The Initial Mass Function of stars

1st lecture

17th Russbach School
14-19 March 2022

Pavel Kroupa
Helmholtz-Institute for Radiation und Nuclear Physics (HISKP)
University of Bonn

Astronomical Institute,
Charles University in Prague

c/o Argelander-Institut für Astronomie
University of Bonn

The IMF : $\xi(m)$ $\quad dN = \xi(m)\, dm$

= number of stars with initial $m \in [m, m+dm]$

One of the most important distribution functions in astrophysics

Important for :

- the luminosity of a stellar system
- its mass locked up in long lived late-type stars
- the rate of CCSN per late-type star
- the rate of merging white dwarf binaries (SN1a) per late-type star
- the rate of merging neutron star binaries per late-type star
- the rate of merging BHs per late-type star

The IMF : $\xi(m)$ $dN = \xi(m)\, dm$

= number of stars with initial $m \in [m, m+dm]$

One of the most important distribution functions in astrophysics

How do we constrain the shape of the IMF ?

Does the shape vary ?

Prelude

Galaxies

Clusters

Star Formation

Star Formation

The outcome of the star-formation process appears to be messy and very complicated.

It is easy to argue that star formation is *stochastic*,

but the challenge is to find *patterns* and *rules*.

One such effort is to constrain the shape of the initial distribution function of stellar masses, the IMF.

Related to the above issue is the question wether the IMF is a *probability* density distribution function
or
an *optimally* sampled distribution function ?

(Kroupa et al., 2013, arXiv:1112.3340)

Finding the shape of the IMF

The Galactic-field LF

We have $dN = \Psi(M_V)\, dM_V$ = # of stars with $M_V \in [M_V, M_V + dM_V]$

$dN = \xi(m)\, dm$ = # of stars with $m \in [m, m + dm]$

since
$$\frac{dN}{dM_V} = -\frac{dm}{dM_V}\,\frac{dN}{dm}$$

follows
$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

the **observable** — the **obstacle** — the **target**

the stellar luminosity function (**LF**)

the stellar mass function (**IMF** if initial)

There are ***two luminosity functions*** for the solar neighbourhood

I. Count stars nearby to Sun

Obtain M_V and d from trigonometric parallax

⟶ Well observed individual stars but *small numbers at faint end* ($\Psi_{\rm near}$)

There are ***two luminosity functions*** for the solar neighbourhood

I. Count stars nearby to Sun

Obtain M_V and d from trigonometric parallax

⟶ Well observed individual stars but *small numbers at faint end* (Ψ_{near})

II. Deep (100 - 300 pc) pencil-beam photographic/CCD surveys

Formidable data reduction (10^5 images $\longrightarrow \approx 100$ stars)

Obtain M_V and d from photometric parallax

⟶ Large # of stars but *poor resolution* (2”-3”) and *Malmquist bias* (Ψ_{phot})

The possibility of *dark matter* in the *Galactic disk*

(Bahcall 1984)

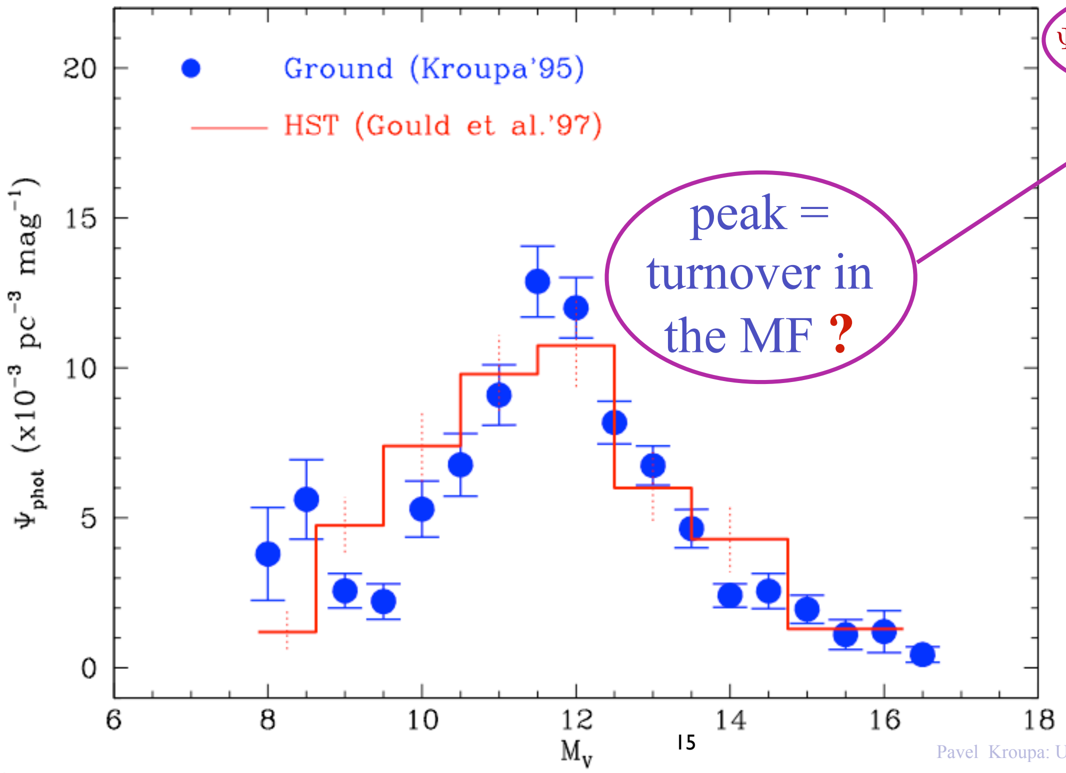
⟶ Many surveys of type II (pencil-beams) to constrain the LF :

ground	Reid & Gilmore	1982
	Gilmore, Reid & Hewett	1985
	Hawkins & Bessell	1988
	Leggett & Hawkins	1988
	Stobie, Ishida & Peacock	1989
	Tinney, Reid & Mould	1993
	Kirkpatrick et al.	1994
HST	Gould, Bahcall & Flynn	1997
	Zheng, Flynn, Gould et al.	2001

Ψ_{phot}

- independent of direction
- maximum (peak) at $M_V \approx 12$

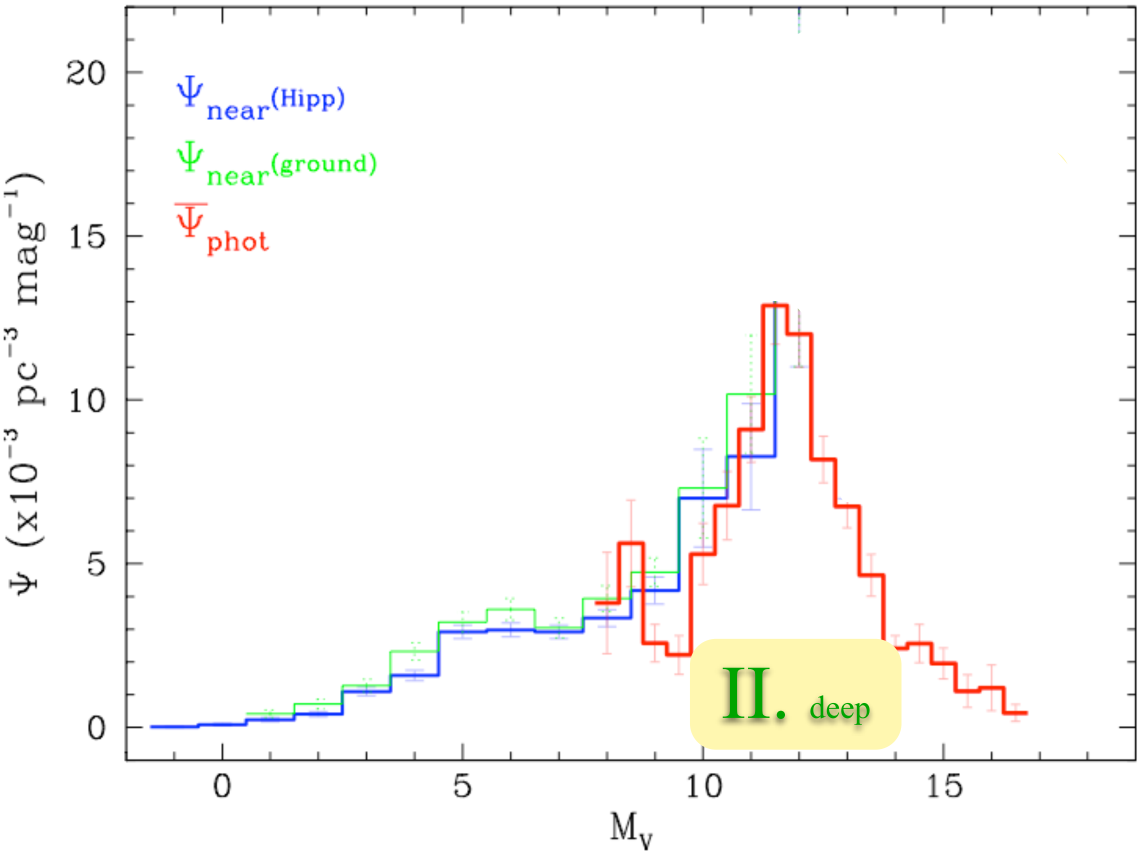
$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



see also Bochanski et al. (2010, AJ)

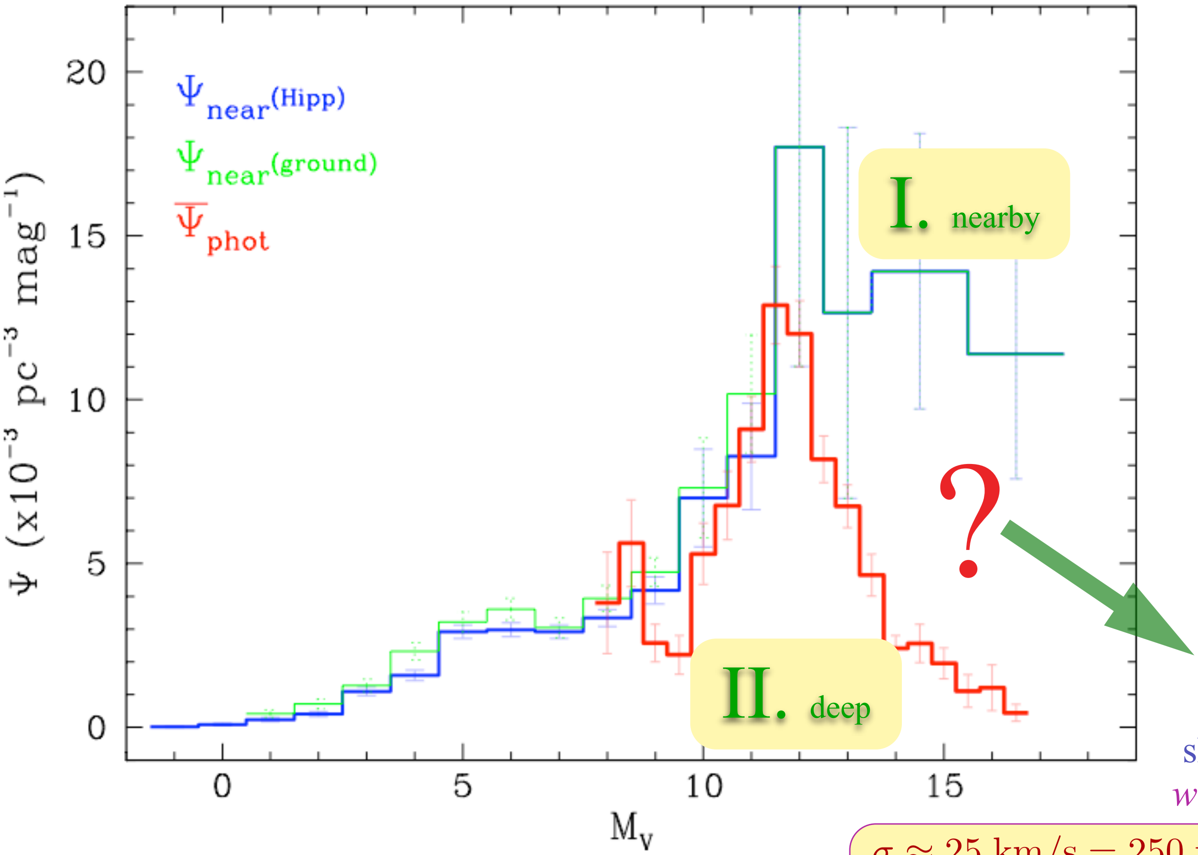
Two solar-neighbourhood samples:

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



BUT: two solar-neighbourhood samples:

$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$



Stellar samples should be *well-mixed*:

$\sigma \approx 25$ km/s $= 250$ pc/10Myr

Problem :

The nearby and deep LFs are not equal.

→ ***Which*** *LF* do we use to calculate the *MF* ?

$$\xi(m) = -\left(\frac{dm}{dM_V}\right)^{-1} \Psi(M_V)$$

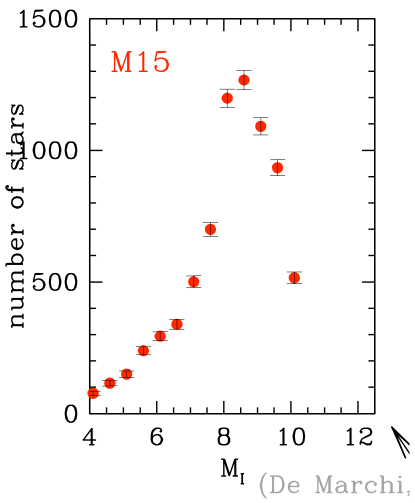
$\Psi_{\rm I}$ OR $\Psi_{\rm II}$??

The LF of stars in star clusters

coeval, equidistant stars!

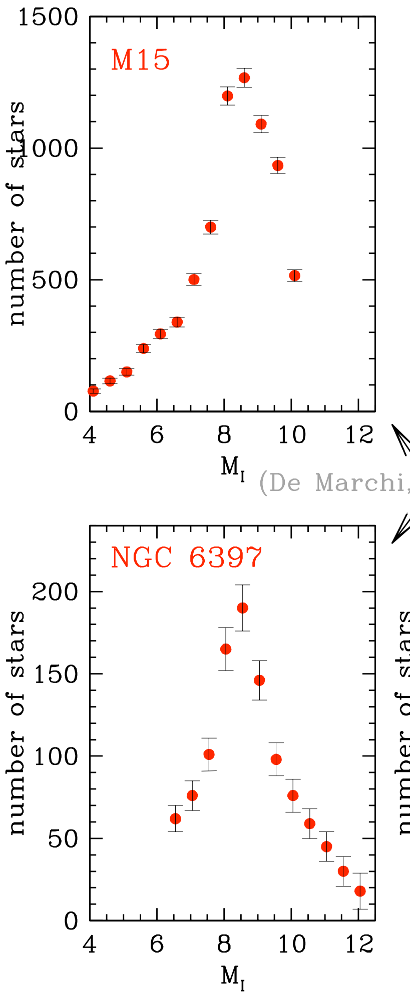
Kroupa 2002
Science

$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$



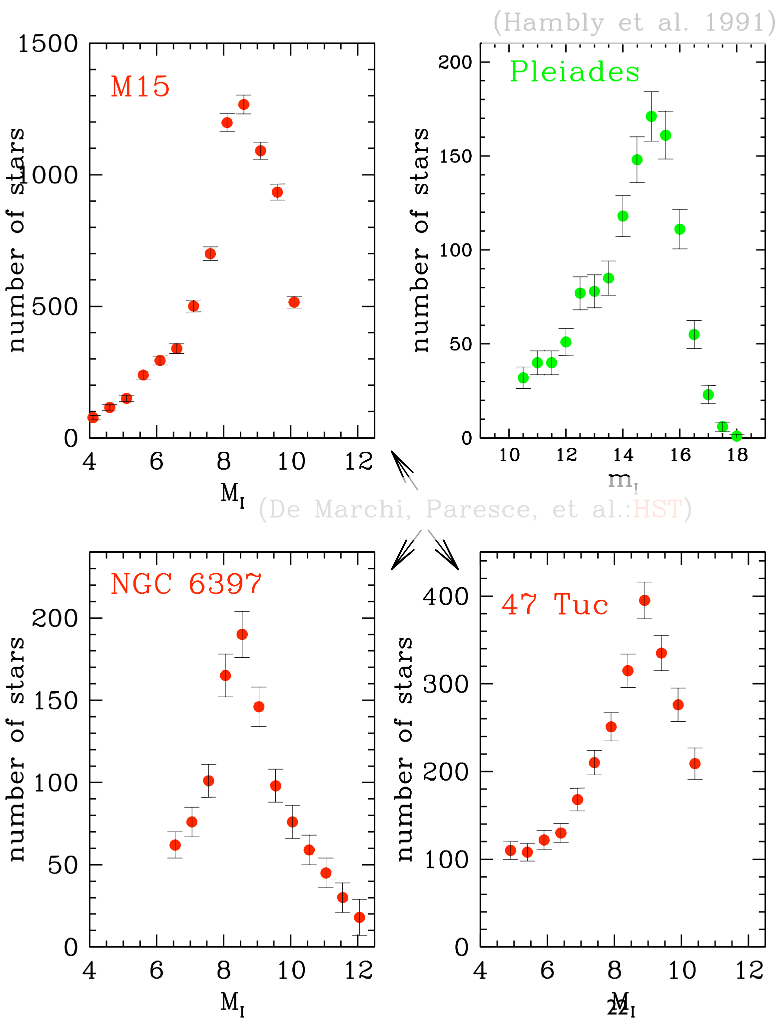
Kroupa 2002
Science

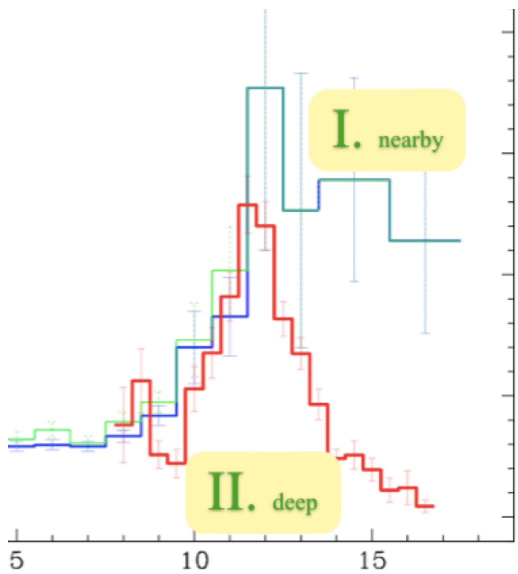
$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$



Kroupa 2002
Science

$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$



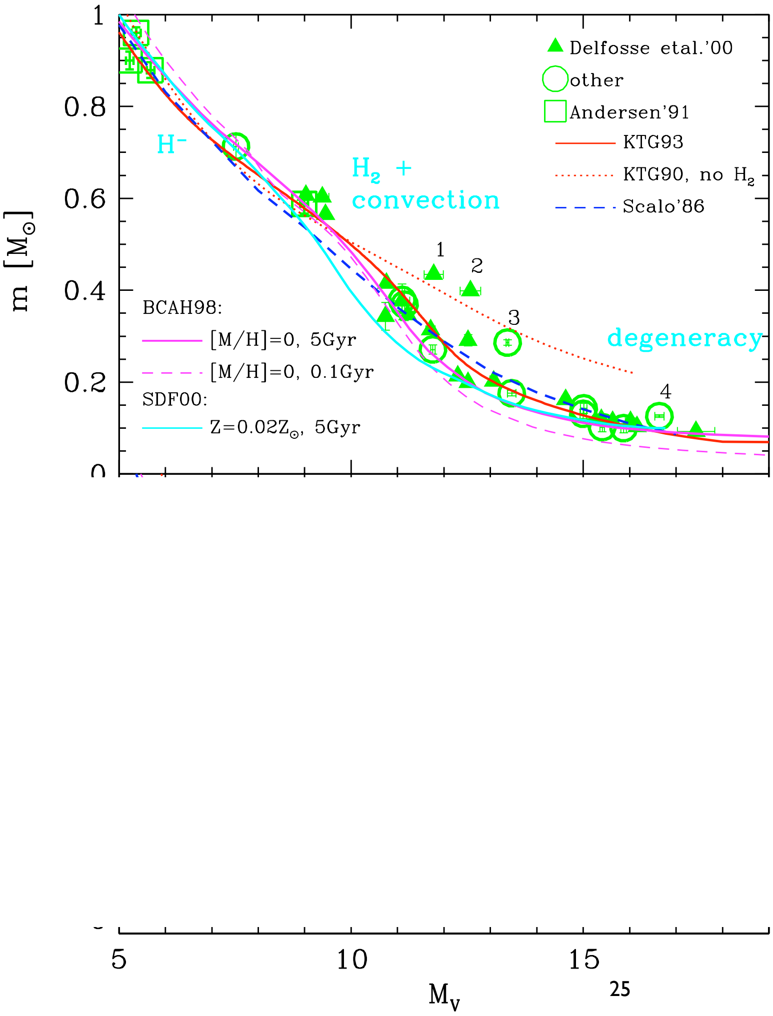


Looking
into
the stars
by
just counting them

The mass-luminosity relation of low-mass stars

Kroupa, Tout & Gilmore 1990;
Kroupa, 2002, *Science*

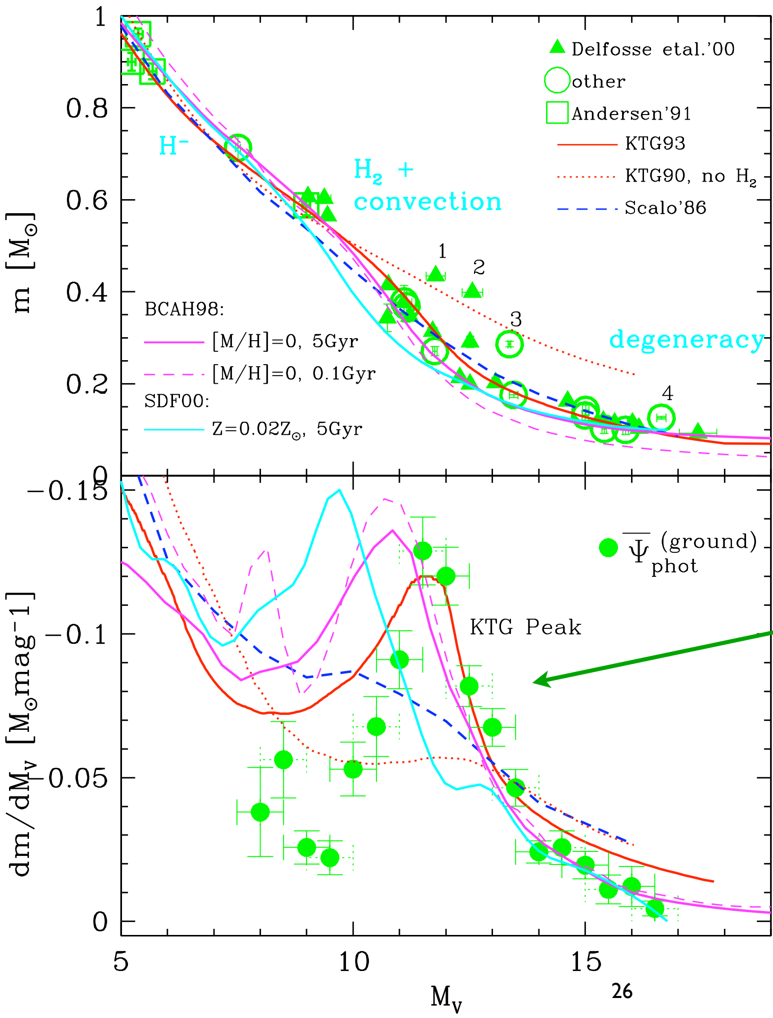
$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$



The mass-luminosity relation of low-mass stars

Kroupa, Tout & Gilmore 1990;
Kroupa, 2002, *Science*

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



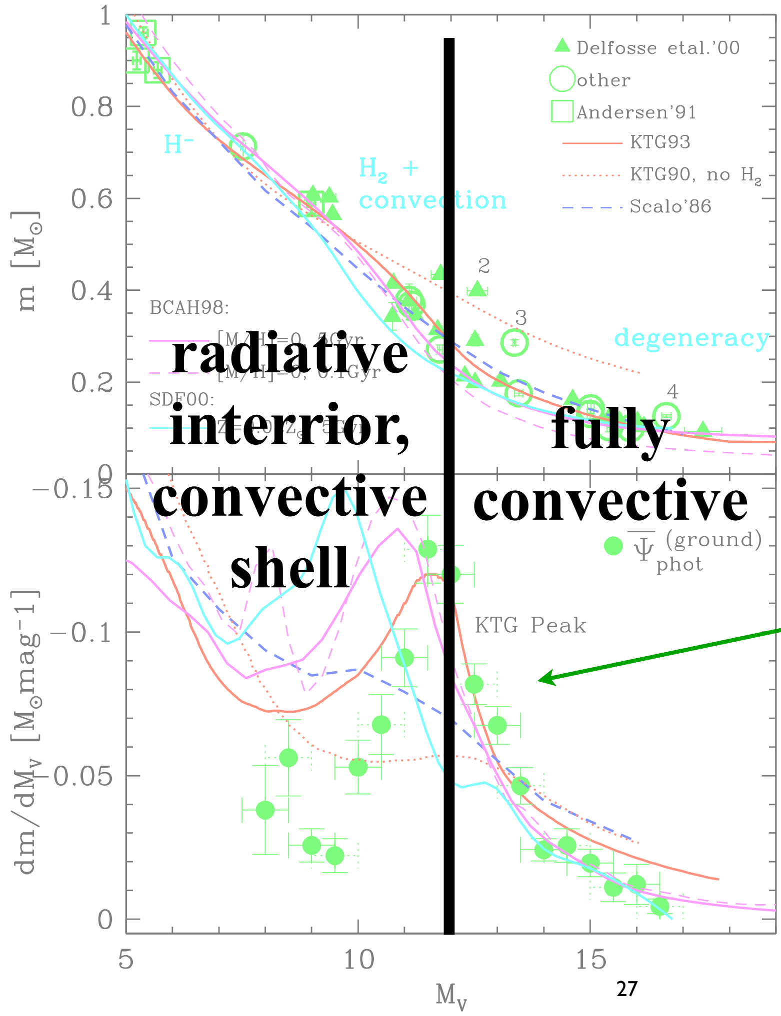
The mass-luminosity relation of low-mass stars

Kroupa, Tout & Gilmore 1990;
Kroupa, 2002, *Science*

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

1. position
2. width
3. amplitude

all agree **!**



The mass-luminosity relation of low-mass stars

Kroupa, Tout & Gilmore 1990;
Kroupa, 2002, *Science*

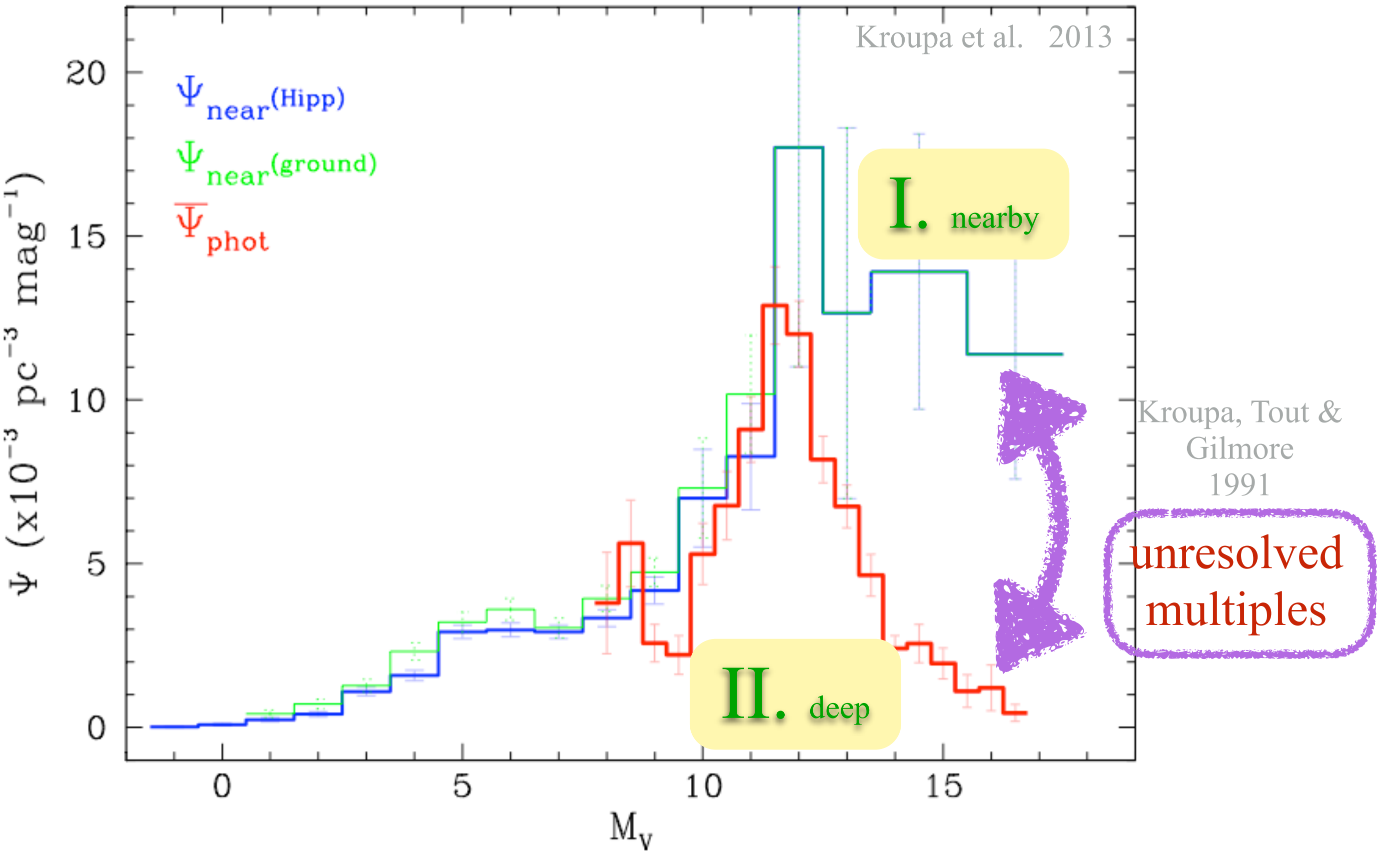
$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

1. position
2. width
3. amplitude

all agree !

Two solar-neighbourhood samples:

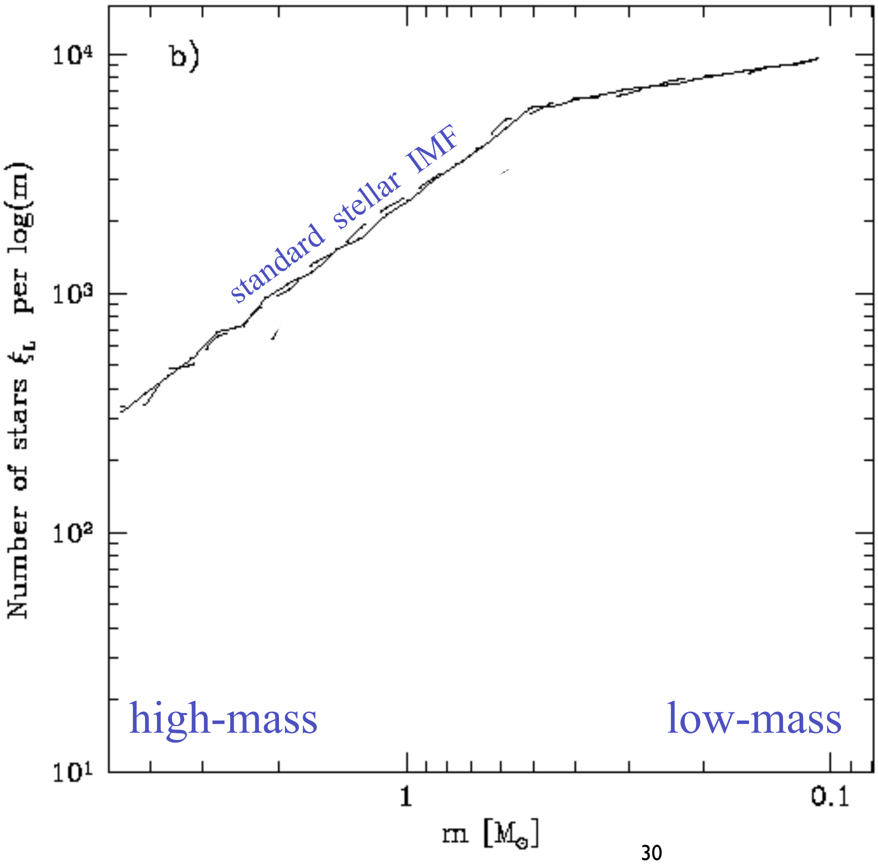
$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



Inferring the IMF in star clusters

MF(t) due to cluster evolution

(Baumgardt & Makino 2003)



$f = 0$

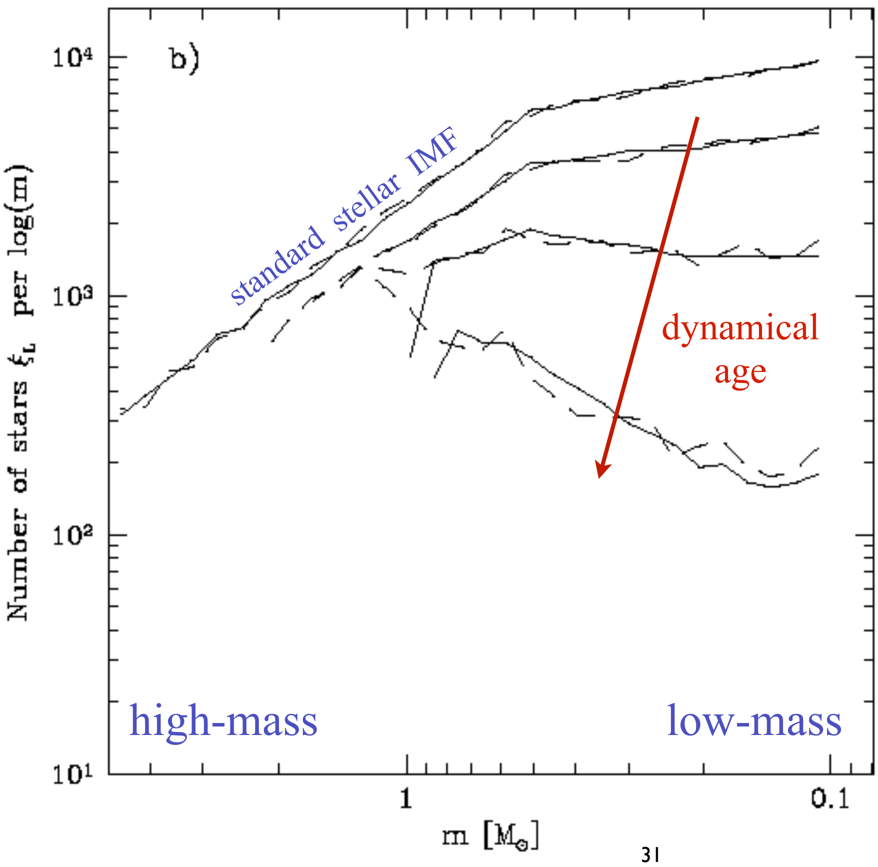
———— $N = 1.28 \times 10^5$

– – – – – $4 \times (N = 8000)$

t = 0
t = 0.3 Tdiss
t = 0.6 Tdiss
t = 0.9 Tdiss

MF(t) due to cluster evolution

(Baumgardt & Makino 2003)



$f = 0$

$N = 1.28 \times 10^5$

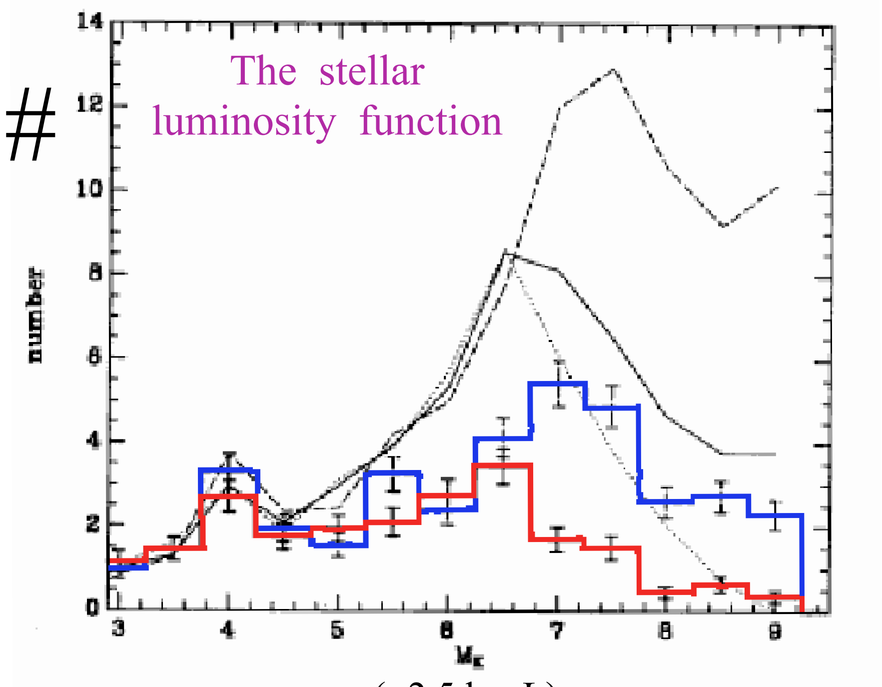
$4 \times (N = 8000)$

t = 0
t = 0.3 Tdiss
t = 0.6 Tdiss
t = 0.9 Tdiss

N-body Models of Binary-Rich Clusters

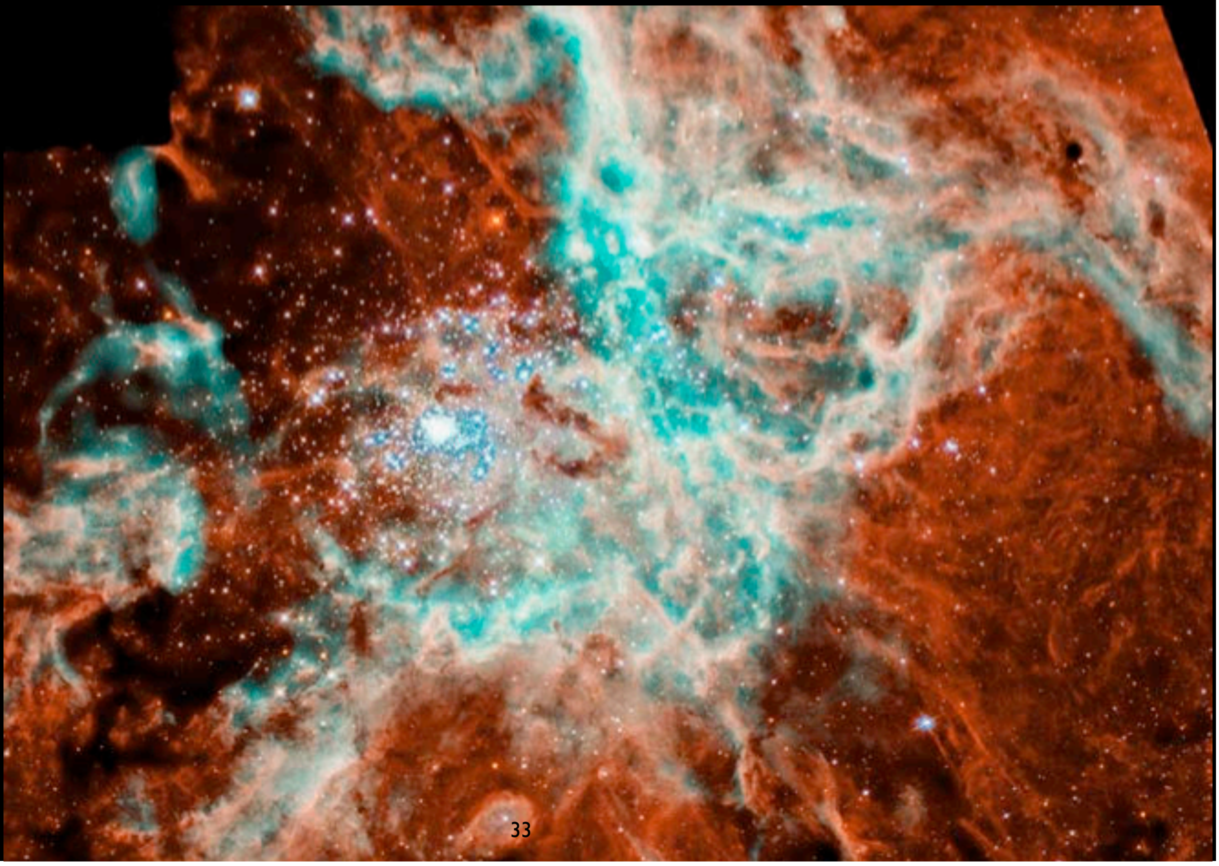
(Kroupa 1995)

$20 \times (N = 400\,\text{stars})$
$f = 1$



$\xi_{obs}(m) \neq \xi_{true}(m)$

Massive stars in very young clusters

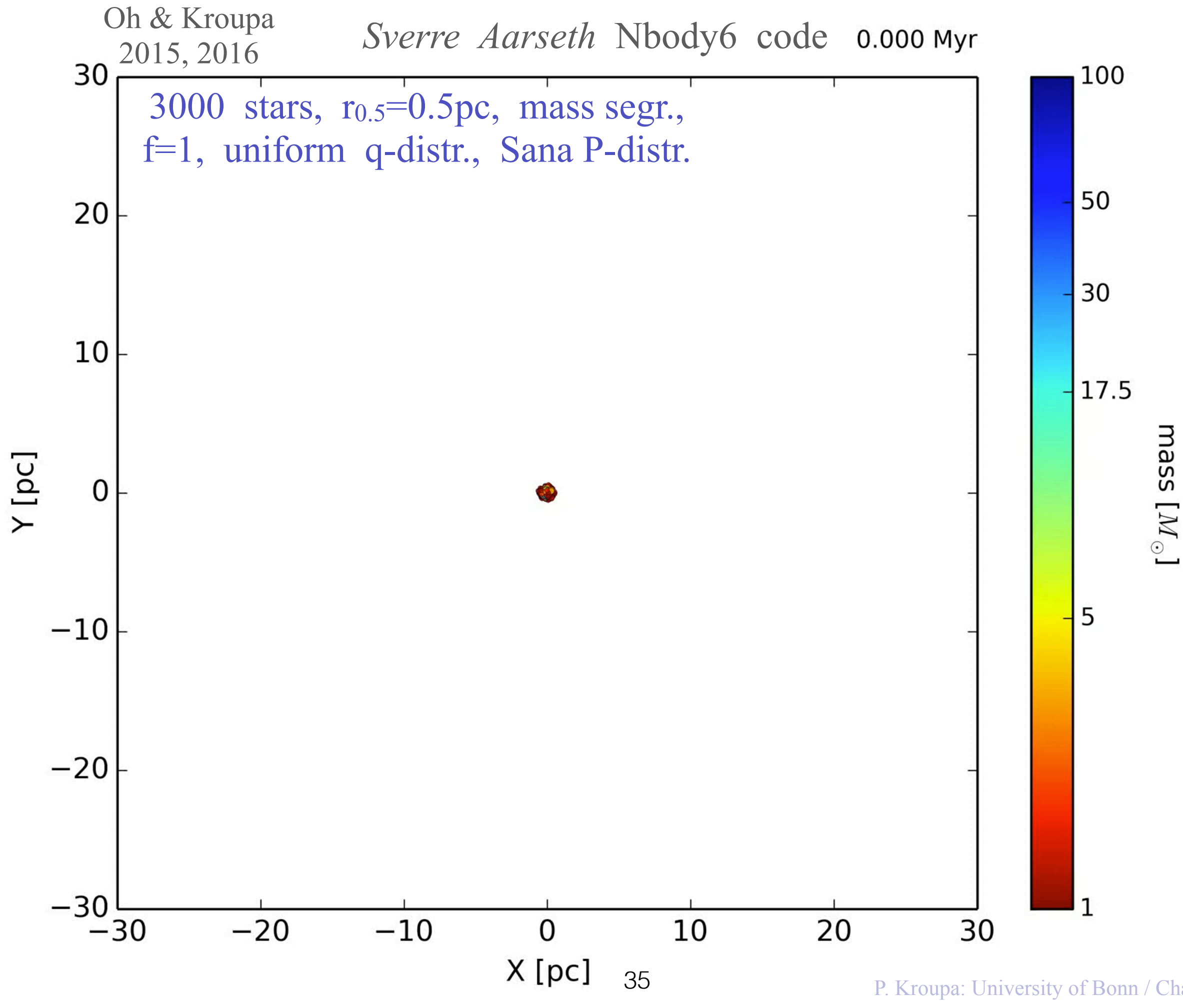


OB stars in clusters / HII regions

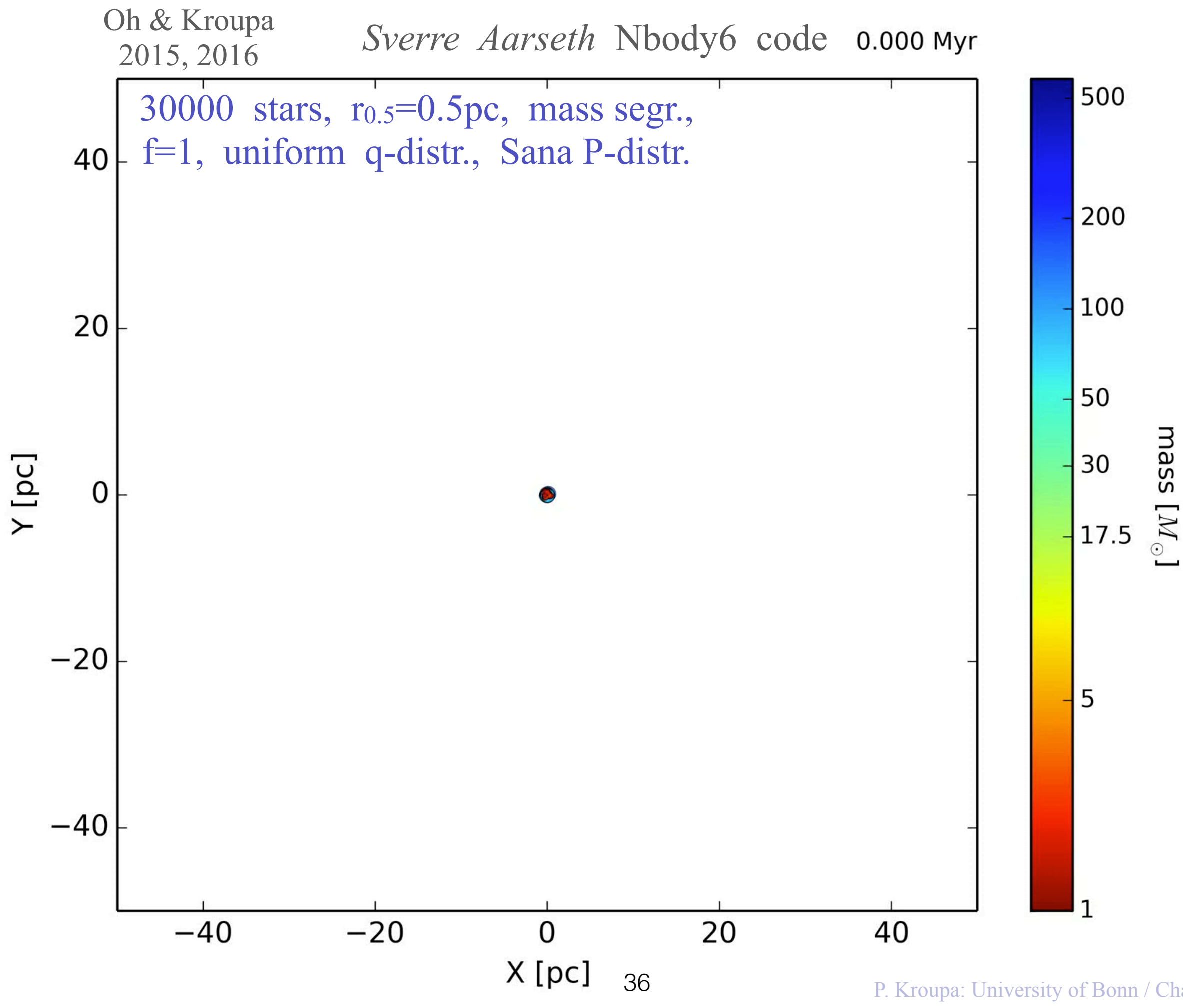
Two competing processes:

Mass segregation $t_{\rm msgr} \approx 2 \left(\frac{m_{\rm av}}{m_{\rm massive}}\right) t_{\rm relax}$ $t_{\rm relax} = \frac{21}{\ln(0.4N)} \left(\frac{M_{\rm ecl}}{100\, M_\odot}\right)^{\frac{1}{2}} \left(\frac{1\, M_\odot}{m_{\rm av}}\right) \left(\frac{R_{0.5}}{1\,{\rm pc}}\right)^{\frac{3}{2}}$	***e.g.*** $t_{\rm relax} \approx 0.6\,{\rm Myr}$ for pre-exposed ONC ↓ $t_{\rm msgr} \approx 0.12\,{\rm Myr} \ll$ age of ONC
Core decay $t_{\rm decay} \approx N_{\rm m} \times t_{\rm core,cross}$ $t_{\rm cross}^{\rm core} \approx 5 \left(\frac{M^{\rm core}}{100\, M_\odot}\right)^{-\frac{1}{2}} \left(\frac{R_{0.5}^{\rm core}}{1\,{\rm pc}}\right)^{\frac{3}{2}}$	***e.g.*** $R_{\rm core} \approx 0.02\,{\rm pc},\ M_{\rm core} \approx 150\, M_\odot$ $t_{\rm cross}^{\rm core} \approx 1.2 \times 10^4\,{\rm yr}$ ↓ $t_{\rm decay} \approx 10^4 - 10^5\,{\rm yr} \ll$ age of ONC

(Pflamm-Altenburg & Kroupa 2006)

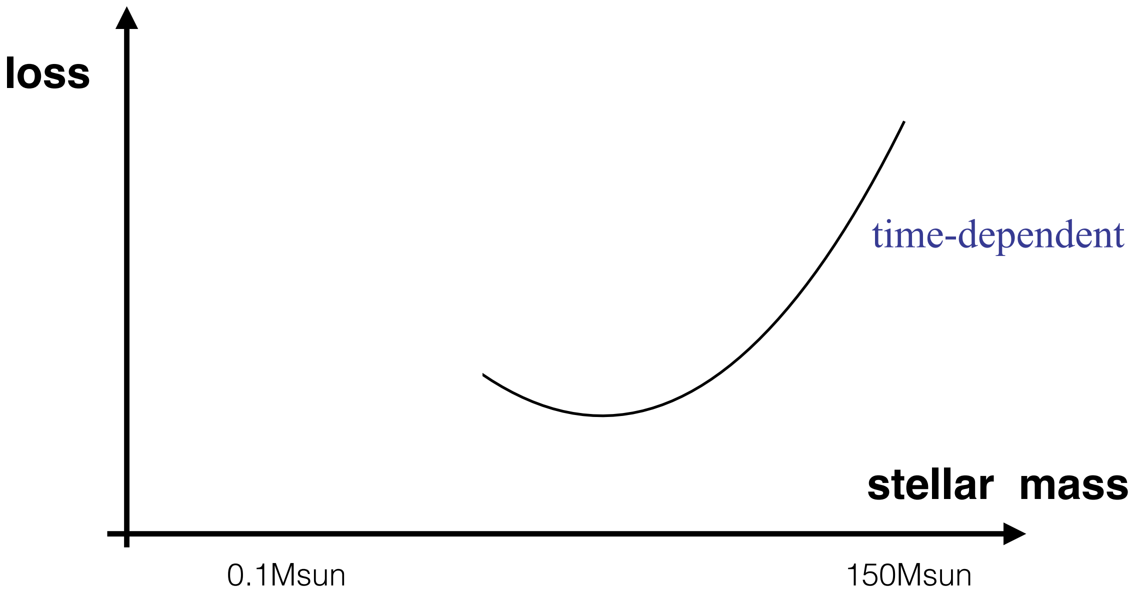

Oh & Kroupa
2015, 2016
Sverre Aarseth Nbody6 code 0.000 Myr
3000 stars, r0.5=0.5pc, mass segr.,
f=1, uniform q-distr., Sana P-distr.
Y [pc]
X [pc]
mass [$M_\odot$]
100
50
30
17.5
5
1

35


Oh & Kroupa
2015, 2016
Sverre Aarseth Nbody6 code 0.000 Myr
30000 stars, r0.5=0.5pc, mass segr.,
f=1, uniform q-distr., Sana P-distr.
Y [pc]
X [pc]
mass [$M_\odot$]
500
200
100
50
30
17.5
5
1

36

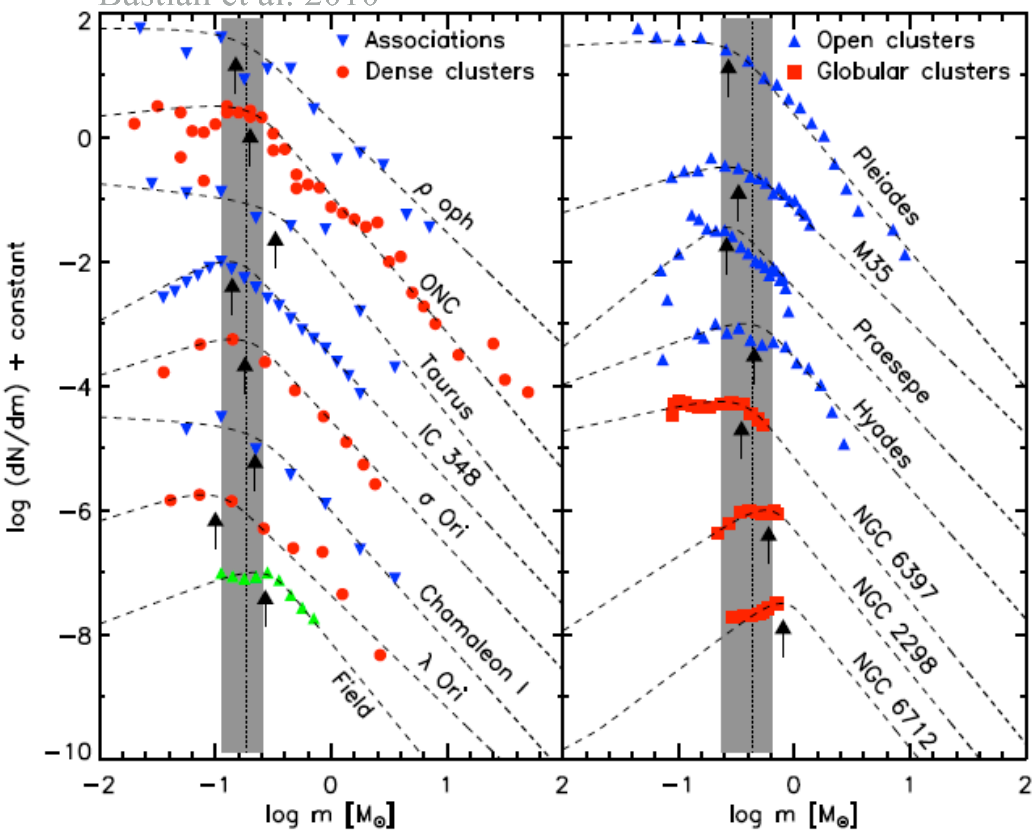
Clusters depopulate themselves
off low-mass stars and high mass stars.



Thus, stellar-dynamical processes
are
extremely important
when determining the IMF shape!!

An invariant canonical IMF

Bastian et al. 2010



The IMF appears largely invariant (in MW CSFEs / embedded clusters)

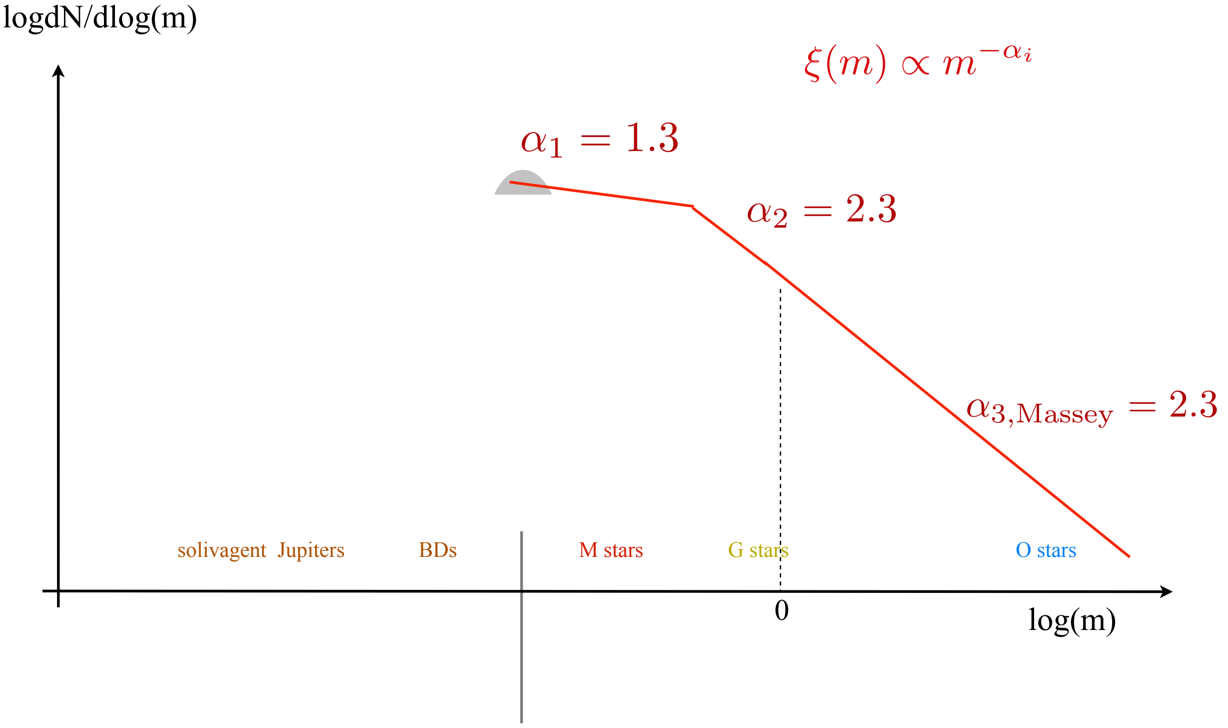
Kroupa 2001, 2002

Bastian, Covey, Meyer, 2010, ARAA

Kroupa et al., 2013

Offner et al., 2014, PPVI

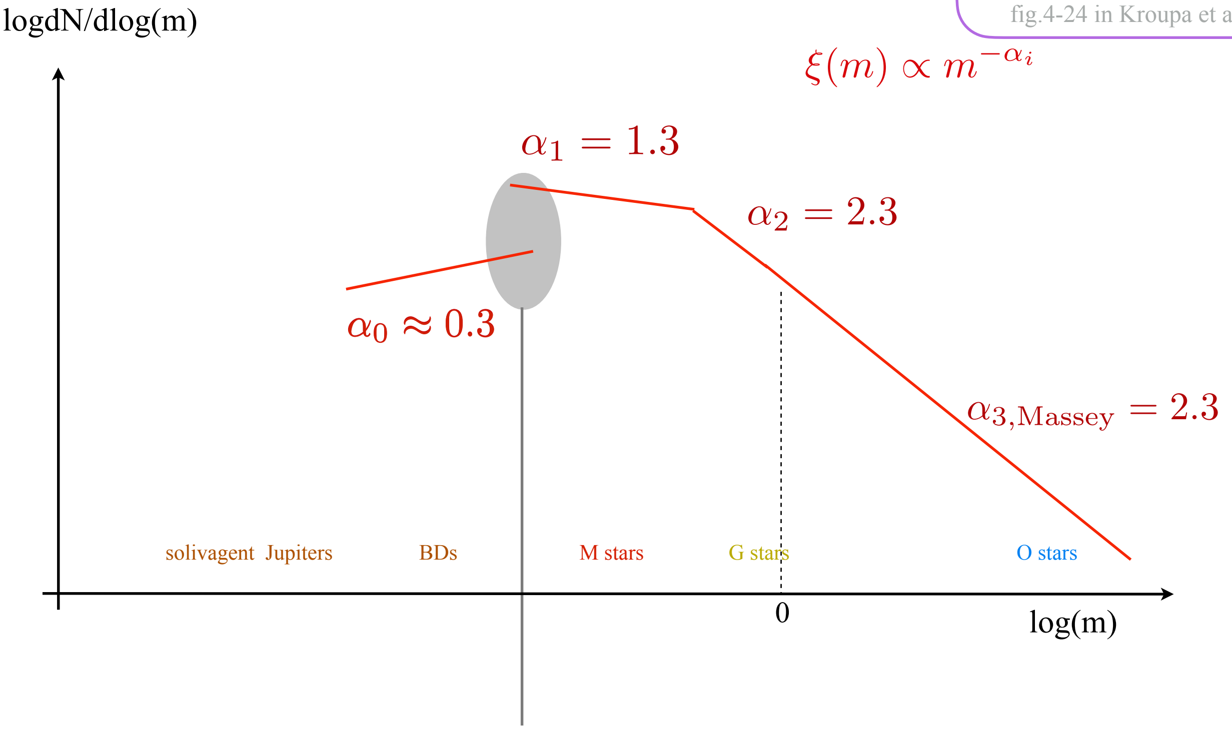
the canonical IMF :



discontinuity: Thies & Kroupa (2007, 2008), Parker & Goodwin (2010)

the canonical IMF :

Rederived later by Chabrier
---
essentially identical result to the canonical IMF
fig.4-24 in Kroupa et al. 2013



discontinuity: Thies & Kroupa (2007, 2008), Parker & Goodwin (2010)

Extreme environments: evidence for a systematically-varying IMF

evidence began emerging since about 2010

R136

2 Myr old

Some subtle hints for a systematically varying IMF are available at high masses

Star-counts: Correct star-counts in R136 for ejected stars

→ IMF in R136 top-heavy (Banerjee & Kroupa 2012)

Excess of massive stars in whole 30Dor region (Schneider et al. 2018)

Ultra-violett spectroscopy of seven nearby (< 120 Mpc) star-bursting regions imply the same (Senchyna et al. 2021).

Top-heavy IMF in Magellanic Bridge cluster NGC796 (Kalari et al. 2018)

GCs in M31: more top-heavy IMF at lower metallicity (Zonoozi et sl. 2016; Haghi et al. 2017)

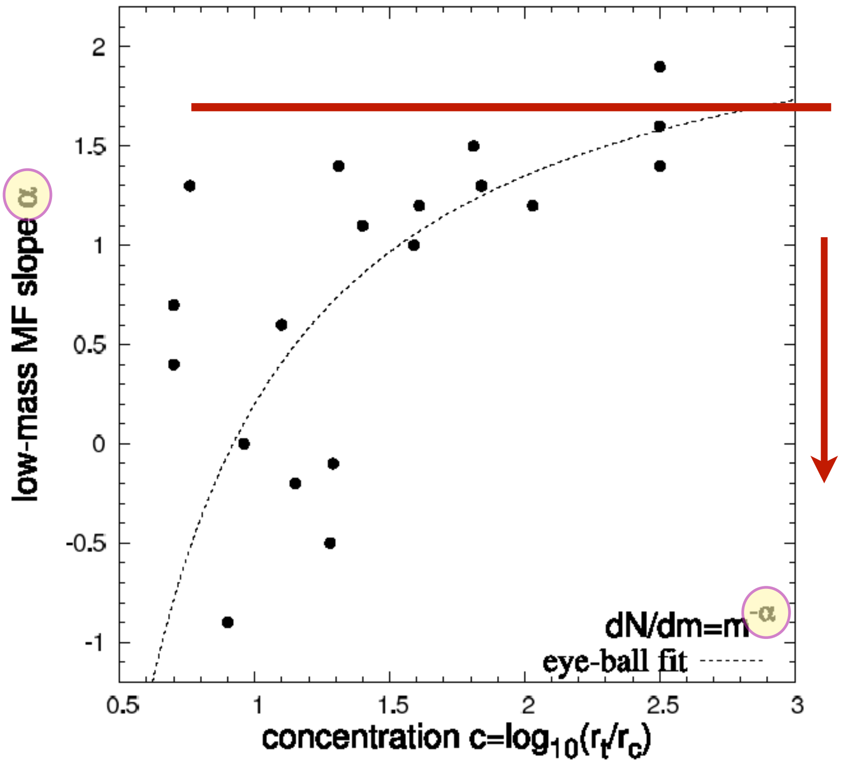
What we know from observation :

Globular clusters : deficit of low-mass stars increases with decreasing concentration

→ disagrees with dynamical evolution (Marks et al. 2012)

A sample of 20 Galactic GCs with solid global MF measurements from deep HST or VLT data.

(de Marchi, Paresce & Pulone 2007)

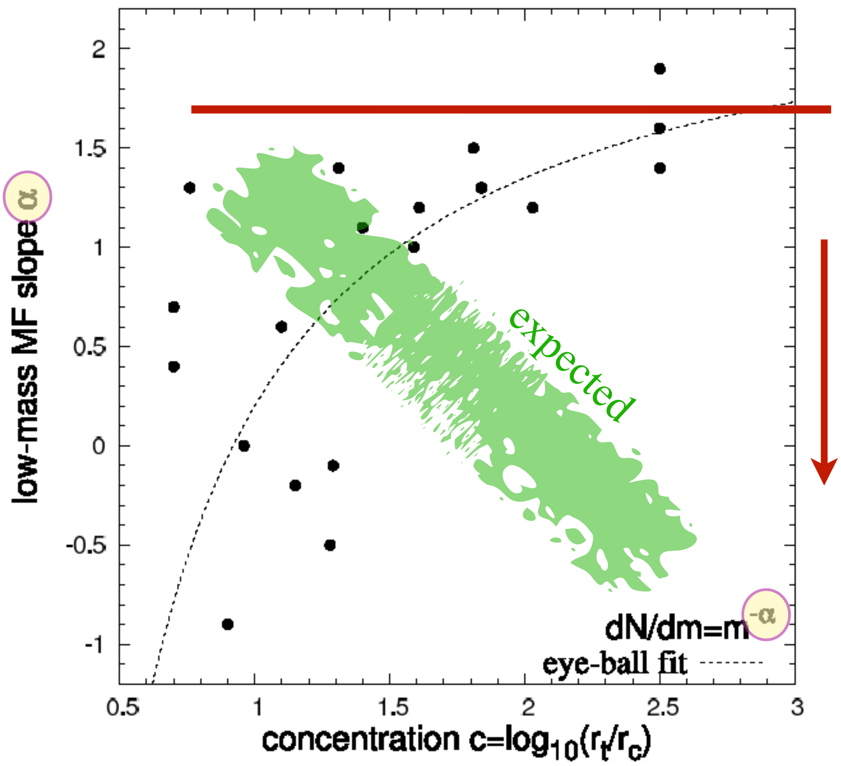


canonical (universal) IMF

low-mass star deficient MF

A sample of 20 Galactic GCs with solid global MF measurements from deep HST or VLT data.

(de Marchi, Paresce & Pulone 2007)



canonical (universal) IMF

low-mass star deficient MF

GCs
(extreme star burst "clusters")

gas expulsion

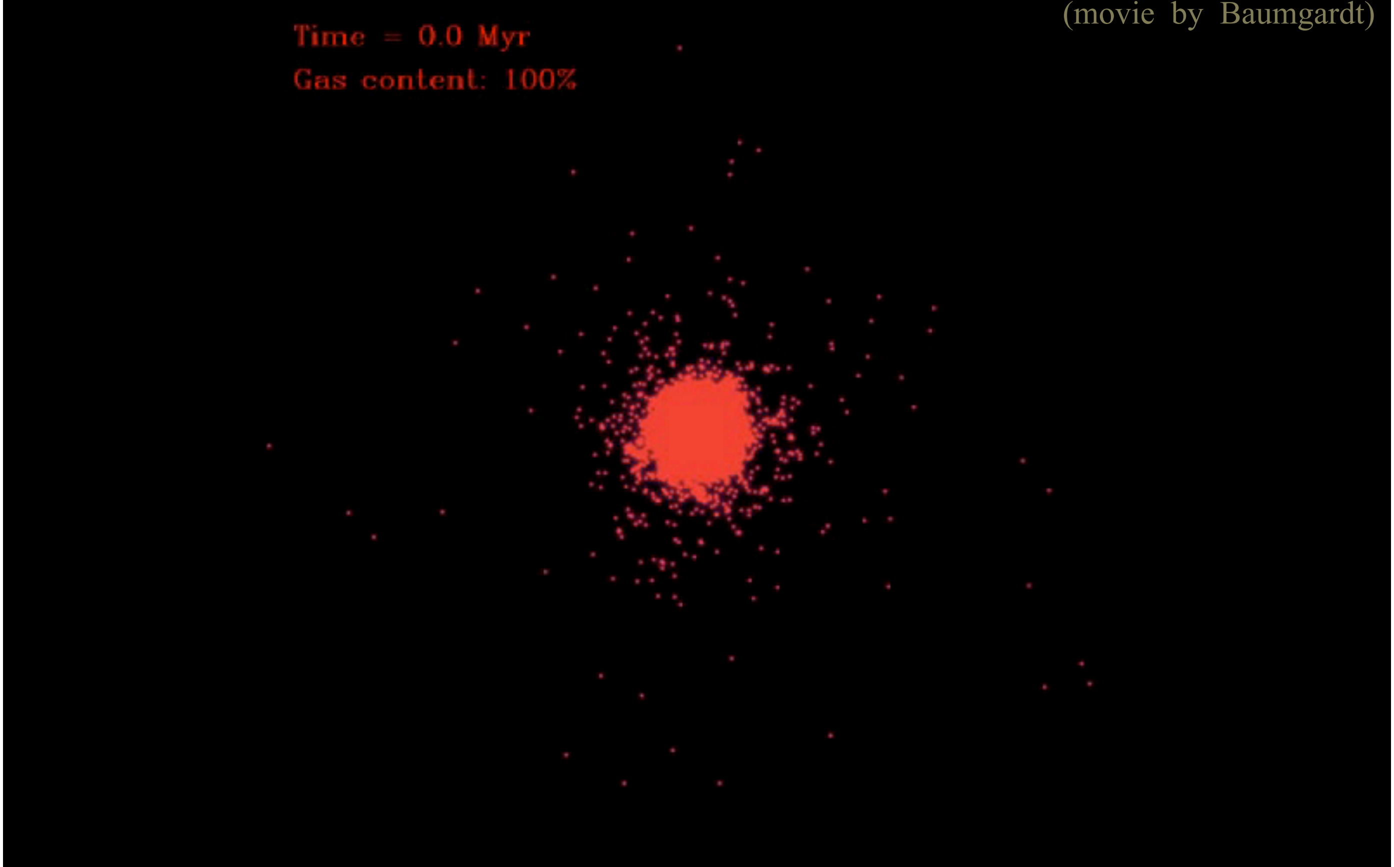
+

mass segregation

!

Cluster reaction to thermal gas removal :

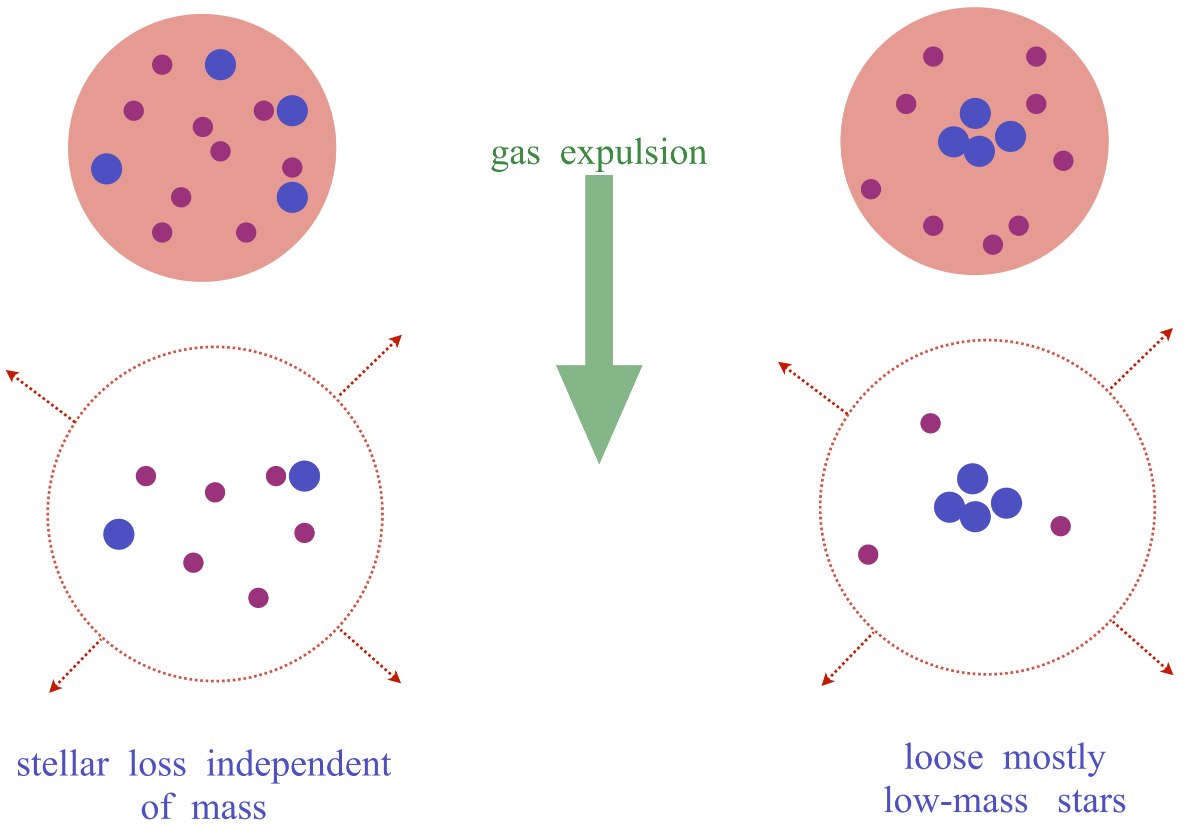
(movie by Baumgardt)



Baumgardt & Kroupa 2007

Nbody models of binary rich initially mass segregated clusters with redisual gas expulsion after birth

(Marks, Kroupa & Baumgardt 2008)



What we know from observation :

Globular clusters : deficit of low-mass stars increases with decreasing concentration

→ disagrees with dynamical evolution (Marks et al. 2012)
(need gas expulsion from mass-segregated clusters)

UCDs : higher dynamical M/L ratios (Dabringhausen et al. 2009)

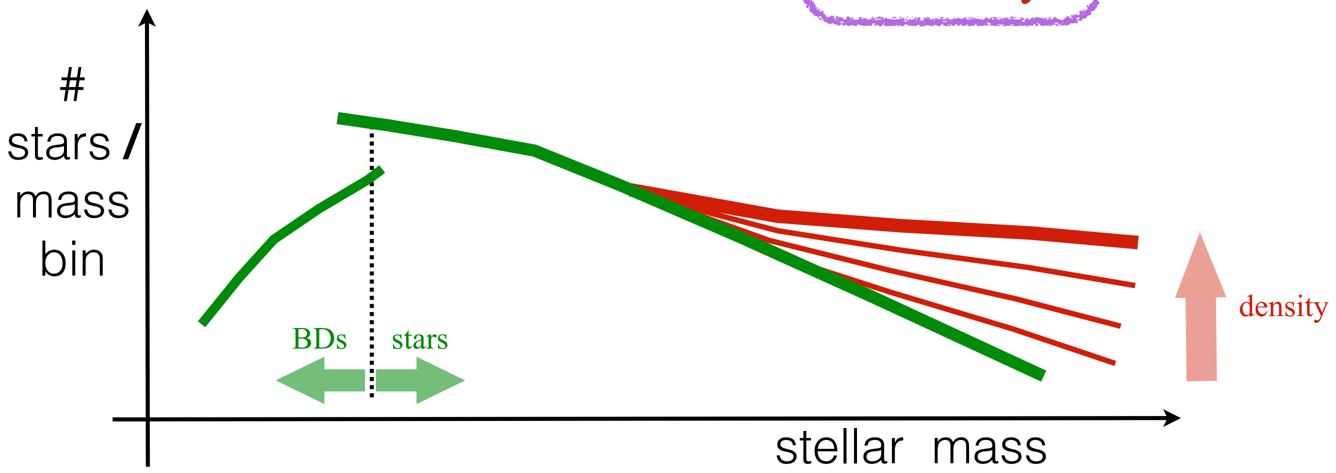
→ cannot be exotic dark matter

UCDs : larger fraction of X-ray sources than expected (Dabringhausen et al. 2012)

→ no explanation other than many remnants

mutual consistency !!

What this implies :



IMF = IMF(Z, $SFRD$)

Z=metallicity, $SFRD$=star-formation rate density

Variation of IMF in pc-scale, 1-Myr time-scale
(molecular cloud core = embedded cluster).

Top-heavy IMF in extreme-density environments : $\xi(m) \propto m^{-\alpha_i}$

THE STELLAR IMF DEPENDENCE ON DENSITY AND METALLICITY: Resolved stellar populations show an invariant IMF (Eq. 55), but for $SFRD \gtrsim 0.1\, M_\odot/(\mathrm{yr\ pc}^3)$ the IMF becomes top-heavy, as inferred from deep observations of GCs. The dependence of α_3 on cluster-forming cloud density, ρ, (stars plus gas) and metallicity, [Fe/H],can be parametrised as

$$\alpha_3 = \alpha_2, \qquad m > 1\, M_\odot \;\wedge\; x < -0.89,$$
$$\alpha_3 = -0.41 \times x + 1.94, \qquad m > 1\, M_\odot \;\wedge\; x \geq -0.89,$$

$$x = -0.14\,[\mathrm{Fe/H}] + 0.99 \log_{10}\left(\rho/\left(10^6\, M_\odot\, \mathrm{pc}^{-3}\right)\right). \qquad (65)$$

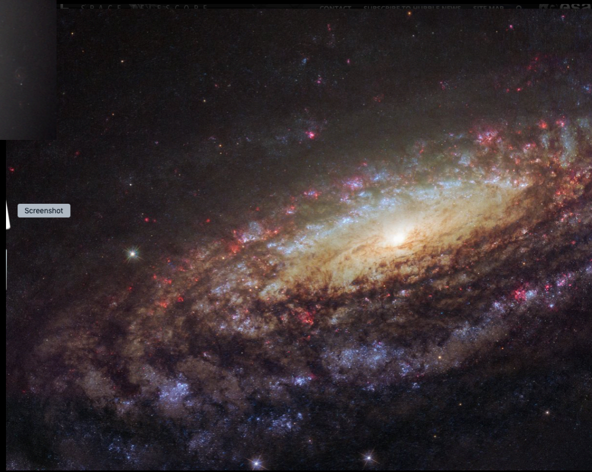
Marks et al. 2012
Kroupa et al. 2013
Yan et al. 2017
Jerabkova et al. 2018
Yan, Jerabkova et al. 2021

newest, most up-to-date formulation

Whole galaxies are made of all their IMFs

Embedded star clusters are the fundamental building blocks of galaxies

(Lada & Lada 2003; Kroupa 2005, ESASP; Dinnbier et al. 2021)



Composite Stellar Populations

Stars form in a clustered mode (Lada & Lada 2003; Megeath et al. 2016).
Thus, the **I**ntegrated **G**alactic **IMF** follows from

$$\textit{IGIMF} = \sum \textit{ of IMFs (in all new clusters)}$$

The IGIMF theory

Kroupa & Weidner 2003
Kroupa et al. 2013

$$\xi_{\rm IGIMF}(m,t) = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}(SFR(t))} \xi(m \leq m_{\rm max}(M_{\rm ecl}))\ \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)
Yan, Jerabkova & Kroupa (2017); Jerabkova et al. (2018)

Natural explanation of the mass-metallicity relation, *radial Halpha cutoff vs UV extended disks* of galaxies and many other problems in understanding galaxies.

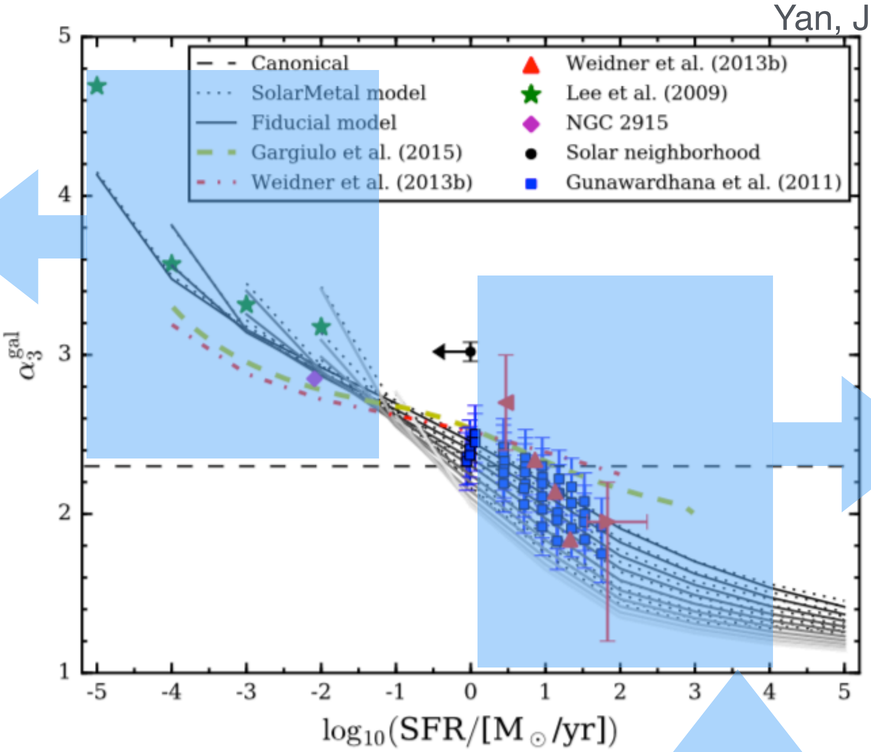
Jerabkova et al. 2018; Yan, Jerabkova et al. 2017; Fontanot et al. 2017;
and recent papers by Simone Recchi et al.; Sylvia Ploeckinger et al.
and previous papers by Koeppen et al., Pflamm-Altenburg et al., Weidner et al.

Consistency Check

Calculate galaxy-wide IMF (= gwIMF) by summing all embedded clusters

Yan, Jerabkova+ 2017

2009 *Janice Lee*: Star forming dwarf galaxies have top-light gwIMF

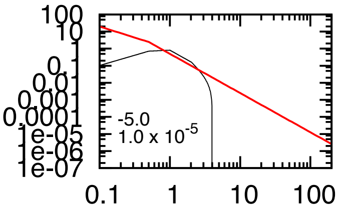


2011 *Madusha Gunawardhana*: Star forming massive galaxies have top-heavy gwIMF

1994 *Francesca Matteucci*: formation of ellipticals --> top-heavy gwIMF

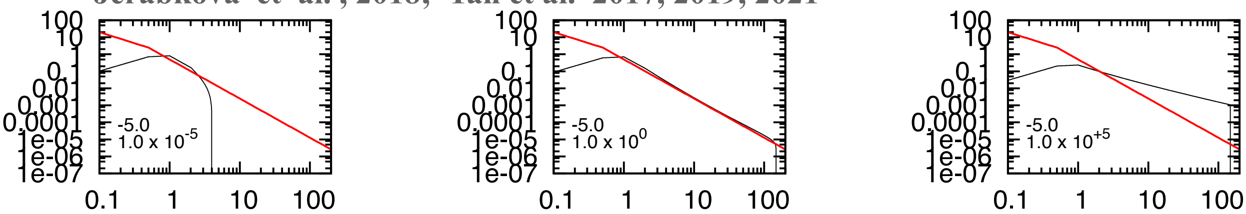
The IGIMF for galaxies with different SFRs & [Fe/H]

Jerabkova et al. , 2018; Yan et al. 2017, 2019, 2021



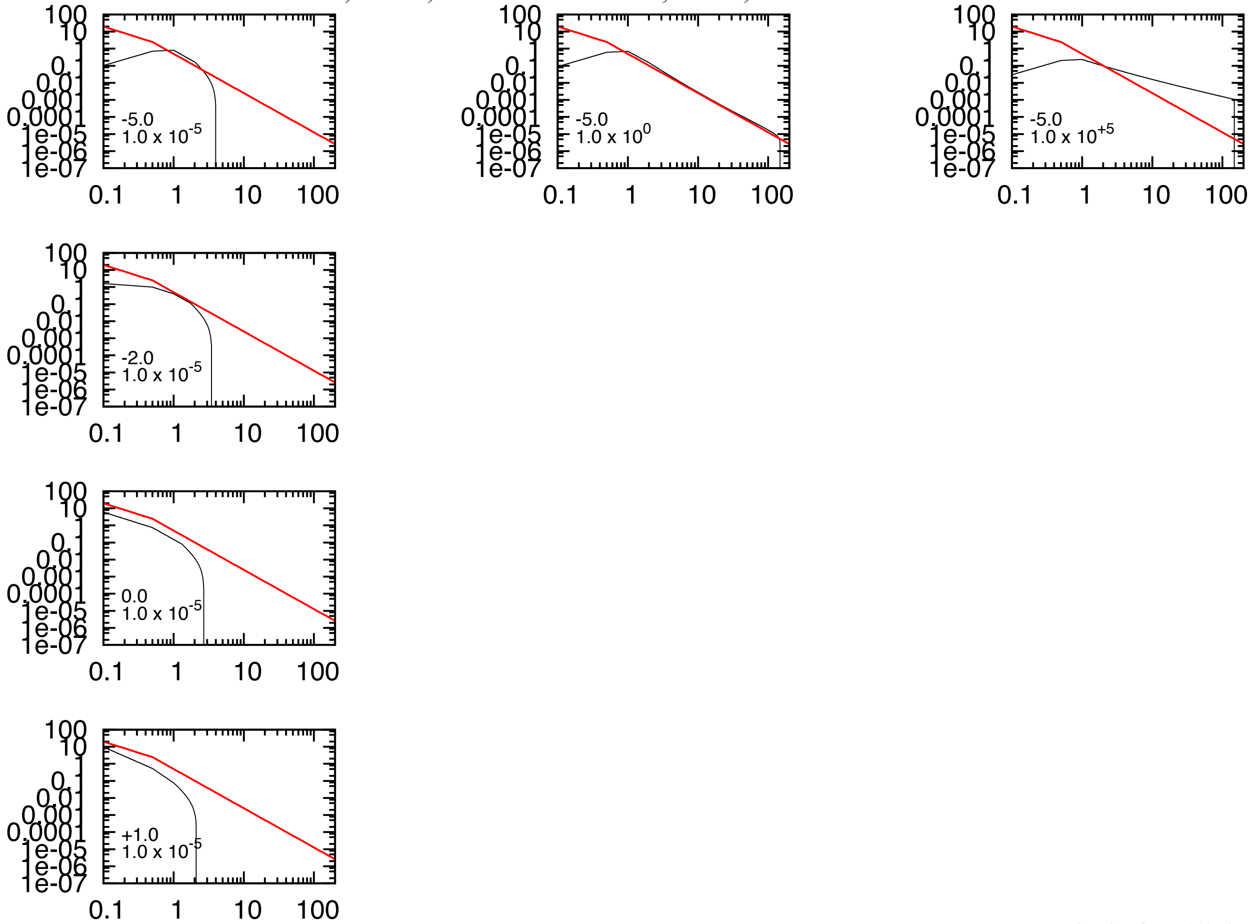
The IGIMF for galaxies with different SFRs & [Fe/H]

Jerabkova et al. , 2018; Yan et al. 2017, 2019, 2021



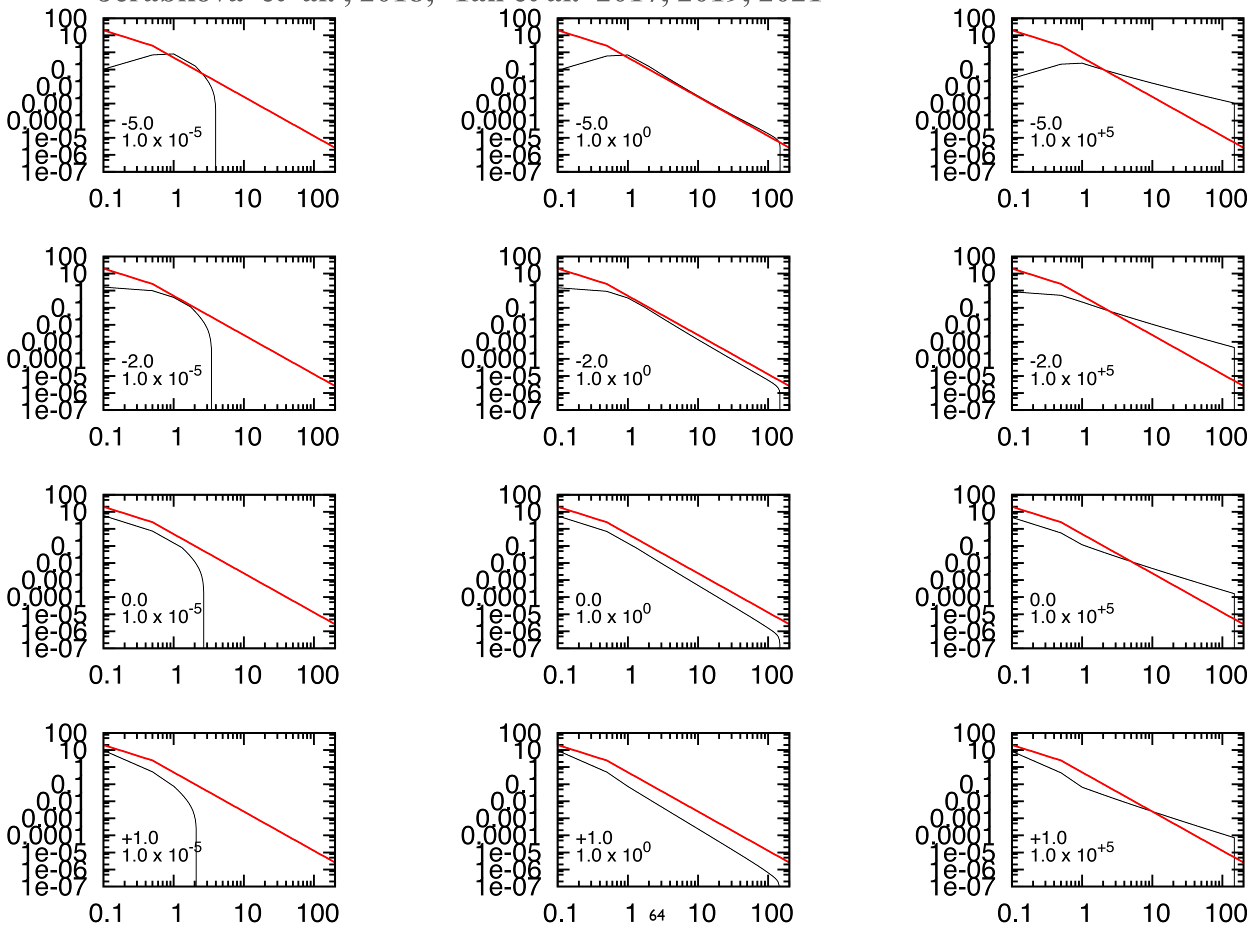
The IGIMF for galaxies with different SFRs & [Fe/H]

Jerabkova et al. , 2018; Yan et al. 2017, 2019, 2021



The IGIMF for galaxies with different SFRs & [Fe/H]

Jerabkova et al. , 2018; Yan et al. 2017, 2019, 2021

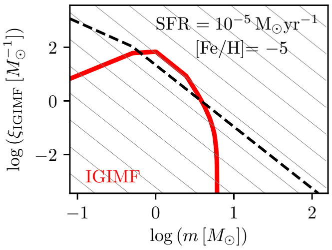


How would an elliptical galaxy evolve in this grid?

Jerabkova et al., A&A,2018

1. low [Fe/H] and start of SFR burst

top-light IMF at the very beginning



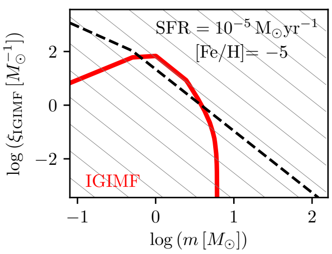
slide courtesy to Jerabkova

How would an elliptical galaxy evolve in this grid?

Jerabkova et al., A&A,2018

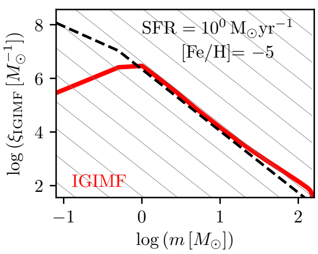
1. low [Fe/H] and start of SFR burst

top-light IMF at the very beginning



2. low [Fe/H] and SFR goes up

MW IMF



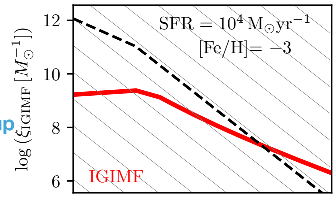
slide courtesy to Jerabkova

How would an elliptical galaxy evolve in this grid?

Jerabkova et al., A&A,2018

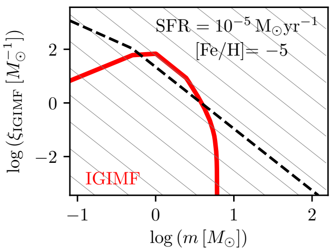
3. [Fe/H] goes up and SFR peaks the burst

top-heavy gwIMF - producing metals



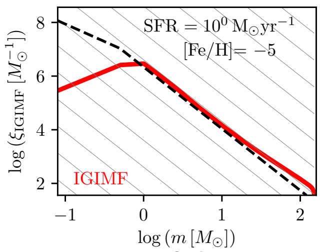
1. low [Fe/H] and start of SFR burst

top-light gwIMF at the very beginning



2. low [Fe/H] and SFR goes up

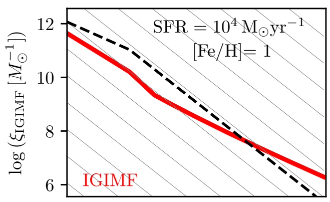
MW gwIMF



slide courtesy to Jerabkova

How would an elliptical galaxy evolve in this grid?

Jerabkova et al., A&A,2018

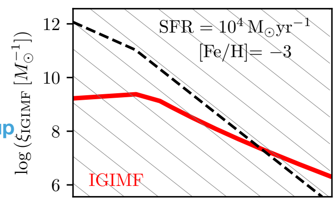


4. [Fe/H] is super solar and SFR peaks the burst

top-heavy gwIMF - producing metals
bottom-heavy gwIMF (observed!)

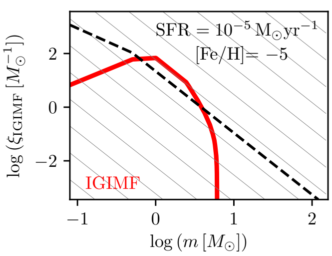
3. [Fe/H] goes up and SFR peaks the burst

top-heavy gwIMF - producing metals



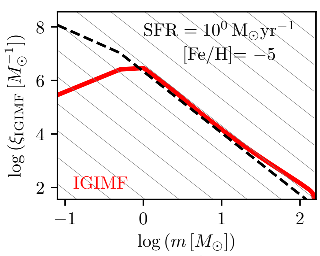
1. low [Fe/H] and start of SFR burst

top-light gwIMF at the very beginning



2. low [Fe/H] and SFR goes up

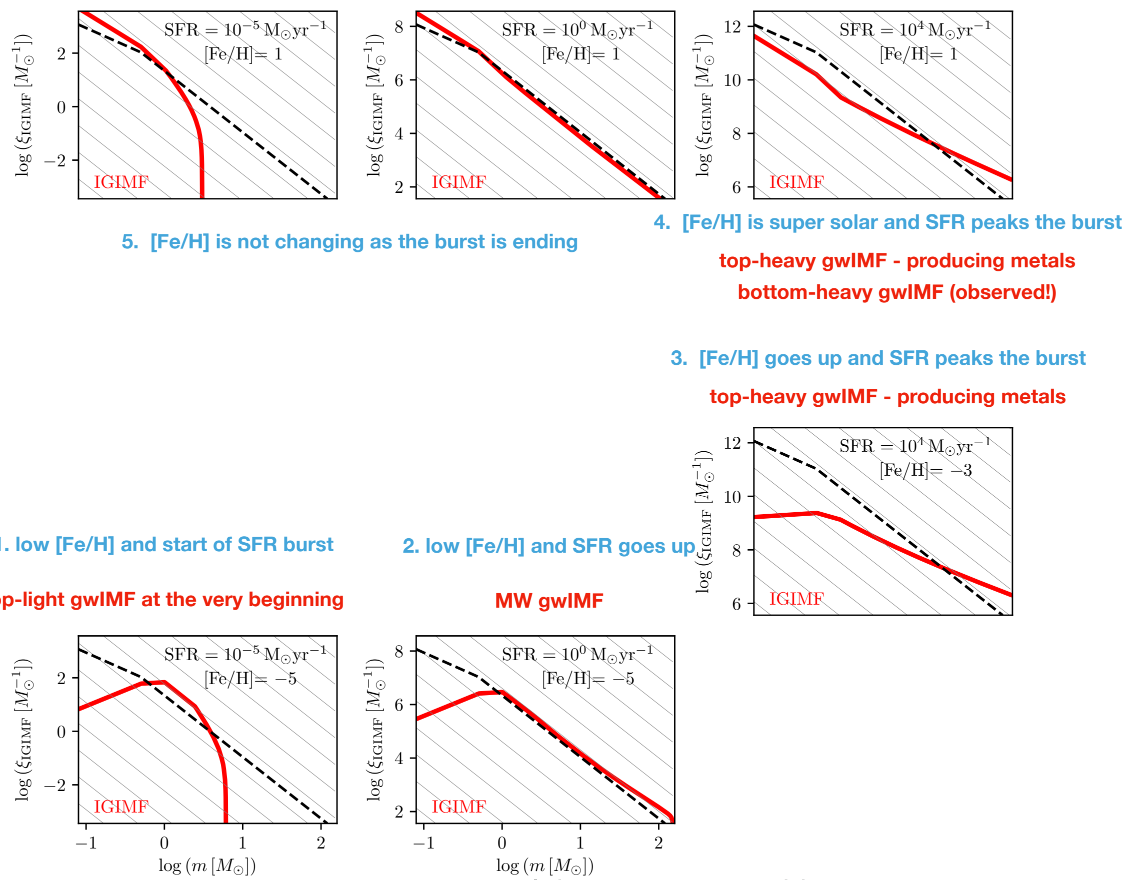
MW gwIMF



slide courtesy to Jerabkova

How would an elliptical galaxy evolve in this grid?

Jerabkova et al., A&A,2018



69

slide courtesy to Jerabkova

P. Kroupa: University of Bonn / Charles University

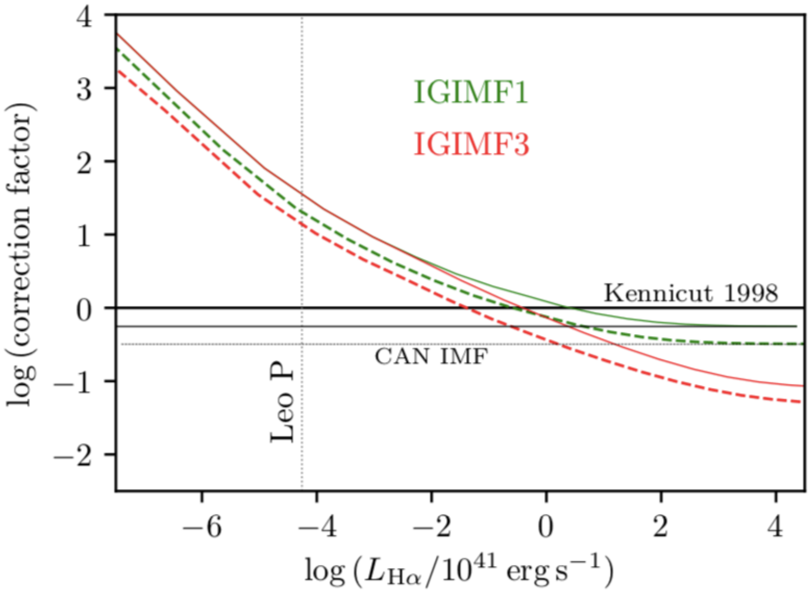
SFRs

Observed SFRs now depend on the SFR

$$\text{correction factor} = \frac{SFR_{\mathrm{IGIMF}}(L_{\mathrm{H}\alpha})}{SFR_{\mathrm{K}}(L_{\mathrm{H}\alpha})}$$

"Kennicutt" - measured assuming invariant standard IMF

Jerabkova et al. 2018



galaxies with high $SFR_{\mathrm{K}}(L_{\mathrm{H}\alpha})$
==> true lower SFR

galaxies with low $SFR_{\mathrm{K}}(L_{\mathrm{H}\alpha})$
==> true higher SFR

==> significant implications on *gas-consumption time scales* and *stellar-mass build-up time scales*

Pflamm-Altenburg & Kroupa (2009, ApJ 706, 516)

Rates of SNIa and neutron star mergers

The shape of the IMF and thus of the gwIMF changes systematically.

Thus the rate of SN1a per long-lived star needs to be recalibrated, given the observed rate.

This has been achieved for the first time correctly in

Downsizing revised: Star formation timescales for elliptical galaxies with an environment-dependent IMF and a number of SNIa

Show affiliations

Yan, Zhiqiang iD ; Jeřábková, Tereza ; Kroupa, Pavel

2021A&A...655A..19Y

Number of SN1a per star must increase with SFR.

Consistent with independent observational data :

Table 1. Observational estimations on the time-integrated number of SNIa per stellar mass formed.

$N_{Ia} \cdot 10^3$ [$M_\odot^{-1}$]	Reference
Volumetric and field galaxies	
2.3 ± 0.6	Maoz et al. (2011)
1.3 ± 0.15	Maoz et al. (2012)
0.485 ± 0.065	Perrett et al. (2012)
$0.43^{+0.04}_{-0.1}$	Graur & Maoz (2013)
$0.98^{+1.3}_{-0.76}$	Rodney et al. (2014)
1.3 ± 0.1	Maoz & Graur (2017)
1.6 ± 0.3	Maoz & Graur (2017)
2 ± 1	Heringer et al. (2019)
4^{+2}_{-1}	Heringer et al. (2019)
Galaxy clusters	
$4.4^{+1.5}_{-1}$	Maoz et al. (2010)
5.4 ± 2.3	Maoz & Graur (2017)
$3.1^{+1.1}_{-1.0}$	Freundlich & Maoz (2021)

The shape of the IMF and thus of the gwIMF changes systematically.

Probably similar for neutron-star -- neutron-star mergers

Number of them per star increases with SFR ?

The process which enhances the population of tight WD--WD and NS--NS binaries is dynamical processing in high-density massive long-lived star clusters

Larger SFR ==> more massive clusters :

THE ASTROPHYSICAL JOURNAL LETTERS, 775:L38 (6pp), 2013 October 1

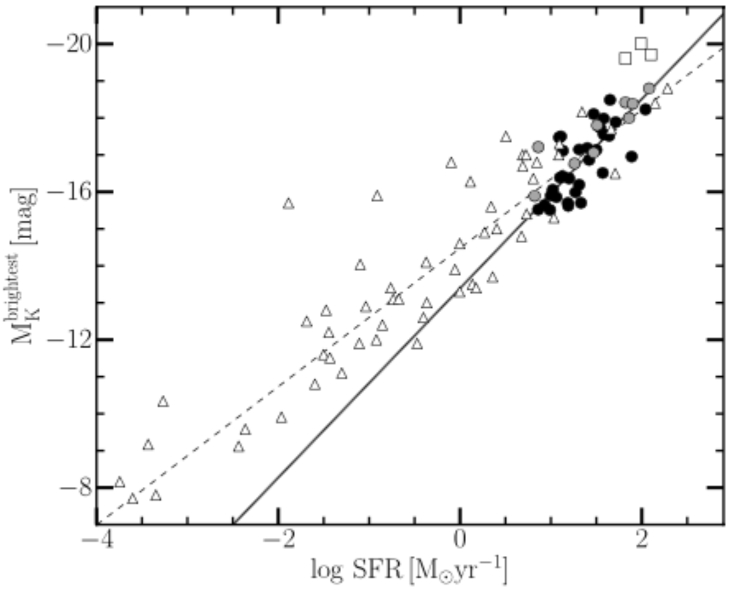


Figure 4. $M^{brightest}$–SFR relation with data from literature (the triangles; Adamo et al. 2011 and references therein) added to the present work (symbols as in Figure 1). The solid line is our best fit of Equation (3) and the dashed line is the fit from Weidner et al. (2004) to the optical V-band data after a constant $V - K = 2$ conversion.

Weidner et al. (2004); Randriamanakoto, Escala et al. (2013)

A good correlation of the most luminous cluster and the number of SSCs with M_K magnitude brighter than −15 shows that a size-of-sample effect is broadly consistent with the observed $M_K^{brightest}$–SFR relation. On the other hand, the observed scatter in the relation is surprisingly small, and we show that it can be explained with random sampling effects *only* if the LF of SSCs is very steep at the bright end, steeper than usually observed. Hence, physical reasons determining the luminosity of the brightest SSC from host properties, and/or internal cluster effects, likely play a role as well. We derived a relation tying

Top-heavy IMF and "quasars"

Consider a UCD with some $10^7\,M_\odot$ today

Initially, it may have had:

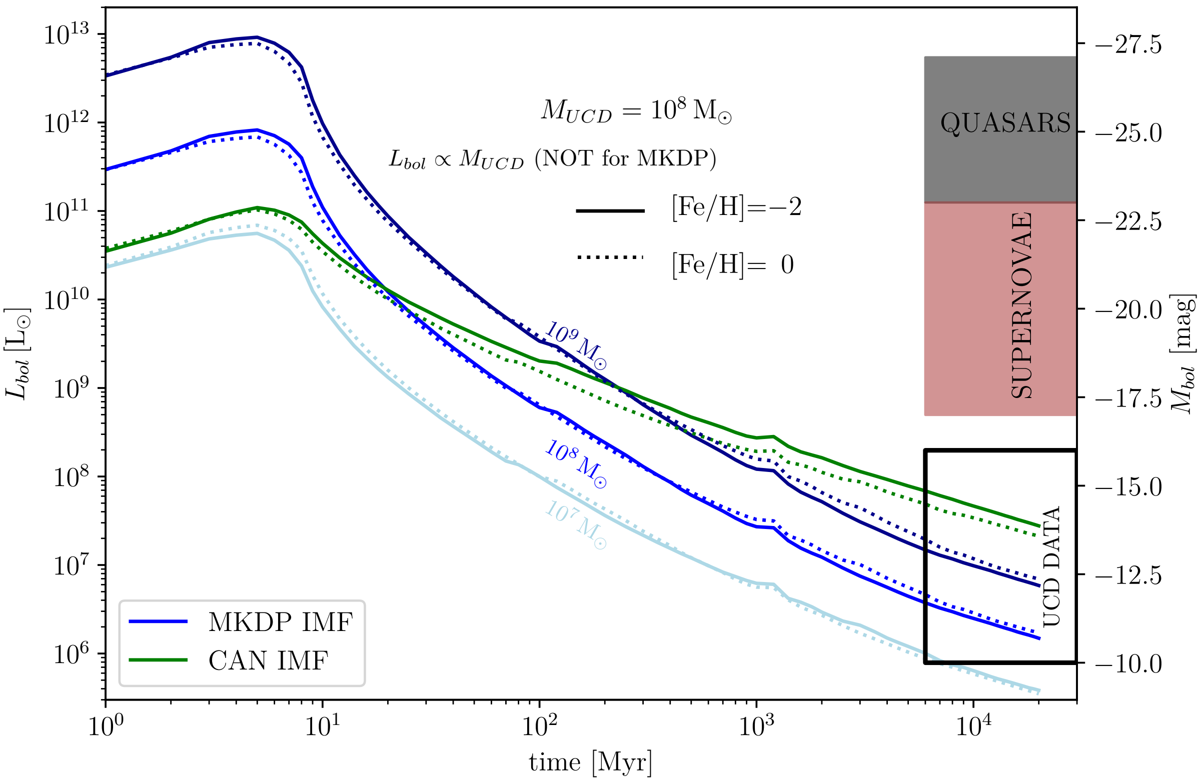
- A mass of some $10^8\,M_\odot \ldots$
- ... but a half-mass radius of only a few pc! (expansion through mass-loss!)
- ... A population of 10^6 O-stars...
- ...with a total luminosity of $>10^{11}\,L_\odot$.

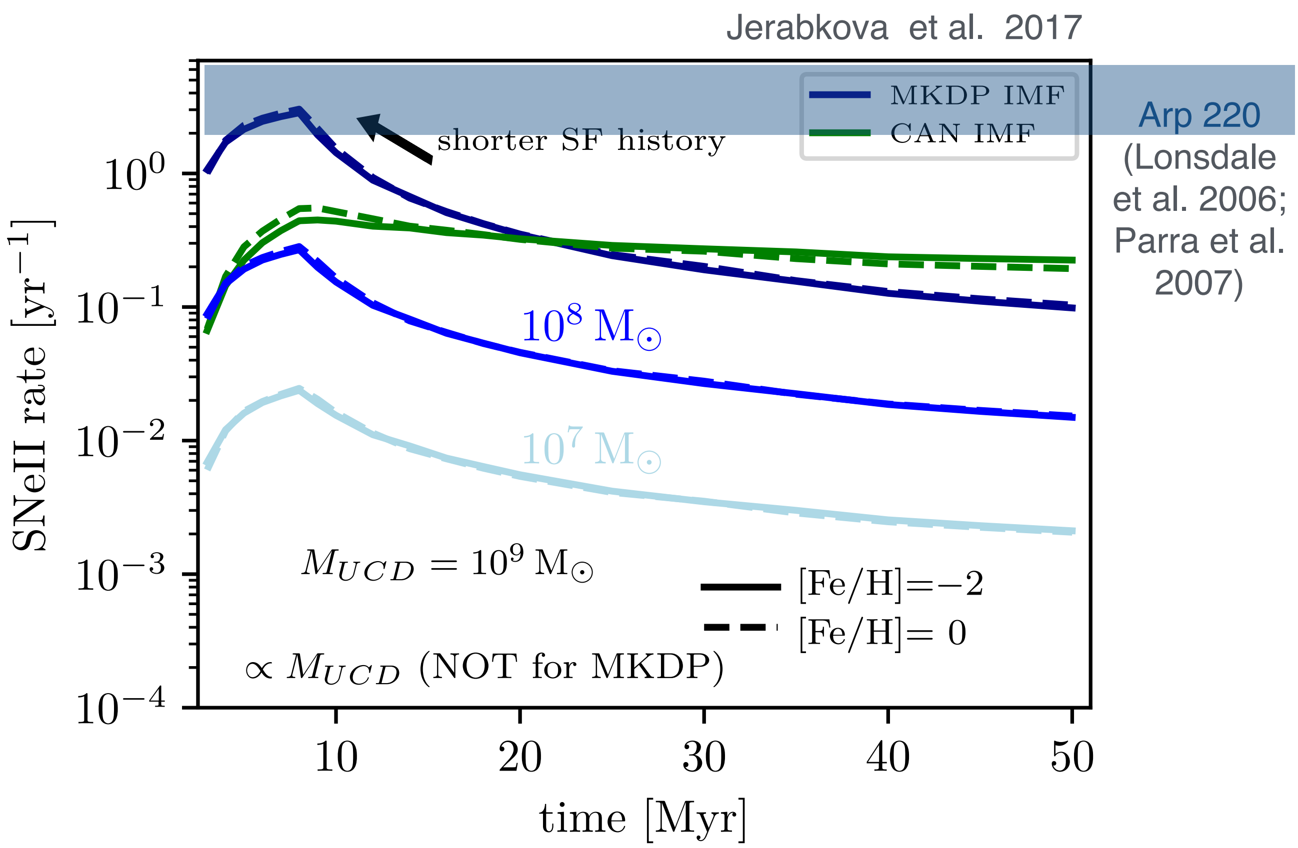
===> Quasar-like objects Jerabkova et al. 2017

luminosities of $10^{12}\,L_{Sun}$,
few-pc radius,
fluctuating on a monthly time-scale,
life-time 50 Myr.

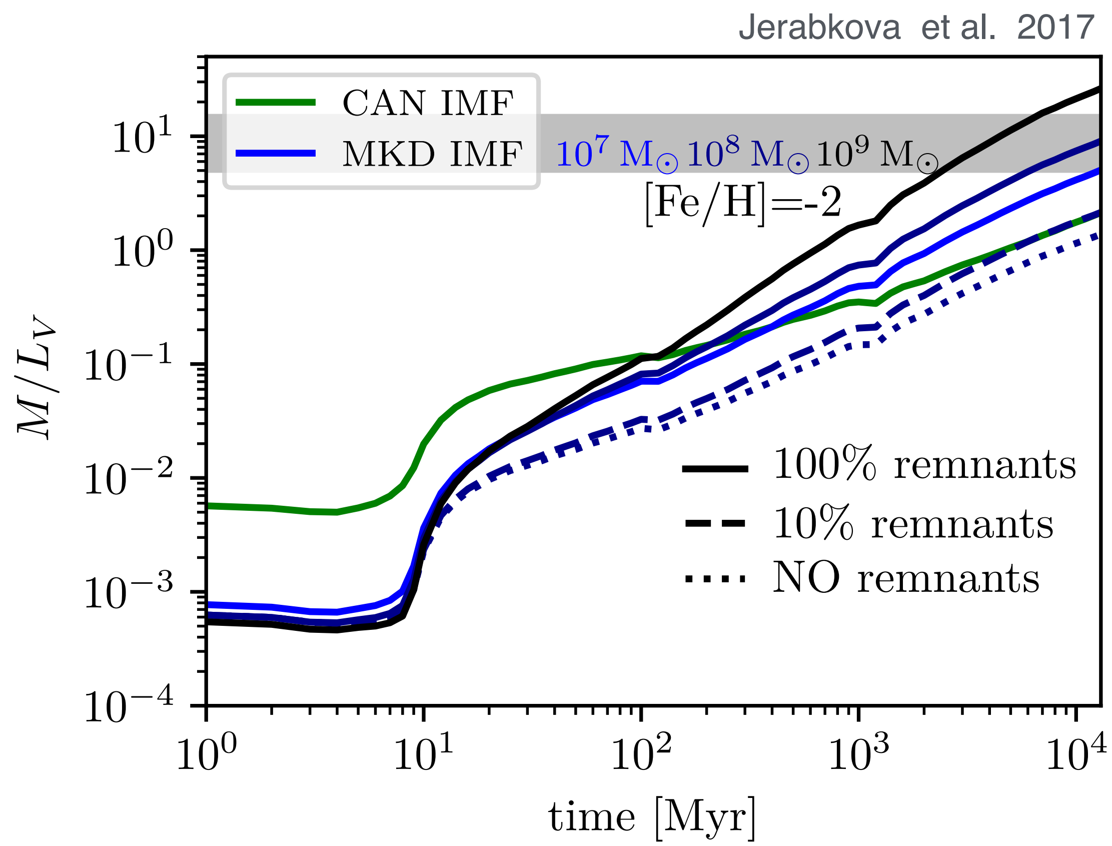
===> Quasar-like objects Jerabkova et al. 2017

The redshift dependent photometric properties are calculated as predictions for *James Webb* Space Telescope (JWST) observations.





Jerabkova et al. 2017
MKDP IMF
CAN IMF
shorter SF history
Arp 220 (Lonsdale et al. 2006; Parra et al. 2007)
10^8 M$_\odot$
10^7 M$_\odot$
$M_{UCD} = 10^9$ M$_\odot$
[Fe/H]=−2
[Fe/H]= 0
$\propto M_{UCD}$ (NOT for MKDP)
SNeII rate [yr^{-1}]
time [Myr]



Jerabkova et al. 2017
CAN IMF
MKD IMF
10^7 M$_\odot$ 10^8 M$_\odot$ 10^9 M$_\odot$
[Fe/H]=-2
100% remnants
10% remnants
NO remnants
M/L_V
time [Myr]

Conclusions

Significant evidence for density and Z dependent IMF
in molecular cloud cores (pc-scale, one Myr-time scale).

The galaxy-wide IMF (the gwIMF) changes with SFR,
as expected. IGIMF theory agrees with observations.

Significant implication for measured SFRs
(and gas-consumption- and stellar-mass build-up times) !

Testbed: extremely star-bursting clusters (UCDs) at high-z.

Are some/most quasars at very high z merely young UCDs ?
Jerabkova et al. 2017

Formation of SMBHs Kroupa et al. 2020
Lecture tomorrow