R-matrix calculations for nuclear astrophysics

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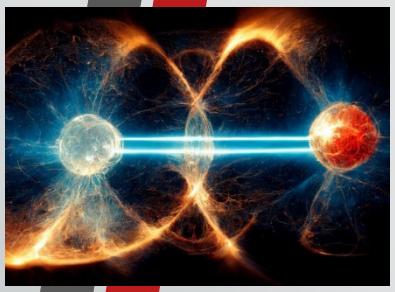
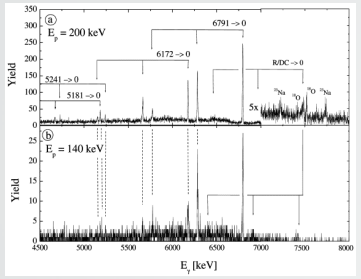
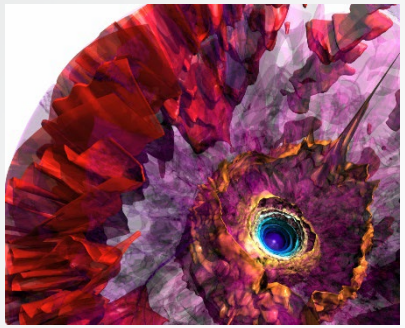
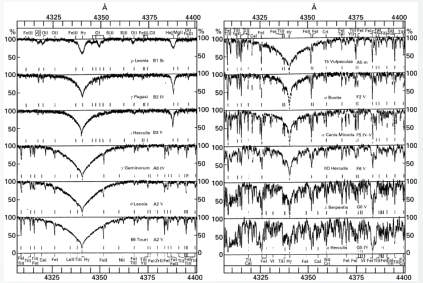
20th Russbach School on Nuclear Astrophysics, March 16-22 2025

Circle of Nuclear Astrophysics Life









Nuclear Experiment and Theory

Nuclear Reaction Rates

Astrophysics models

Astronomical Data

Stellar Observation

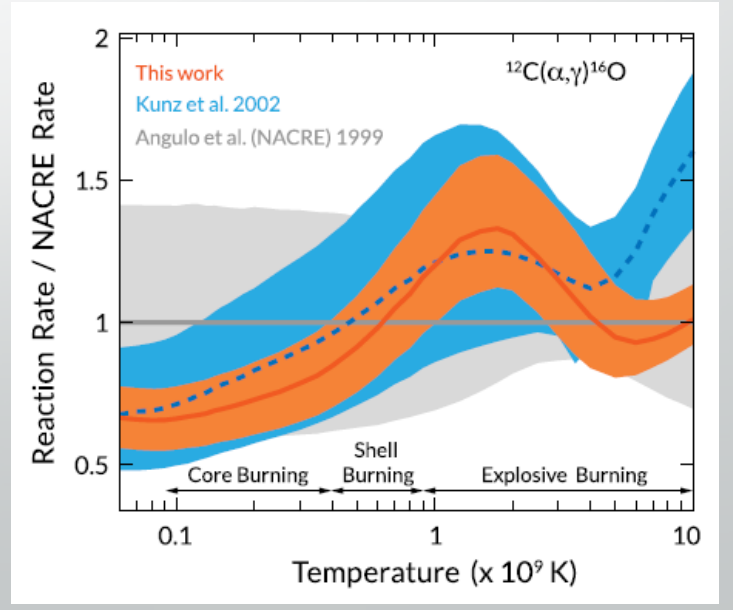
Nuclear Reaction Rates

NACRE (1999)

$$N_A\langle\sigma v\rangle_r = N_A\left(\frac{2\pi}{\mu k_B}\right)^{3/2}\hbar^2\omega\gamma T^{-3/2}\exp(-E_r/k_BT)$$

$$N_A\langle\sigma v\rangle = N_A\frac{(8/\pi)^{1/2}}{\mu^{1/2}(k_BT)^{3/2}}\int_0^\infty \sigma E\exp(-E/k_BT)dE,$$

- Nuclear Reaction rates are largely calculated from
  - Resonance strengths --- for narrow resonances
  - Nuclear **cross sections** --- for cross sections where resonance interference is important and the cross section varies more slowly with energy
- Well defined uncertainties?



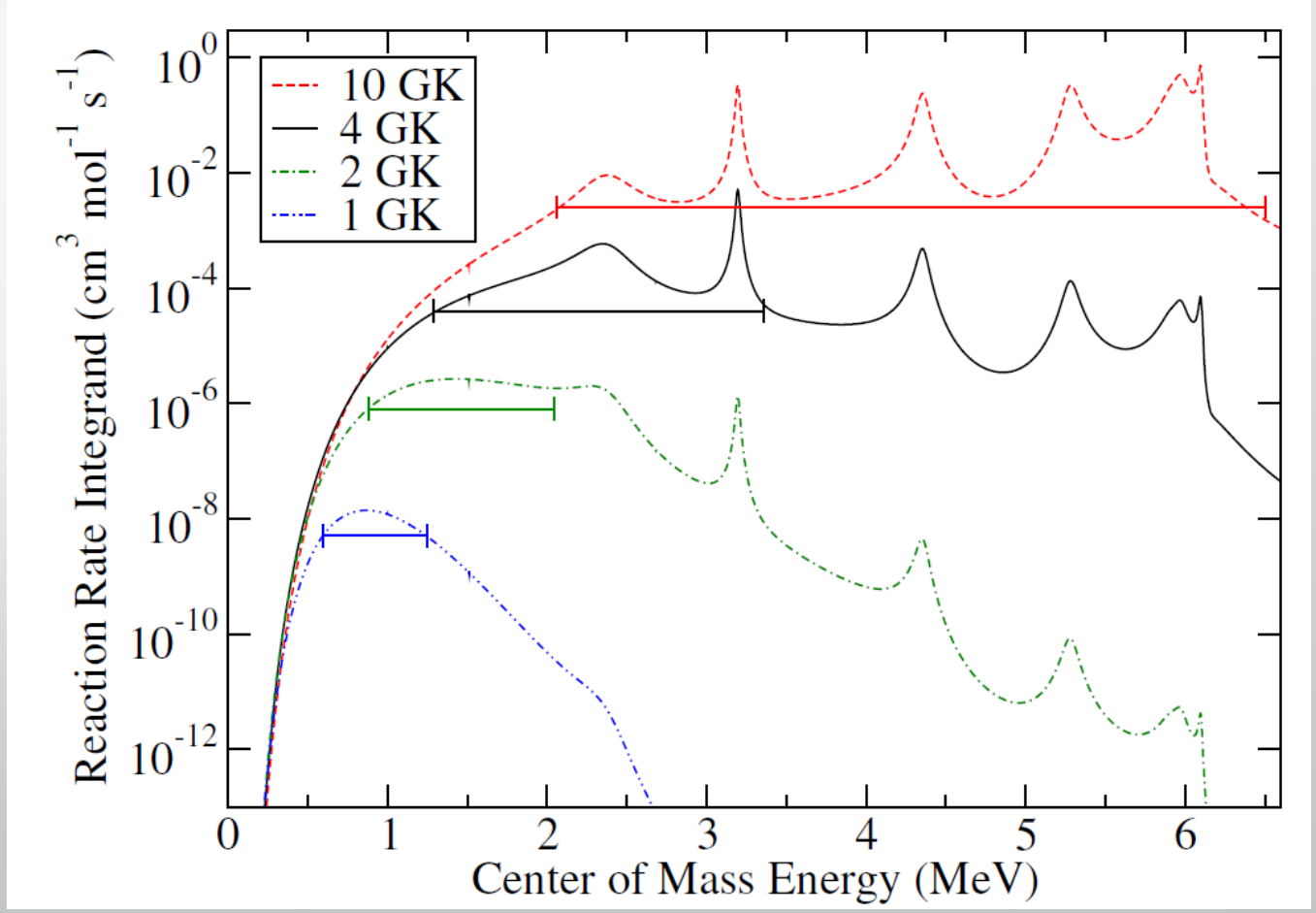
Reaction rate integrand

$$N_A\langle\sigma v\rangle = N_A \frac{(8/\pi)^{1/2}}{\mu^{1/2}(k_B T)^{3/2}} \int_0^\infty \sigma E \exp(-E/k_B T)\, dE,$$

- What energy ranges are important at a given temperature?
- Gamow energy range estimate isn't always so accurate

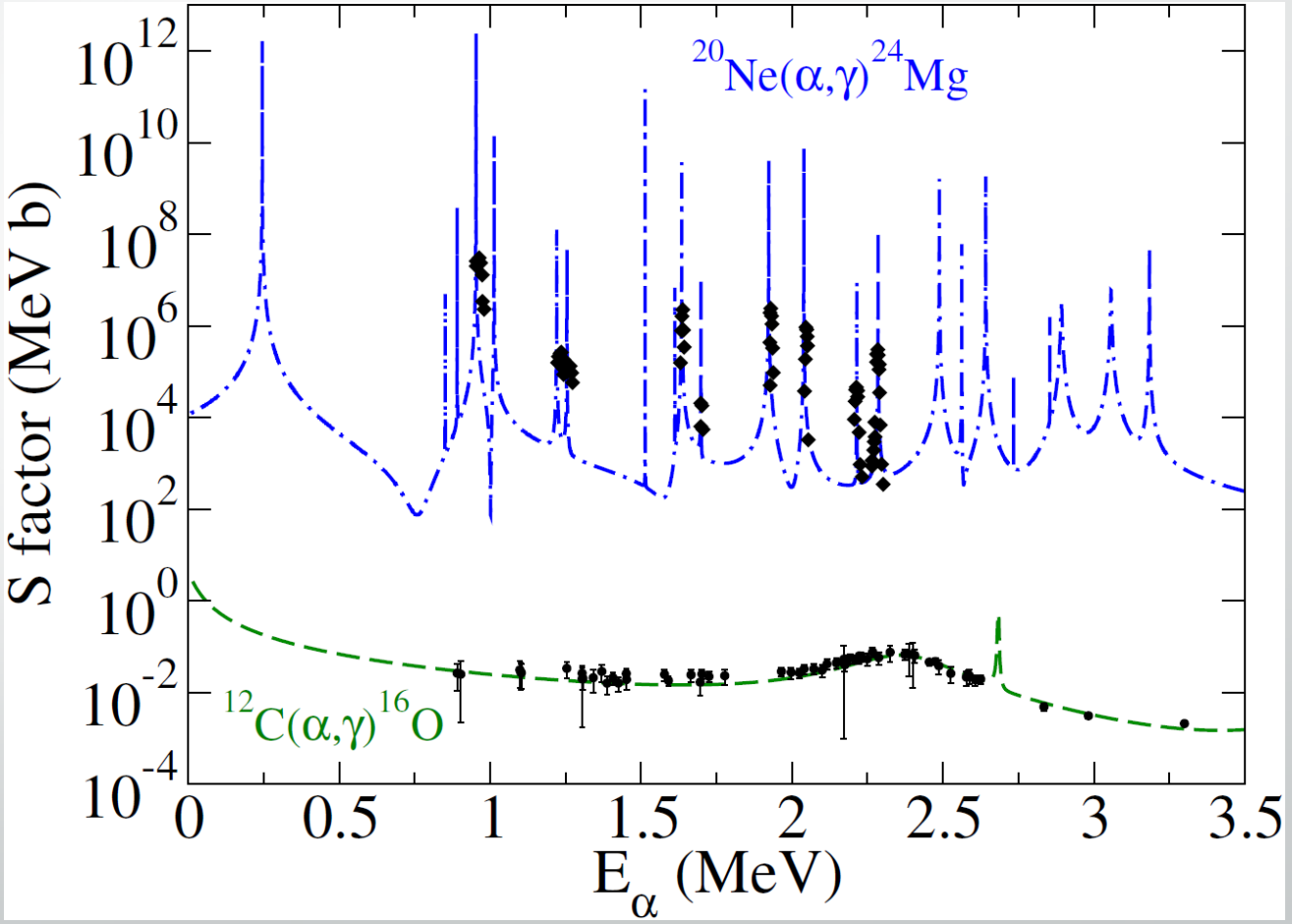
$$E_0 = 0.122(Z_1^2 Z_2^2 \hat{\mu} T_9^2)^{1/3}$$

$$\Delta E = 0.236(Z_1^2 Z_2^2 \hat{\mu} T_9^5)^{1/6}$$



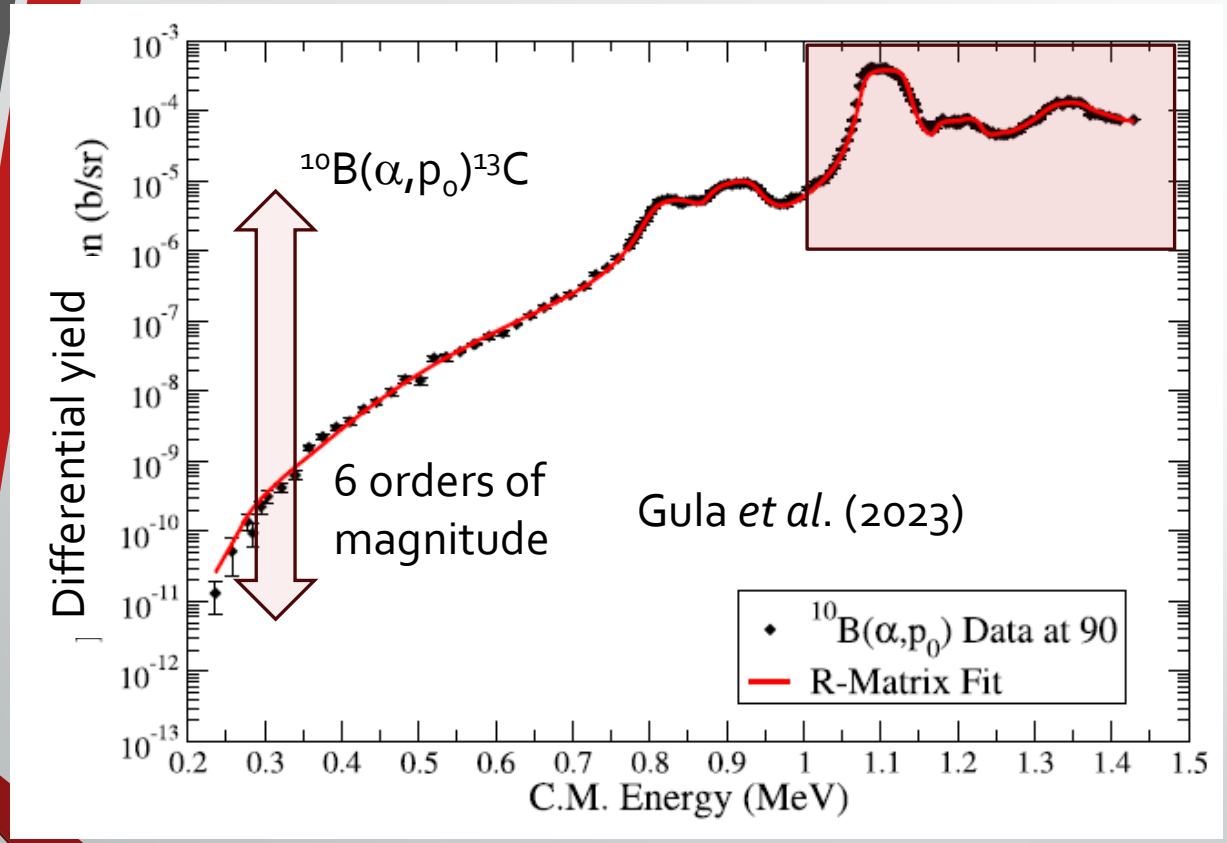
Experimental resolution

- This distinction is in part due to experimental resolution
- Experimental nuclear physics experiments are always compromising between count rate and resolution



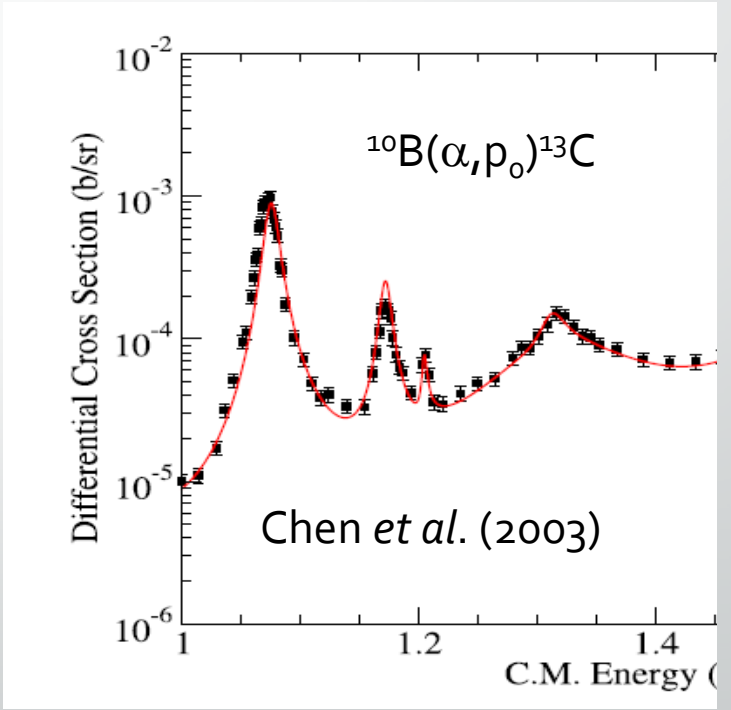
Resolution due to target thickness

Thick target, 50 µg/cm²



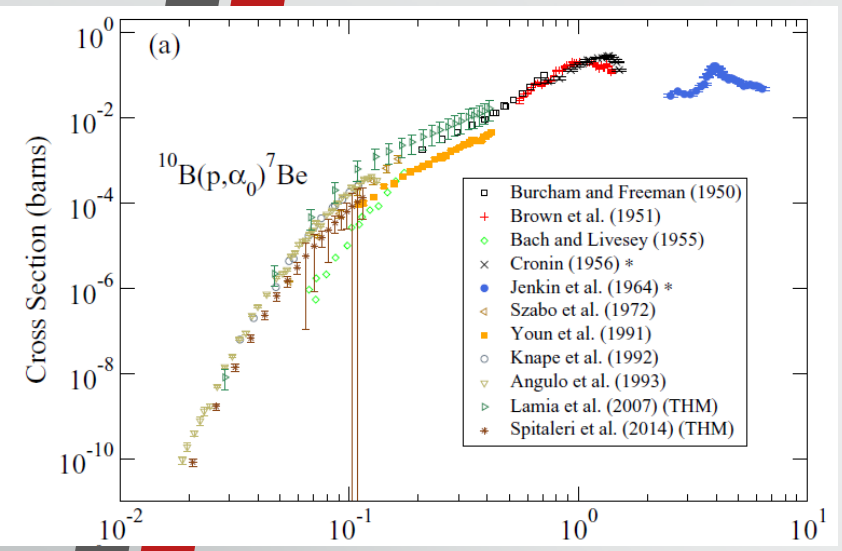
Significant resolution distortion

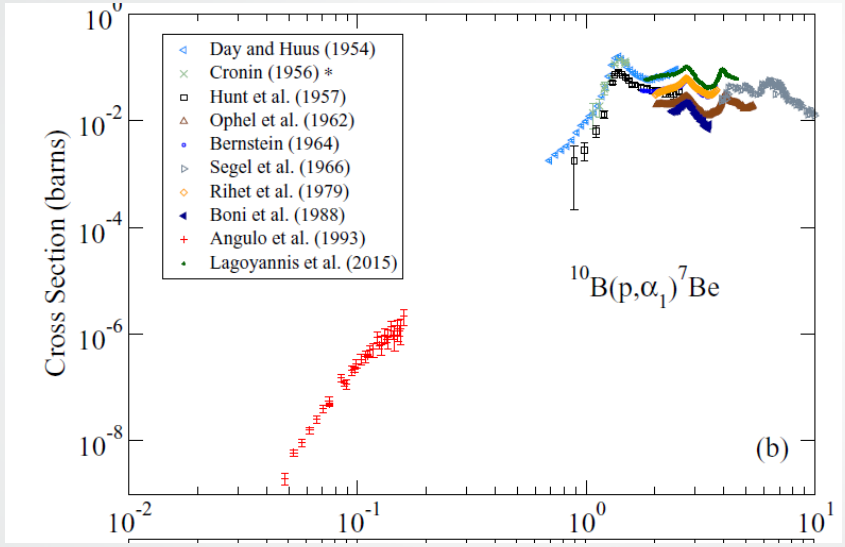
Thin target, 0.035 µg/cm²

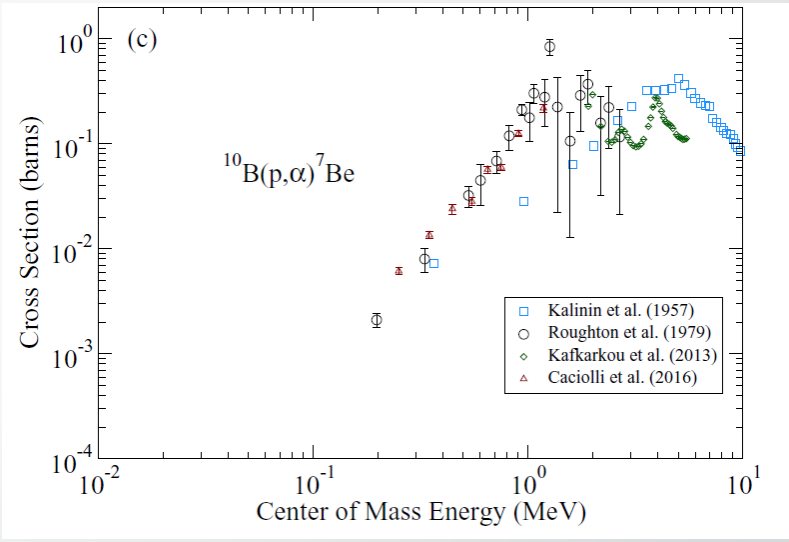


Negligible resolution distortion

Uncertainty, uncertainty, uncertainty



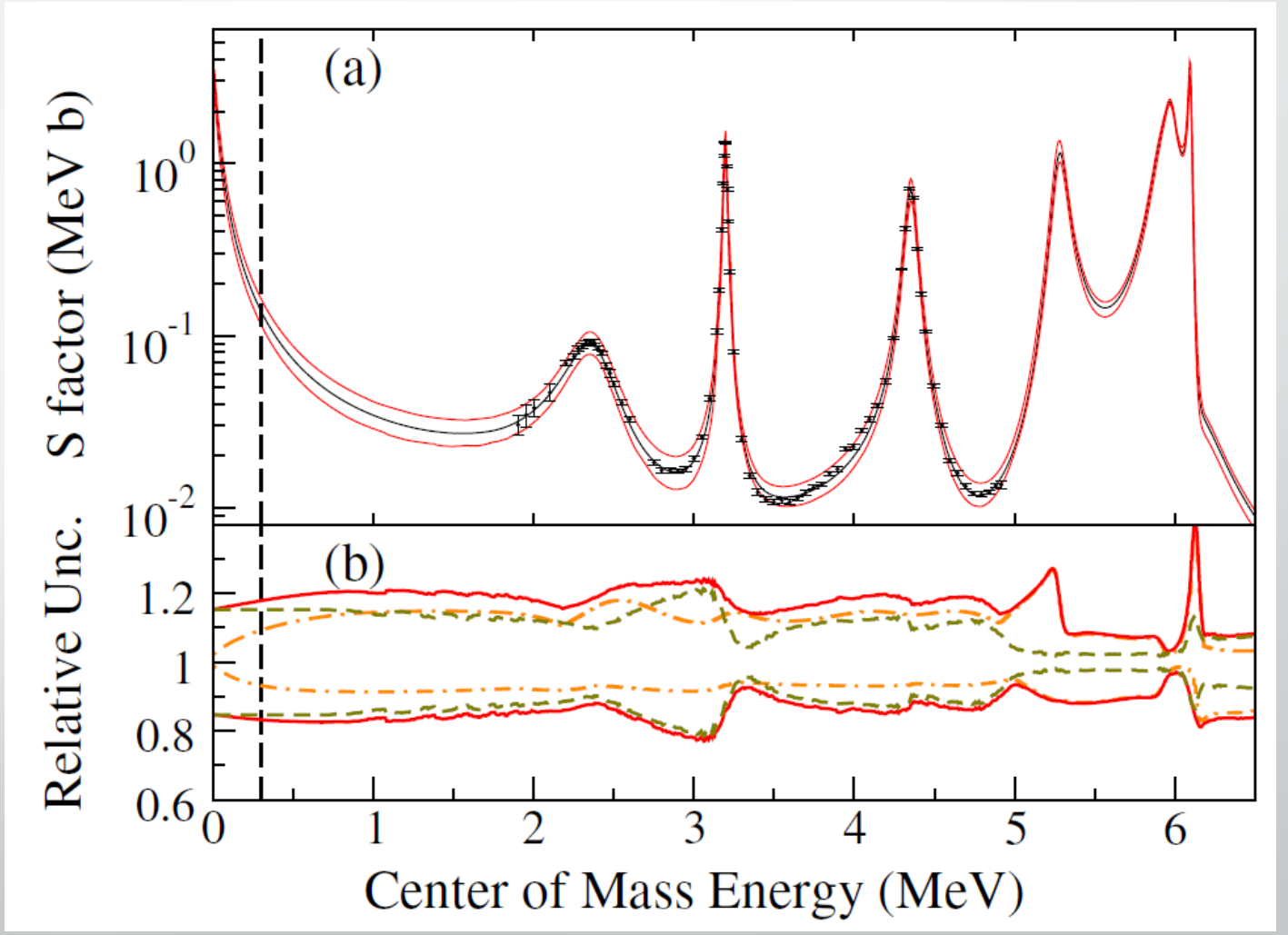




- Uncertainty in the data
  - Systematic differences in overall scale of the cross section (usually target related)
  - Difference in the energy dependance of the data

Uncertainty, uncertainty, uncertainty

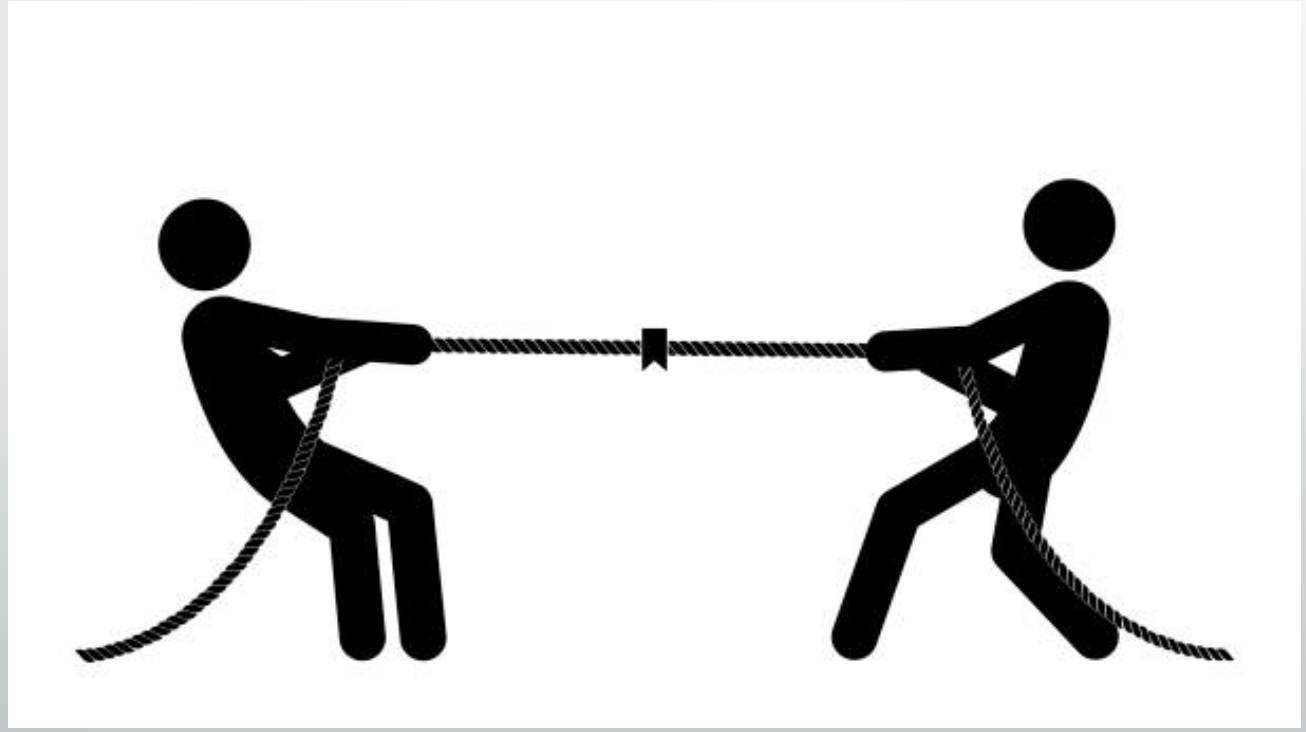
- If data uncertainties weren't bad enough, then we must often extrapolate the cross section
- Model uncertainties!



Nuclear models

phenomenological ⟷ first principles

Tunable parameters to match experimental data



Fundamental physics constraint

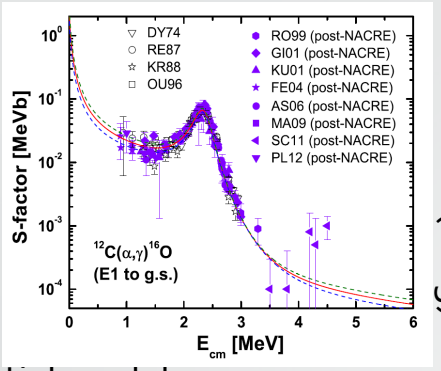
Nuclear models

phenomenological  first principles

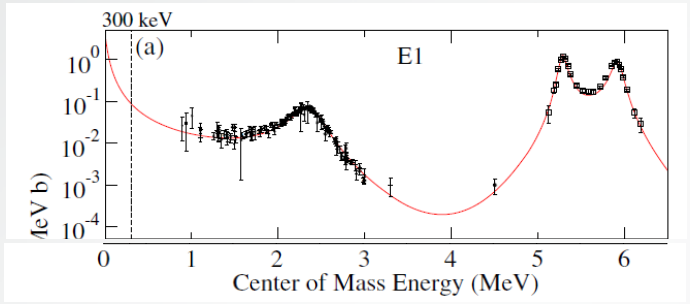
- Implements a few fundamental physical constraints
- Relies heavily on tuning parameters to match experimental data

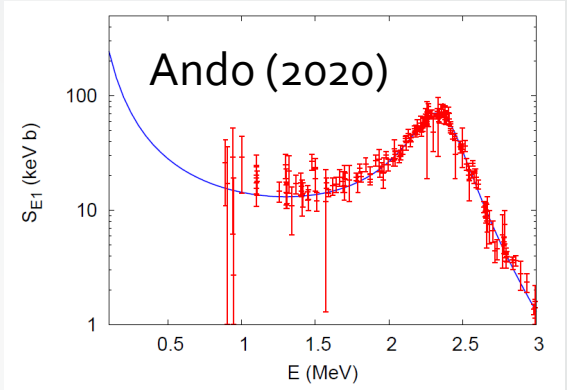
- Implementation of accurate fundamental nuclear theory to calculate the cross section
- Very computationally expensive
- Can only be done for light nuclei

Example phenomenological models



NACRE2 (2013)

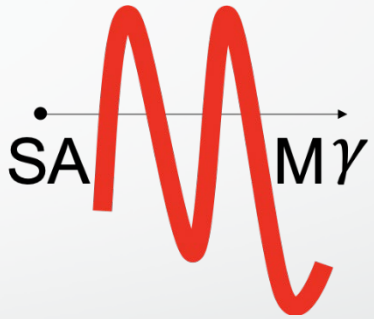


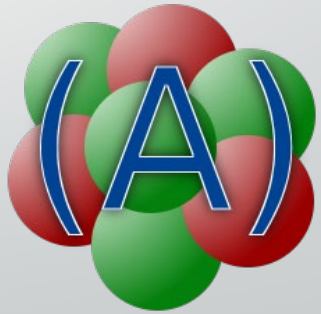


- Potential models
  - Different (simple) functional forms for the potential are used
  - Often uses a Coulomb part plus Woods-Saxon potential and a surface absorption term
  - **Reproduces many resonances and direct part of the cross section directly from the potential**
  - **Additional resonances may have to be added ad hoc using individual Breit-Wigner functions**
  - **If an absorption term is used, it doesn't conserve unitarity. Usually just fits one reaction at a time.**
  - Tunable parameters: potential strengths (V_i), radii (r_i) and diffuseness (a_i) and possibly BR parameters as well
- R-matrix models
  - No specific potential is defined
  - **Allows for maximum flexibility to fit data**
  - **In the ideal case, all channels are modeled data from each is fit and unitarity is conserved**
  - **All resonances are added ad hoc**
  - **Resonance interference is accurately modeled**
  - **Background terms need to be included to model direct reaction part**
  - Each resonance has a tunable energy (E), reduced width amplitude (γ_i) and each channel has a radius parameter (a_c)
- Effective field theory
  - Derived as an expansion of momentum, where the momentum is small compared to the binding energies of the interacting particles
  - **Model accuracy (uncertainty) in the calculation can be estimated from the number of terms used in the expansion**
  - Fit parameters are the effective range parameters (ERP). **Some of these ERPs, can be related to fit parameters of other theories, such as level energies and asymptotic normalization coefficients (ANCs)**
  - **Still under development**

Why R-matrix?

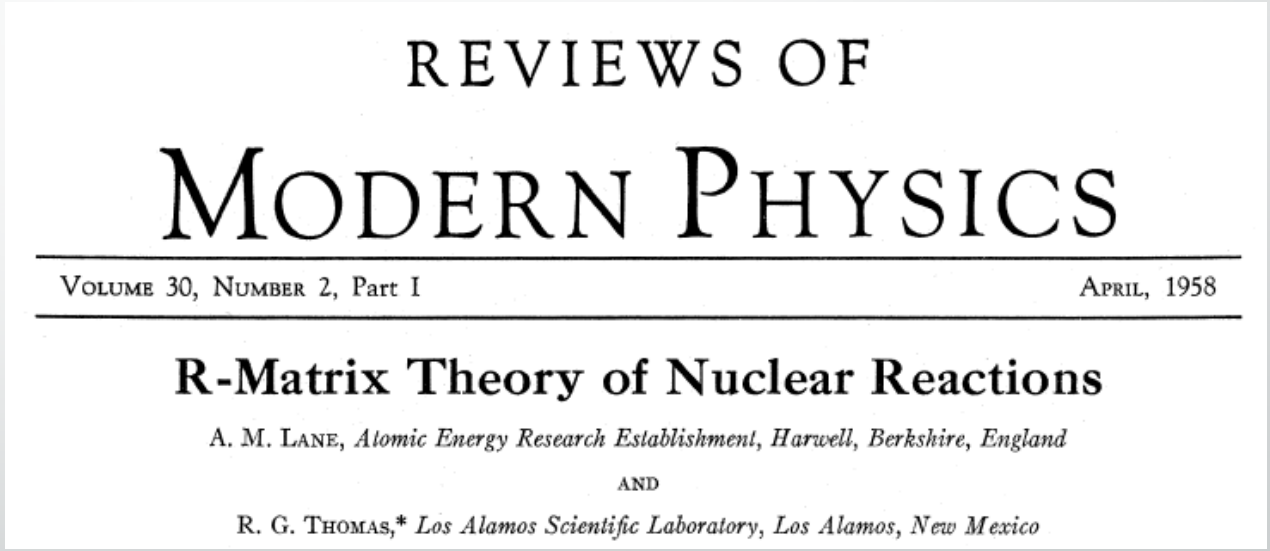
- Assumption: the data is correct
- **Maximum flexibility** → can most accurately reproduce experimental data
- **Maximum flexibility** → easiest to mess up / most influenced by incorrect data
- Track record is pretty good
- It's been used for a long time, a lot of the details of the formalism have been derived and are accessible in the literature
- **Computer codes are available and accessible**





R-matrix theory

- Includes “fundamental” quantum mechanics information



REVIEWS OF

MODERN PHYSICS

VOLUME 30, NUMBER 2, Part I — APRIL, 1958

R-Matrix Theory of Nuclear Reactions

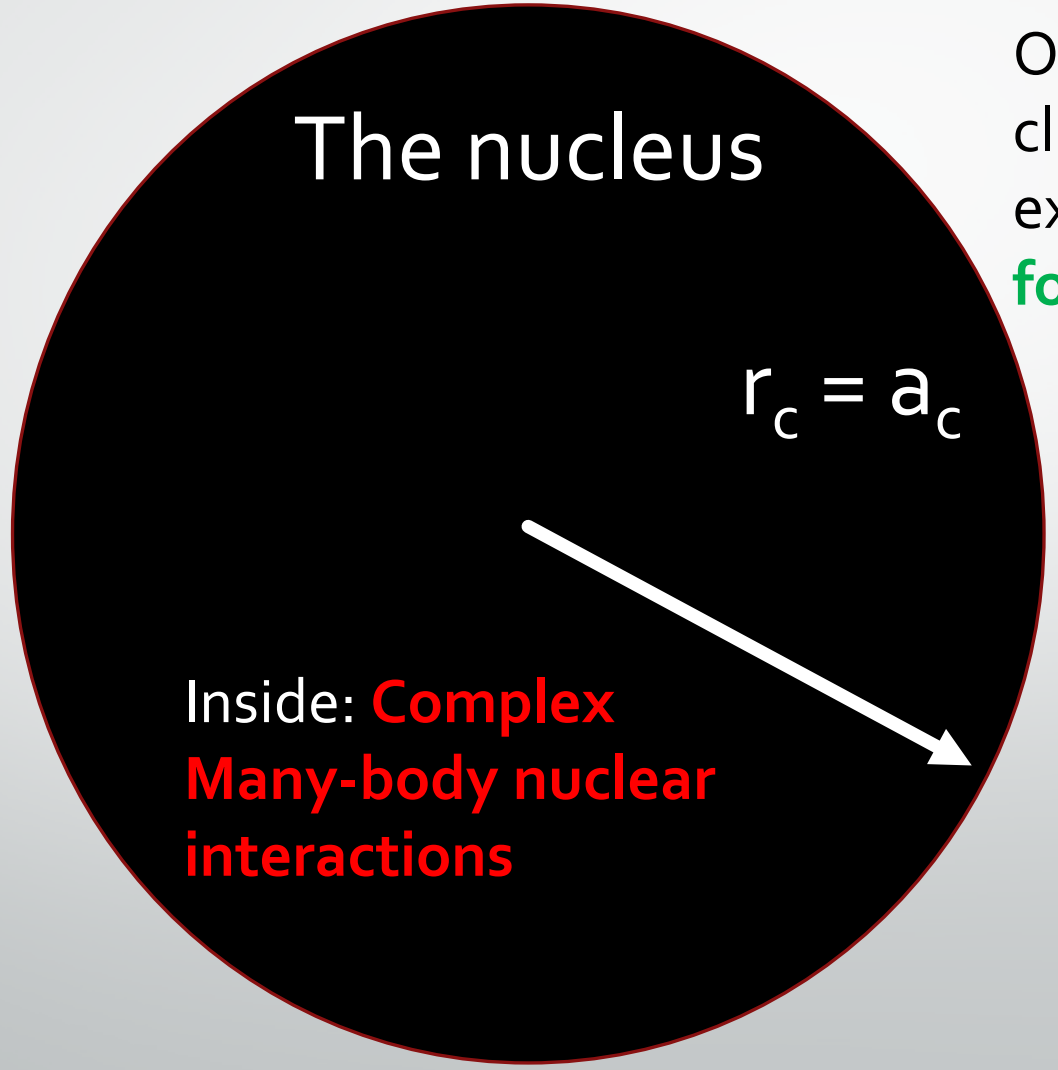
A. M. LANE, *Atomic Energy Research Establishment, Harwell, Berkshire, England*

AND

R. G. THOMAS,* *Los Alamos Scientific Laboratory, Los Alamos, New Mexico*

R-matrix bible

Classic boundary value problem



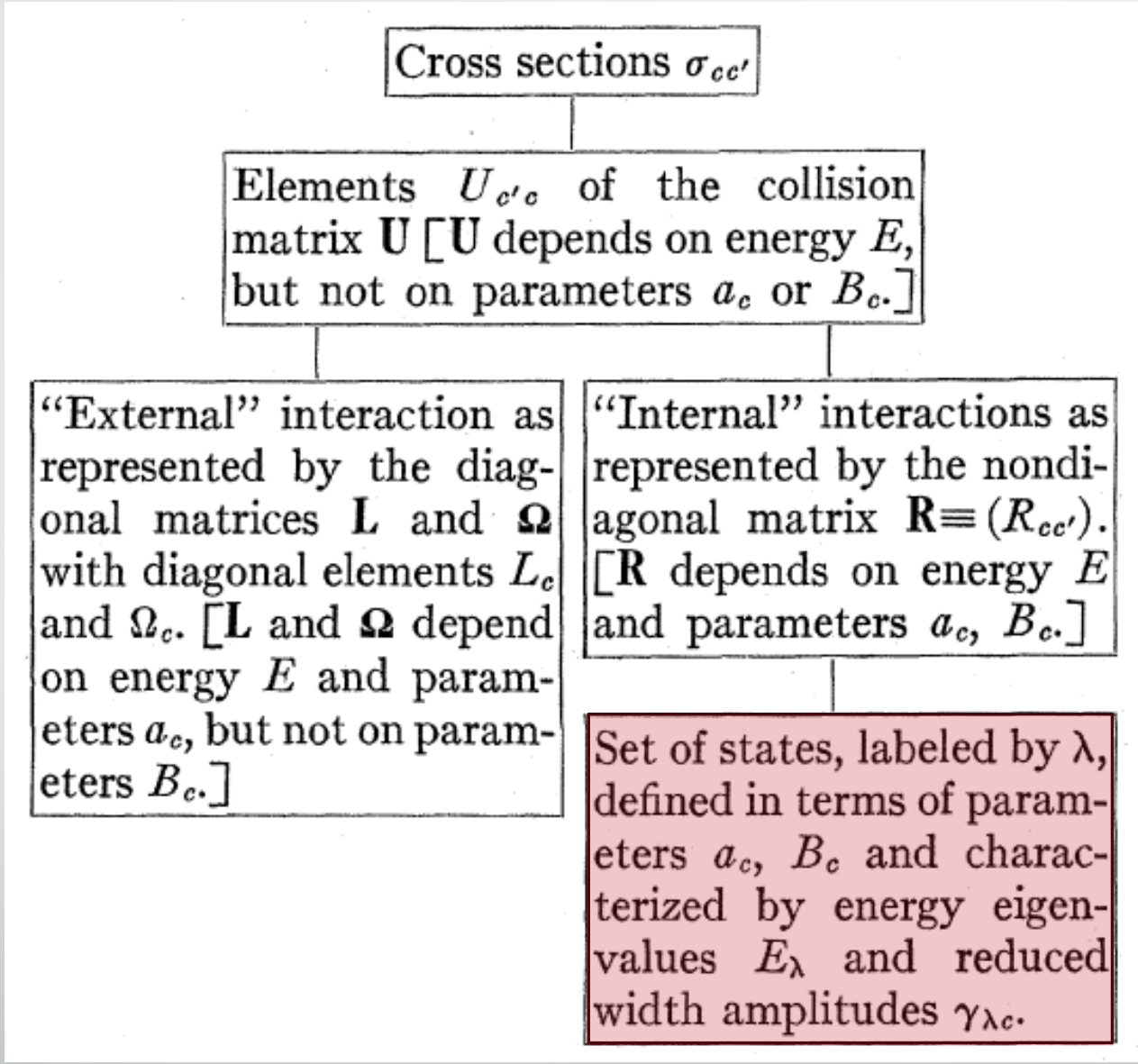
Outside: two particle clusters that do not interact except through the **Coulomb force**

At $r_c > a_c$

$V = V_{Coulomb}$

At $r_c < a_c$

$V = V_?$



This is the only assumption about the form of the potential in the internal region

L&T flow chart to the problem

Probably what you did in classes, but with Coulomb outside and an unknown potential inside

- Solve the Schrödinger equation assuming
  - Point like particles, 3D
  - Two particles in, two particles out
  - Angular momentum and parity conservation
  - Long-range Coulomb interaction
  - Probability conservation (unitarity)
  - Time-reversal invariance
  - **No specific potential specified!**

Wave functions of relative motion

$$H_c\chi = \mathcal{E}\chi,$$

The wave function of relative motion has the form

$$\chi \sim r_\alpha^{-1} u_{\alpha s l}(r_\alpha)(i^l Y_m^{(l)}(\Omega_\alpha)), \qquad (2.5)$$

$$\left[\frac{d^2}{dr_\alpha^2} - \frac{l(l+1)}{r_\alpha^2} - \frac{2M_\alpha}{\hbar^2}(V_{\alpha s l} - E_\alpha)\right] u_{\alpha s l}(r_\alpha) = 0.$$

Match internal and external wave functions and their derivatives at the surface

$$\Psi_{JM} = \sum_\lambda A_{\lambda J} X_{\lambda J M},$$

$$H X_{\lambda J M} = E_{\lambda J} X_{\lambda J M}.$$

$$\gamma_{\lambda c} \equiv (\hbar^2/2M_c a_c)^{\frac{1}{2}} \int \varphi_c^* X_{\lambda J M} dS,$$

Connection with general reaction theory or the collision matrix (U)

Green's theorem, single channel

$$\left(u_2\frac{du_1}{dr}-u_1\frac{du_2}{dr}\right)_{r=a}+\frac{2M}{\hbar^2}(E_1-E_2)\int_0^a u_1u_2dr=0,$$

Boundary condition

$$(du_\lambda/dr)_{r=a}=0.$$

More Green's theorem

$$G(r,a)=\frac{\hbar^2}{2Ma}\sum_\lambda\frac{u_\lambda(r)u_\lambda(a)}{E_\lambda-E}$$

The R-function

$$R=G(a,a)=\sum_\lambda \gamma_\lambda^2/(E_\lambda-E),$$

Generalized to the multichannel case, The R-matrix

$$R_{c'c}=\sum_\lambda \gamma_{\lambda c'}\gamma_{\lambda c}/(E_\lambda-E).$$

Connection to the collision matrix

$$\mathbf{U}^J=\boldsymbol{\rho}^{\frac{1}{2}}\mathbf{O}^{-1}(1-\mathbf{R}^J\mathbf{L}^0)^{-1}(1-\mathbf{R}^J\mathfrak{L}^0)\mathbf{I}\boldsymbol{\rho}^{-\frac{1}{2}}$$

All these other matrices besides R can be calculated from special functions

Transition Matrix (T-matrix)

$$T_{\alpha's'l',\alpha sl}{}^J=e^{2i\omega_{\alpha'l'}}\delta_{\alpha's'l',\alpha sl}-U_{\alpha's'l',\alpha sl}{}^J.$$

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E},$$

this squared

The (unpolarized differential) cross section

$$(2s+1)\frac{k_\alpha^2}{\pi} d\sigma_{\alpha s,\,\alpha' s'} d\Omega_{\alpha'} = (2s+1)\left|C_{\alpha'}(\theta_{\alpha'})\right|^2 \delta_{\alpha' s',\,\alpha s}$$

$$+ \sum_{\substack{J_1 J_2 M_1 M_2 \\ l_1 l_2 l_1' l_2' \\ \nu\nu' m_1' m_2'}} (2l_1+1)^{\frac{1}{2}}(2l_2+1)^{\frac{1}{2}}(sl_1\nu 0\,|\,J_1 M_1)$$

$$\times (sl_2\nu 0\,|\,J_2 M_2)(s' l_1' \nu' m_1'\,|\,J_1 M_1)(s' l_2' \nu' m_2'\,|\,J_2 M_2)$$

$$\times (T_{\alpha' s' l_1',\,\alpha s l_1}{}^{J_1} Y_{m_1'}{}^{(l_1')}(\Omega_{\alpha'}))$$

$$\times (T_{\alpha' s' l_2',\,\alpha s l_2}{}^{J_2} Y_{m_2'}{}^{(l_2')}(\Omega_{\alpha'}))^*$$

$$+ \sum_{\substack{JMll' \\ m'\nu\nu'}} (2l+1)^{\frac{1}{2}}(sl\nu 0\,|\,JM)(s' l' \nu' m'\,|\,JM)$$

$$\times \delta_{\alpha' s' \nu',\,\alpha s \nu} 2\,\mathrm{Re}\left[i T_{\alpha' s' l',\,\alpha s l}{}^{J} Y_{m'}{}^{(l')}(\Omega_{\alpha'}) C_{\alpha'}(\theta_{\alpha'})\right].$$

Coulomb term

Resonance term

Interference term

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E},$$

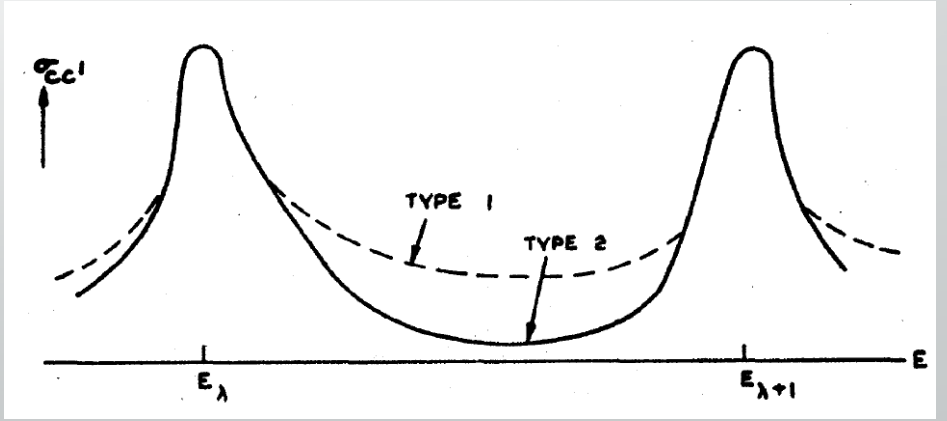
The parameters of the *R*-matrix are what we are normally varying as fit parameters to experimental data.

$\gamma_{\lambda c}$ Reduced width amplitudes

E_λ Pole energies

Remember λ is the level index and c is the channel index.

Also, $\gamma_{\lambda c}$ can be positive or negative and its sign determines the interference pattern between multiple resonances!



Lane and Thomas, XII.5

Primary γ-ray transitions

- Often implemented using perturbation theory
  - Works in cases where the electromagnetic part of the interaction is small compared to the hadronic part
  - Alternative approaches include Reich-Moore and relativistic wave functions
- Can include a direct capture model
  - External (potential) Capture

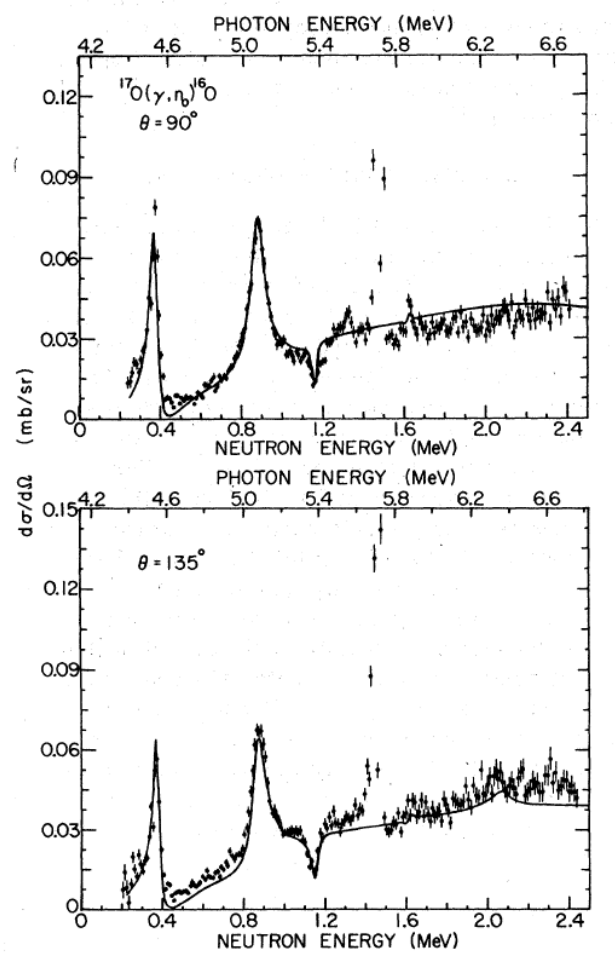


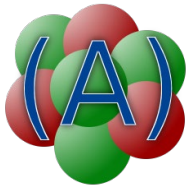
FIG. 5. Final R-matrix analysis of the observations of the $^{17}O(\gamma,n_0)^{16}O$ reaction. The parameters used in this analysis, Tables III and IV, also describe the $^{16}O(n,n)^{16}O$ reaction. The interference minimum at 5.38 MeV is not as deep as that given by the simple hard-sphere model.

- First used by Holt *et al*. (1978) to fit $^{17}O(n,\gamma)^{16}O$ data
- Used by Barker and Kajino (1991) to fit $^{12}C(\alpha,\gamma)^{16}O$
- Used by Angulo and Descouvemont (2001) to fit $^{14}N(p,\gamma)^{15}O$
- Detailed description in deBoer *et al*. (2017) by Carl Brune
- **Mathematic implementation is rather cumbersome**

The AZURE2 code: an R-matrix code for nuclear astrophysics

- Originally developed by R.E. Azuma under the Joint Institute for Nuclear Astrophysics (JINA), published 2010

- AZURE2 developed by Ethan Uberseder, made publically available in 2012

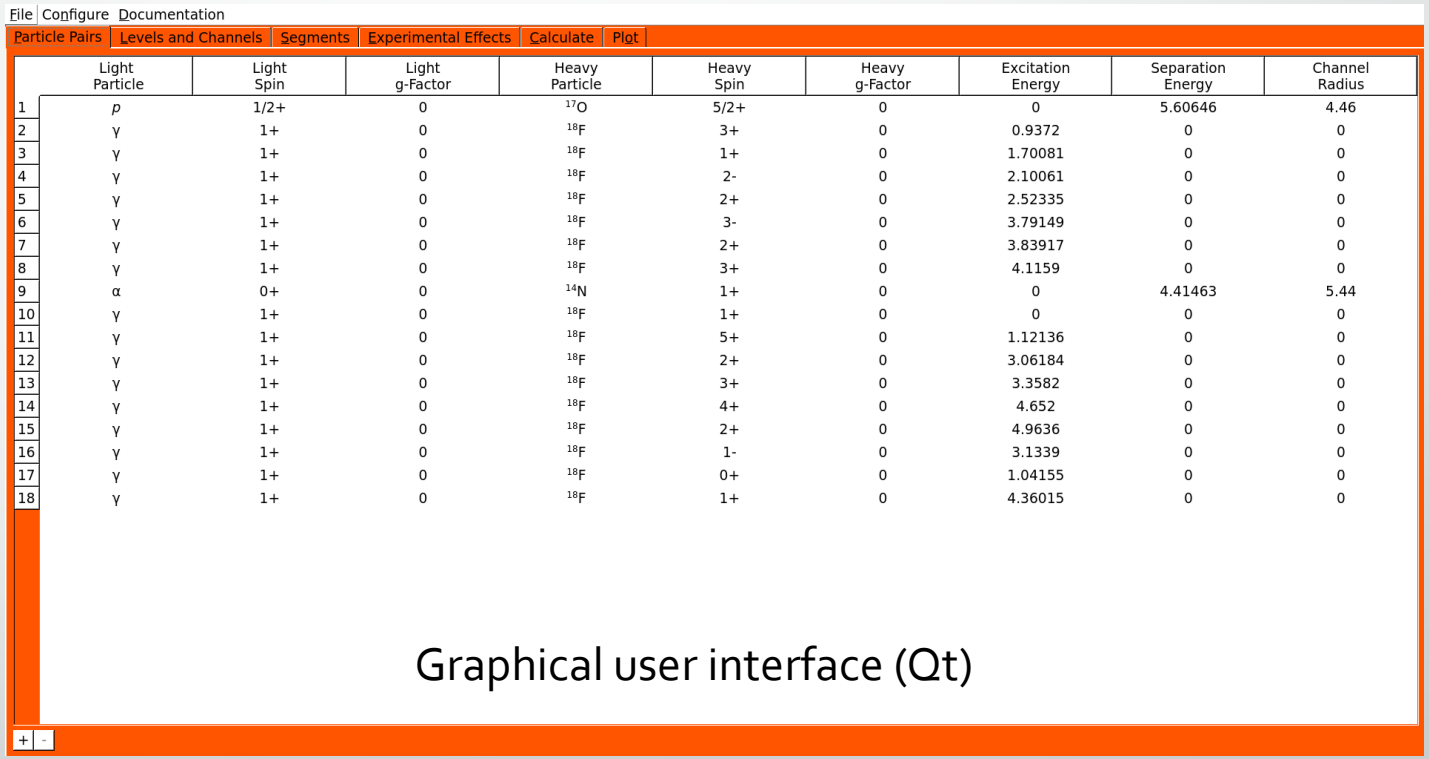


- azure.nd.edu

- **Developed benchmark to increase the fidelity of R-matrix calculations in the community**

What are AZURE2's main capabilities?

- Designed originally for charged particle reactions and for extrapolations to low energy
  - $^{12}C(\alpha,\gamma)^{16}O$ was the benchmark reaction
- Radiative capture
- Simultaneous multiple exit and entrance channel fits (AZURE2)



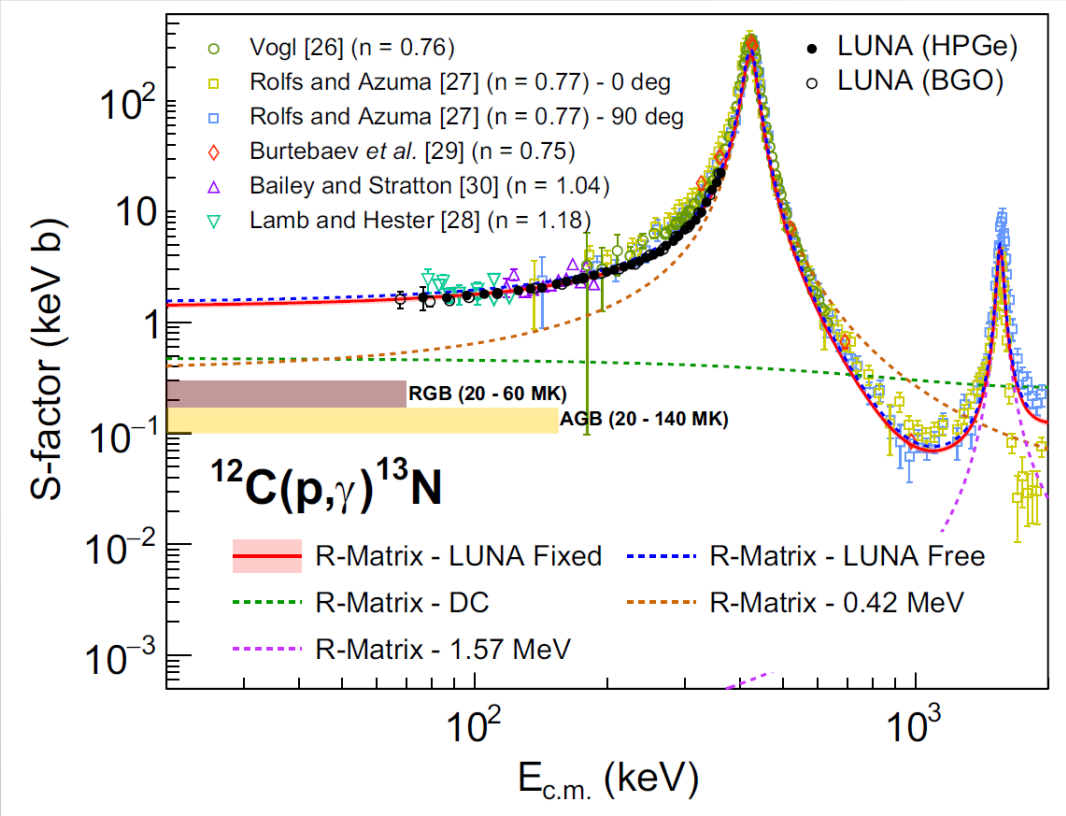
	Light Particle	Light Spin	Light g-Factor	Heavy Particle	Heavy Spin	Heavy g-Factor	Excitation Energy	Separation Energy	Channel Radius
1	*p*	1/2+	0	^{17}O	5/2+	0	0	5.60646	4.46
2	γ	1+	0	^{18}F	3+	0	0.9372	0	0
3	γ	1+	0	^{18}F	1+	0	1.70081	0	0
4	γ	1+	0	^{18}F	2-	0	2.10061	0	0
5	γ	1+	0	^{18}F	2+	0	2.52335	0	0
6	γ	1+	0	^{18}F	3-	0	3.79149	0	0
7	γ	1+	0	^{18}F	2+	0	3.83917	0	0
8	γ	1+	0	^{18}F	3+	0	4.1159	0	0
9	α	0+	0	^{14}N	1+	0	0	4.41463	5.44
10	γ	1+	0	^{18}F	1+	0	0	0	0
11	γ	1+	0	^{18}F	5+	0	1.12136	0	0
12	γ	1+	0	^{18}F	2+	0	3.06184	0	0
13	γ	1+	0	^{18}F	3+	0	3.3582	0	0
14	γ	1+	0	^{18}F	4+	0	4.652	0	0
15	γ	1+	0	^{18}F	2+	0	4.9636	0	0
16	γ	1+	0	^{18}F	1-	0	3.1339	0	0
17	γ	1+	0	^{18}F	0+	0	1.04155	0	0
18	γ	1+	0	^{18}F	1+	0	4.36015	0	0

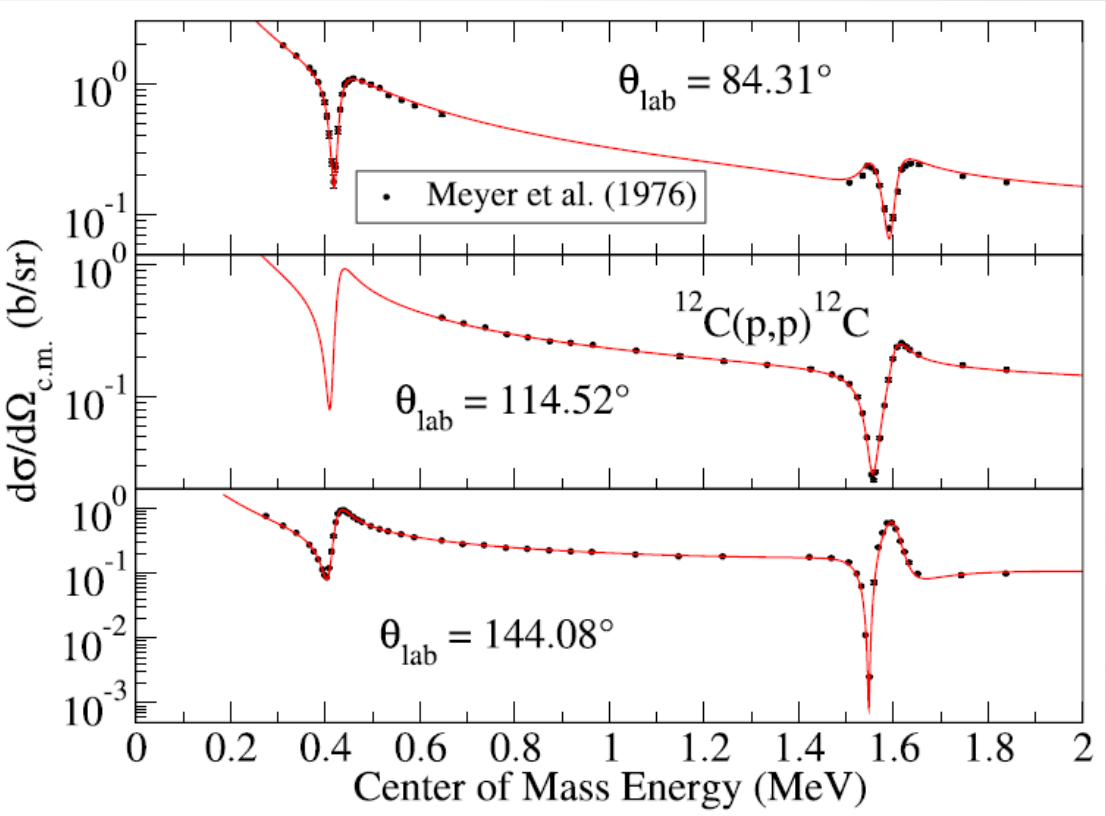
Graphical user interface (Qt)

What kind of reactions are applicable for R-matrix?

- Usually Compound Nucleus
  - Two step reactions (a+A → C* → b+B)
    - No three body yet…
  - Usually nucleon – nucleon reactions
    - But can also be extended to capture (a+A → C* + γ)
    - And even (C* + β → b+B)
- Usually low energy
  - Low angular momentum dominates
- Usually low level density
  - Practical limitations
- Useful when broad resonances are present in the cross section

Example, ^{12}C+p





- Skowronski *et al*. (2023) and Kettner *et al*. (2023)
- A few well separated resolved resonances
- Well determined spin-parities (J^{π})

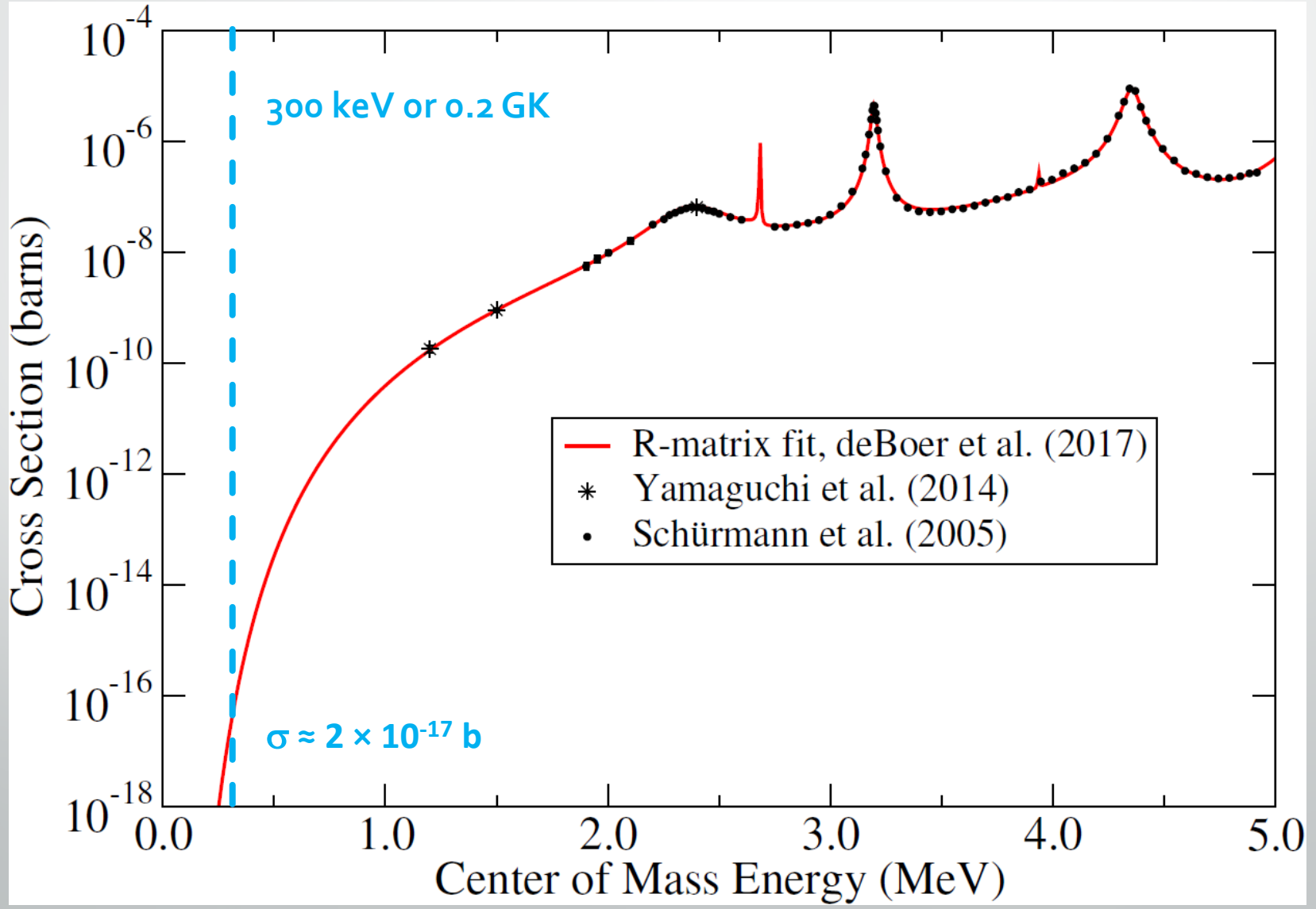
Level Structure of ^{13}N

E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m *4* % ε = 100
2364.9 *6*	DEFGHI K	1/2+	31.7 keV *8* % IT = 0.00158 *13* % p = 100
3502 *2*	A DEFGHIJK	3/2-	62 keV *4* % IT = 0.0011 % p = 100
3547 *4*	D FGHI K	5/2+	47 keV 7 % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100
7155 *5*	CD FGH K	7/2+	9.0 keV *5* % p = 100
7376 *9*	A CD FGHIJKL	5/2-	75 keV *5* % p = 100
7900	FG	3/2+	≈ 1500 keV % p = 100
8918 *11*	A D FG IJ L	1/2-	230 keV % p = 100
9000	C GH J	9/2+	280 keV *30*
9476 *8*	A CD FGH J	3/2-	30 keV % p = 100
1025E+1 *15*	E	(1/2+)	≈ 280 keV % IT = ? % p = ?
10360	A CD FGH	5/2-	30 keV % p = 100

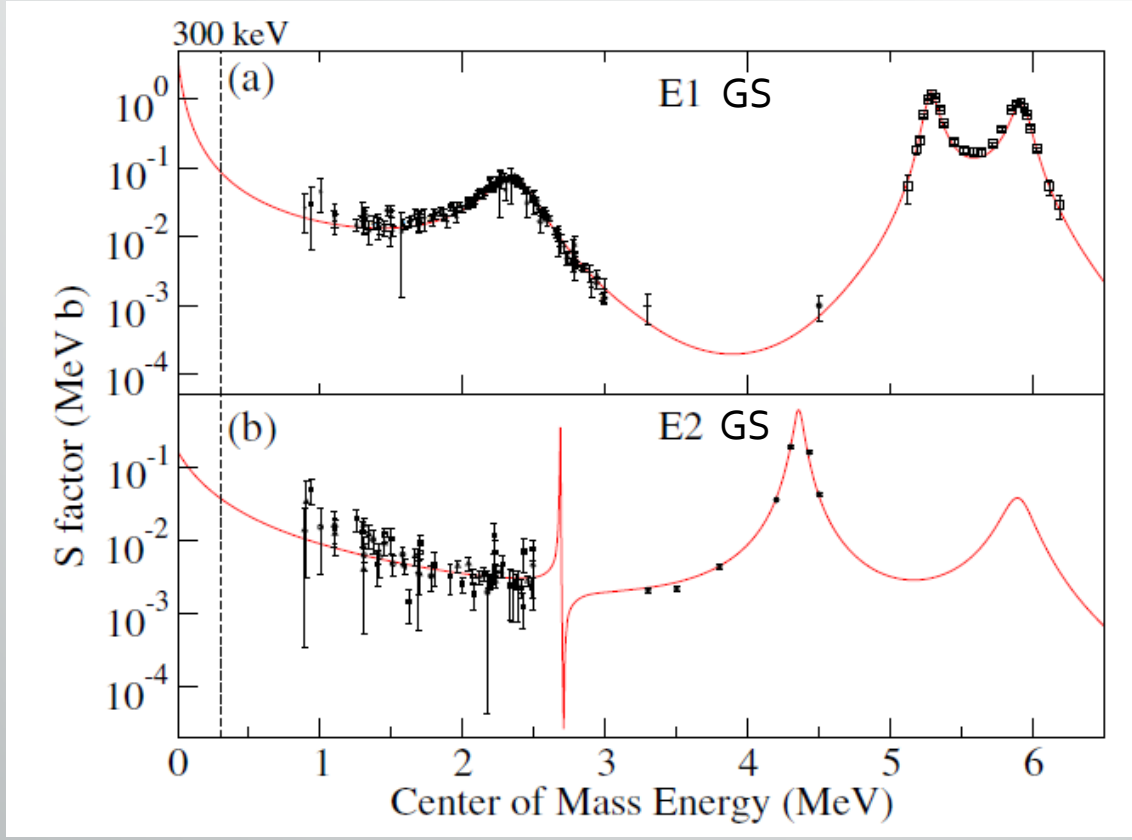
Almost 3 MeV gap

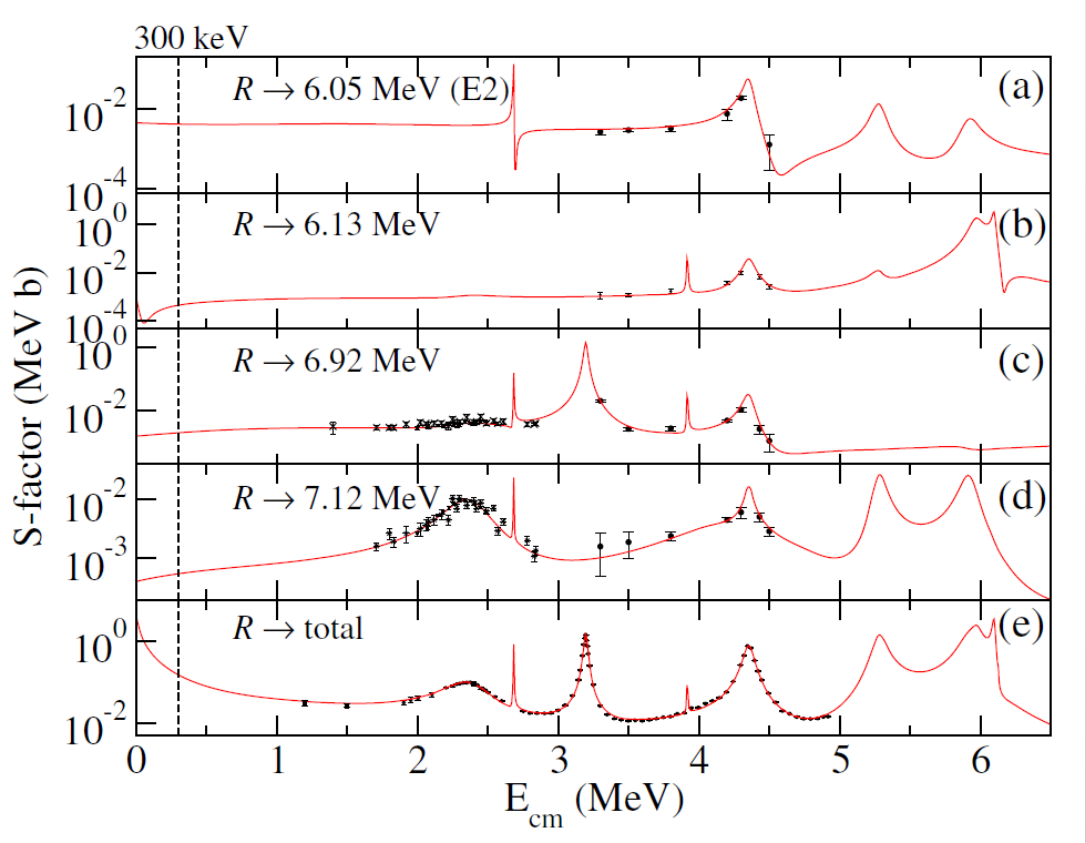
- **The ideal case for R-matrix**
- Low level density
- Isolated set of levels near threshold
- Resonances are broad enough to easily measure but not too broad so that their shape can't be easily discerned

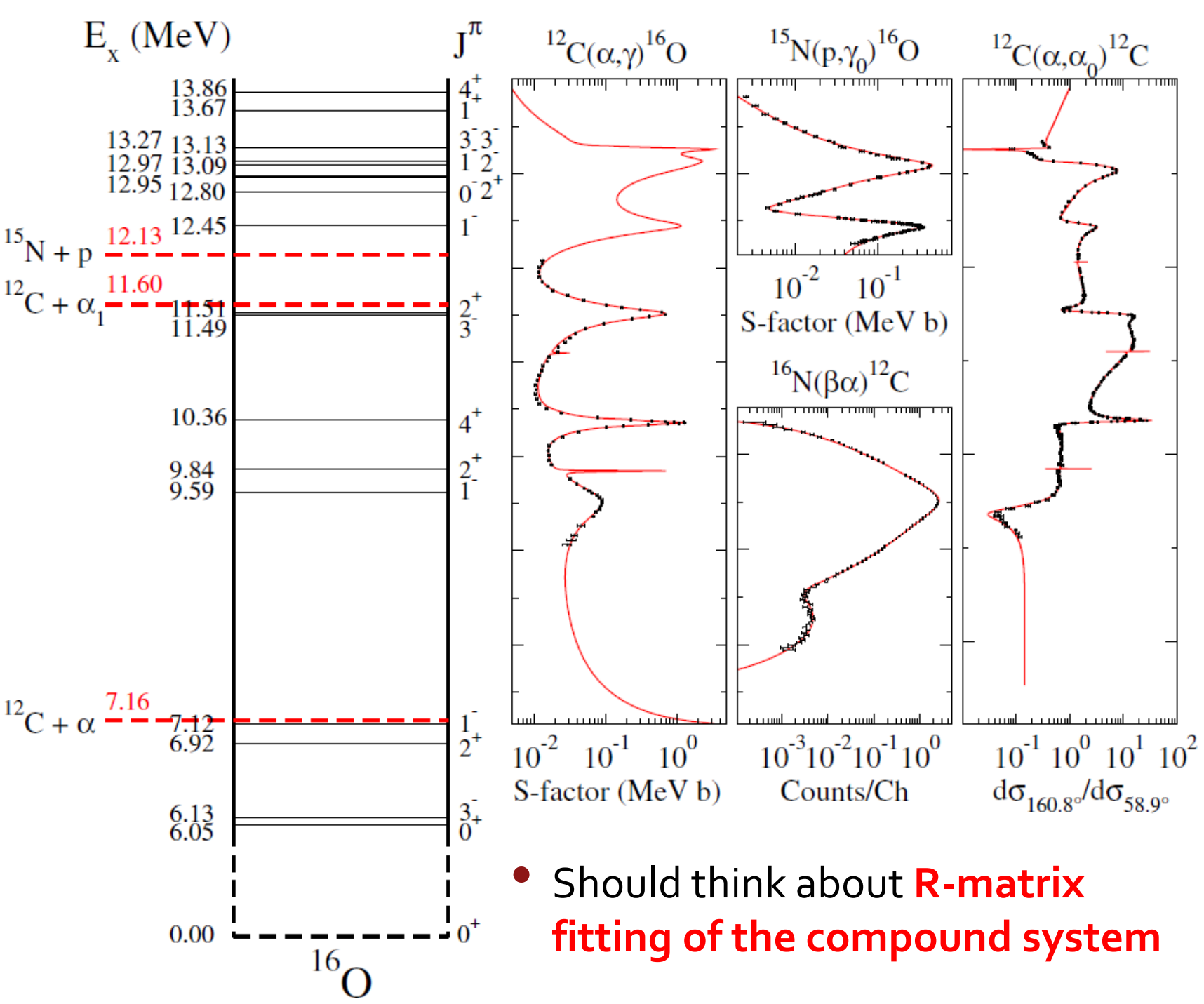
Example: $^{12}C(\alpha,\gamma)^{16}O$ extrapolation to low energy



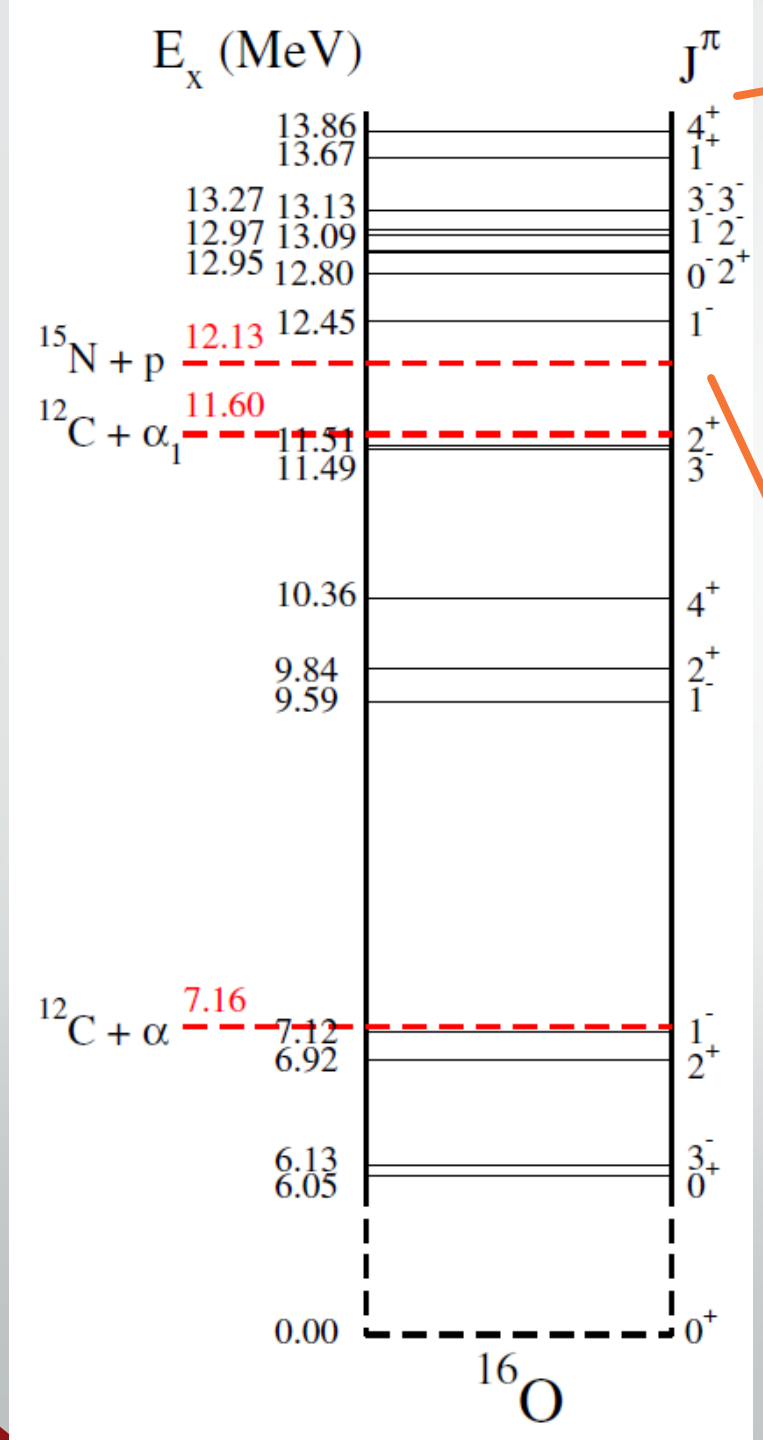
Several γ-decay channels



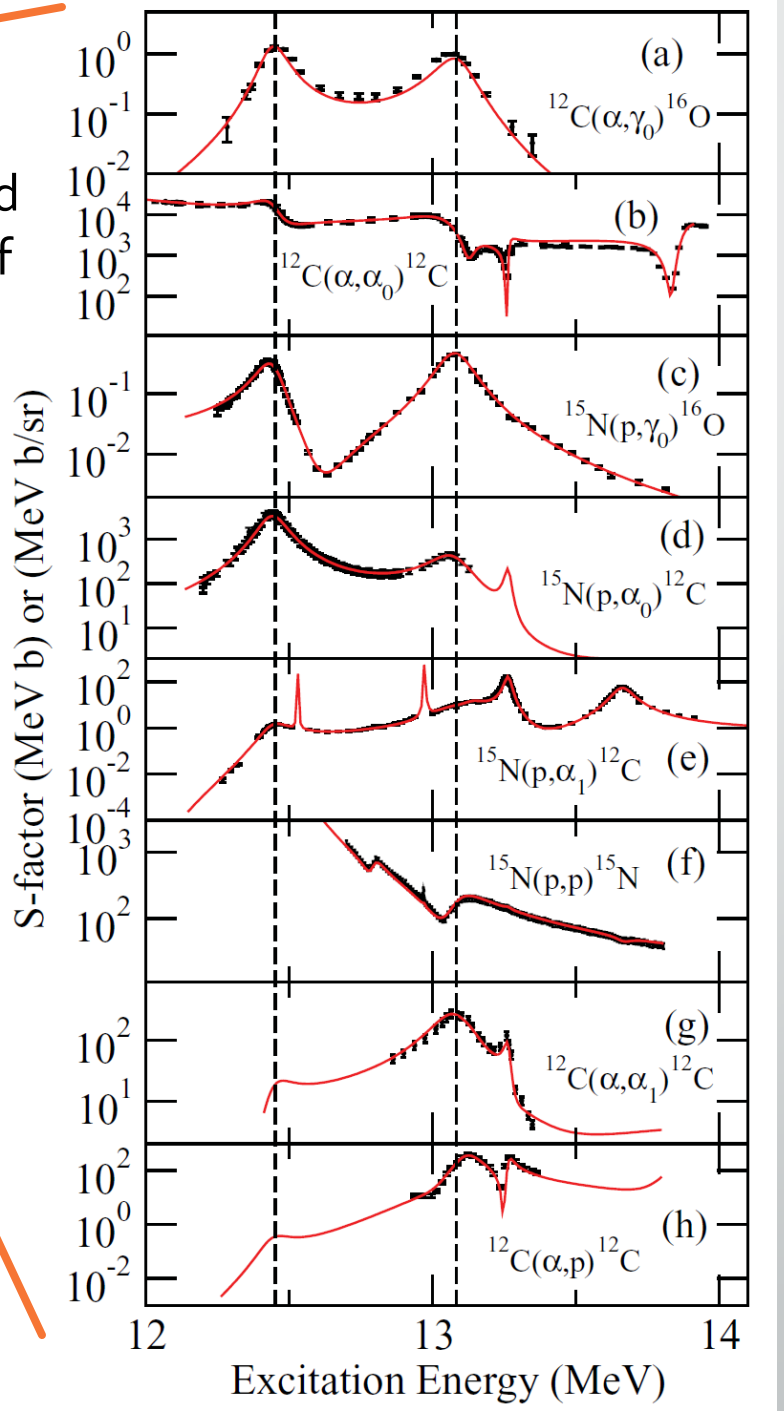




- Should think about **R-matrix fitting of the compound system**

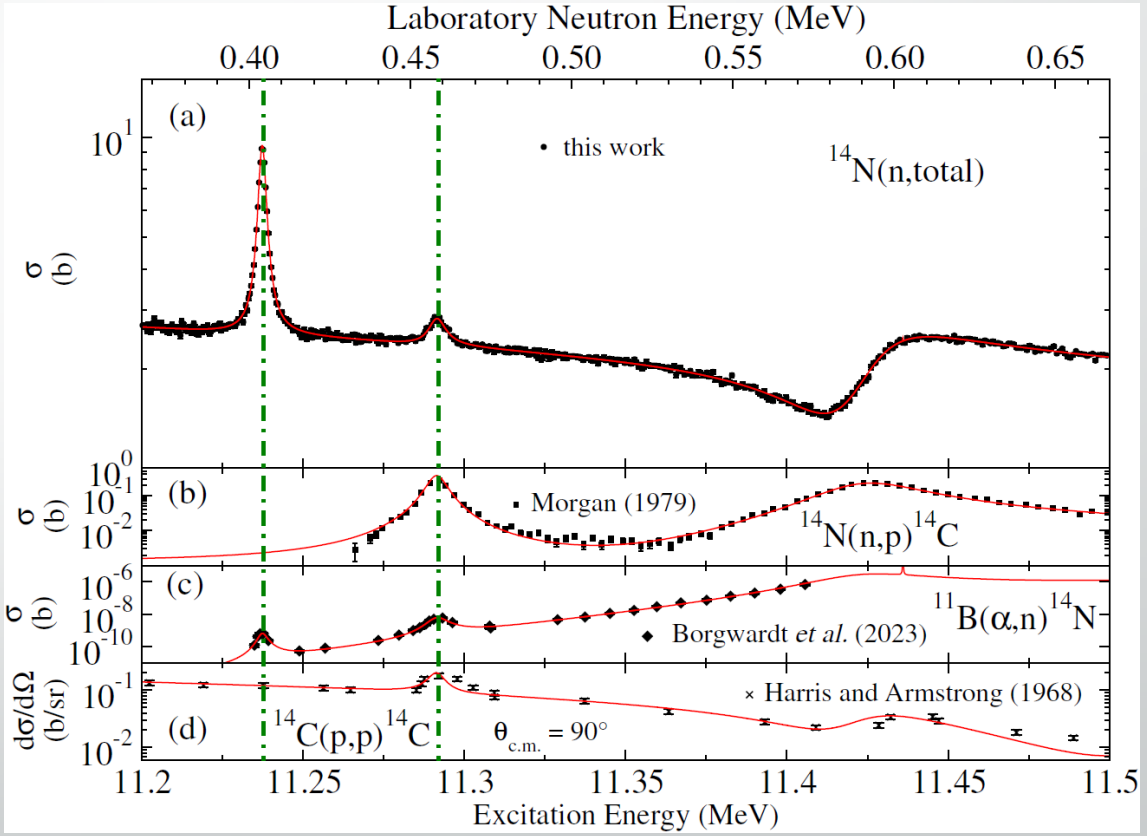


Same level parameters used to describe all of this data simultaneously!



Uncertainty, uncertainty, uncertainty

- When we fit multiple data sets simultaneously it helps us **understand systematic uncertainties**
  - Do we have the energy scale calculated correctly (shared excitation energy)?
  - Do we have the cross section scale correct (unitarity)?
- Of course, the price is that it is a lot more work to fit all of this data together



Uncertainty, uncertainty, uncertainty

BRICK

The Bayesian R-matrix Inference Code Kit (BRICK) mediates between the R-matrix code AZURE2 and the MCMC sampling software emcee, in order to facilitate uncertainty quantification in R-matrix calculations.

D. Odell, R. J. deBoer, C. R. Brune, D. R. Phillips

BRICK tutorial

BAND

Bayesian Analysis of Nuclear Dynamics

NSF, CSSI program

ORIGINAL RESEARCH article

Front. Phys. , 12 June 2022

Sec. Nuclear Physics

Volume 10 - 2022 | https://doi.org/10.3389/fphy.2022.888476

This article is part of the Research Topic
Uncertainty Quantification in Nuclear Physics

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Performing Bayesian Analyses With AZURE2 Using BRICK: An Application to the ^{7}Be System

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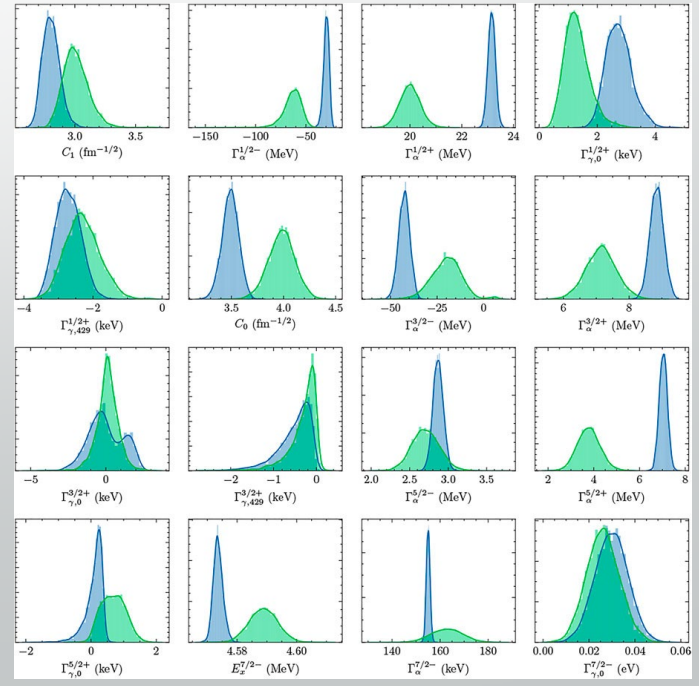
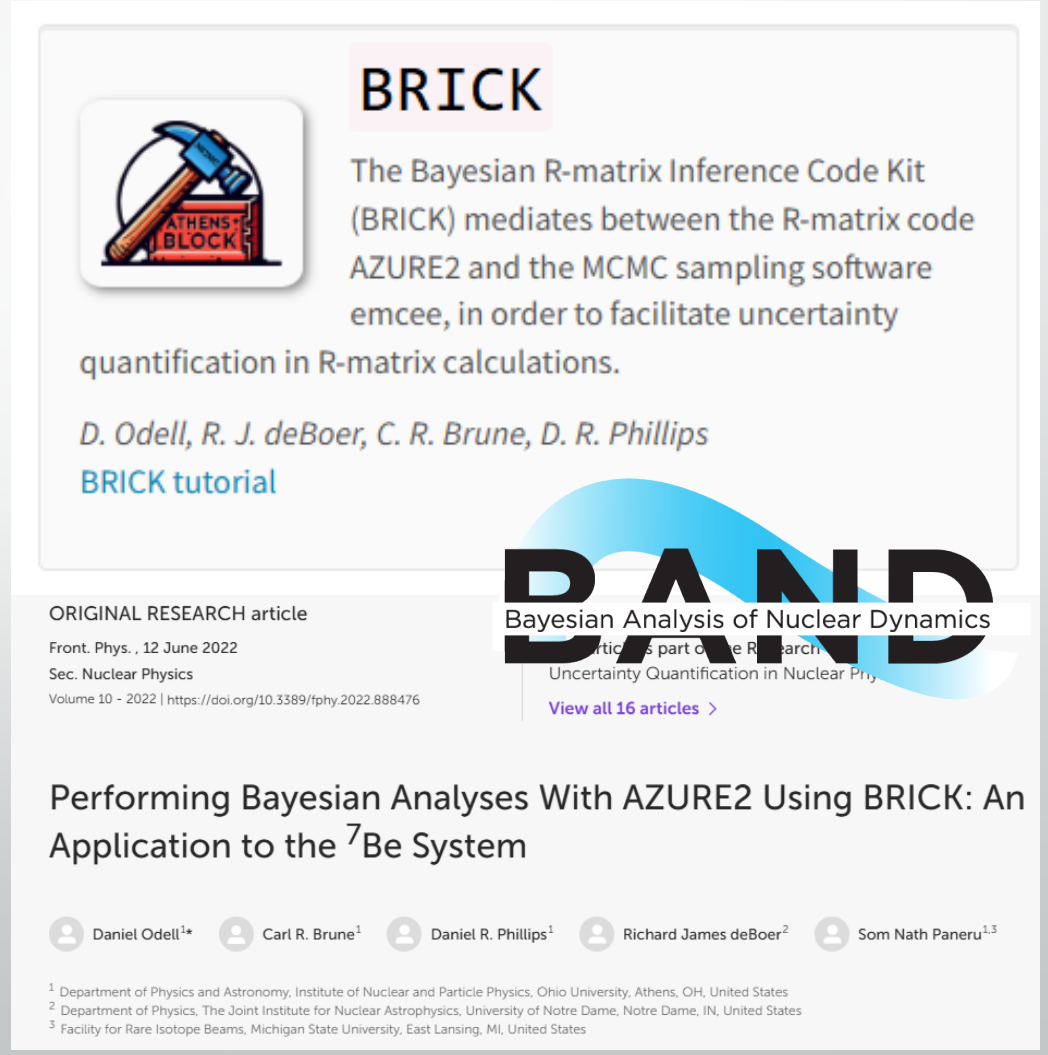
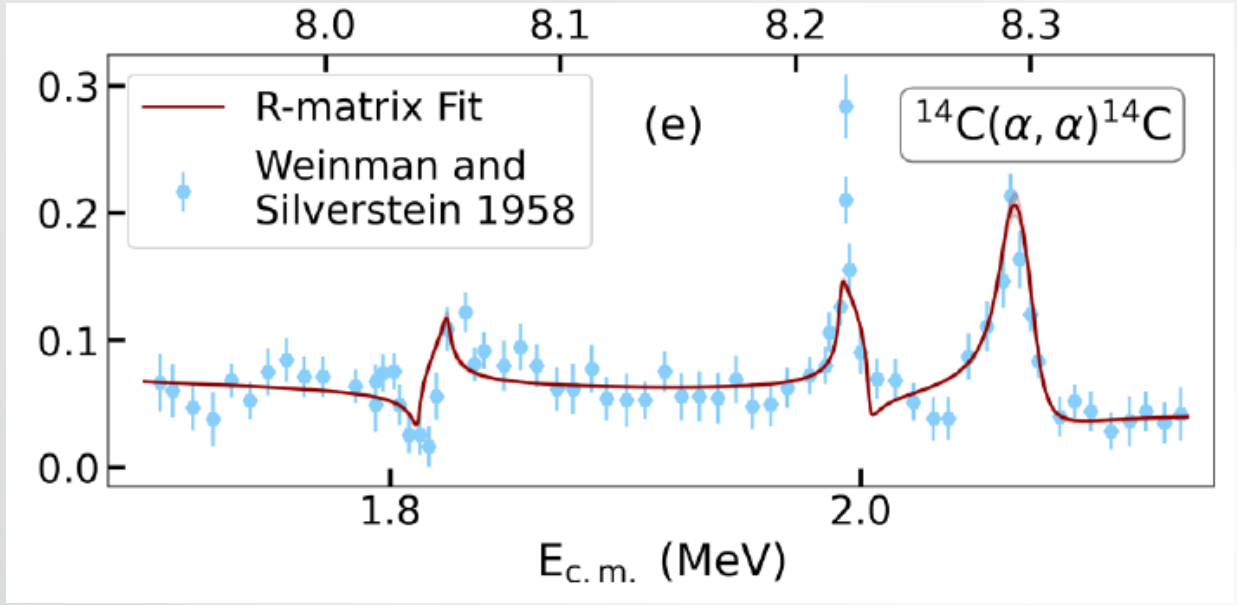
[2] Department of Physics, The Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, IN, United States
[3] Facility for Rare Isotope Beams, Michigan State University, East Lansing, MI, United States

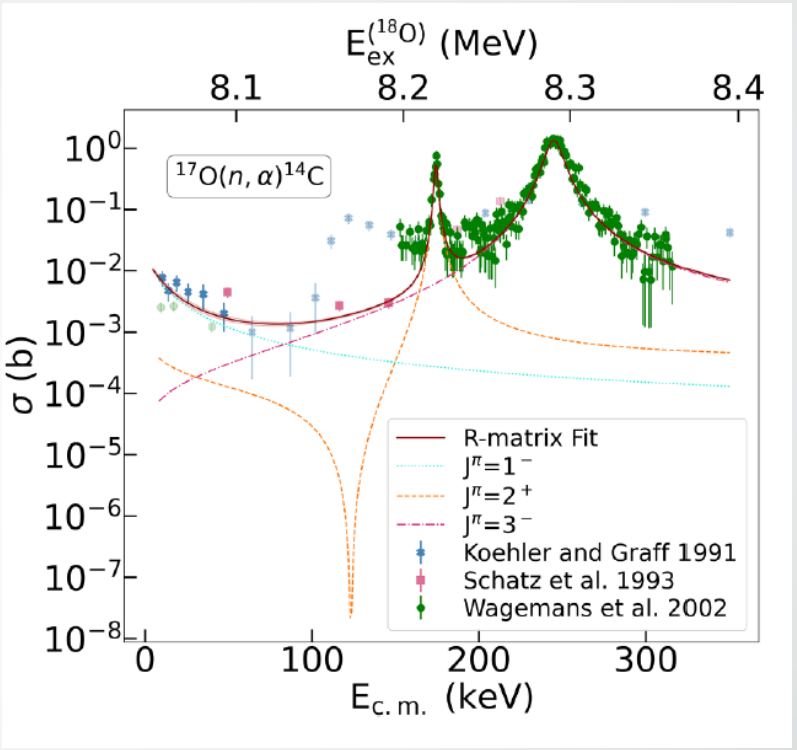


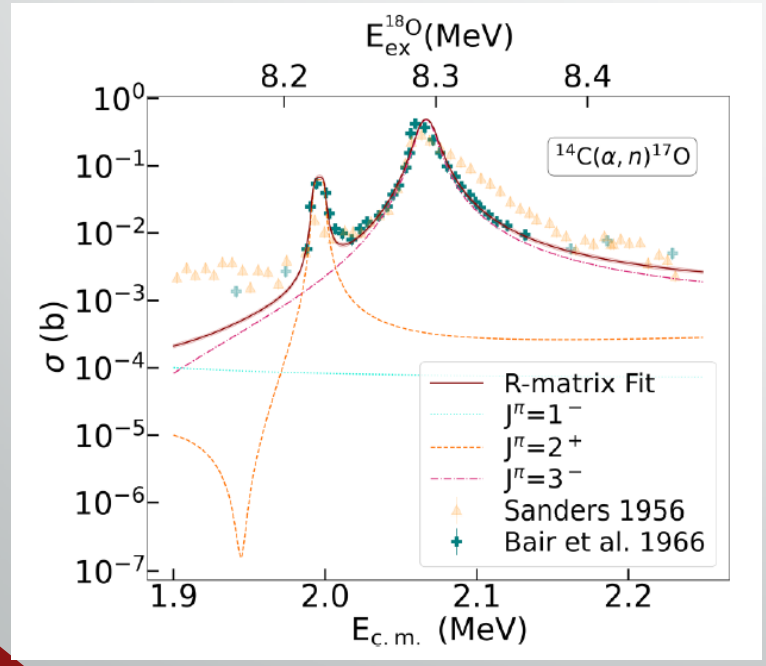
Improving uncertainty estimation for R-matrix fits

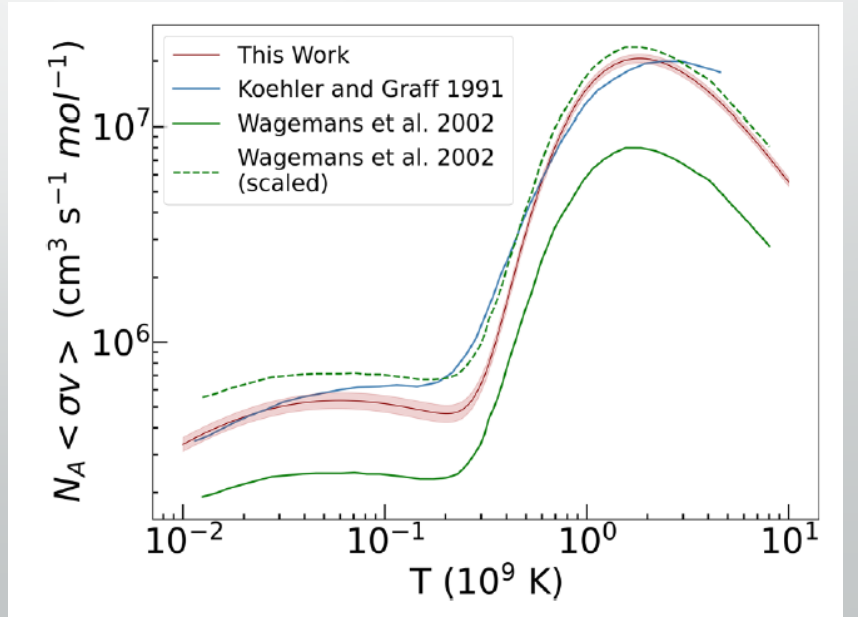
- **Bayesian** methods provide a way to improve and gain more detailed information
- See de **Souza et al. (2020)** for an application to $^{3}H(d,n)^{4}He$
- Computationally intensive, but probably doable
- **Daniel Odell** at Ohio University has developed the Bayesian R-matrix Inference Code Kit (**BRICK**) for use with the **AZURE2** R-matrix code




R-matrix Fit
Weinman and Silverstein 1958
(e)
$^{14}C(\alpha,\alpha)^{14}C$
$E_{c.m.}$ (MeV)


$E_{ex}^{(18O)}$ (MeV)
$^{17}O(n,\alpha)^{14}C$
R-matrix Fit
$J^{\pi}=1^{-}$
$J^{\pi}=2^{+}$
$J^{\pi}=3^{-}$
Koehler and Graff 1991
Schatz et al. 1993
Wagemans et al. 2002
σ (b)
$E_{c.m.}$ (keV)


E_{ex}^{18O}(MeV)
$^{14}C(\alpha,n)^{17}O$
R-matrix Fit
$J^{\pi}=1^{-}$
$J^{\pi}=2^{+}$
$J^{\pi}=3^{-}$
Sanders 1956
Bair et al. 1966
σ (b)
$E_{c.m.}$ (MeV)


This Work
Koehler and Graff 1991
Wagemans et al. 2002
Wagemans et al. 2002 (scaled)
$N_A < \sigma v > (cm^3\ s^{-1}\ mol^{-1})$
T (10^9 K)

Priors

Parameter	Distribution
$E_{ex}(1^-, 0)$	$N(8.04, 0.02)$
$C_n(1^-, 0)$	$U(0.0001, 0.1)$
$\Gamma_\alpha(1^-, 0)$	$N(2000, 700)$
$C_n(2^+, 0)$	$U(0, 10)$
$C_\alpha(2^+, 0)$	$U(-4000, 4000)$
$E_{ex}(2^+, 1)$	$N(8.22, 0.01)$
$\Gamma_n(2^+, 1)$	$N(200, 100)$
$\Gamma_\alpha(2^+, 1)$	$N(-1700, 100)$
$E_{ex}(3^-, 0)$	$N(8.29, 0.006)$
$\Gamma_n(3^-, 0)$	$N(-5600, 700)$
$\Gamma_\alpha(3^-, 0)$	$N(-2900, 200)$
n_{WAG}	$U(0.1, 50)$
n_{SCH}	$U(0.1, 50)$
n_{KOE}	$N(1, 0.029)$
n_{BAI}	$U(0.1, 50)$
n_{WEI}	$U(0.1, 50)$
TT_{BAI}	$U(3.8 \times 10^{+17}, 2 \times 10^{+17})$
TT_{WEI}	$U(3 \times 10^{+17}, 5 \times 10^{+17})$
ES_{WEI}	$U(-0.2, 0.4)$

Posteriors

Parameter	Posterior median ± C.I.
$E_{ex}(1^-, 0)$	$8.03885\ (+4.4 \times 10^{-4}, -4.6 \times 10^{-4})$
$C_n(1^-, 0)$	$4.92 \times 10^{-3}\ (+6.5 \times 10^{-4}, -5.6 \times 10^{-4})$
$\Gamma_\alpha(1^-, 0)$	$2040\ (+220, -220)$
$C_n(2^+, 0)$	$0.47\ (+0.17, -0.10)$
$C_\alpha(2^+, 0)$	$-1440\ (+400, -450)$
$E_{ex}(2^+, 1)$	$8.218552\ (+9.4 \times 10^{-5}, -9.5 \times 10^{-5})$
$\Gamma_n(2^+, 1)$	$219\ (+17, -16)$
$\Gamma_\alpha(2^+, 1)$	$-1925\ (+75, -75)$
$E_{ex}(3^-, 0)$	$8.28921\ (+2.0 \times 10^{-4}, -2.1 \times 10^{-4})$
$\Gamma_n(3^-, 0)$	$-8400\ (+330, -330)$
$\Gamma_\alpha(3^-, 0)$	$-3250\ (+130, -130)$
n_{WAG}	$2.87\ (+0.13, -0.13)$
n_{SCH}	$1.29\ (+0.21, -0.16)$
n_{KOE}	$1.007\ (+2.9 \times 10^{-2}, -2.8 \times 10^{-2})$
n_{BAI}	$1.234\ (+7.1 \times 10^{-2}, -6.4 \times 10^{-2})$
n_{WEI}	$0.9231\ (+8.1 \times 10^{-3}, -7.7 \times 10^{-3})$
TT_{BAI}	$4.68 \times 10^{+17}\ (+2.4 \times 10^{+16}, -2.5 \times 10^{+16})$
TT_{WEI}	$6.26 \times 10^{+17}\ (+2.9 \times 10^{+16}, -3.0 \times 10^{+16})$
ES_{WEI}	$2.43 \times 10^{-3}\ (+3.8 \times 10^{-4}, -4.1 \times 10^{-4})$

Correlation matrix for fit parameters

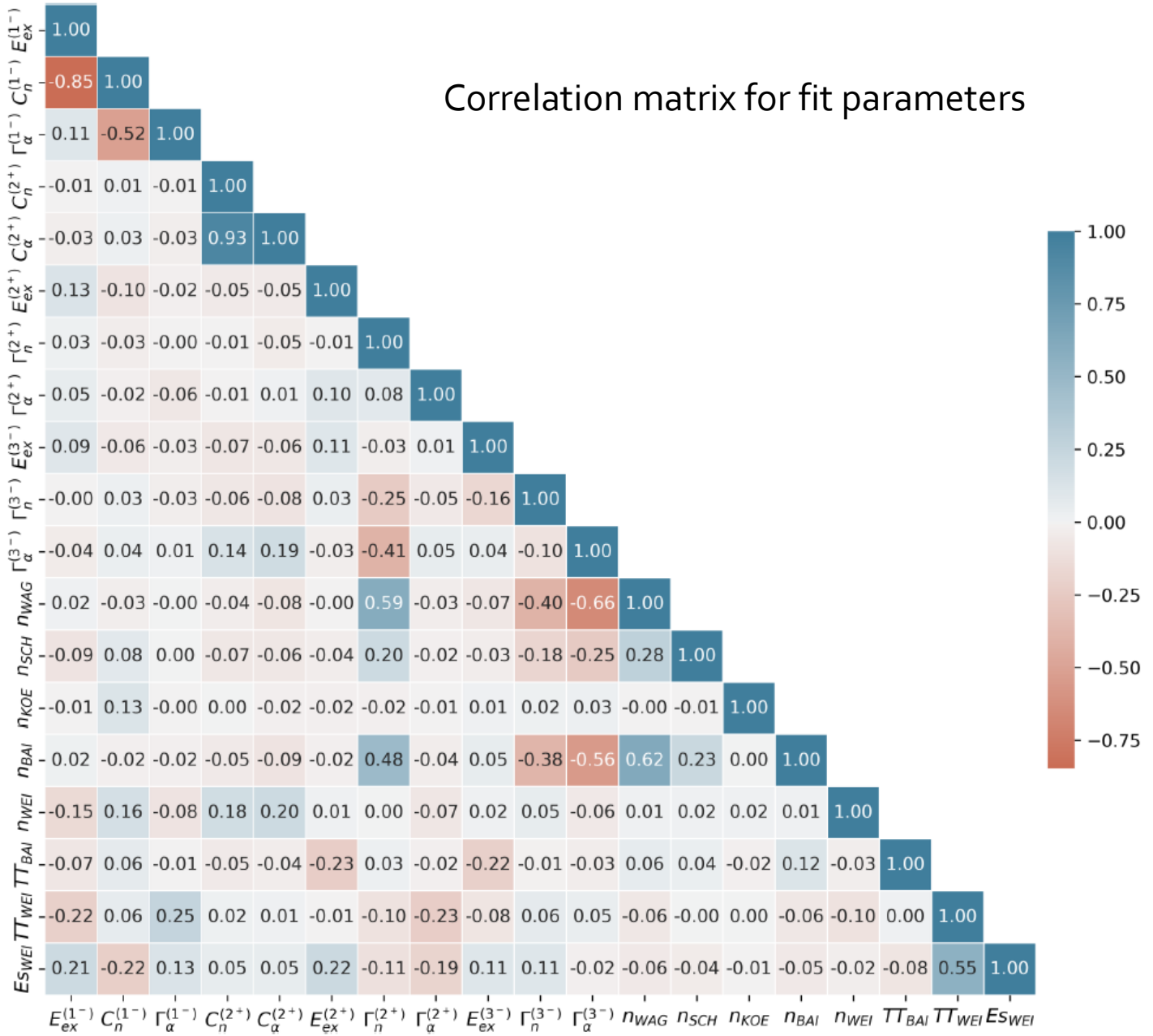


FIG. 7. Covariance matrix for the *R*-matrix fit parameters.

What's there left to do? A lot! Parameter transformations

PHYSICAL REVIEW C **66**, 044611 (2002)

Alternative parametrization of *R*-matrix theory

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(Receved 15 July 2002; published 24 October 2002)

An alternative parametrization of ***R***-matrix theory is presented which is mathematically equivalent to the standard approach, but possesses features that simplify the fitting of experimental data. In no level shifts and no boundary-condition constants which allow the positions and partial w number of levels to be easily fixed in an analysis. These alternative parameters can be c ***R***-matrix parameters by a straightforward matrix diagonalization procedure. In additic express the collision matrix directly in terms of the alternative parameters.

Definite complete invariant parametrization of *R*-matrix theory

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partment of Nuclear Engineering, University of Tennessee, 53 Neyland Drive, Knoxville, Tennessee 37996-2300, USA

R-matrix theory with level-dependent boundary condition parameters

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(Received 16 October 2020; revised 18 October 2021; accepted 30 November 2021; published 13 December 2021)

I present a new formalism of the *R*-matrix theory where the formal parameters for the resonance energies and widths are identical to the observed values. By allowing the boundary condition parameters to vary from level to level, the freedom required to adjust the formal parameters for the pole positions to the observed values is obtained. The basis of the resulting theory becomes nonorthogonal, and I describe the procedure to construct a consistent *R*-matrix theory with such a nonorthogonal basis. And by adjusting the normalization of the states that form the basis, the formal parameters for the reduced decay widths also become the same as those observed, leaving no formal parameters that are different from the observed ones. A demonstration of the developed theory to the elastic $^{12}C+p$ scattering data is presented.

What's there left to do?
Secondary γ-rays following particle emission

Secondary γ-ray decays from the partial-wave T matrix with an R-matrix application to $^{15}\mathrm{N}(p,\alpha_1\gamma)\,^{12}\mathrm{C}$

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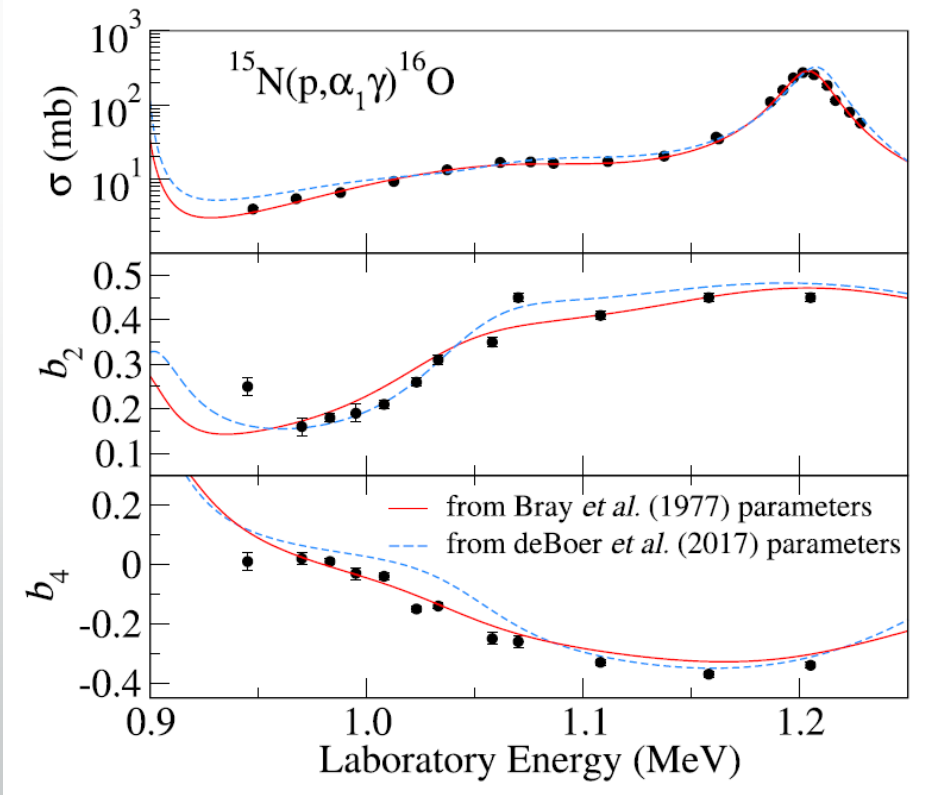
(Received 8 April 2020; accepted 13 August 2020; published 31 August 2020)

The secondary γ rays emitted following a nuclear reaction are often relatively straightforward to detect experimentally. Despite the large volume of such data, a practical formalism for describing these γ rays in terms of partial-wave T-matrix elements has never been given. The partial-wave formalism is applicable when R-matrix methods are used to describe the reaction in question. This paper supplies the needed framework, and it is demonstrated by the application to the $^{15}\mathrm{N}(p,\alpha_1\gamma)\,^{12}\mathrm{C}$ reaction.

DOI: 10.1103/PhysRevC.102.024628

- A natural extension would be to secondary γ-rays following primary γ-ray decay



What's there left to do?
Polarization cross sections

Analyzing power measurement for the $^{14}N(\vec{p}, \gamma)^{15}O$ reaction at astrophysically relevant energies

S. O. Nelson,[1,2] M. W. Ahmed,[1,2] B. A. Perdue,[1,2] K. Sabourov,[1,2] A. L. Sabourov,[1,2] A. P. Tonchev,[1,2] R. M. Prior,[3,2] M. Spraker,[3,2] and H. R. Weller[1,2]

[1]*Department of Physics, Duke University, Durham, North Carolina 27708, USA*

[2]*Triangle Universities Nuclear Laboratory, Duke Station, Durham, North Carolina 27708, USA*

[3]*Department of Physics, North Georgia College and State University, Dahlonega, Georgia 30597, USA*

(Received 18 August 2003; published 18 December 2003)

The $^{14}N(\vec{p}, \gamma)^{15}O$ reaction has been investigated by measuring the angular distributions of its cross section and analyzing power using a 270-keV polarized proton beam. Calculations using a direct-capture-plus-resonance model were compared with the data. The results indicate the presence of γ transition amplitudes which were not considered in previous extrapolations of the astrophysical S factor to low energies. The impact on the zero-energy S factor of the $^{14}N(\vec{p}, \gamma)^{15}O$ reaction is discussed.

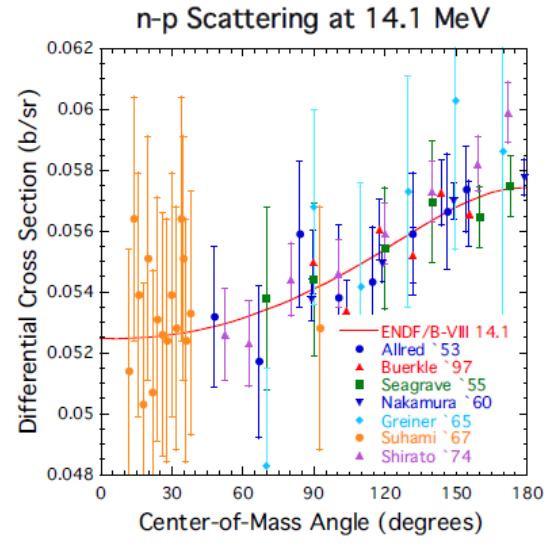
DOI: 10.1103/PhysRevC.68.065804 PACS number(s): 26.20.+f

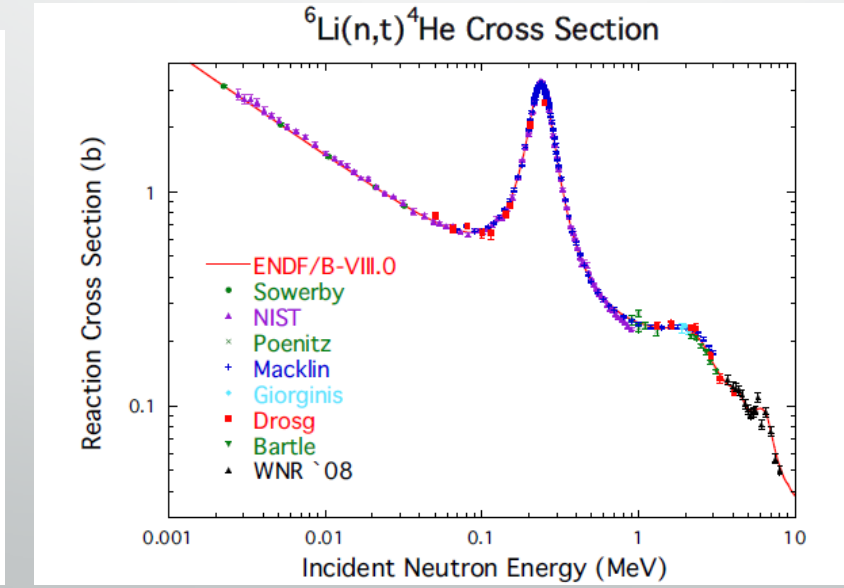
- Polarization cross sections give more unique constraints on the actual spin-angular momentum channels that are contributing to the cross section
- Lots of (underutilized) data already in the literature
- Polarization measurements are challenging
- General implementation of the mathematics is also very challenging

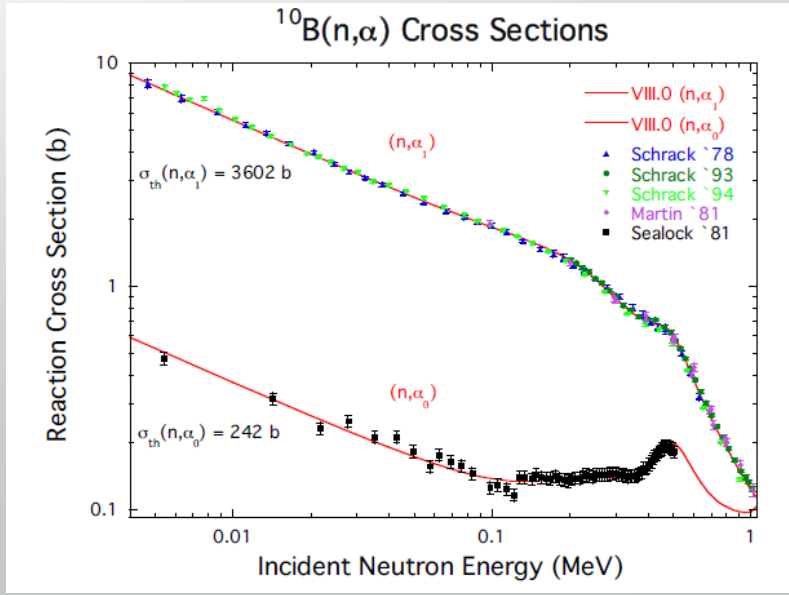
Nuclear Data Evaluation

ENDF/B-VIII.0: The 8^{th} Major Release of the Nuclear Reaction Data Library with CIELO-project Cross Sections, New Standards and Thermal Scattering Data

D. A. Brown,[1] M. B. Chadwick,[2,*] R. Capote,[3] A. C. Kahler,[2] A. Trkov,[3] M. W. Herman,[1] A. A. Sonzogni,[1] Y. Danon,[4] A. D. Carlson,[5] M. Dunn,[6] D. L. Smith,[7] G. M. Hale,[2] G. Arbanas,[8] R. Arcilla,[1] C.R. Bates,[2] B. Beck,[9] B. Becker,[10] F. Brown,[2] R. J. Casperson,[9] J. Conlin,[2] D. E. Cullen,[9] M.-A. Descalle,[9] R. Firestone,[11] T. Gaines,[12] K. H. Guber,[8] A. I. Hawari,[13] J. Holmes,[14] T. D. Johnson,[1] T. Kawano,[2] B. C. Kiedrowski,[15] A. J. Koning,[3] S. Kopecky,[16] L. Leal,[17] J. P. Lestone,[2] C. Lubitz,[18] J. I. Márquez Damián,[19] C. M. Mattoon,[9] E. A. McCutchan,[1] S. Mughabghab,[1] P. Navratil,[20] D. Neudecker,[2] G. P. A. Nobre,[1] G. Noguere,[21] M. Paris,[2] M. T. Pigni,[8] A. J. Plompen,[16] B. Pritychenko,[1] V. G. Pronyaev,[22] D. Roubtsov,[23] D. Rochman,[24] P. Romano,[7] P. Schillebeeckx,[16] S. Simakov,[25] M. Sin,[26] I. Sirakov,[27] B. Sleaford,[9] V. Sobes,[8] E. S. Soukhovitskii,[28] I. Stetcu,[2] P. Talou,[2] I. Thompson,[9] S. van der Marck,[29] L. Welser-Sherrill,[2] D. Wiarda,[8] M. White,[2] J. L. Wormald,[13] R. Q. Wright,[8] M. Zerkle,[14] G. Žerovnik,[16] and Y. Zhu[13]







Final comments

- The phenomenological *R*-matrix method is a powerful tool for nuclear data evaluation and extracting reaction rates and their uncertainties from cross section data
  - Only as good as the input data, both cross sections and knowledge of the nuclear level structure
- **Its flexibility makes it very popular for the fitting of a wide variety of experimental data**
- **However, its flexibility can also make uncertainties more difficult to quantify**
  - Lots of work being done in this area recently
- There's a lot that can still be done and improved on
  - Many reactions to look at or improve on
  - Many types of data yet to be fully utilized