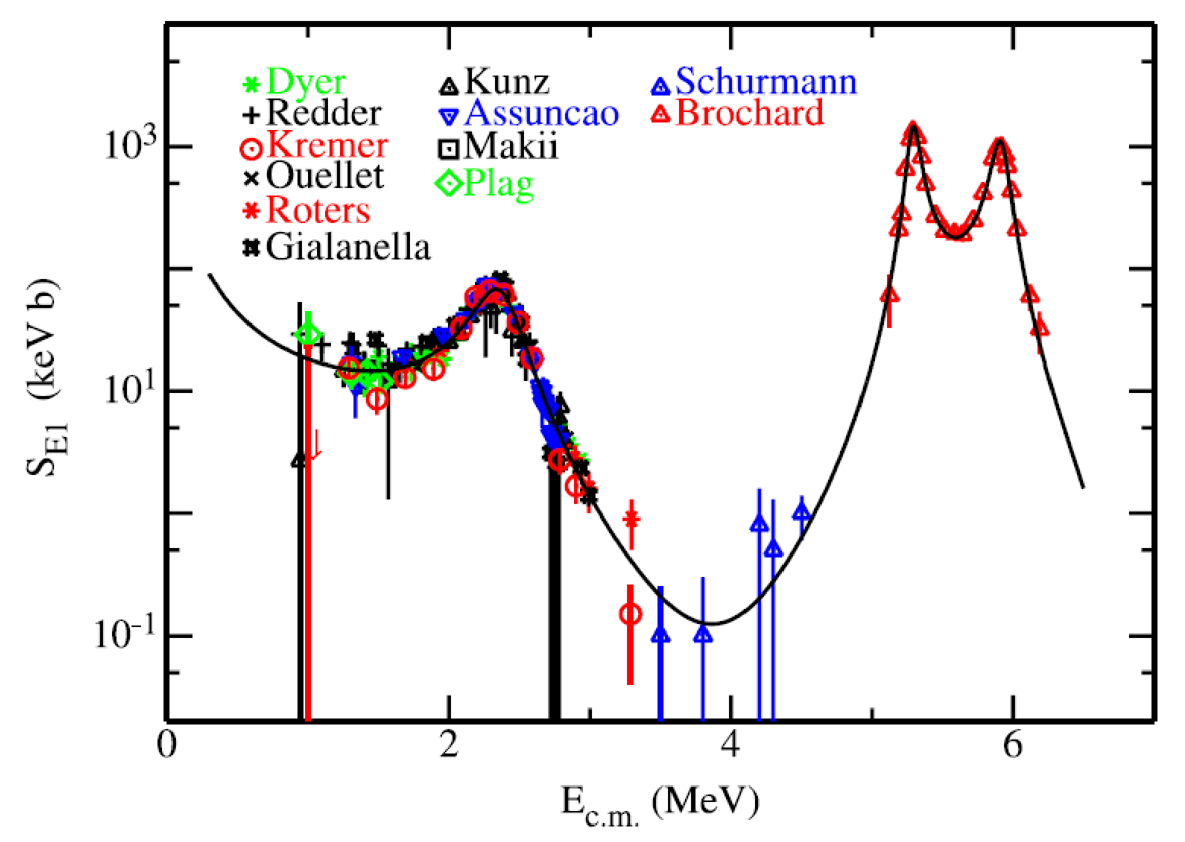
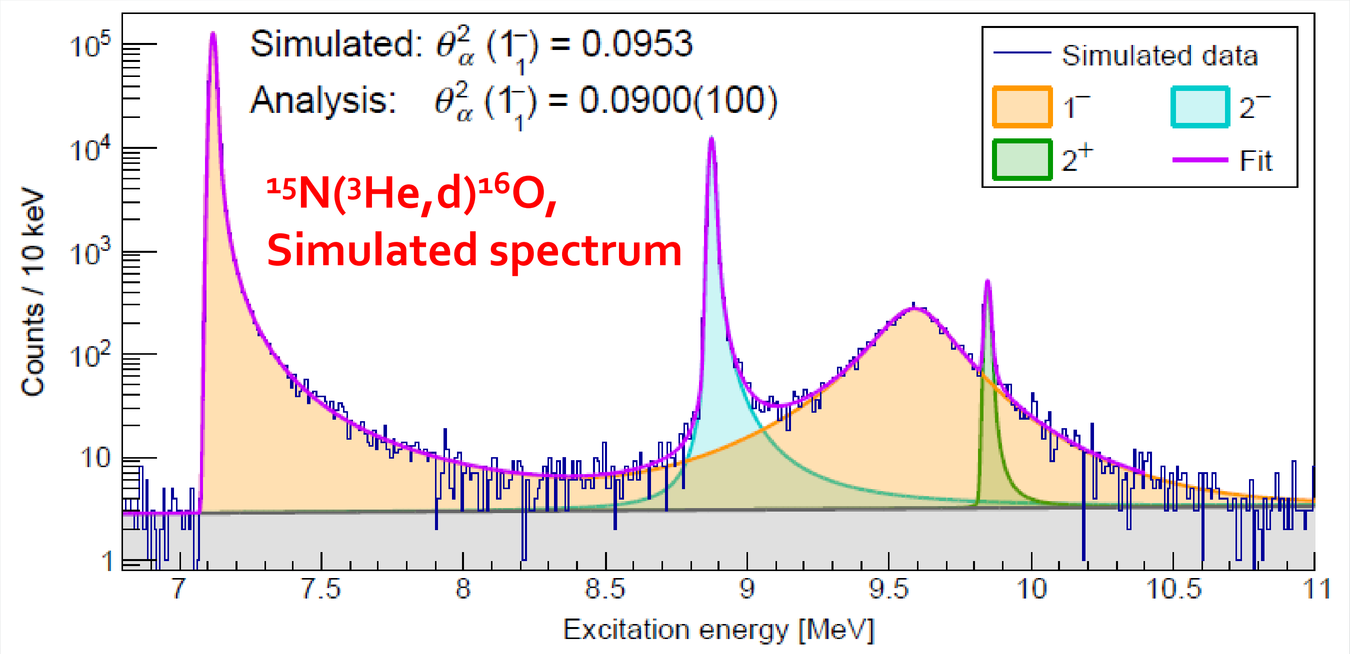
[2] Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA
[3] Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia

- **Missed this paper in review!**
- Bayesian uncertainty estimates for proposed data
- Emphasizes that both low and high energy data can better constrain the extrapolated S-factor



On the horizon: $^{12}C(\alpha,\gamma)^{16}O$, $^{15}N(^{3}He,d)^{16}O$ and $^{16}O(p,p')^{16}O$

- **Kevin C.W. Li** (University of Oslo)
- $^{12}C(\alpha,\gamma)^{16}O$ at "high" energies at iThemba in South Africa
- $^{15}N(^{3}He,d)^{16}O$ at IJCLab in France
- $^{16}O(p,p')^{16}O$ at RCNP in Japan



$$N_{ab,c}(E) = P_c \left| \sum_{\lambda,\mu}^{N} G^{\frac{1}{2}}_{\lambda ab} \gamma_{\mu c} A_{\lambda\mu} \right|^2$$

Like β-delayed α-emission spectral fitting

On the horizon: *ab initio* calculations of ^{16}O

- Shen et al. (2023), Nature Comm.
- Ab initio calculations of ^{16}O are underway

^{12}C

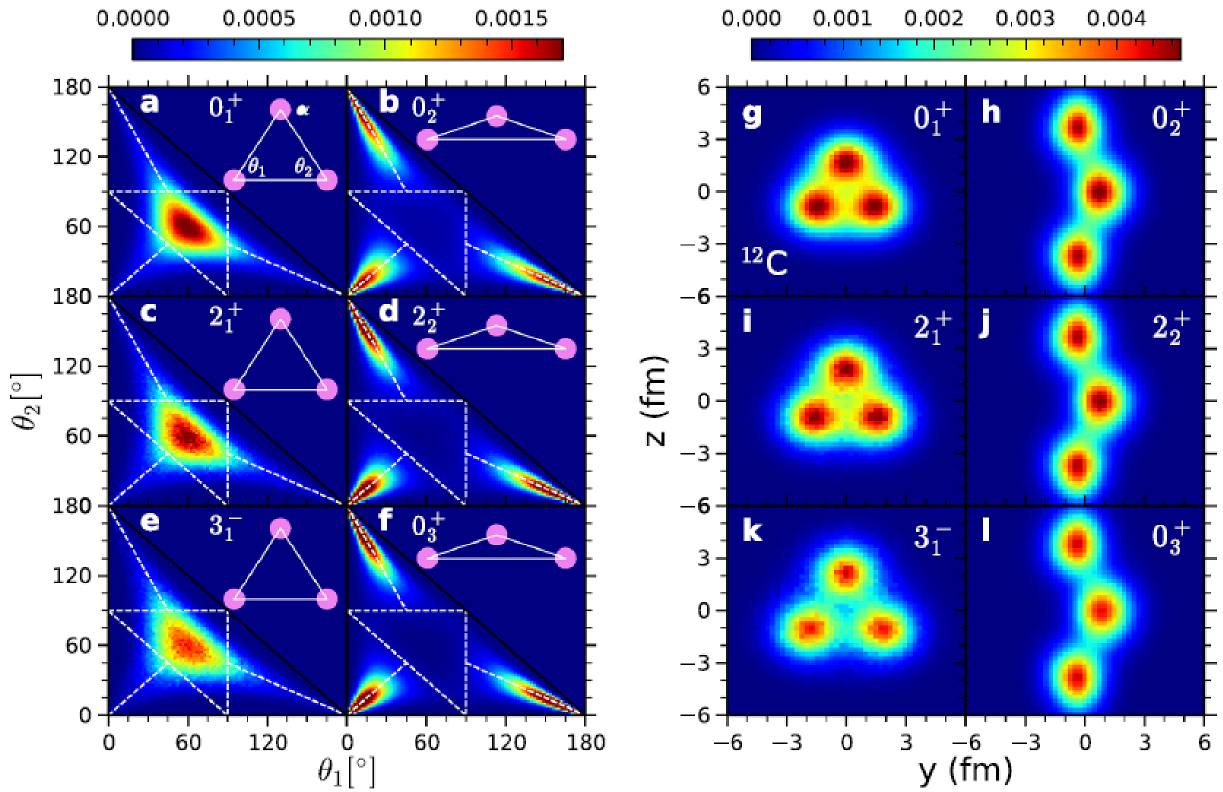
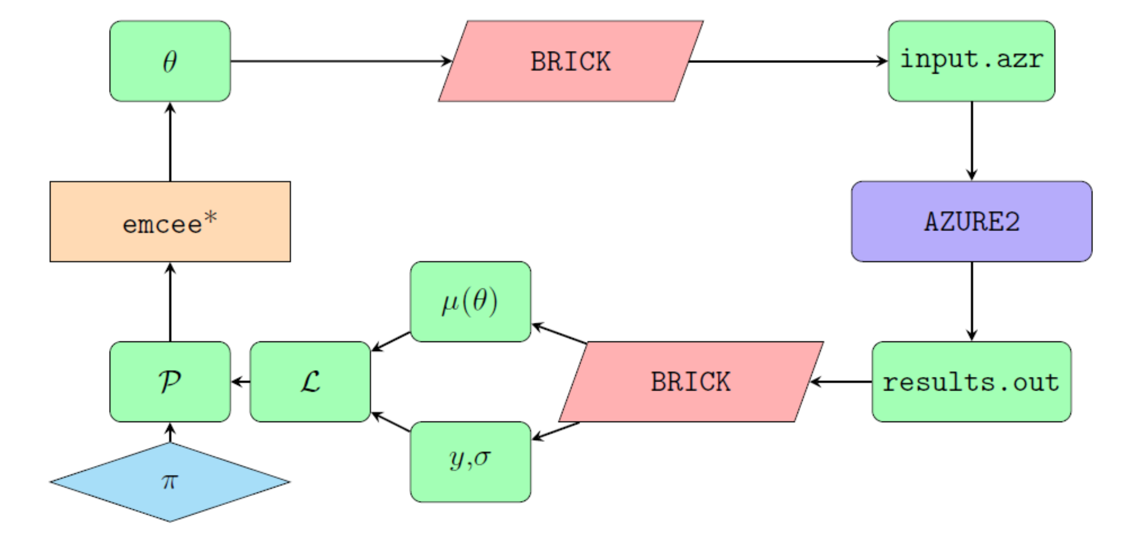
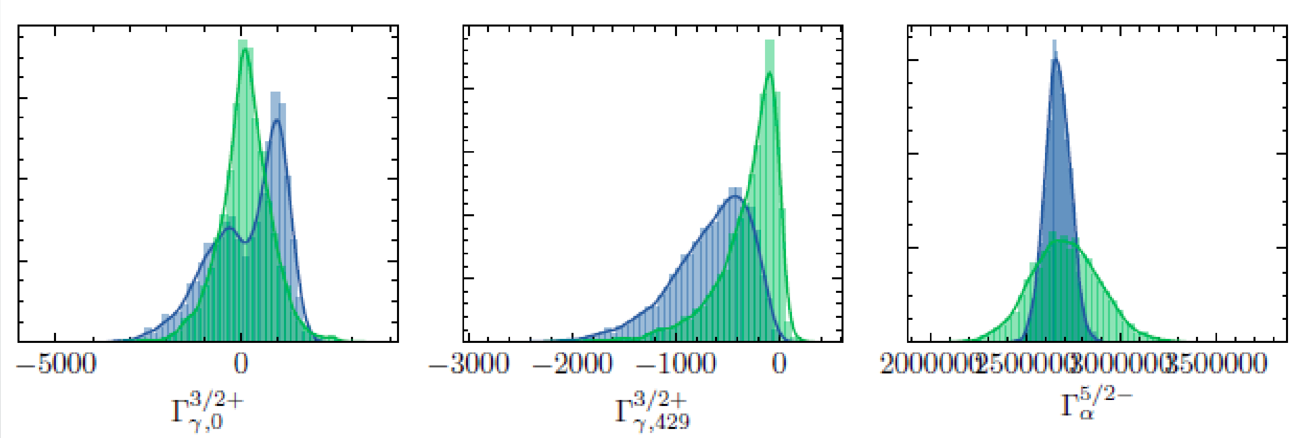


Fig. 2 | Nuclear density distributions for several ^{12}C states using the SU(4) interaction. a–f Results for the density distribution of the two inner angles of the triangle formed by the three alpha clusters for the 0_1^+, 0_2^+, 2_1^+, 2_2^+, 3_1^-, 0_3^+ states respectively. The two axes are for the two inner angles θ_1 and θ_2 measured in degrees. **g–l** Results for the two-dimensional projection of the nuclear density for the 0_1^+, 0_2^+, 2_1^+, 2_2^+, 3_1^-, 0_3^+ states respectively. In each case the orientation of the shortest root-mean-square direction is aligned with the x axis.

On the horizon: Improving uncertainty estimation for R-matrix fits

- A more general problem
- **Bayesian** methods provide a way to improve and gain more detailed information
- See de **Souza *et al.* (2020)** for an application to $^3H(d,n)^4He$
- Computationally intensive, but probably doable
- **Daniel Odell** at Ohio University has developed the Bayesian R-matrix Inference Code Kit (**BRICK**) for use with the **AZURE2** R-matrix code
- New capabilities being developed by **Jakub Skowronski**





Figures courtesy of Daniel Odell

Summary

- Improvements in the uncertainty of the low energy S-factor are hampered by **inconsistent data**
  - **Newer data are much more consistent!** We're on the right track, but measurements are very challenging and issues remain
  - **Ground state E2 data is in the worst shape**, because you usually need to measure a more complete angular distribution to obtain it
  - **Recoil separator measurements provided a LOT more constraint for the R-matrix fit because they covered a wide energy range.** More of these type measurements would be very useful!
  - Asymptotic normalization coefficients **(ANCs) are key**, but quantifying their **uncertainties is challenging**
- New measurements using new techniques are being investigated
  - $^{16}O(\gamma,\alpha)^{12}C$ (HIγS, Jefferson Lab, ELI-NP)
  - Virtual photon exchange at FAIR
  - Additional types of transfer measurements
  - Improved traditional measurements, but in low background environments with very high beam intensities (reduce statistical / outlier uncertainties)
    - **JUNA, LUNA, Dresden**
- Include more experimental uncertainties in R-matrix analysis
  - Energy uncertainty, experimental resolution
  - **Bayesian** uncertainty estimation
  - Improved computational resources

Questions?

My 2017 estimate of S(300 keV):

140 ± 21 (MC) +18/-11 (model)

But see Shen *et al.* (2020)

Assumes ANC uncertainties are accurate