

# Second harmonic generation of acoustic waves in a nonlinear elastic solid

**Fernando Lund**

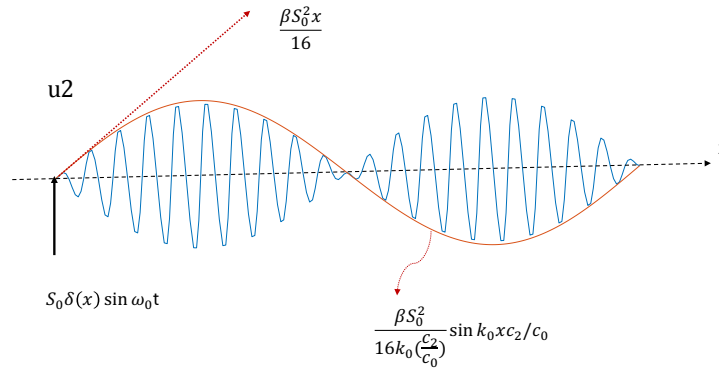
Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile  
email: flund@dfi.uchile.cl

## ABSTRACT

Nonlinear ultrasound higher harmonics have become increasingly useful as a nonintrusive probe of both microstructure as well as damage of solid materials [1]. The current theoretical underpinning of these efforts rely on a formula for the second harmonic that is proportional to the square of the linear wave and grows linearly with distance away from the source [2]. This formula holds only for small distances, since otherwise there would be a violation of the conservation of energy. This restriction is here lifted.

Consider the one-dimensional problem of wave propagation in a weakly nonlinear elastic solid with a time harmonic loading of frequency localized at the origin, for a displacement  $u(x, t)$  as a function of position  $x$  and time  $t$  on the whole real line. Using a successive approximation scheme starting with the linear case, and a multiple time analysis, an explicit formula is found for the second harmonic that is valid for all  $x$  [1]. The second harmonic so obtained is an amplitude-modulated wave. As indicated in the figure, for small distances from the source the usual formula found in the literature [2] is recovered.

Consequences of this result for non-destructive testing will be discussed.



## References

- [1] F. Lund, Second and third harmonic generation of acoustic waves in a nonlinear elastic solid in one space dimension, *J. Sound Vib.* **600** (2025) 118895, <https://doi.org/10.1016/j.jsv.2024.118895>.
- [2] C. M. Kube et al., A unifying model of weakly nonlinear elastic waves; large on large theory, *J. Acoust. Soc. Am.* **151** (2022) 1294-1310, <http://dx.doi.org/10.1121/10.0009101>.