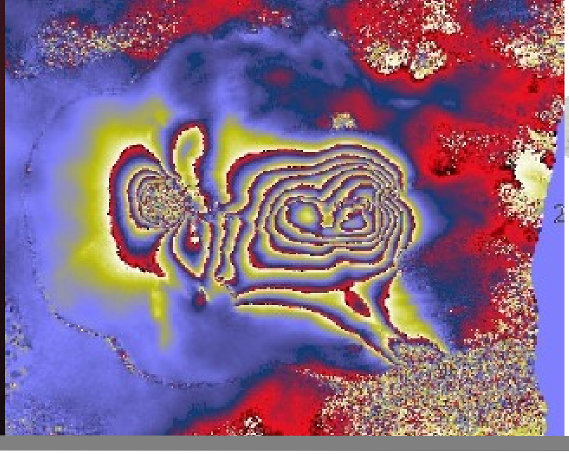
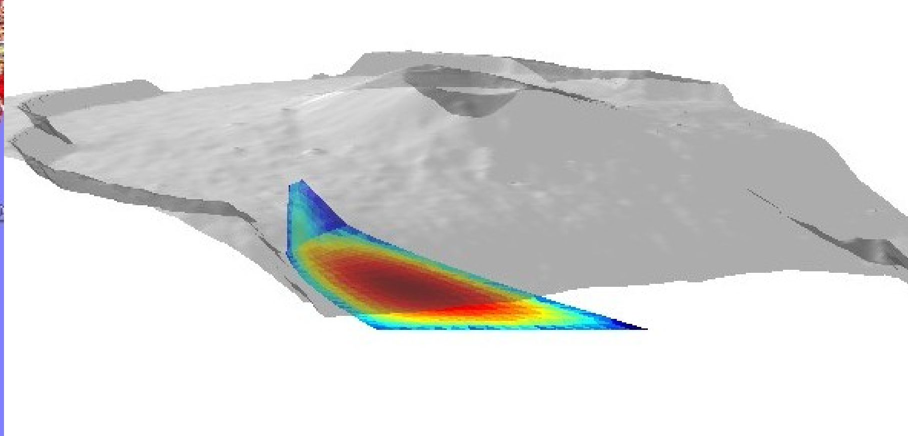




Towards physics-based inversions of deformation data

Valérie Cayol, valerie.cayol@uca.fr

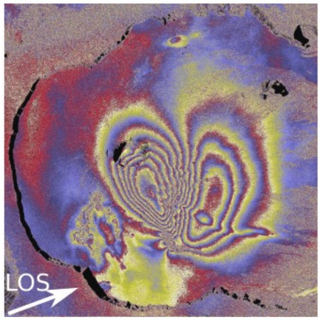
Also with Quentin Dumont[1], Marine Tridon[1], Christelle Wauthier[2], Yo Fukushima[3], Adriana Iozzia[4], Jean-Luc Froger[1], Gilda Currenti[4]

1. Laboratoire Magmas et Volcans, Clermont-Ferrand, France
2. Pennsylvania State University, Pennsylvania, USA
3. International Research Institute of Disaster Science, Tohoku University, Tohoku, Japan
4. Instituto Nazionale di di Geofisica e Vulcanologia, Catania, Italy
4. European Center for Geodynamics and Seismology, Walferdange, Luxembourg
5. National Museum of Natural History, Walferdange, Luxembourg
6. Royal Museum for Central Africa, Tervuren, Belgique

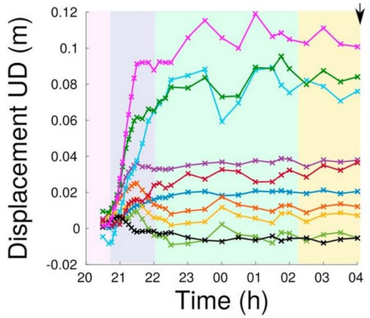
May, 2024 Improve Training network, Potsdam

Why do we model deformation ?

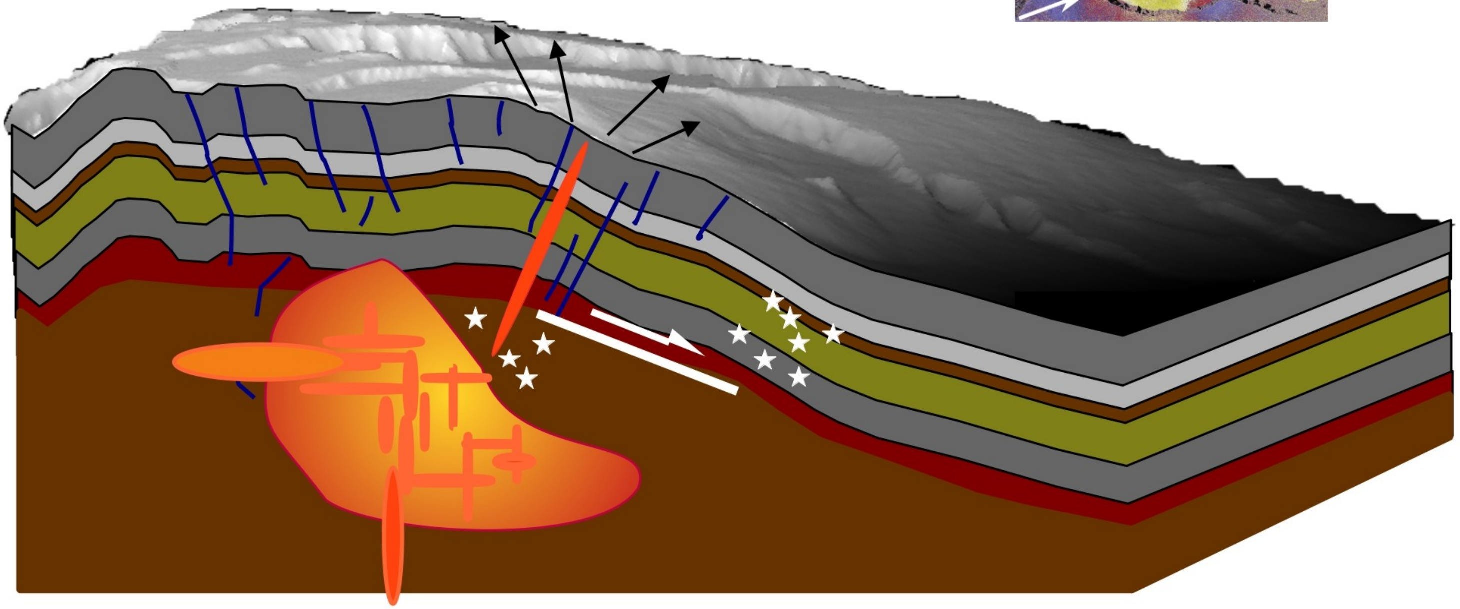
InSAR



GNSS



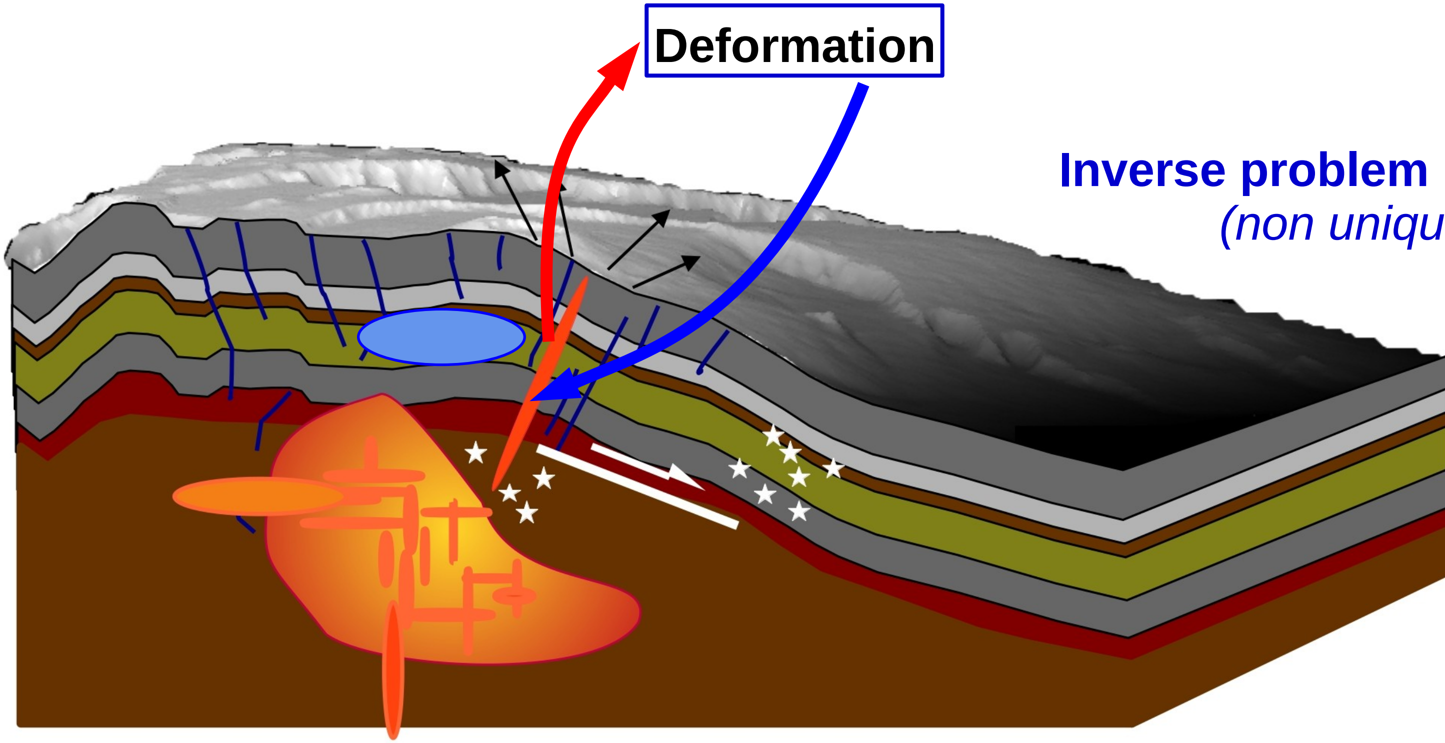
Deformation



- **Where is magma stored ?**
- **What are the physical and mechanical parameters controlling magma transfer ?**
- **How do edifices grow and collapse ?**

Forward versus inverse models

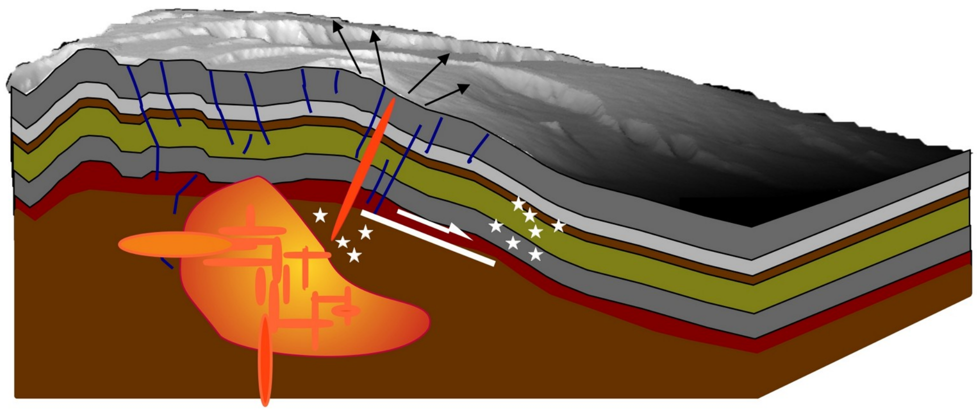
Direct problem : $\boldsymbol{m} \rightarrow \boldsymbol{u} = G(\boldsymbol{m})$, $\boldsymbol{m}$ = *parameters*
(unique) u = *observations*

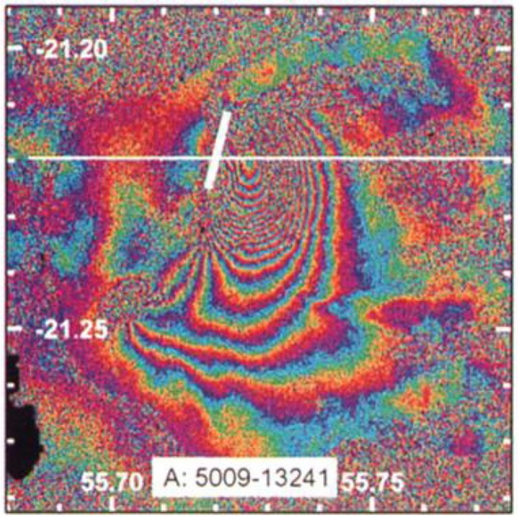


Inverse problem : $\boldsymbol{u} \rightarrow \boldsymbol{m}$ **?**
(non unique)

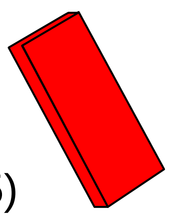
Classical elastostatic analytic models

Reality





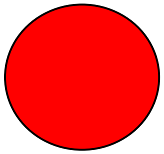
Analytic Models



Constant displacement

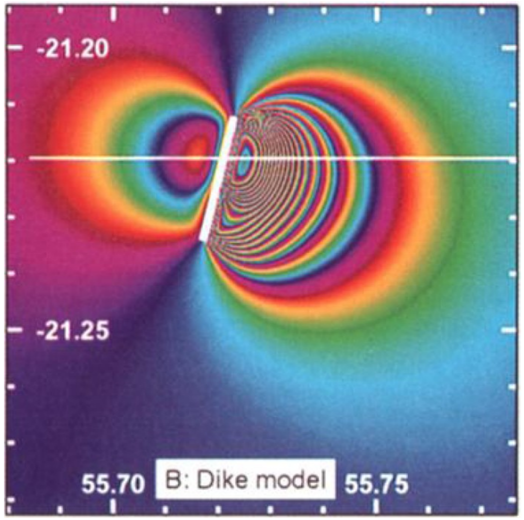
(Okada, 1985)

Models



Constant overpressure

(Mogi, 1958)

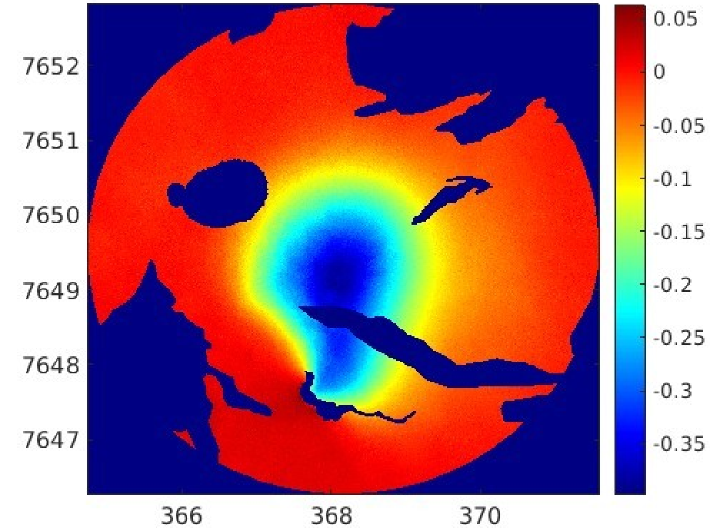


(*Sigmundsson et al., GRL, 1999*)

InSAR data call for more realistic modelds

Inversions:
Definition of a cost-function

Data point undersampling $\rightarrow$ $\boldsymbol{u}_o$



- The simplest cost-function :

$$\chi^2 = \sum_{i=1}^{N} \left(u_o^i - u_m^i\right)^2 = \|\boldsymbol{u}_o - \boldsymbol{u}_m\|^2$$

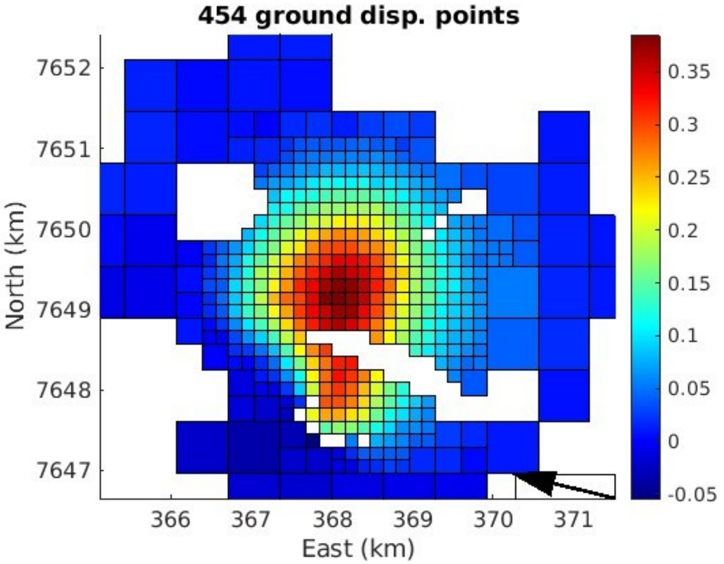
where u_o^i is the i[th] observed displacement

u_m^i is the i[th] modelled displacements

- Normalized cost function:

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(u_o^i - u_m^i\right)^2}{\sigma_i^2}$$

$\boldsymbol{\sigma}_i$ Standard deviation or error on the i[th] data



- Taking the data correlation into account:

$$\chi^2 = (u_o - u_m)^T C_d^{-1} (u_o - u_m)$$

where C_d is the full covariance matrix

Inversions : linear versus non-linear inversions

Linear inversions: there is a linear relation between the parameters ***m*** and the observations, $\boldsymbol{u_m}$

$$\boldsymbol{u}_m = \boldsymbol{G}\,\boldsymbol{m}$$

Exemple: Okada's Model (1985, 1992); Mogi's solution (1958) are linear models

Typically, the location of a source is known, and the amplitude of the source is searched for.

To minimize the cost function: $\chi^2 = \|\boldsymbol{u}_o - \boldsymbol{u}_m\|^2 = \|\boldsymbol{u}_o - \boldsymbol{G}\,\boldsymbol{m}\|^2$

We seek ***m*** such that : $\dfrac{\partial \chi^2}{\partial \boldsymbol{m}} = 0$

Which leads to solving a linear system of equations: $\boldsymbol{m} = \left(\boldsymbol{G}^T \boldsymbol{G}\right)^{-1} \boldsymbol{G}^T \boldsymbol{u}_o$

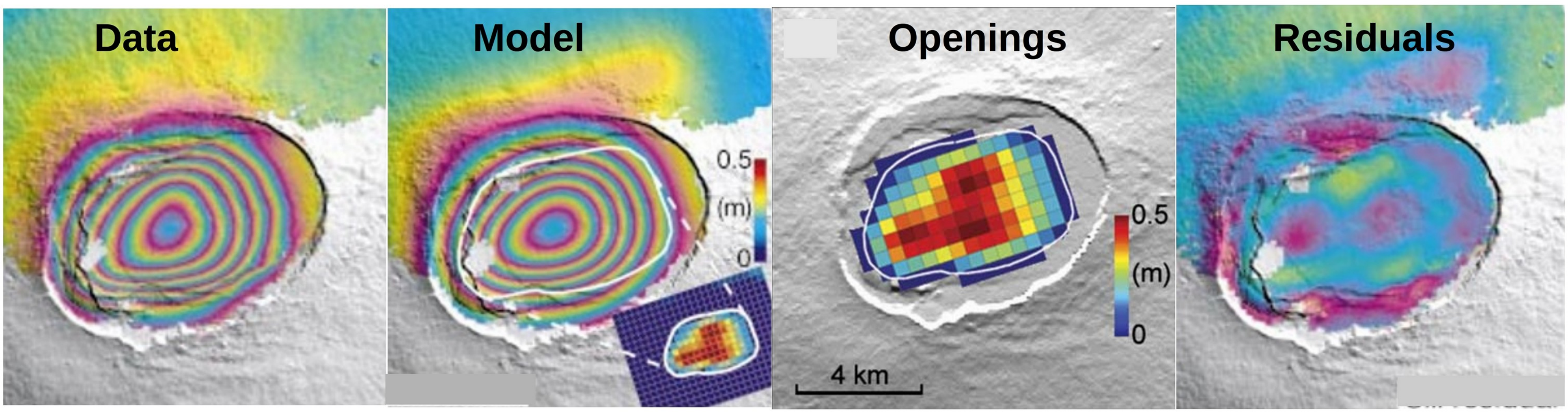
Pros: fast method

Cons: the source location has to be knows

Example of a linear inversion: "kinematic" models

Uplift at Sierra Negra volcano in 1998-99 (Galapagos)

Minimization of $\chi^2 = \|\boldsymbol{u}_o - \boldsymbol{G}\,\boldsymbol{m}\|^2 + \beta^2 \|\nabla\,\boldsymbol{m}\|^2$, **where *m* is the opening vector**



Amelung et al., Science, 2000

Still widely used

Science

RESEARCH ARTI

Cite as: Sigmu...

Fracturing and tectonic stress drives ultra
flow into dikes

Freysteinn Sigmundsson[1*], Michelle Parks[2], Halldór Geirsson[1], Andrew Hooper[3], Vincent Drou
G. Ófeigsson[2], Sonja H. M. Greiner[1,4,5], Yilin Yang[1], Chiara Lanzi[1], Gregory P. De Pascale[1], Kristí
Valentyn Tolpekin[7], Hildur María Friðriksdóttir[2], Páll Einarsson[1], Sara Barsotti[2]

AGU ADVANCING EARTH AND SPACE SCIENCE

JGR Solid Earth

RESEARCH ARTICLE
10.1029/2019JB019117

The 24 December 2018 Eruptive Intrusion at Etna Volcano as Revealed by Multidisciplinary Continuous Deformation Networks (CGPS, Borehole Strainmeters and Tiltmeters)

M. Aloisi[1], A. Bonaccorso[1], F. Cannavò[1], G. Currenti[1], and S. Gambino[1]

Key Points:
- Imaging multidisciplinary continuous deformation data to improve dike ascent modeling
- Detailed temporal model of the 2018 intrusion at Etna volcano

Inversions : linear versus non-linear inversions

Non-linear inversions: there is a non-linear relation between the parameters ***m*** and the observations, $\boldsymbol{u_m}$

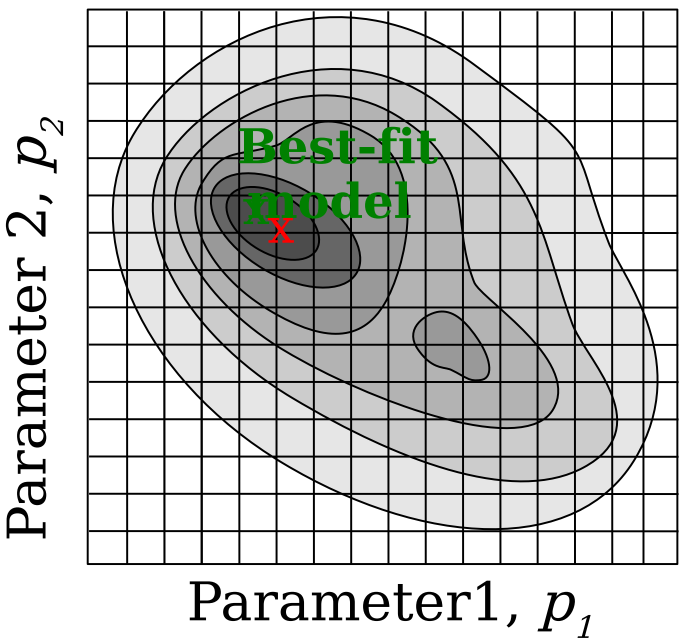
$$\boldsymbol{u}_m = \boldsymbol{G}(\boldsymbol{m})$$

The link between the source location, orientation parameters and the ground displacement is a non linear relation.

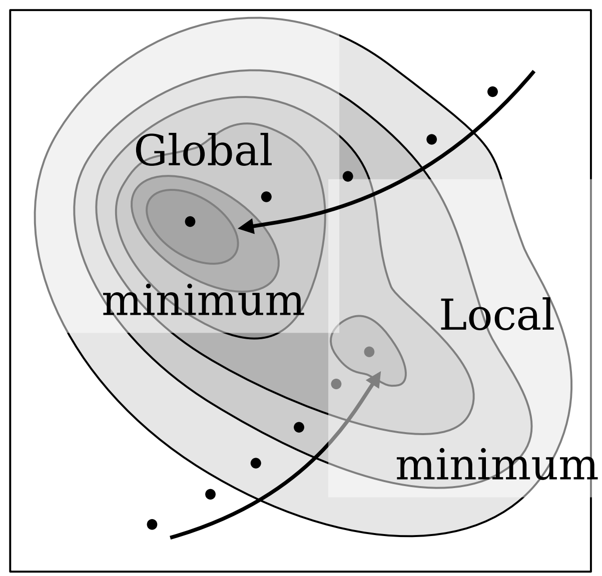
To minimize the cost function: we can use different methods

$$\chi^2 = \|\boldsymbol{u}_o - \boldsymbol{u}_m\|^2 = \|\boldsymbol{u}_o - \boldsymbol{G}(\boldsymbol{m})\|^2 ,$$

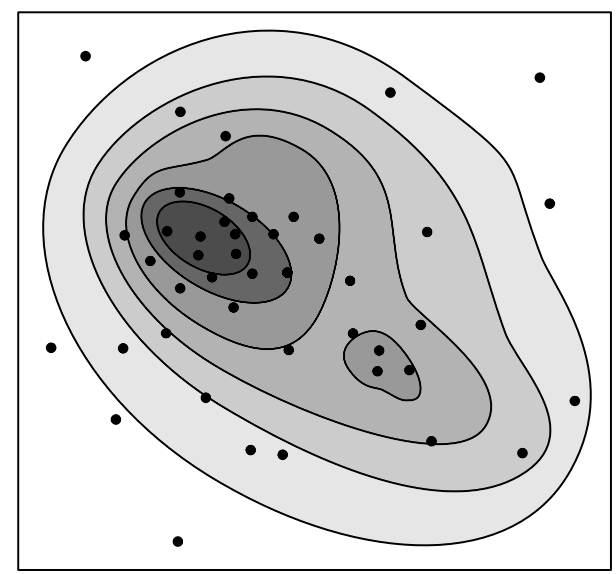
Grid search



Gradient-based method



Monte Carlo search

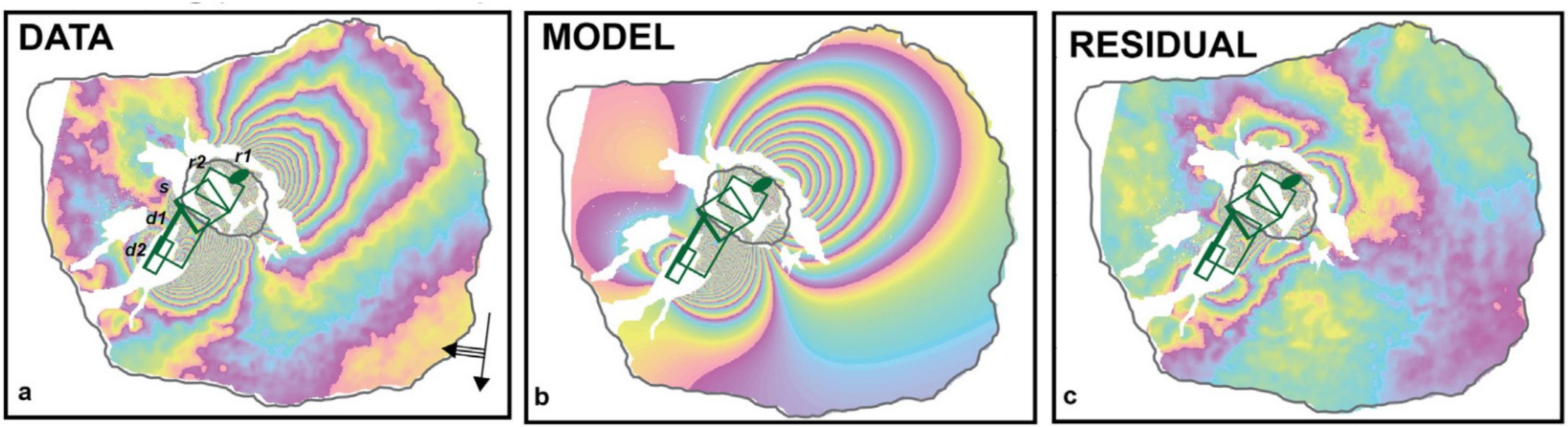


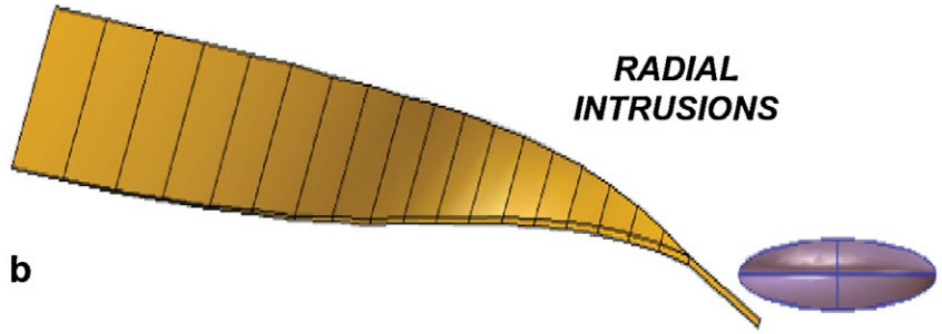
Pros: adapted to non-linear inversion

Cons: slow

Non-linear inversion to capture source geometries

Radial intrusion in 2009 at Fernandina volcano (Galapagos)



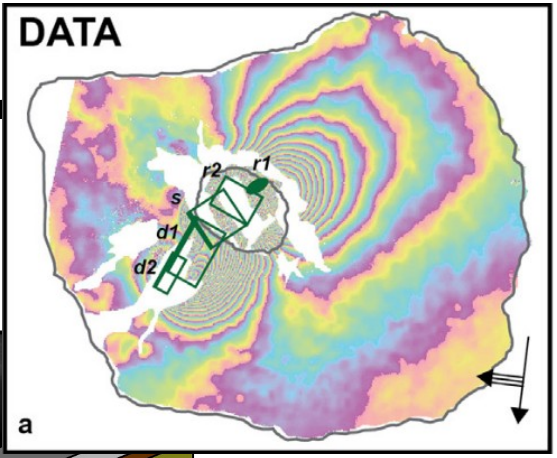


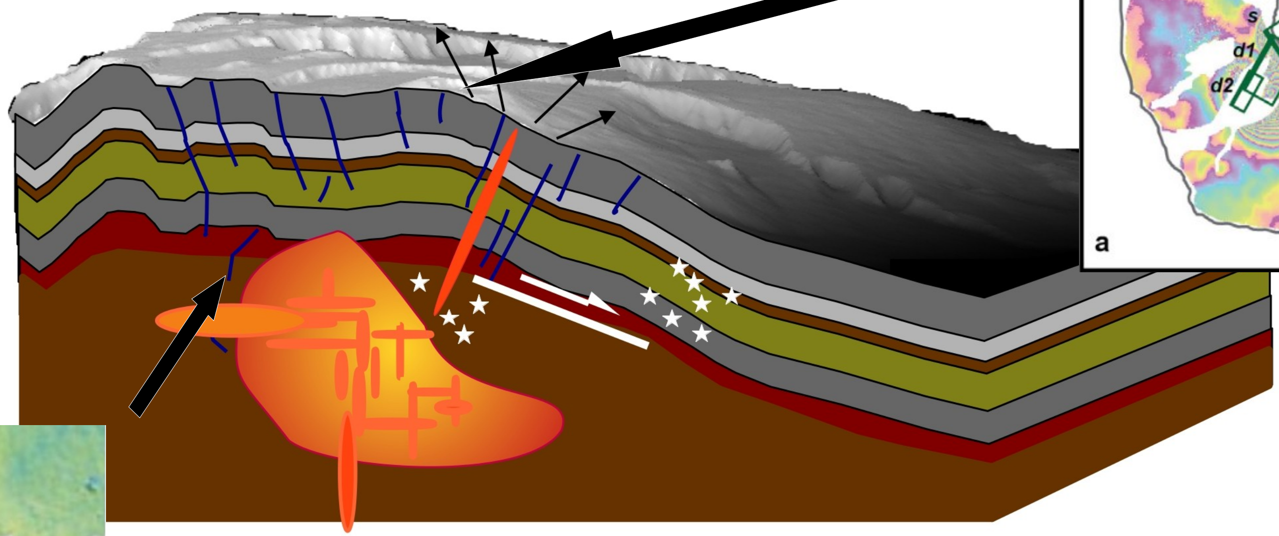
Bagnardi et al., EPSL, 2013

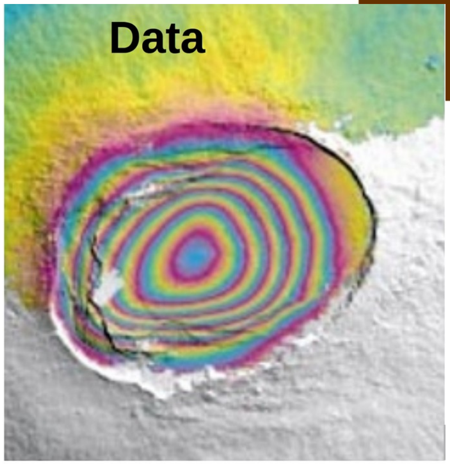
What can be learnt from analytic and kinematic models ?

Intrusion pathways

Bagnardi et al., EPSL, 2013







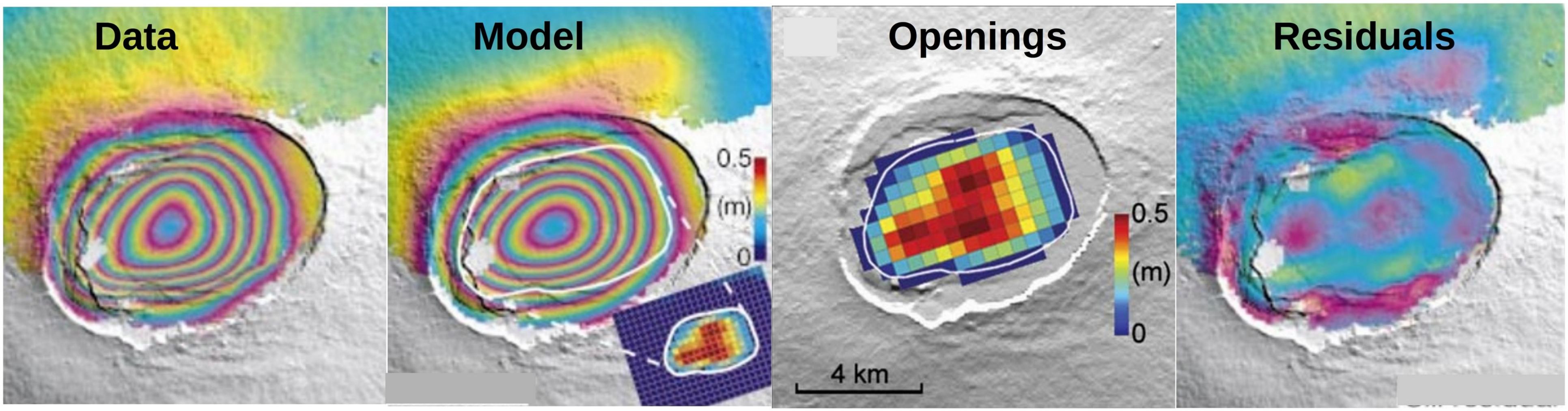
Characteristics of reservoirs

Amelung et al., Science, 2000

Analytic and kinematic models require many parameters

- **To better capture openings: linear inversion of 77 parameters**

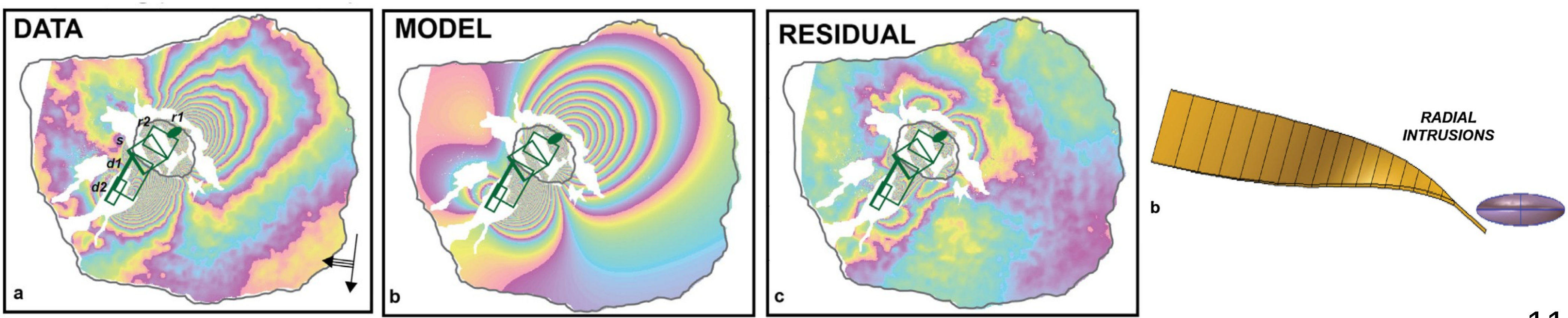
Uplift at Sierra Negra volcano in 1998-99 (Galapagos)



Amelung et al., 2000

- **To better capture complex geometries: non linear inversion of 22 parameters**

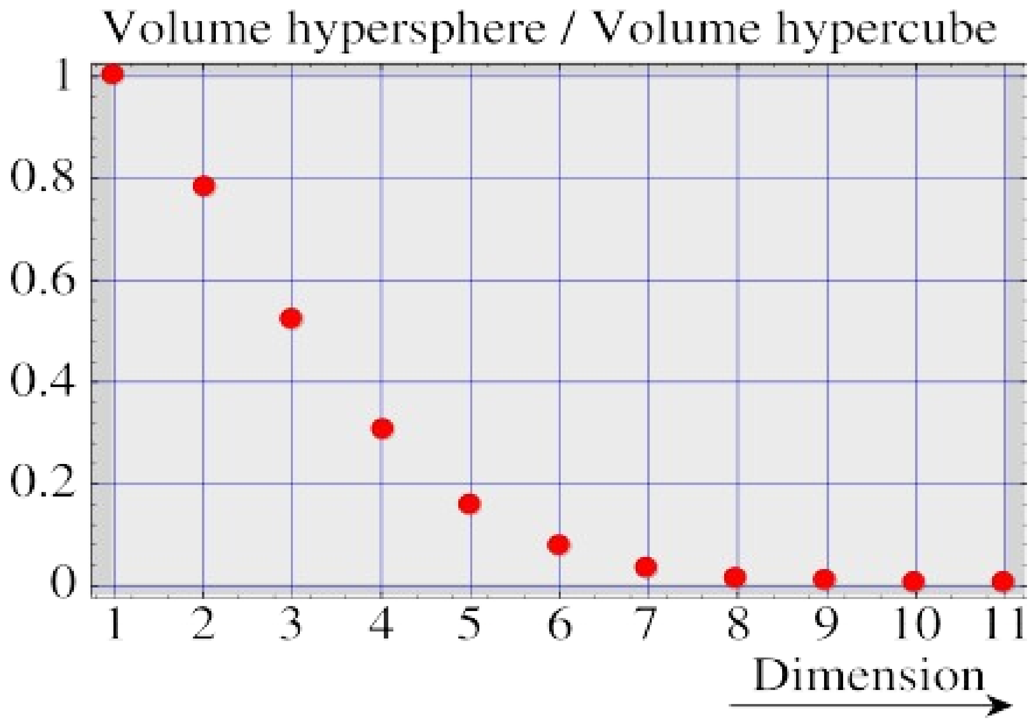
Radial intrusion in 2009 at Fernandina volcano (Galapagos)



Bagnardi et al., EPSL, 2013

Why should large numbers of parameters be avoided ?

- The probability of finding the best-fit solution decreases with the dimension of the search space ;



Tarantola, Inverse Problem Theory, 2005

- With non linear inversions, the search time increases exponentially with the number of parameters ;
- There is a risk of overfitting the data.

➡ **Use of Akaike Information Criteria**

AIC = 2*k + χ^2 + cst with k = Nb parameters and χ^2 = cost-function

3D Mixed Boundary Elements

Cayol et Cornet, Int. J. Rock Mech. Min. Sc., 1997;
Cayol and Cornet, JGR, 1998; Cayol et al., JGR 2014

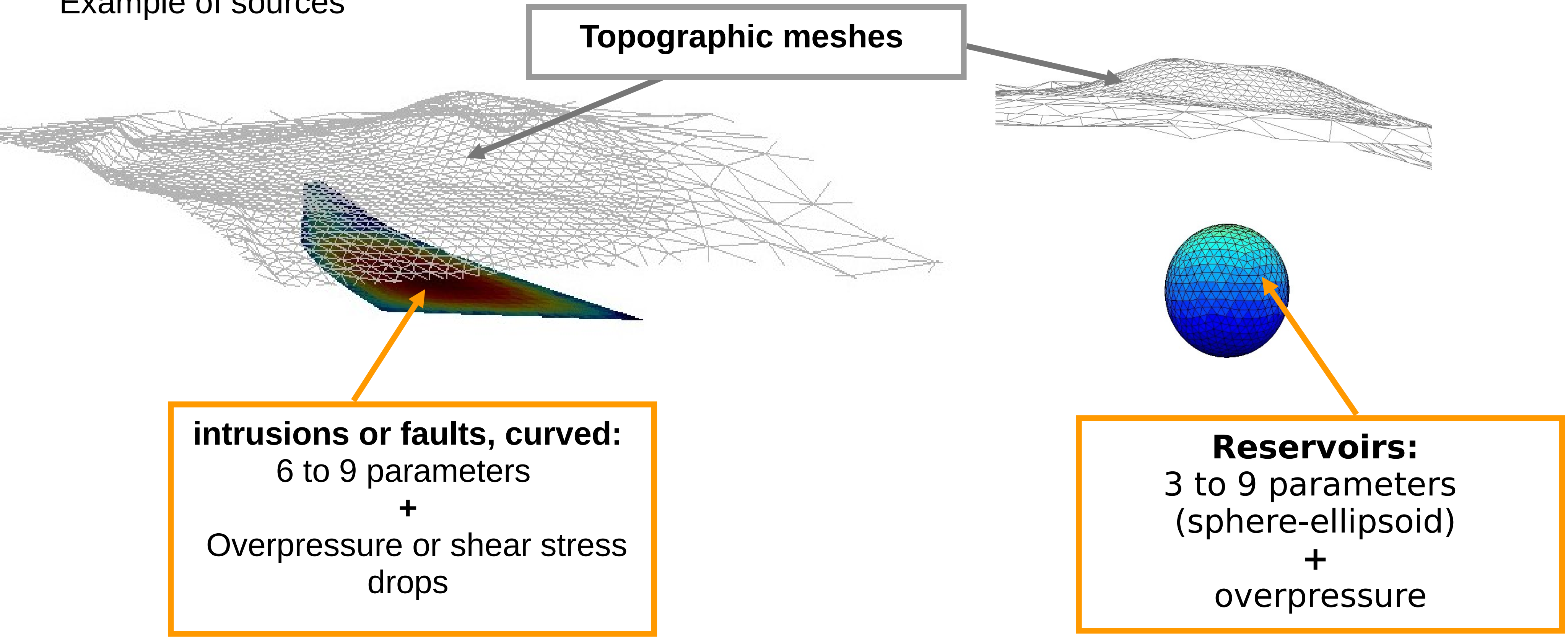
3D Numerical method:

- **Realistic topographies;**
- **Any number and geometry of fractures and pressure sources;**
- **Treats more than one source appropriately (interactions are taken into account);**

Assumptions:

- **intrusions, faults, reservoirs are submitted to constant stress changes;**
- **Fractures may be curved.**

Example of sources

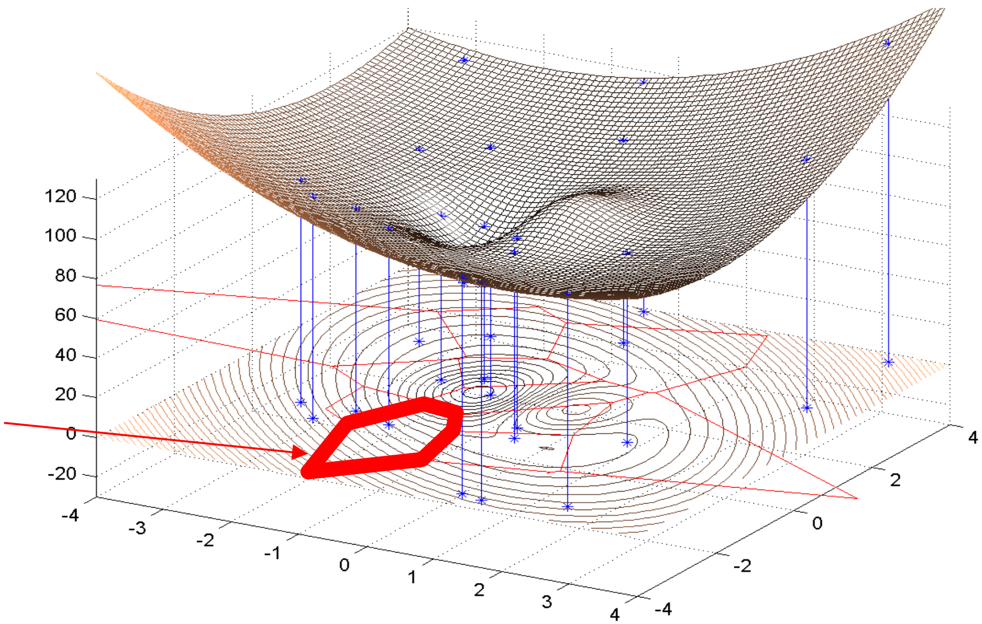


Combined with Non-linear inversions relying on a Neighborhood Algorithm

Fukushima et al., JGR,2005

Near-neigborhood non-linear inversion (*Sambridge, JGI, 1999a*) **to invert for geometrical parameters**

Voronoi cell (neighborhood): region closest to a point than any other point



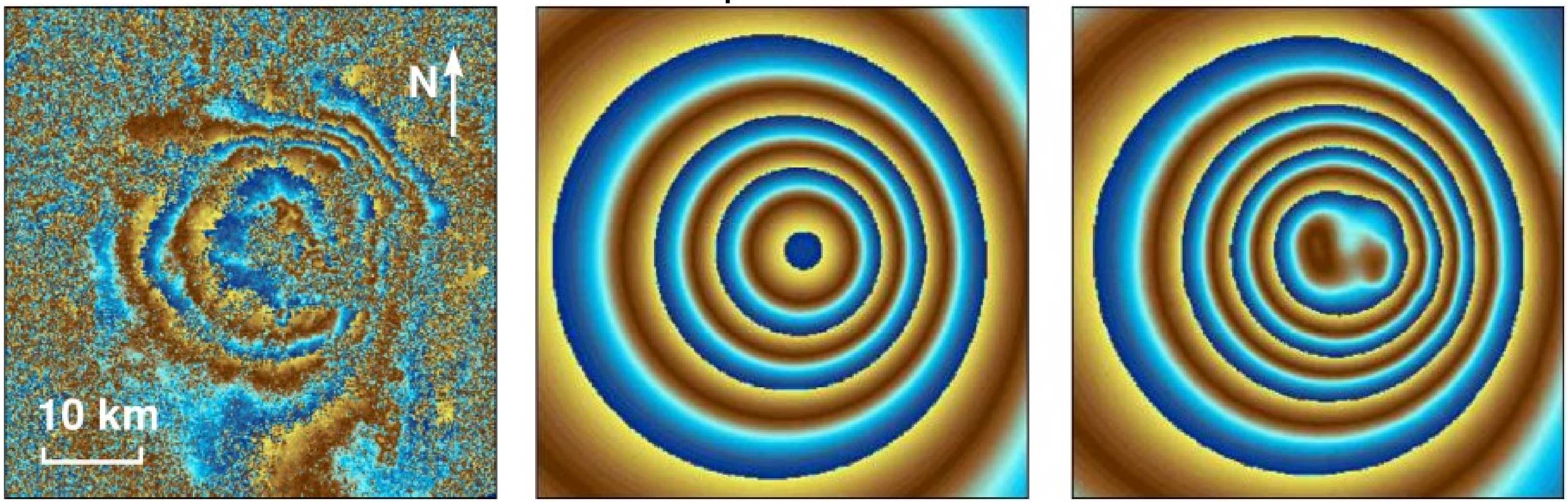
Misfit function in a two parameters space

- **Linear inversions of pressure;**
- **Appraisal of model using Bayesian inference → confidence intervals and trade-offs between parameters** (*Sambridge, JGI, 1999b*)
- **Available to registered used at** http://www.opgc.fr/defvolc

Topography is taken into account

Topographies have an influence of computed displacements

Data | 1/2 space | Mixed BEM

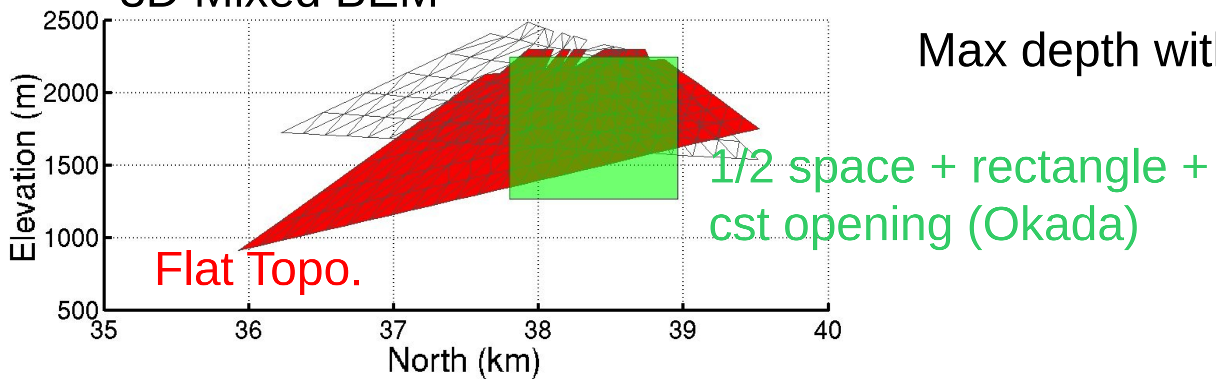


Etna, 1992-1993 eruption,
Massonnet et al., Nature, 1995

Cayol and Cornet, GRL, 1998

Neglecting topographies bias results : volume errors, depths errors

3D Mixed BEM



Volume with Okada (1985) : 80 % overestimation

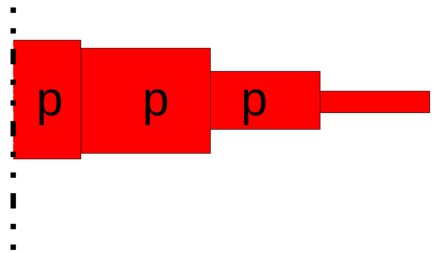
Max depth with Okada : 40 % overestimation

Fukushima et al., JGR, 2005

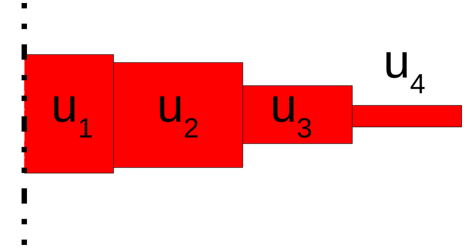
Crozier et al., Bull. Volc., 2023

Boundary conditions are homogeneous stress

Fied observation

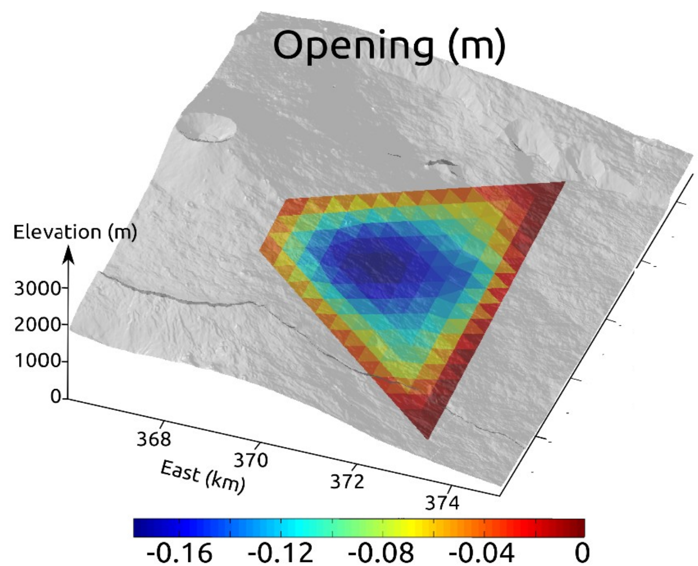


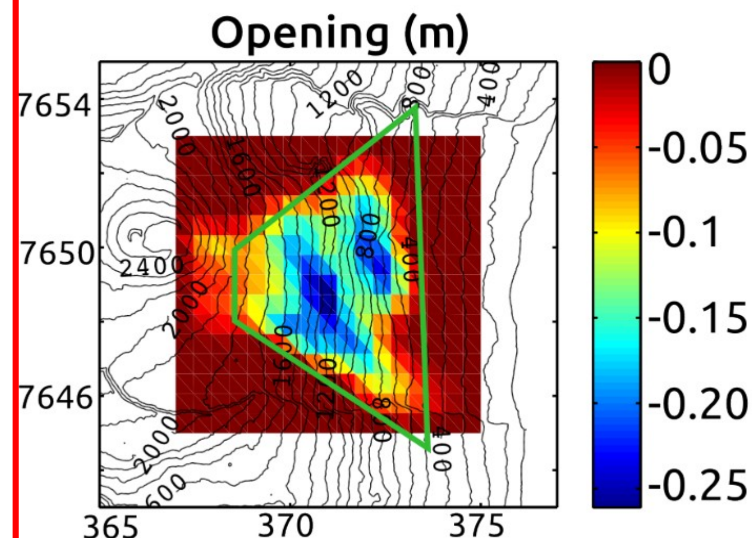
Pressure boundary condition



Displacement boundary condition : kinematic models

Inverted openings





Tridon et al., JGR, 2016

One parameter

~ 5 % of inverse models

500 parameters

~ 95 % of inverse models

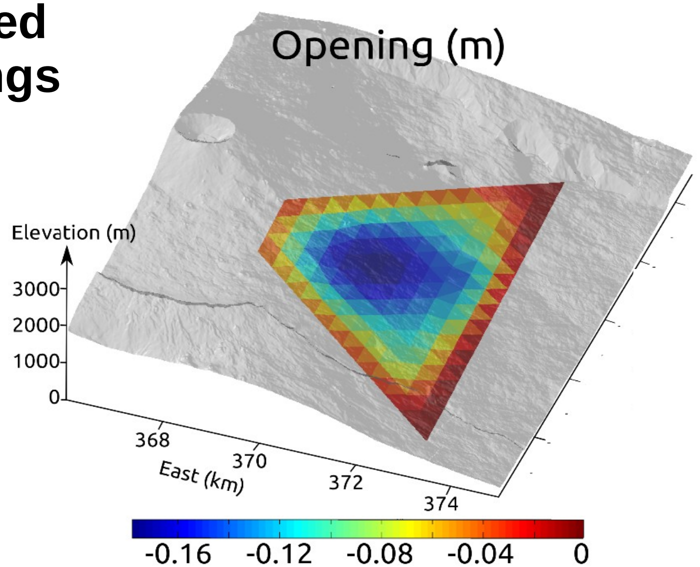
These studies

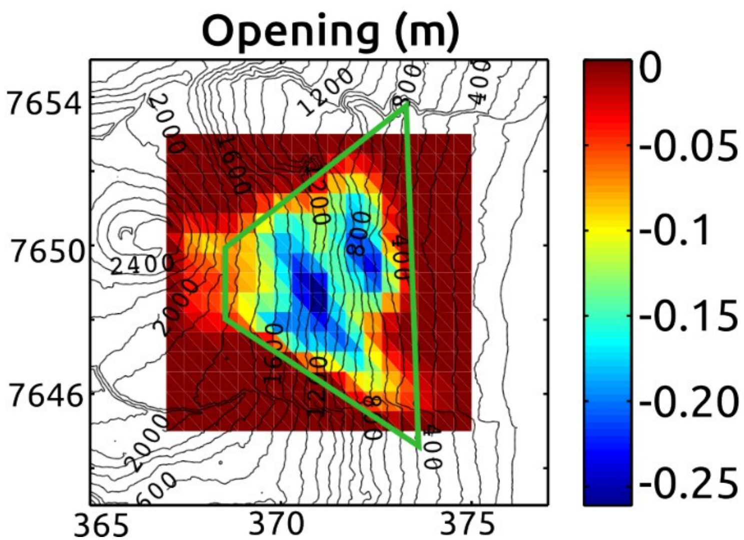
Stress boundary conditions leads to better models

AIC = 2*k + χ^2 + cst with k = Nb parameters and $\chi^2 = (u_o - u_m)^T C_d^{-1} (u_o - u_m)$

Pressure boundary condition | **Displacement boundary condition**

Inverted openings





One parameter

$\chi^2 \approx 2784$

$\boxed{\textbf{AIC} \approx 2800}$

500 parameters

$\chi^2 \approx 2300$

AIC ≈ 3300

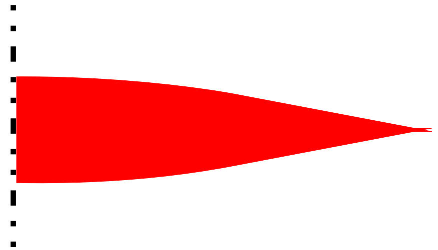
Tridon et al., JGR, 2016

- **Models with stress boundary conditions require less inversion parameters**
- **Inverting for stress leads to better models than inverting for dislocation amplitudes**

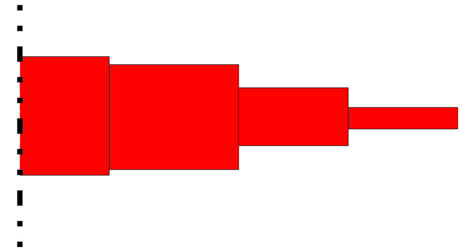
Models with stress boundary conditions are closer to the physics

Field observation

Models



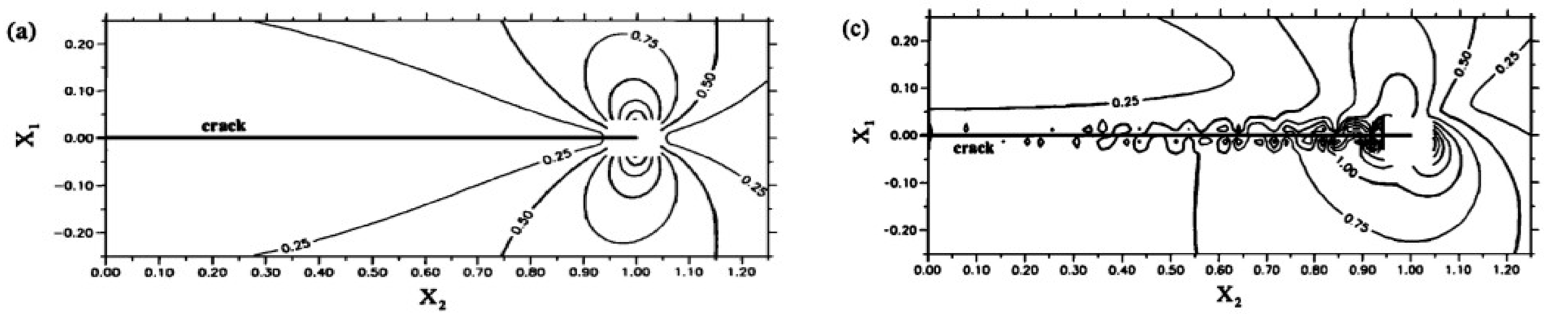
Pressure boundary condition



Displacement boundary condition : kinematic models

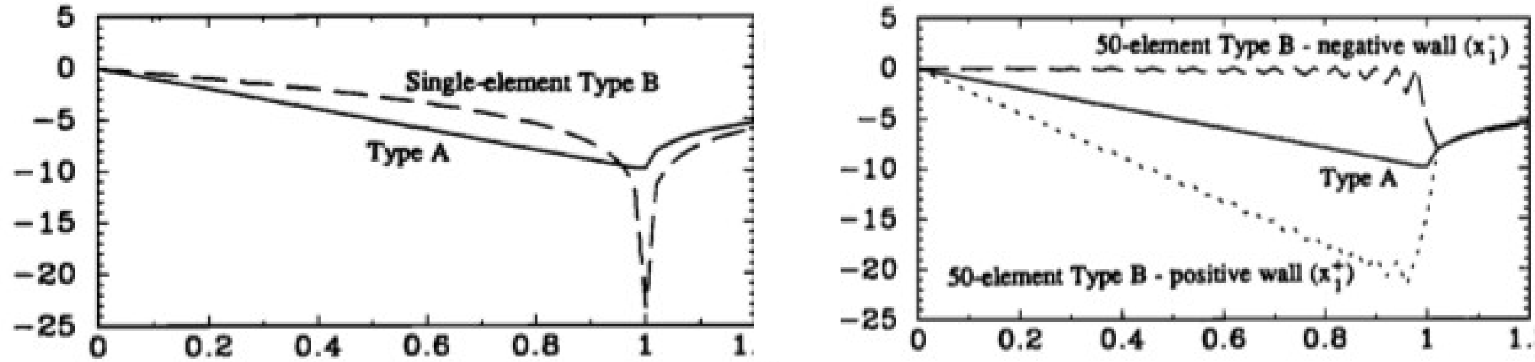
Fracture stress

Less stress singularities



Fracture shear displacement

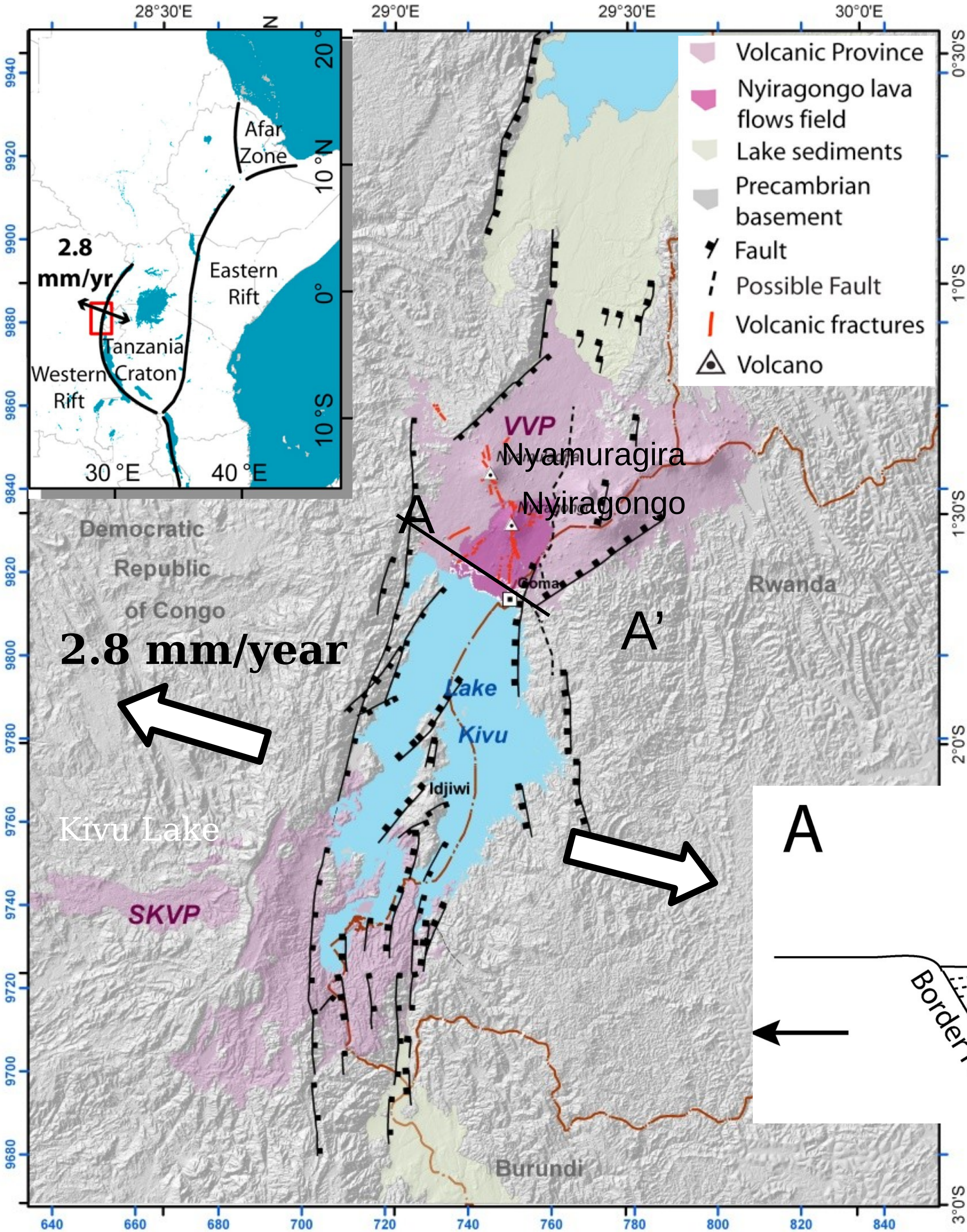
Shear displacements Are determined



Zeller and Pollard, JGR, 1992

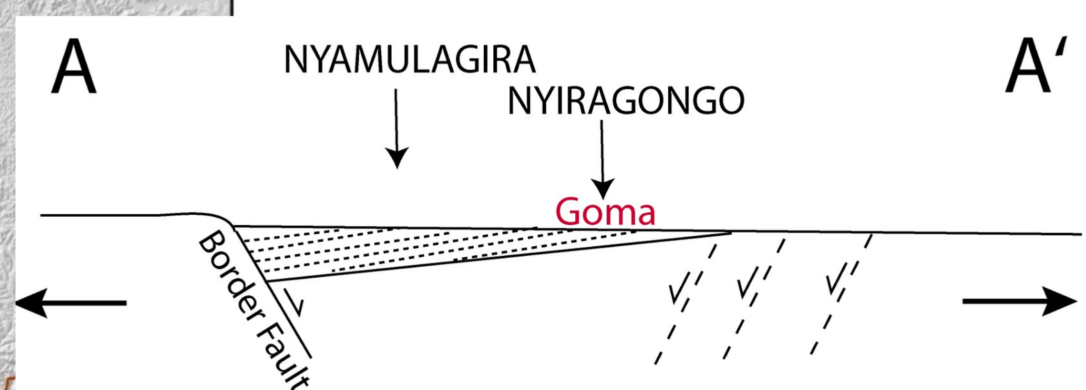
Stress change inversion as crustal stress gauge

What drives and accommodates rift extension in Kivu ?



The Virunga volcanic Province (VVP)

Nyiragongo, Goma and its surburb of 2,1 M inhabitants

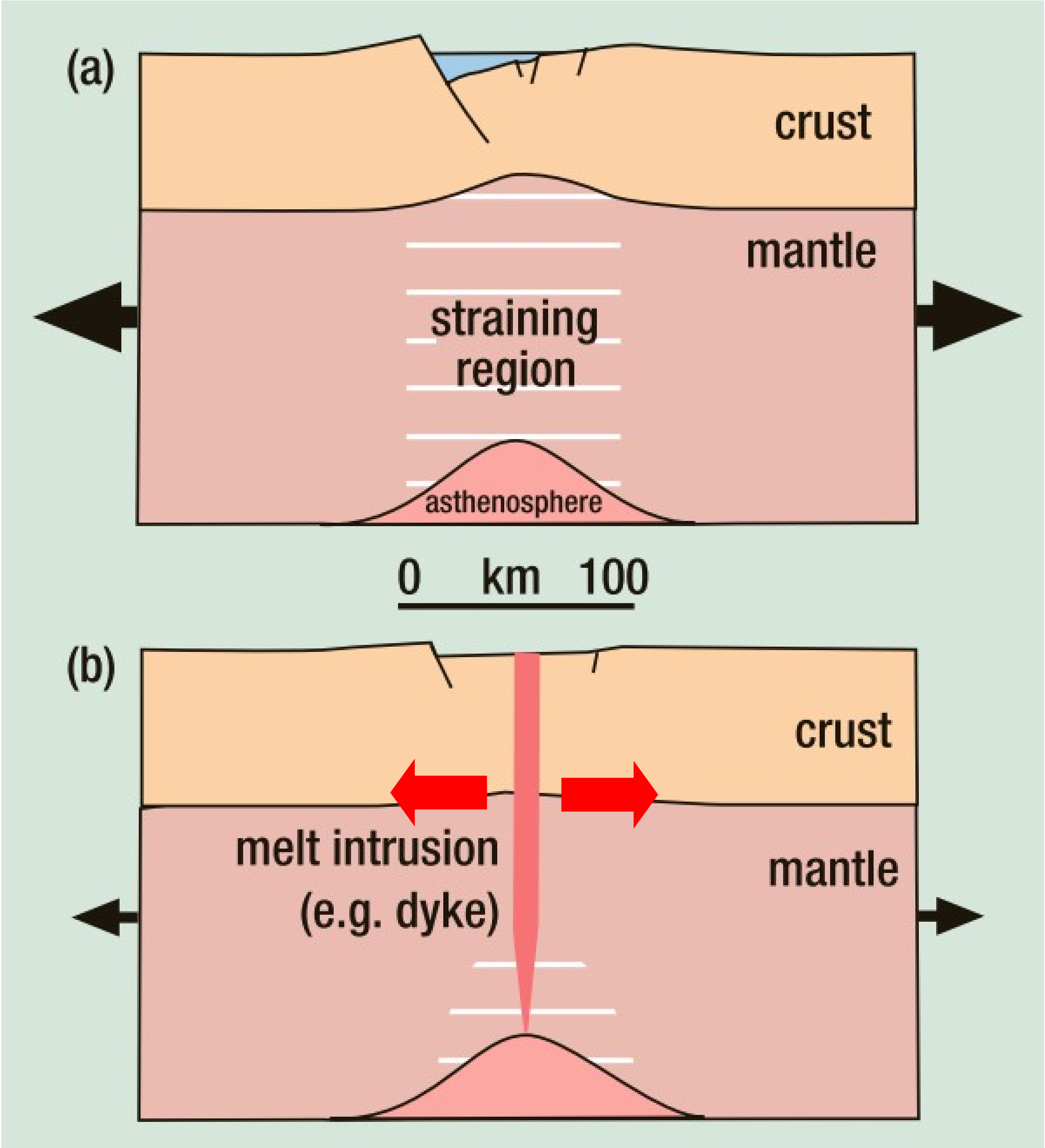


Wauthier et al., JGR, 2012

What drives and accommodates rift extension in Kivu ?

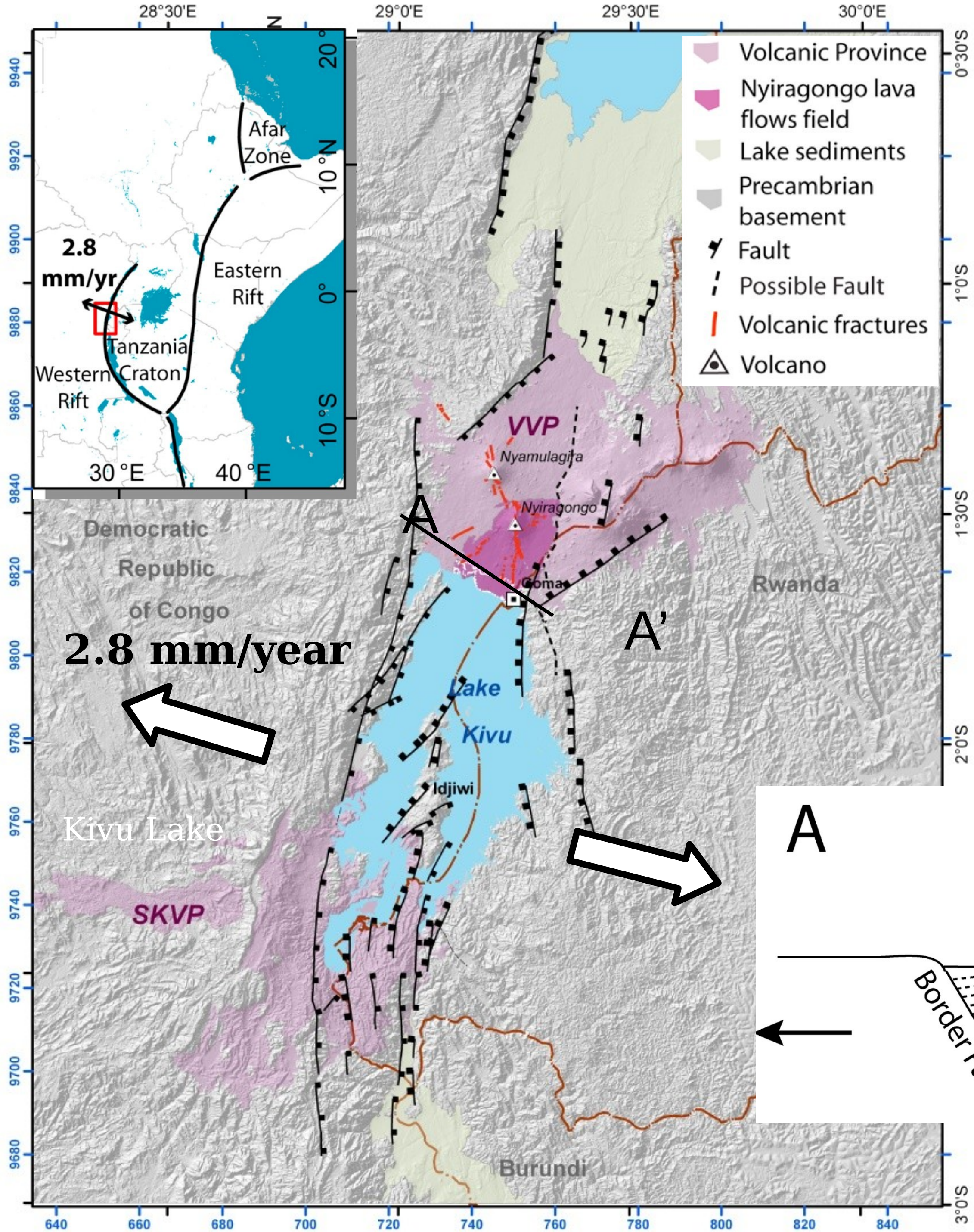
Tectonic stresses and Faulting assisted extension ?

Magma assisted extension ?



(*Ebinger, Astronomy and Geophysics, 2005*)

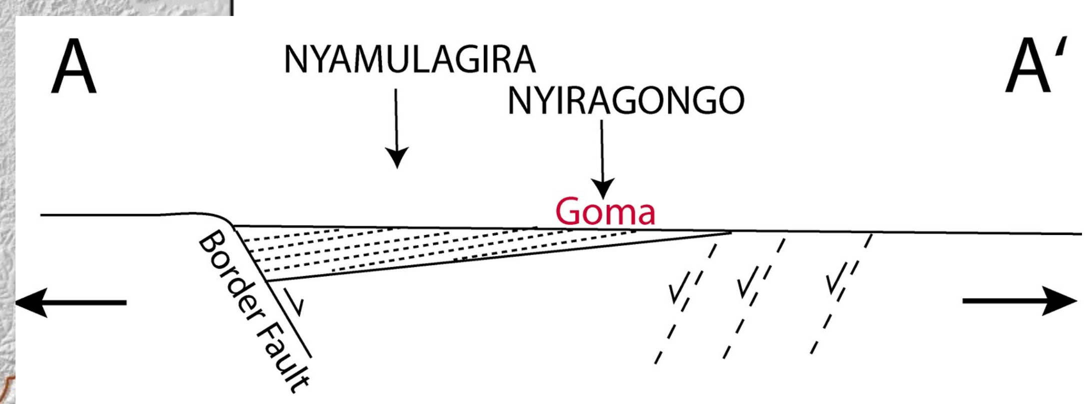
What drives and accommodates rift extension in Kivu ?



In the Virunga Basin:

- Only 15% of crustal extension
- Extension is accommodated by western border detachment faults,

Ebinger, Geol. Soc. Am. Bull, 1989

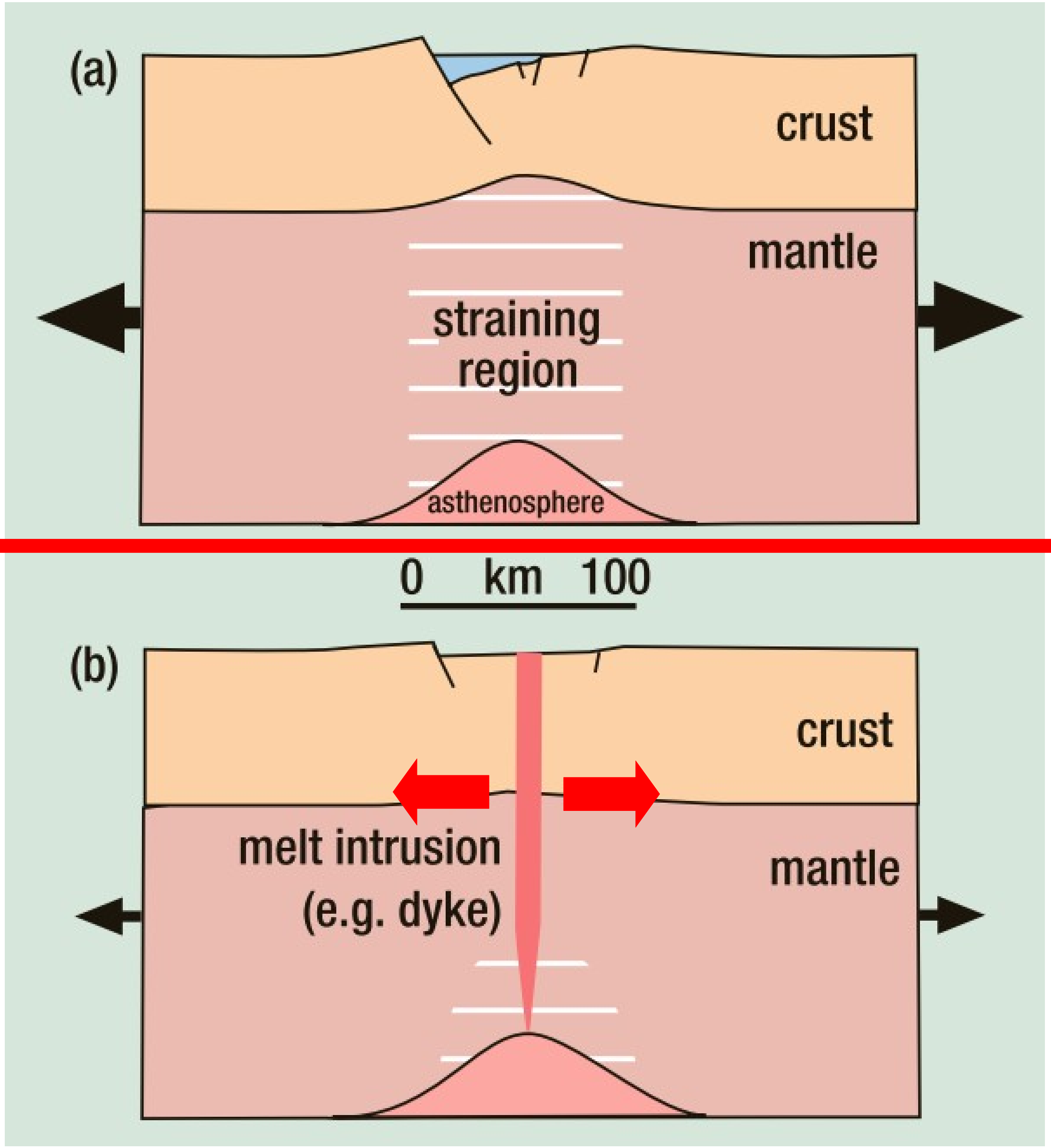


Wauthier et al., JGR, 2012

What drives and accommodates rift extension in Kivu ?

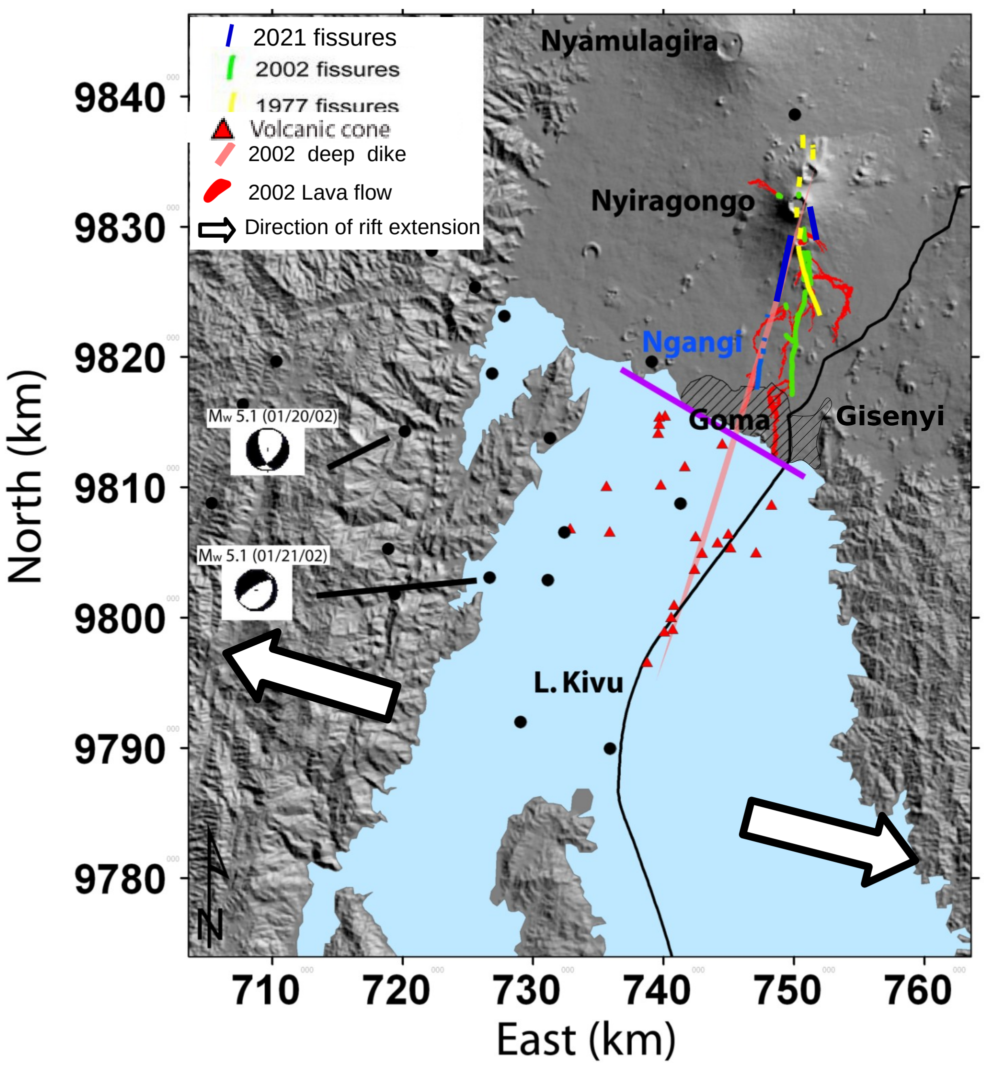
Tectonic stresses ?

Magma assisted extension ?



(*Ebinger, Astronomy and Geophysics, 2005*)

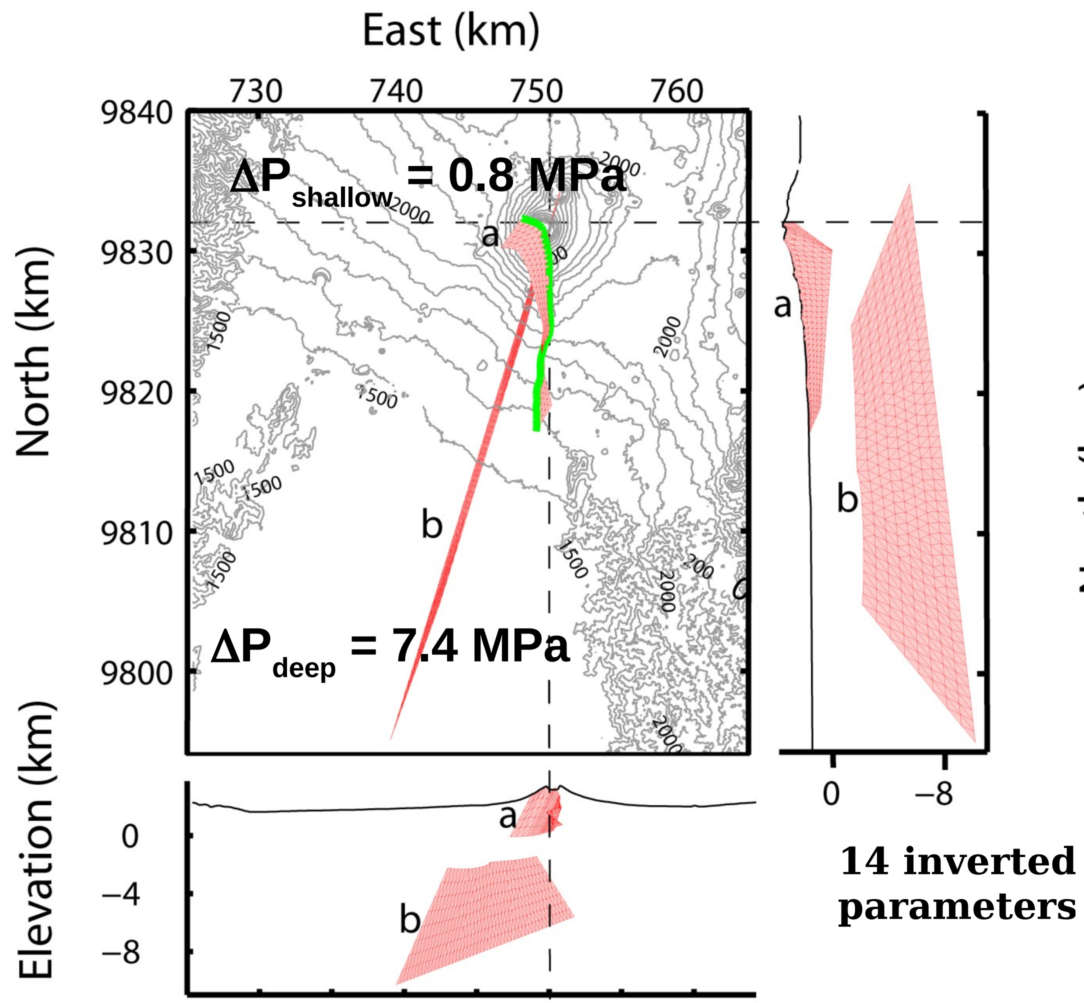
Nyiragongo 2002 and 2021 eruption
Democratic Republic of the Congo

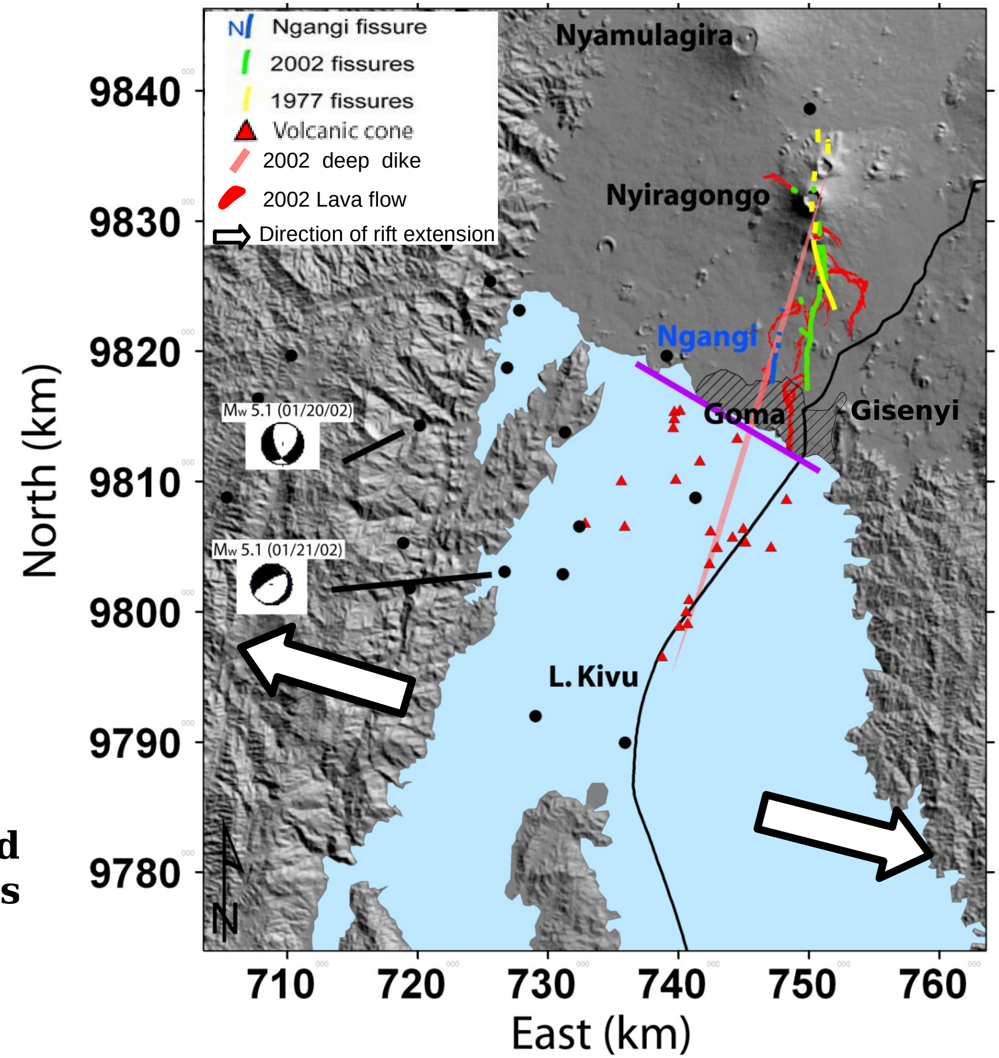


- A strato-volcano with a crater lava lake

- Three historical eruptions in 1977, 2002 and 2021
- Associated fissures trend NS

Model for Nyiragongo 2002 eruption *(Wauthier et al., JGR, 2012)*

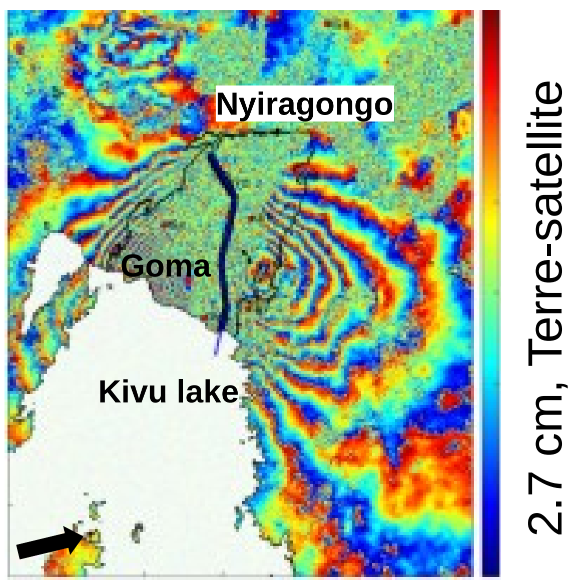




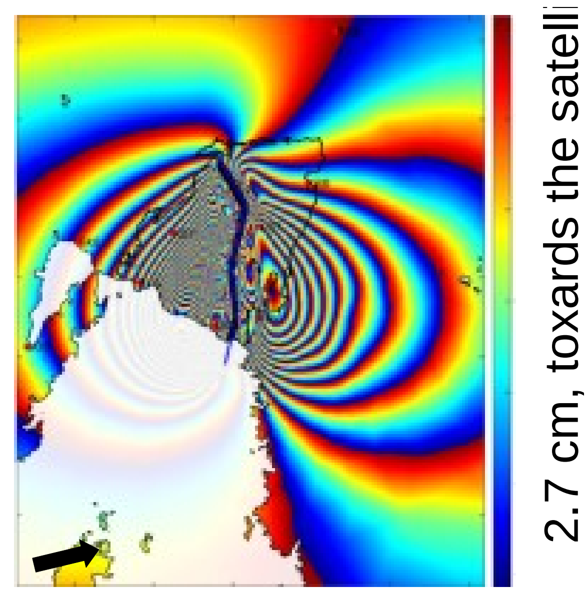
- The deep dike is perpendicular to the rift extension direction → Injection direction guided by the rift extension

The may 2021 eruption confirms the small overpressure

InSAR data

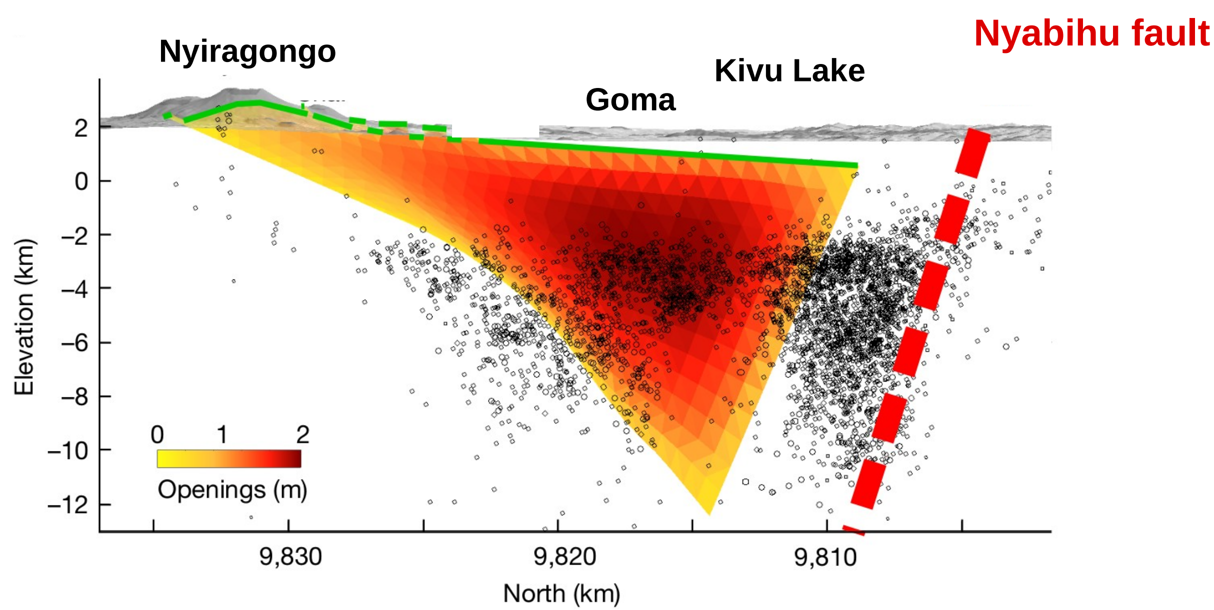


Model



7 inverted parameters

$\Delta P_{shallow}$ = 0.8 MPa

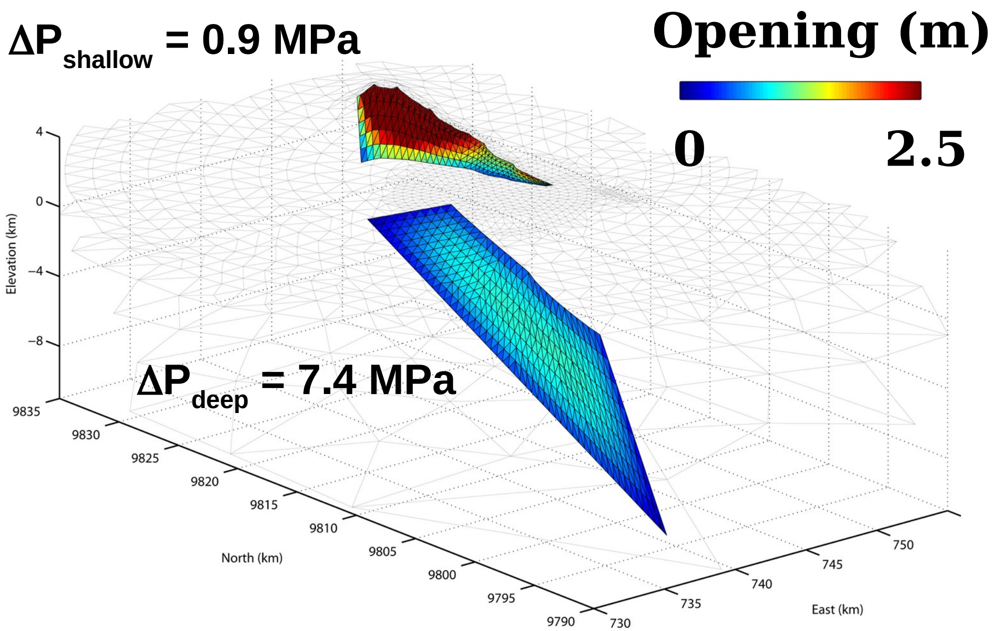


Smittarello et al., Nature, 2022

A magma-assisted rift rextension

Overpressure from InSAR data inversion:

Wauthier et al., JGR, 2012

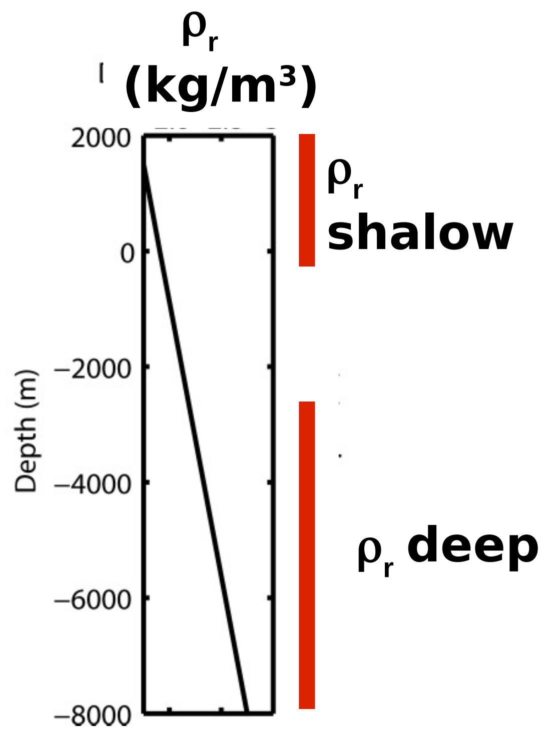


Overpressure theoretical model:

Assumption: crustal stresses are lithostatic ($\sigma_h \sim \sigma_v \sim P_{rock}$),

$$\Delta P\left(z_{dike}\right) = P_{magma} - P_{rock} = \int \left(\rho_m - \rho_r\left(z\right)\right) g dz,$$

with:



we get $\Delta P_{Shallow} \sim 1$ MPa and $\Delta P_{Deep} = 4.5$ MPa

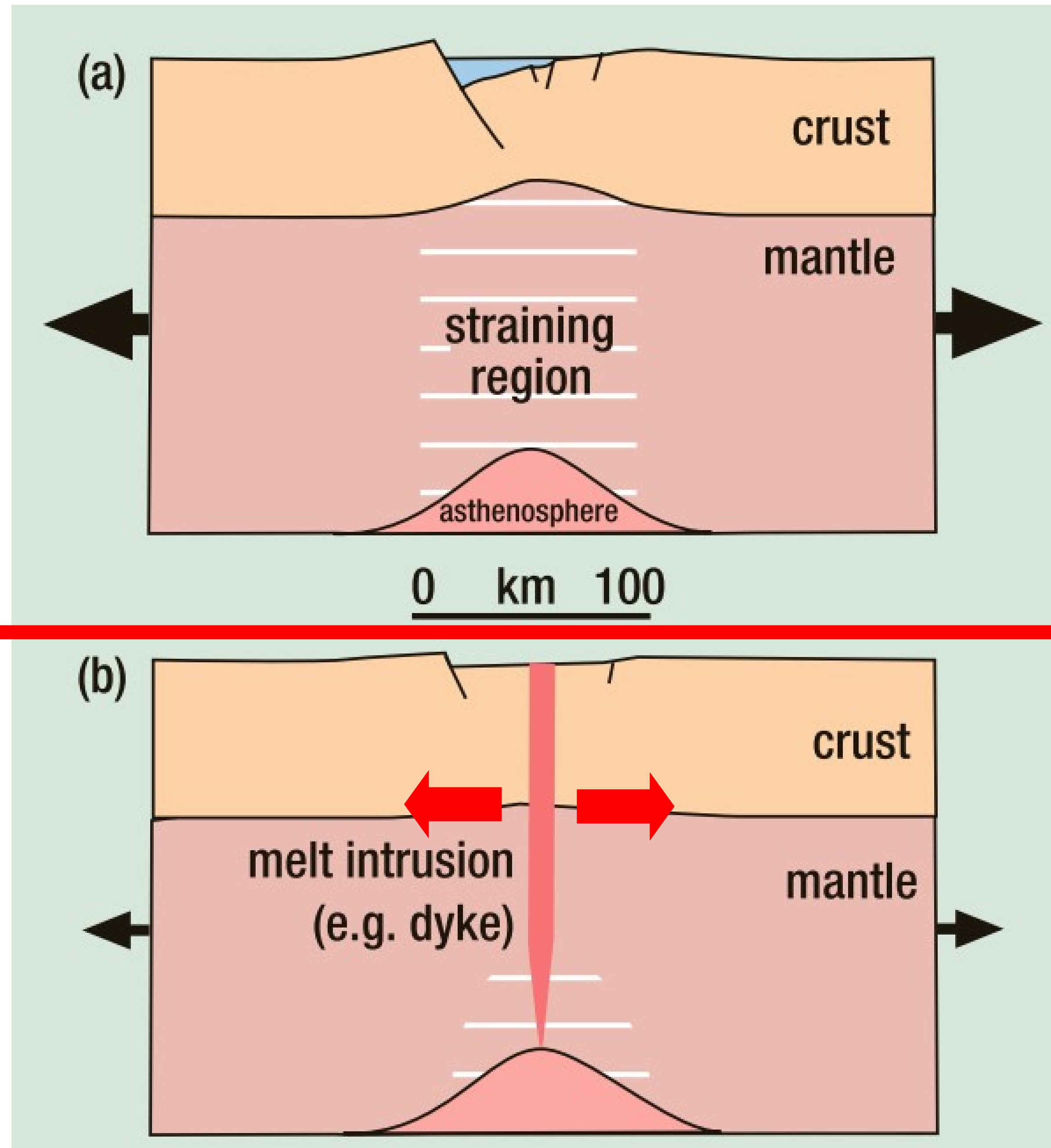
The crust is at a lithostatic stress state :
Unconsistent with a rift extension driven by plate separation

The rift extension is driven by the magmatic activity

What drives and accommodates rift extension in Kivu ?

Tectonic stresses ?

Magma assisted extension ?

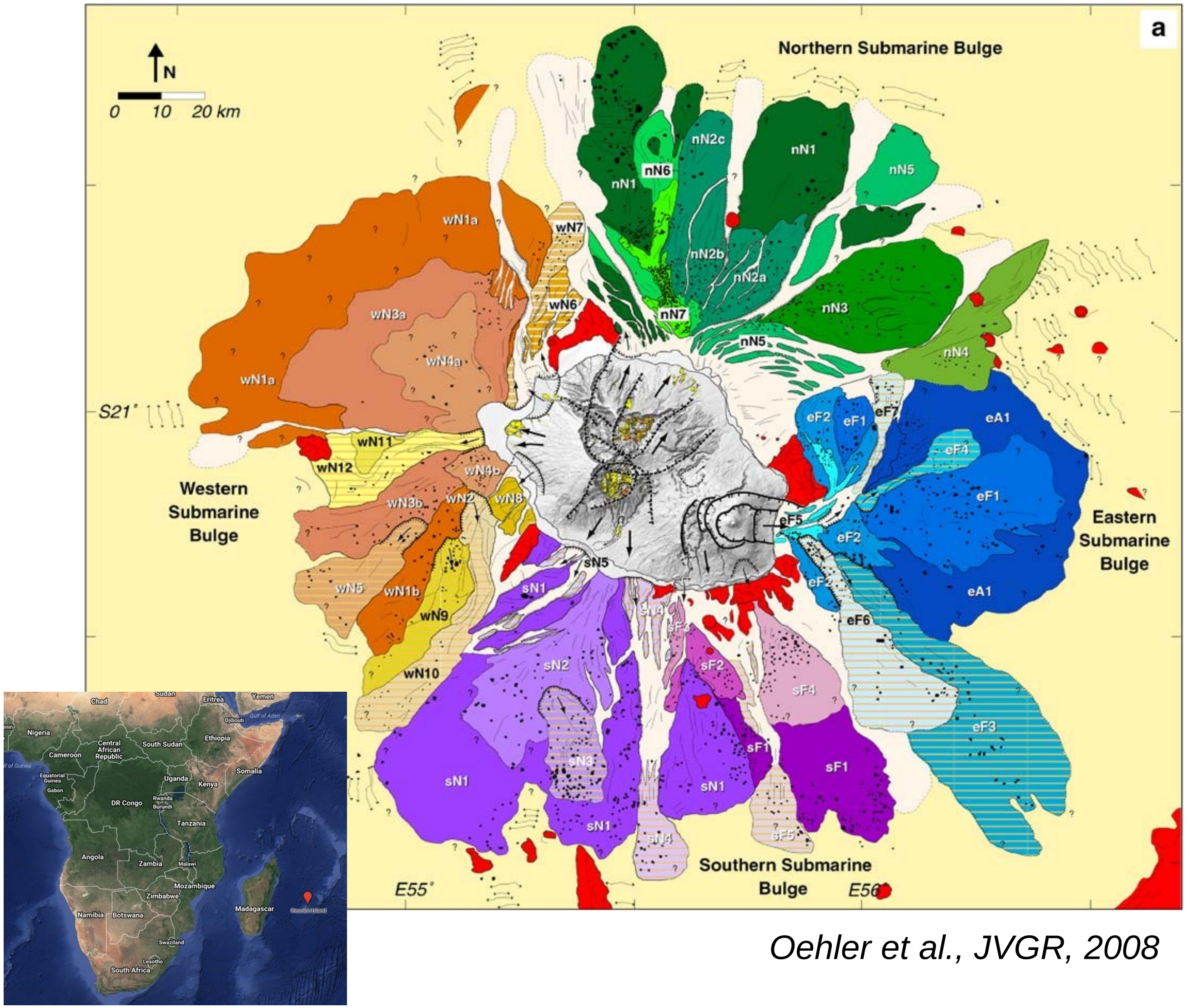


Ebinger, Astronomy and Geophysics, 2005

Stress change inversion for flank failure mechanisms

Flank failures at Réunion Island

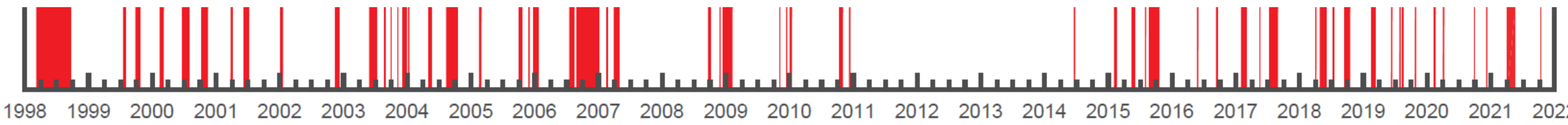
- Induce 24 % of volcano casualties (tsunamis and large earthaquakes)
- Ubiquous at Réunion Island



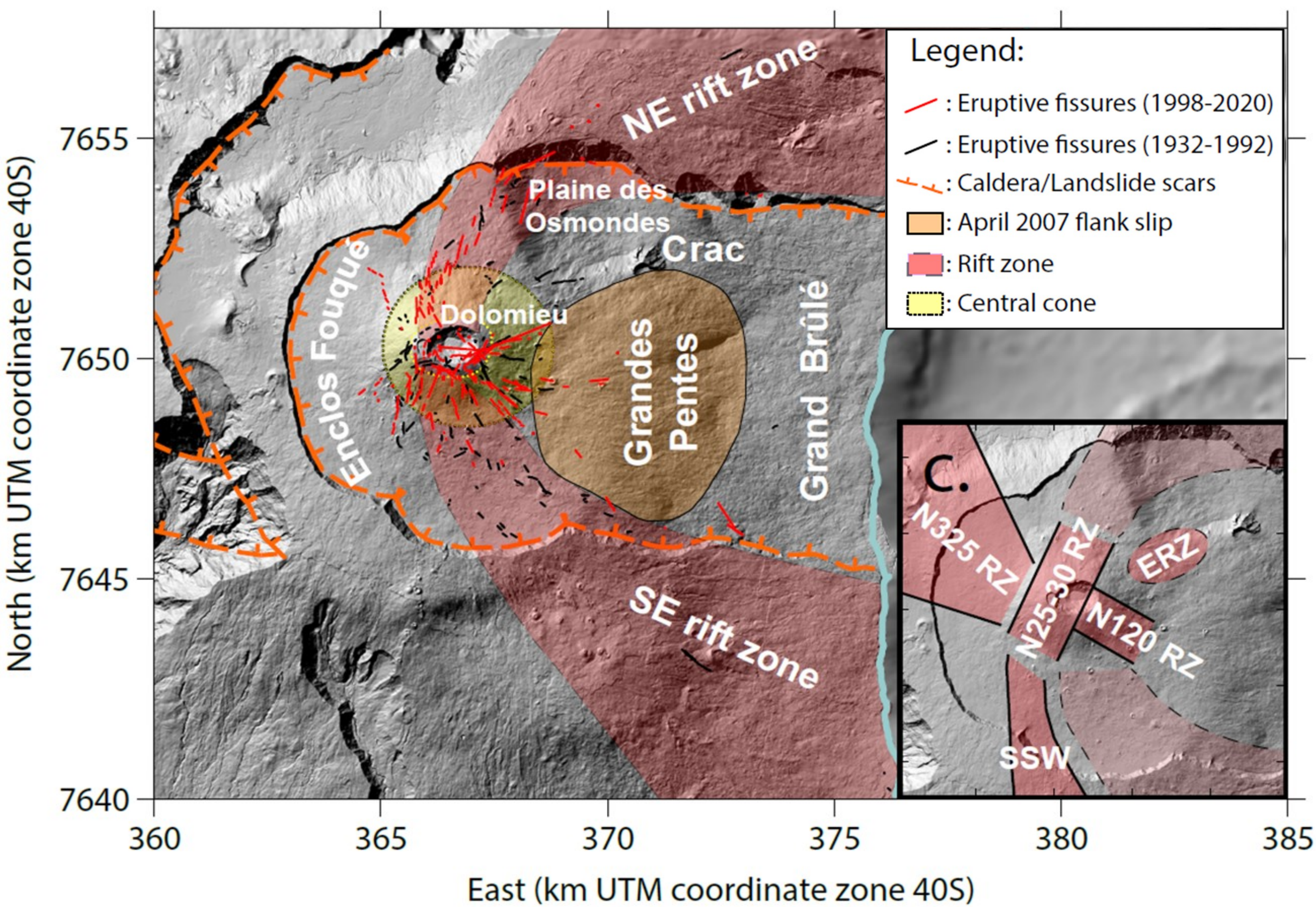
Oehler et al., JVGR, 2008

47 flank failure events
Largest 100 km^3
Oldest 2 My

Piton de la Fournaise is very active: 59 intrusions since 1998 (2.3/year)

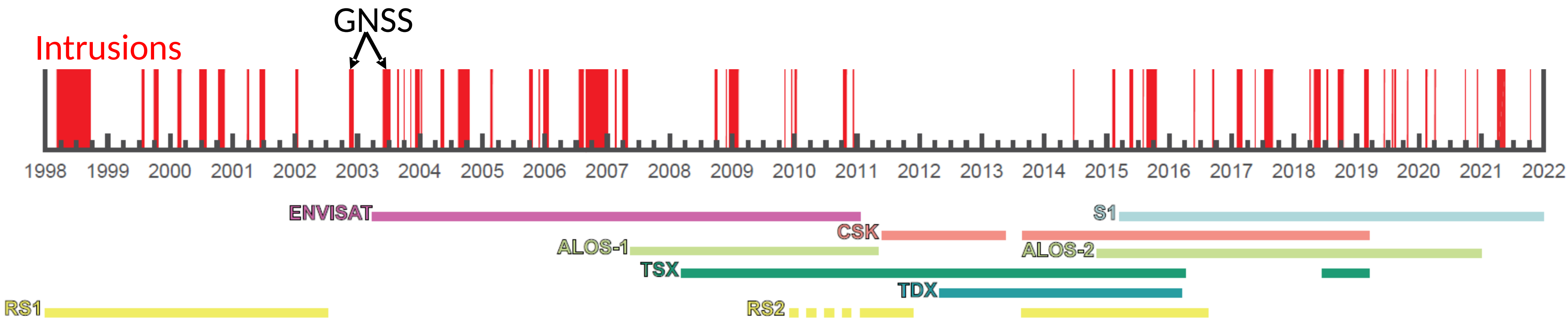


Eruptive fissures : 1932 - 2020

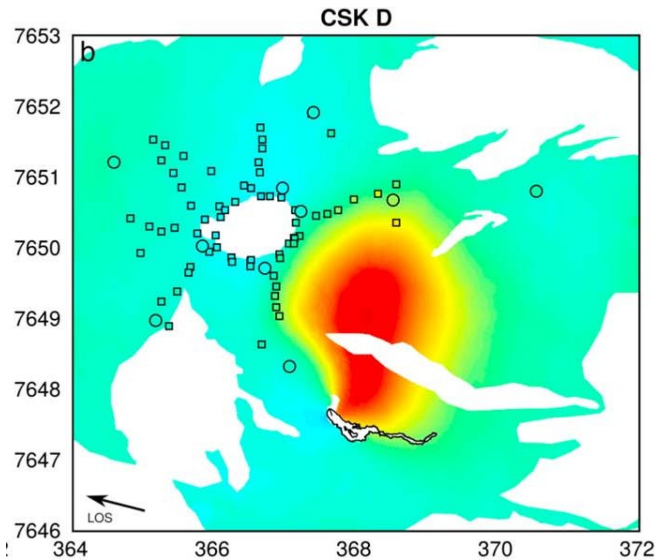


Piton de la Fournaise is one of the best monitored volcanoes

- **since 1998, 57/59 intrusions imaged by at least one InSAR data**

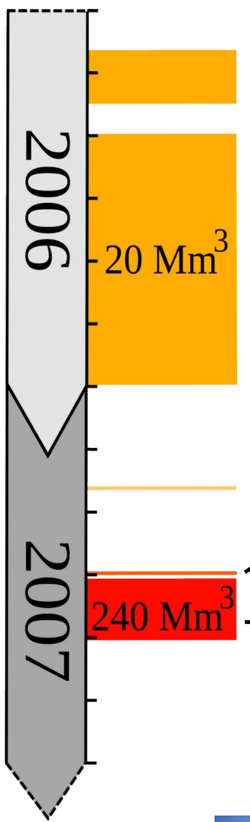


- **GNSS campain + continuous data can be used for non imaged eruptions**

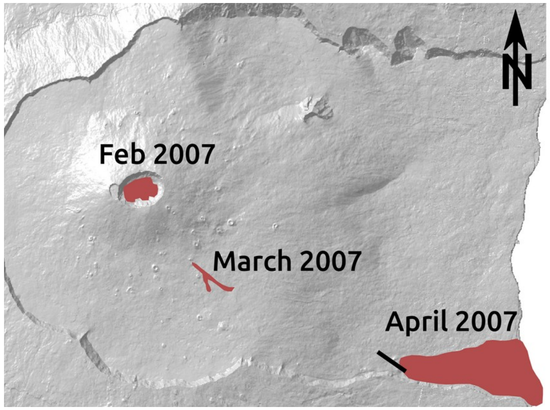


Smittarello et al., JGR, 2019

An unusual flank displacement in 2007



3 summit eruptions

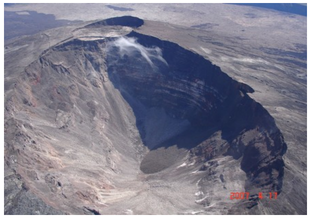


March 30 summit eruption

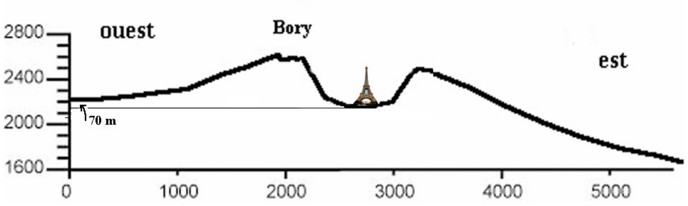
April 2007: distal eruption

- largest emitted volume of XX, XXI century= 240 Mm^3
- 300 meters high caldera collapse on April 6

September 2006

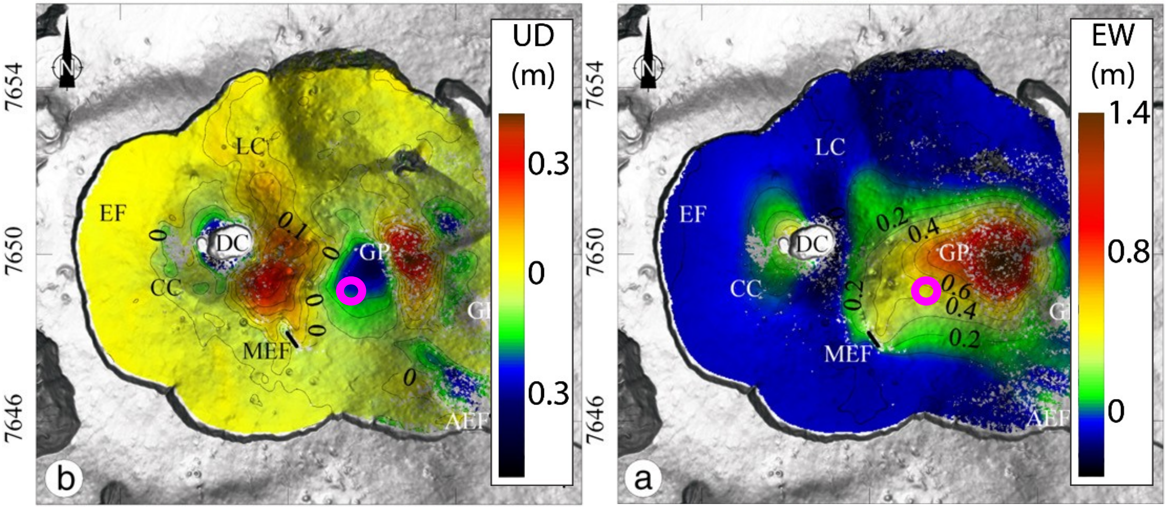


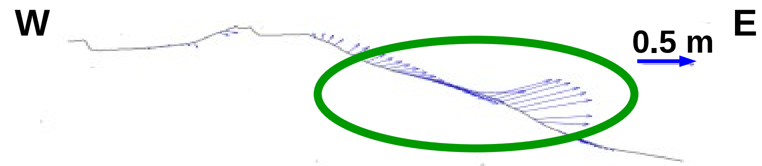
April 2007



An unusual flank displacement in 2007

Co-eruptive displacement

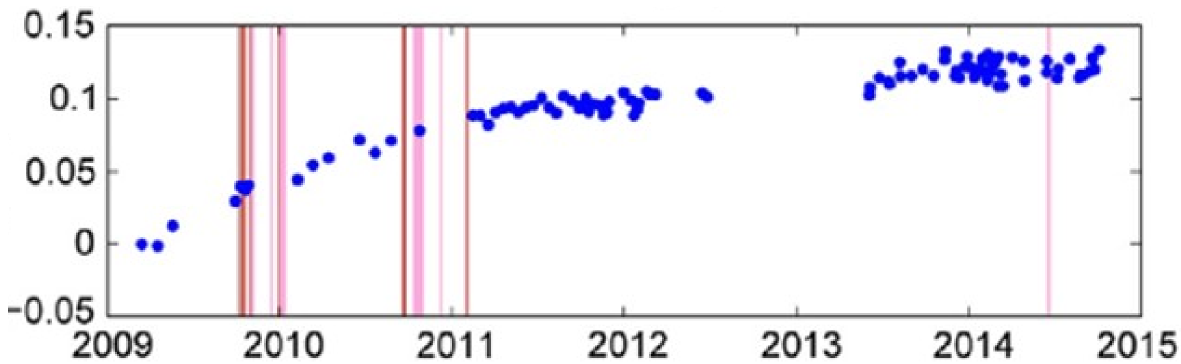




Froger et al., JVGR, 2015

1.4 m eastward / 0.37 m uplift

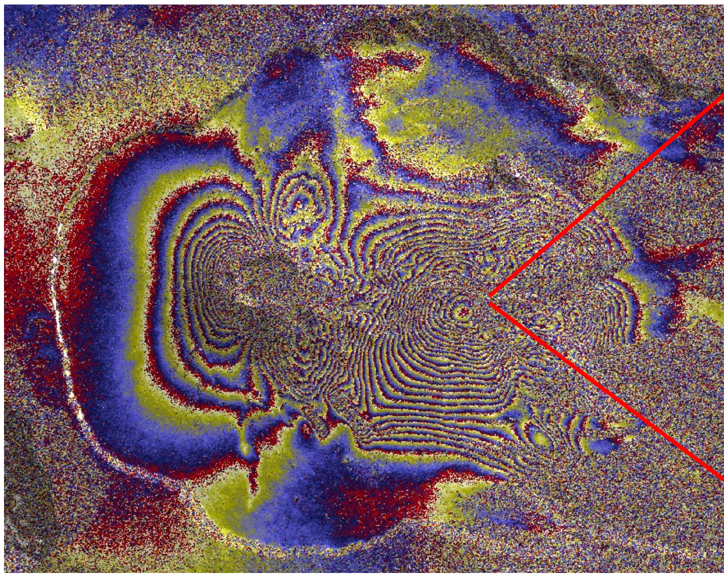
Long-term EW displacement

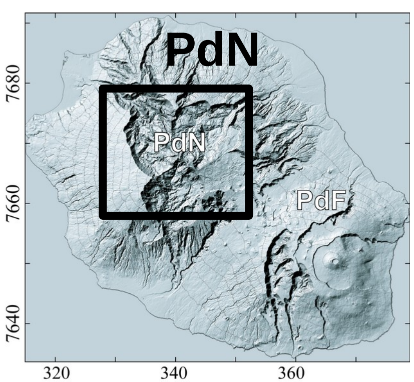


1-2 cm/yr eastward and subsidence

Chen et al., Rem. Sens. Envir., 2017

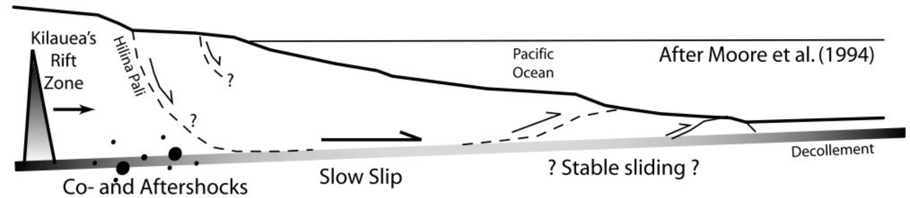
Origin of the 2007 flank displacement





System of faults ?

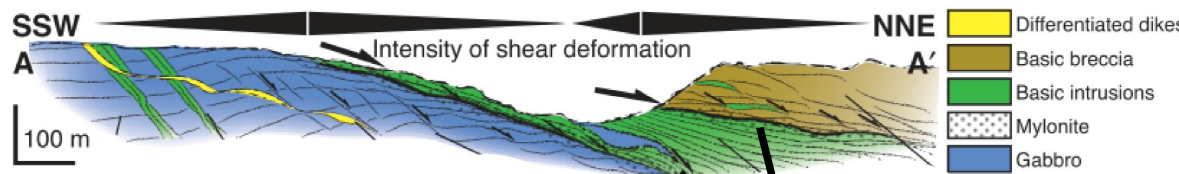
Hawaï



Montgomery-Brown et al., JGR, 2009

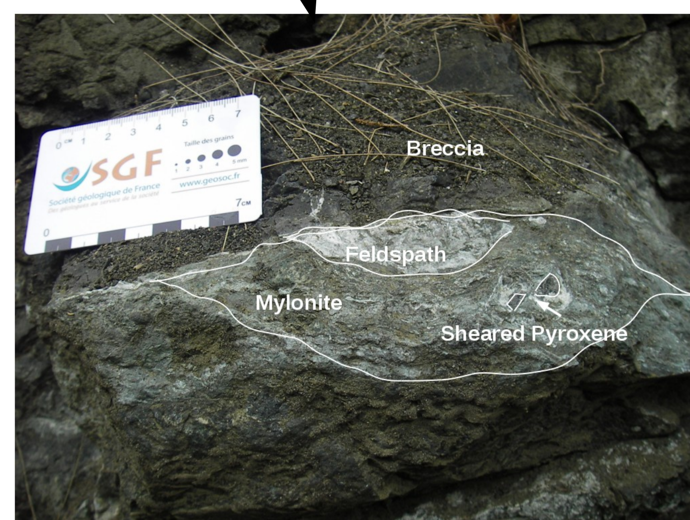
Intrusion and shearing of a sill ?

Piton des Neiges



Famin and Michon, Geology, 2010

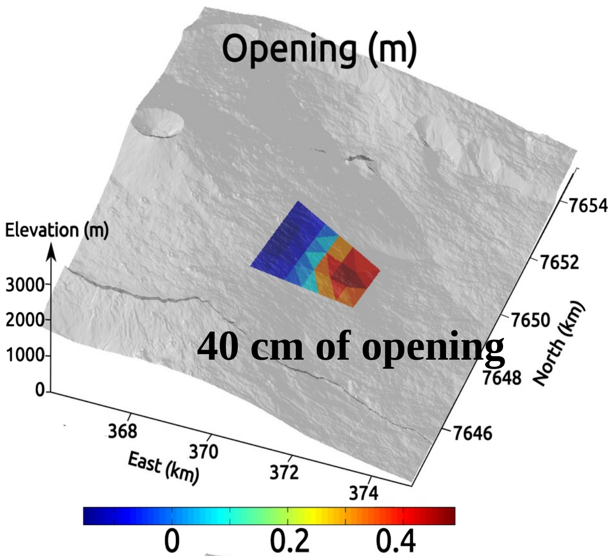
Evidence of shear ductile and brittle deformation



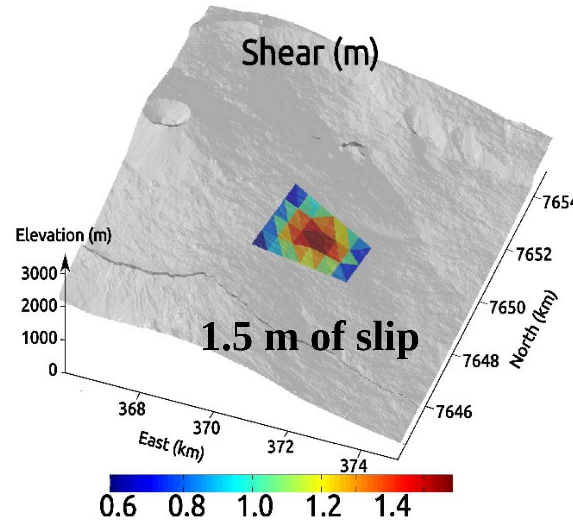
Inversion of geometry and stress changes for the co-eruptive displacement

Tridon et al., IAVCEI, 2017

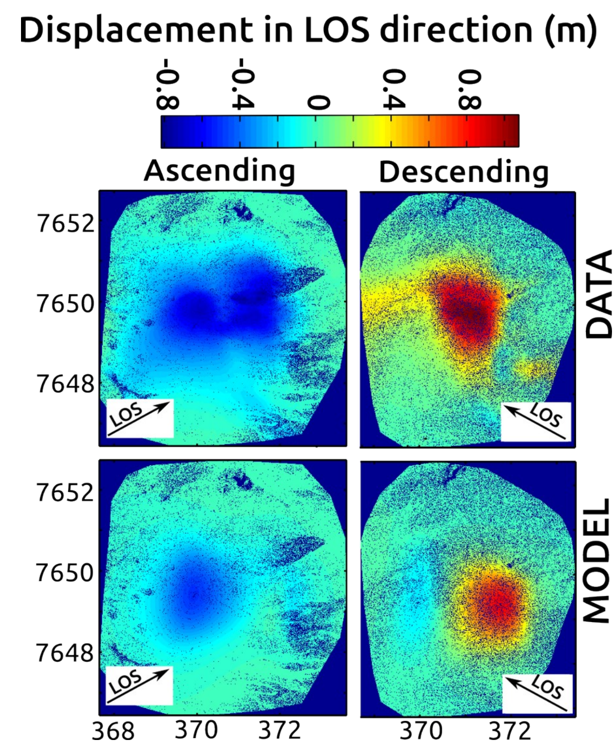
Inversion of 8 geometrical parameters + a pressure and a shear stress change



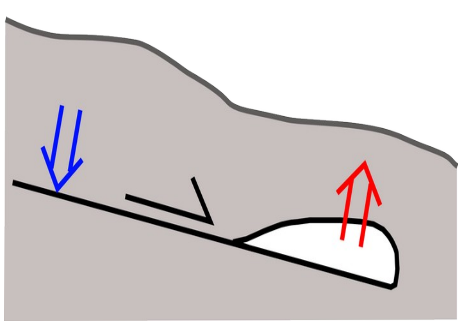
Pressure change ~ 0



Shear stress drop ~ 2 MPa



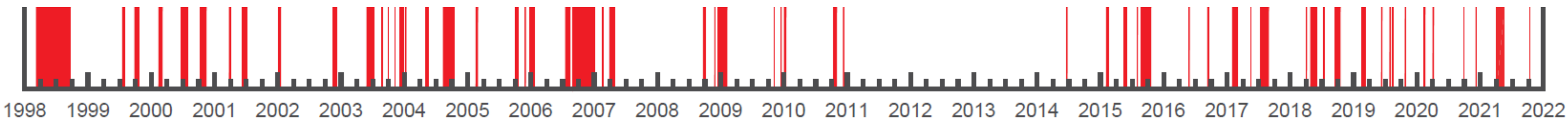
A FAULT !

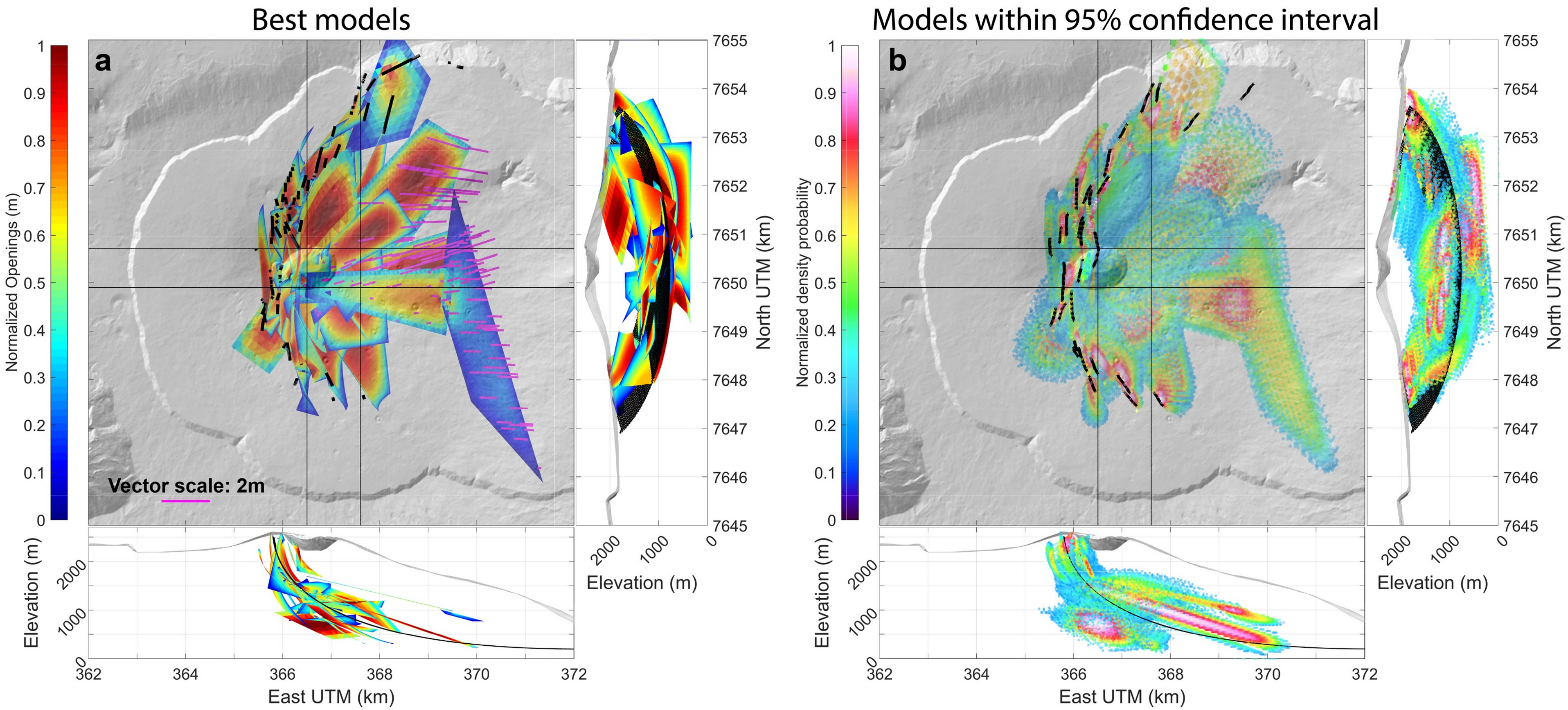


85% of displacement explained

Displacements are characteristic of a detachment fold

Inverse modeling of 22 years of InSAR and GNSS data



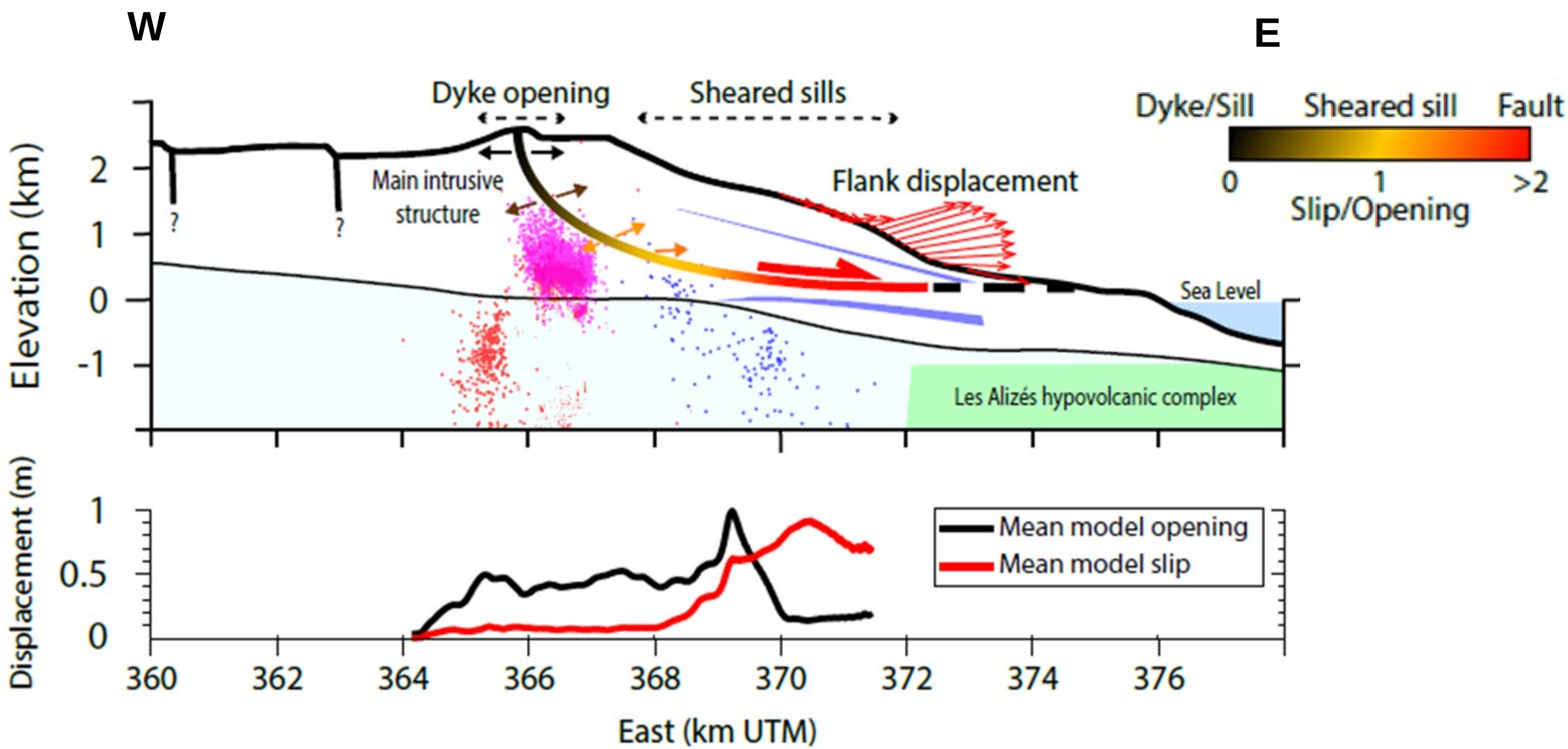


Dumont et al., Nature Communication, 2023 ; EPSL, 2024

➡ **80% of the magma intrudes in a spoon-shaped collapse structure**

A major spoon-shaped collapse structure

Dumont et al., Nature Communication, 2023 ; EPSL, 2024



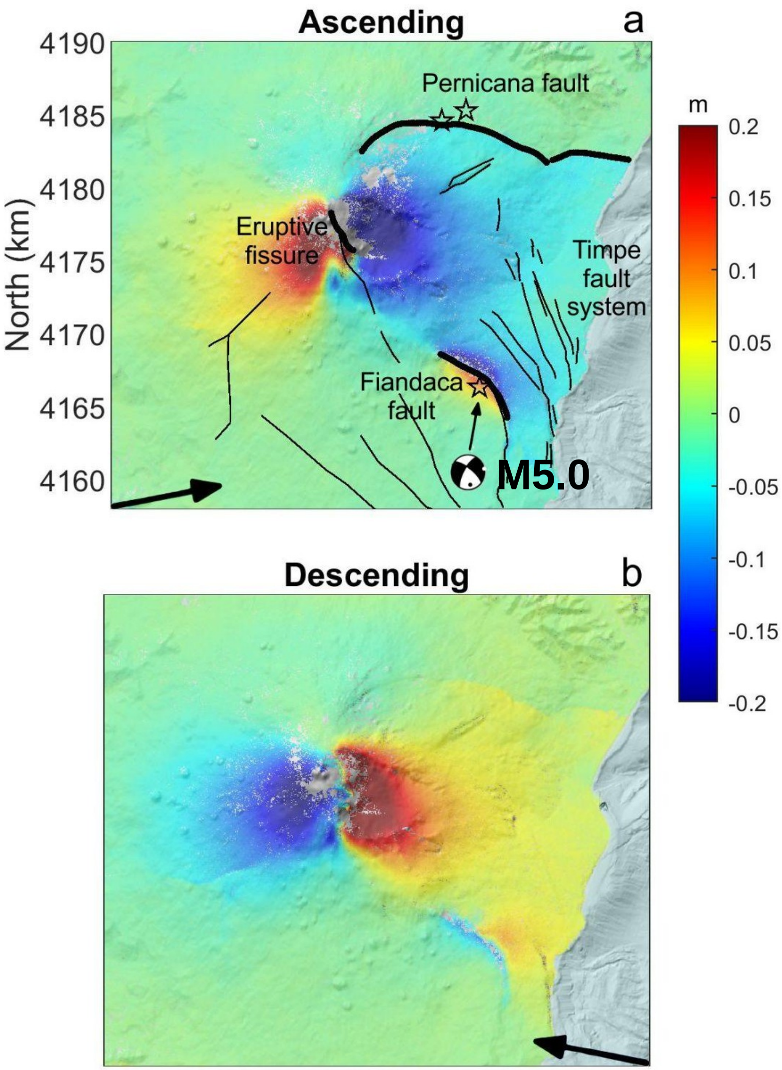
Continuum of displacements from west to east:

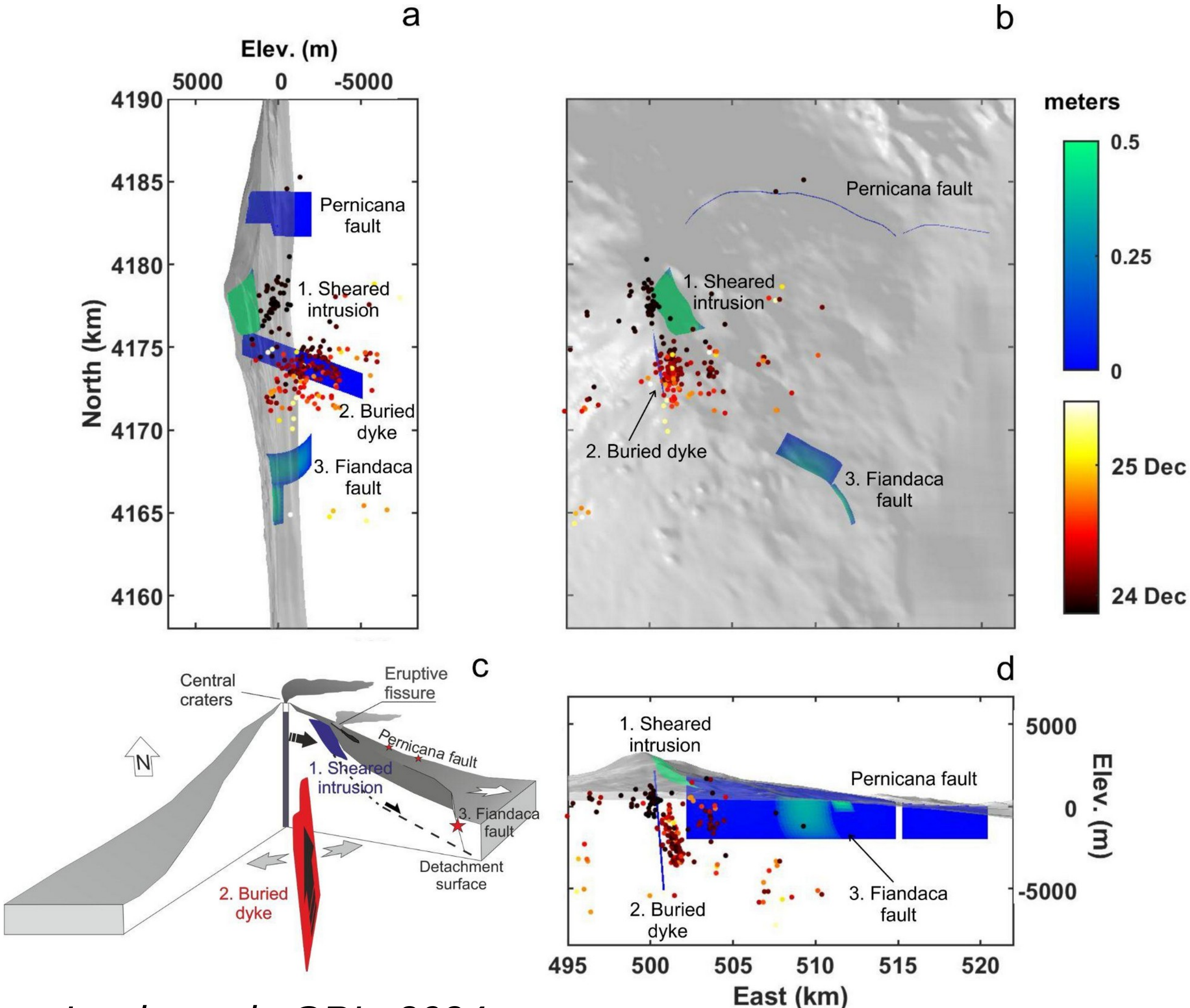
- **Pure opening** of subvertical curved dykes
- **Curved sheared sills**
- **Fault slip** in the easternmost part (in 2007)

➡ **Hybrid between previously assumed models;**

➡ **Could accommodate flank failure**

A similar structure may be active at Etna as evidence by the 2018 Christmas event





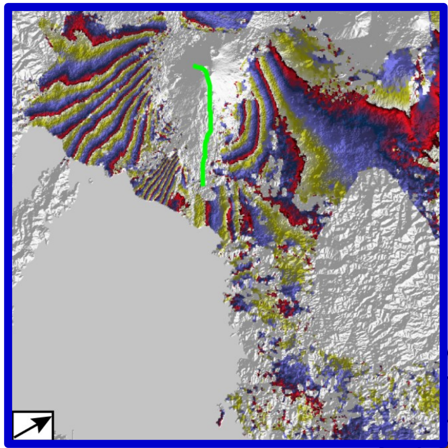
Iozzia et al., GRL, 2024

- A curved sheared intrusion and a buried dyke explain displacement close to the summit;
- Pernicana fault responded passively; Fiandaca fault released accumulated stress;

What can be learnt from the Inverse modelling of InSAR data ?

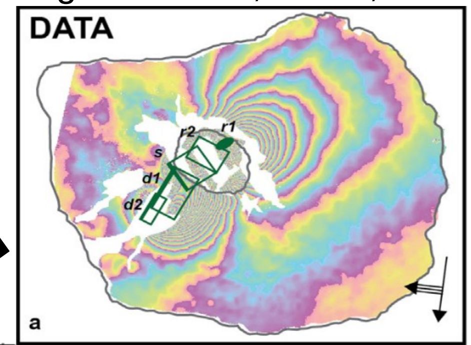
Rift extension drive

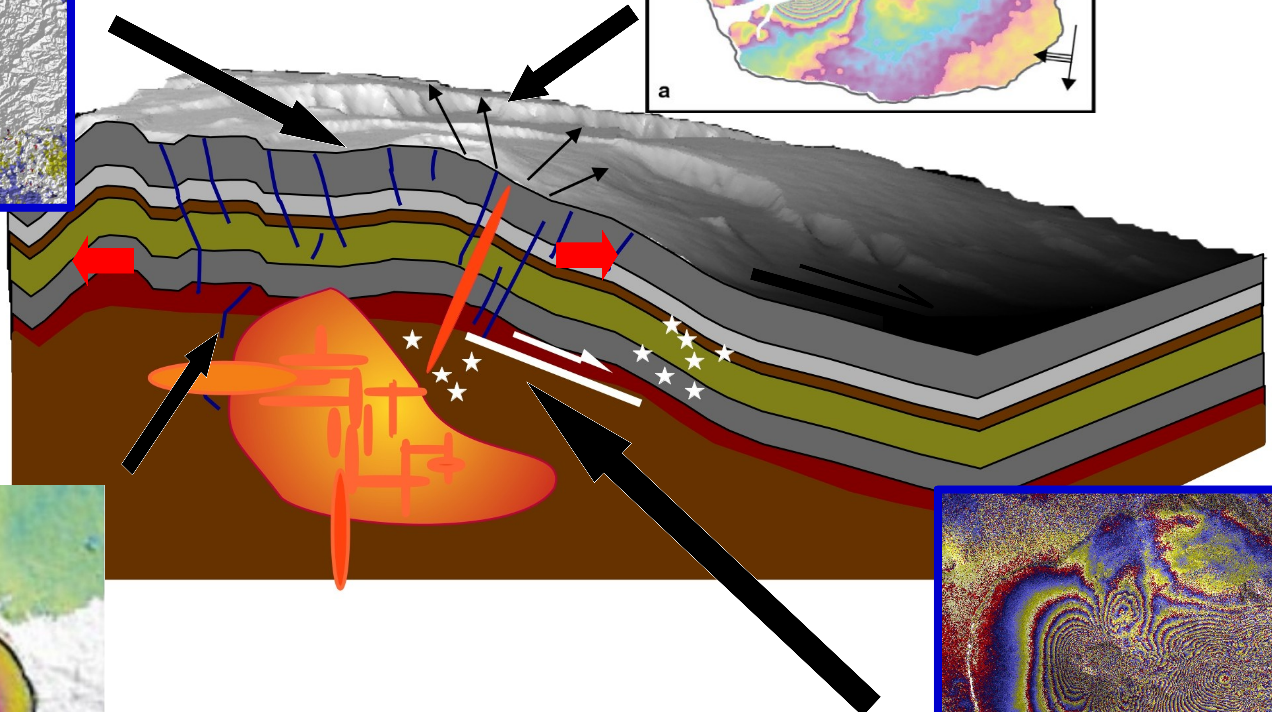
Wauthier et al., 2012

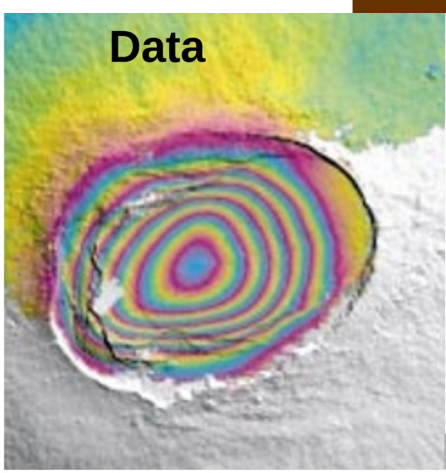


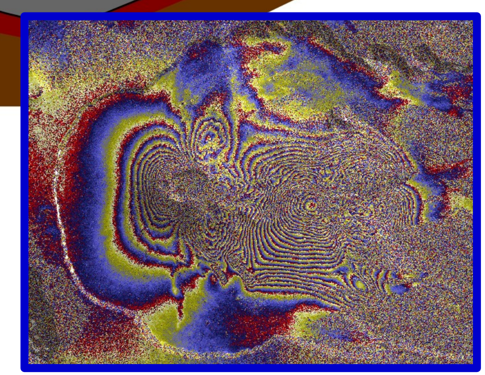
Intrusion pathways

Bagnardi et al., EPSL, 2013









Flank slip mechanism

Tridon et al., JGR, 2016;
Dumont et al., Nat. comm., 2022.

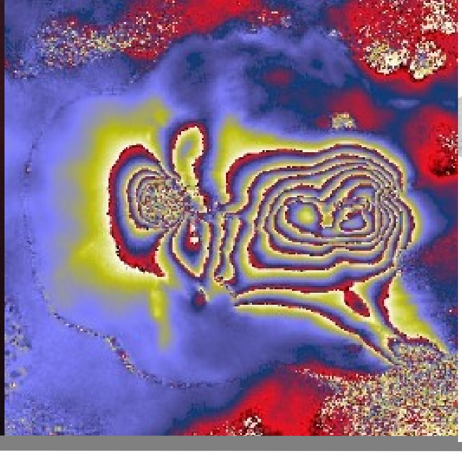
Characteristics of reservoirs

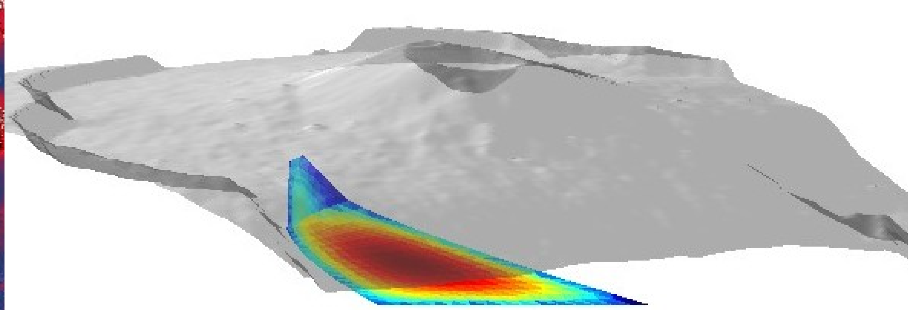
Amelung et al., Science, 2000

Conclusions

- **Inverting for stress changes:**
  - **is more physical than kinematic inversions;**
  - **leads to more likely models;**
  - **is more informative.**

- **In the Virunga Volcanic Province, the rift extension is driven by magmatic activity rather than plate extension;**
- **At Piton de la Fournaise, we find a continuum of fracture displacement: dike intrusion -> sheared intrusions -> fault slip that accommodates magma intrusions;**
- **Sheared intrusions also seem to be active at Etna.**
- **Sheared intrusions should be searched at other shield volcanoes with evidence of flank slip**





Thank you for your attention !

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Quentin Dumont

Yo Fukushima

Gilda Currenti

Christelle Wauthier

Marine Tridon

IMPROVE European Training Network