

# Microscopic nuclear theory for the equation of state of neutron stars

Ingo Tews, Los Alamos National Laboratory

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LA-UR-25-22641



#### 10/16/2017

#### The New York Times

#### LIGO Detects Fierce Collision of Neutron Stars for the First Time

GW170817, Aug 17, 2017

#### **Neutron stars:**

- Remnants of core-collapse supernovae
- Typical masses of 1.4 M<sub>sol</sub>
- Typical radii of only O(10) km

Neutron star mergers:

- Coalescence of two neutron stars
- Can be detected in gravitational waves and EM spectrum (Multimessenger astrophysics)
- Explore highest densities in the Cosmos!

Credit: ESO/L. Calçada

- **1932**: Discovery of the neutron by Chadwick (Nobel Prize).
- **1933/34**: Proposition of the existence of neutron stars by Baade and Zwicky as engines for supernovae.

#### COSMIC RAYS FROM SUPER-NOVAE

By W. BAADE AND F. ZWICKY

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON AND CALI-FORNIA INSTITUTE OF TECHNOLOGY, PASADENA

Communicated March 19, 1934

In addition, the new problem of developing a more detailed picture of the happenings in a super-nova now confronts us. With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a *neutron star*, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density. As neutrons can be packed much more closely than ordinary nuclei and electrons, the "gravitational packing" energy in a *cold* neutron star may become very large, and, under certain circumstances, may far exceed the ordinary nuclear packing fractions. A neutron star would therefore represent the most stable configuration of matter as such. The consequences of this hypothesis will be developed in another place, where also will be mentioned some observations that tend to support the idea of stellar bodies made up mainly of neutrons.



#### Walter Baade



Fritz Zwicky



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- **1939**: Tolman, Oppenheimer and Volkoff calculate neutronstar mass limit of **0.7** M<sub>sol</sub> for cold, degenerate neutron gas.



J. Robert Oppenheimer George Volkoff

**Richard Tolman** 

#### On Massive Neutron Cores

J. R. OPPENHEIMER AND G. M. VOLKOFF Department of Physics, University of California, Berkeley, California (Received January 3, 1939)

#### V. Discussion—Application to Stellar Matter

We have seen that for a cold neutron core there are no static solutions, and thus no equilibrium, for core masses greater than  $m \sim 0.7 \odot$ . The corresponding maximum mass  $M_0$  before collapse is some ten percent greater than this. Since neutron cores can hardly be stable (with respect to formation of electrons and nuclei) for masses less than  $\sim 0.1 \odot$ , and since, even after thermonuclear sources of energy are exhausted, they will not tend to form by collapse of ordinary matter for masses under  $1.5 \odot$  (Landau's limit), it seems unlikely that static neutron cores can play any great part in stellar evolution;<sup>18</sup> and



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#### Jocelyn Bell (1967)

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- **1974**: Hewish wins Nobel prize for the discovery of pulsars.
- 1974: Discovery of the Hulse-Taylor pulsar PSR B1913+16, first binary neutron-star system. Tests of General Relativity, e.g., gravitational waves lower orbital frequency → observed!
- 2010, 2013, 2019: Discovery of 2 M<sub>sol</sub> neutron stars.





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- 2010, 2013, 2019: Discovery of 2 M<sub>sol</sub> neutron stars.
- 2017: First discovery of gravitational waves from neutron-star merger, GW170817!







#### What stabilizes Neutron Stars?











#### What stabilizes Neutron Stars?

Neutron stars are stabilized against gravity by pressure of strongly interacting matter.

 Neutron star:
 Atomic nucleus, e.g.,  $^{208}$ Pb:

  $M \sim 1.4 \,\mathrm{M_{sol}} = 3 \cdot 10^{30} \,\mathrm{kg}$   $M \sim 3 \cdot 10^{-25} \,\mathrm{kg}$ 
 $R \sim 10 - 13 \,\mathrm{km}$   $R \sim 6 \,\mathrm{fm} = 6 \cdot 10^{-18} \,\mathrm{km}$ 
 $\rho \sim 10^{14} \,\mathrm{g/cm^3}$   $\rho \sim 10^{14} \,\mathrm{g/cm^3}$  

 Nuclear saturation density
 Image: Neurophysical system

Although the corresponding scales differ by many orders of magnitude, properties of neutron stars and nuclei are strongly connected.

Nuclear interactions exert outward pressure that stabilize both nuclei and neutron stars!





Pressure

Neutron Stars described by Tolman-Oppenheimer-Volkoff (TOV) equations, equation of state (EOS) only ingredient:

relation between **density**, **composition**, **temperature**, **energy**, **pressure**.

- Neutron stars have typical temperatures of  $T{=}10^7{-}10^8\,K \rightarrow E_{th}{=}8~keV \ll E_F$
- Therefore, neutron stars can be considered objects at T=0
- Then, EOS relates pressure p and energy density  $\epsilon$





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3/20/2025



### **NS (multi-messenger) observations**

First neutron-star merger observed on Aug 17, 2017 :

SSS17a

The New York Times

LIGO Detects Fierce Collision of









#### **Microscopic Nuclear Physics**

Neutron-star structure depends on the EOS, given by  $p = p(\epsilon)$ 

- > Baryon density:  $n = \frac{A}{V}$
- > Energy density:  $\epsilon = \frac{E}{V} = n \cdot \frac{E}{A}$

Pressure: 
$$p = -\frac{\partial E}{\partial V} = -\frac{\partial E/A}{\partial V/A} = n^2 \frac{\partial E}{\partial v}$$

In neutron star, we have neutrons, protons, and electrons in beta equilibrium. Therefore, EOS is described by

$$\frac{E}{A}(n,x)$$

where x is the proton fraction,  $x = n_p/n$ .

- x = 0.5: Symmetric nuclear matter: Connection to **laboratory experiments**
- x = 0.0: Pure neutron matter: Connection to **astrophysical observations**.
- Difference is called symmetry energy: Connection to heavy-ion collisions, neutron skins, ...





>

#### **Microscopic Nuclear Physics**

Many different approaches to calculate  $\frac{E}{A}(n,x)$  but I will focus on **microscopic calculations** where we solve

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$



see also Carbone, Drischler, Gandolfi, Hagen, Hebeler, Holt, Lovato, Novario, Piarulli, Schwenk,, ...



# **Microscopic Nuclear Physics**

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We need:

A theory for the strong interactions among nucleons

**Chiral Effective Field Theory** 

$$\mathcal{H} = \sum_i \mathcal{T}_i + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \cdots$$

A computational method to solve the many-body Schrödinger equation:

> e.g., many-body perturbation theory, quantum Monte Carlo, coupled cluster, self-consistent Green's function, ...



see also Carbone, Drischler, Gandolfi, Hagen, Hebeler, Holt, Lovato, Novario, Piarulli, Schwenk,, ...



- Atomic nucleus consists of strongly interacting matter.
- Made up by quarks and gluons (Quantum Chromodynamics).
- Extremely complicated to solve!





- Atomic nucleus consists of strongly interacting matter.
- Made up by quarks and gluons (Quantum Chromodynamics).
- Extremely complicated to solve!
- Probing a nucleus at low energies does not resolve quark substructure of nucleons!
- We can describe the nucleus in terms of **neutrons** (udd) and **protons** (uud) as effective degrees of freedom.





Solve Schrödinger equation:

Hamiltonian is sum of kinetic and interaction parts:

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$

$$\mathcal{H} = \sum_{i} \mathcal{T}_{i} + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \cdots$$

Two-nucleon forces

Three-nucleon forces

- V is hermitian, because the Hamiltonian is hermitian,
- V is symmetric under the permutation of identical particles, i.e.,  $V_{ij} = V_{ji}$ ,
- V is translationally and rotationally invariant,
- V is invariant under translations in time, i.e., time-independent,
- V is Lorentz invariant (for nonrelativistic interactions this reduces to Galilean invariance),
- V is invariant under parity transformations and time reversal,
- V has to conserve baryon and lepton number,
- V has to be approximately isospin symmetric and charge independent,
- and V has to include the properties of spontaneously and explicitly broken chiral symmetry.



Holt et al., PPNP 73 (2013)





Holt et al., PPNP 73 (2013)

	NN	3N	4N
LO $O\left(\frac{Q^0}{\Lambda^0}\right)$ (2 LECs)	ХН	_	_

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...





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Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...



Systematic expansion of nuclear forces in momentum Q over breakdown scale  $\Lambda_{b}$ :

- Based on symmetries of QCD
- Pions and nucleons as explicit degrees of freedom
- Power counting scheme results in systematic expansion, enables uncertainty estimates!
- Natural hierarchy of nuclear forces
- **Consistent interactions**: Same couplings for twonucleon and many-body sector
- Fitting: NN forces in NN system (NN phase shifts), 3N forces in 3N/4N system (Binding energies, radii)

	NN	3N	4N
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Results for chiral EFT calculations of nuclei:



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Excellent description of properties of nuclei up to the medium-mass region (fits to light nuclei).





Present theoretical predictions for nuclear systems are limited by:

- our incomplete understanding of nuclear interactions,
- and our ability to **reliably calculate** these strongly interacting systems.



$$V(\mathbf{q}, \mathbf{k}) = V_{\text{cont}}(\mathbf{q}, \mathbf{k}) + V_{\pi}(\mathbf{q}, \mathbf{k})$$

$$V_{\text{cont}}^{\text{LO}}(\mathbf{q}, \mathbf{k}) = V_{\text{cont}}^{\text{LO}} = \alpha_1 \mathbf{1} + \alpha_2 \sigma_i \cdot \sigma_j + \alpha_3 \tau_i \cdot \tau_j + \alpha_4 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$$

$$V_{\text{cont}}^{\text{NLO}}(\mathbf{q}, \mathbf{k}) = \gamma_1 \mathbf{q}^2 + \gamma_2 \mathbf{q}^2 \sigma_i \cdot \sigma_j + \gamma_3 \mathbf{q}^2 \tau_i \cdot \tau_j + \gamma_4 \mathbf{q}^2 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j + \gamma_5 \mathbf{r}^2 + \gamma_6 \mathbf{r}^2 \sigma_i \cdot \sigma_j$$

$$+ \gamma_7 \mathbf{k}^2 \tau_i \cdot \tau_j + \gamma_8 \mathbf{k}^2 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j + \gamma_9 \mathbf{r}^2 \sigma_i + \sigma_j)(\mathbf{q} \times \mathbf{k}) + \gamma_{10}(\sigma_i + \sigma_j)(\mathbf{q} \times \mathbf{k}) \tau_i \cdot \tau_j$$

$$+ \gamma_{11} \mathbf{r}^2 \mathbf{r}_i \cdot \mathbf{k})(\sigma_j \cdot \mathbf{q}) + \gamma_{12} \mathbf{r}^2 \mathbf{r}_i \cdot \mathbf{r}_j + \gamma_{13} \mathbf{r}^2 \mathbf{r}^2 \mathbf{r}^2 \mathbf{r}^2 \mathbf{k} + \gamma_{14} \mathbf{r}^2 \mathbf{$$

$$V_{\text{OPE}}^{(0)}(\mathbf{q}) = - \underbrace{\frac{g_A^2}{4f_\pi^2}}_{q^2 + m_\pi^2} \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q} \boldsymbol{\sigma}_j \cdot \mathbf{q}}{q^2 + m_\pi^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

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The Hamiltonian depends on a set of parameters: low-energy couplings (LECs).

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<sup>1</sup>S<sub>0</sub> phase nucleon-nucleon scattering phase shift

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70 Schematic! 60 · 50 -Phase Shift [deg] 40 -30 -20 -10 -0 20 40 60 80 0 100 Lab Energy [MeV]

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The uncertainty of nuclear interactions is mapped into the uncertainty of model parameters (LECs)

> More details: Somasundaram et al., PRC 2024





Somasundaram, IT, et al., PRC (2024)



Can work to desired accuracy with error estimates!



Use Machine Learning / Artifical Intelligence to propagate uncertainties

#### **Result for the equation of state**



Armstrong et al. arXiv 2502.03680

Use Machine Learning to propagate 200,000 interactions to EOS!

#### **Result for the equation of state**



Armstrong et al. arXiv 2502.03680

 $\bigotimes$ 

Use Machine Learning to propagate 200,000 interactions to EOS!

#### **Chiral EFT and neutron stars**





#### **Nuclear-physics Multi-Messenger Astrophysics (NMMA)**

#### **Prior construction**



#### NMMA framework:

Pang et al., Nat. Comm. (2023)

- EOS consistent with theory
- Masses and NICER via published posteriors
- Simultaneous full GW and KN analyses
- Available online.

Dietrich, Coughlin, Pang, Bulla, Heinzel, Issa, **IT**, Antier, Science (2020)

BUT: There are still many open questions and problems!

 What is the breakdown scale? Does it change in the many-body system?





Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

BUT: There are still many open questions and problems!

- What is the breakdown scale? Does it change in the many-body system?
- How do results depend on the regularization scheme (explicit form of the interaction) and scale (cutoff necessary in many-body methods)?
- Does this series converge in the many-body system?
- What is the correct **power counting** scheme?
- How to best determine all unknown coefficients?

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#### The future: Cosmic Explorer (CE)



- 3<sup>rd</sup>- generation Gravitational-Wave Detectors will increase sensitivity by at least factor of 10
- US-proposal: Cosmic Explorer
- EU-proposal: Einstein Telescope



#### The future: Cosmic Explorer (CE)



- CE will detect the majority of neutron-star mergers in the universe!
- GW170817 would have been observed with an SNR 100 times higher.
- Will measure neutron-star radius to within a few percent!

#### Thanks

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# Thank you for your attention!