HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

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Dynamically assisted tunneling in the impulse regime

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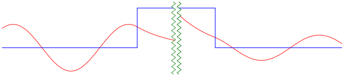
Time-dependent Tunneling

- S. Coleman: *"Every child knows..."*
  $P \sim \exp\left[-2\int dx\,\sqrt{2m[V(x)-E]}/\hbar\right]$
  S. Coleman, *The uses of instantons*, in *Aspects of symmetry* (1985)

- Question: $V(x) \to V(t,x)$?
- Simplified: $V(t,x) = V_0(x) + xV_1(t)$
  - Pre-acceleration
  - Potential deformation
  - Franz-Keldysh effect
  - Displacement effect at rear end



- Adiabatic versus non-adiabatic:
  Büttiker-Landauer "traversal" time $\mathfrak{T} = \sqrt{m}\int dx\,/\sqrt{2[V_0(x)-E]}$

S. Coleman, *The uses of instantons*, in *Aspects of symmetry* (1985)
L. V. Keldysh, Sov. Phys. JETP **34**, 788 (1958)
M. Büttiker and R. Landauer, PRL **49**, 1739 (1982)

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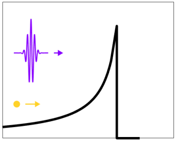
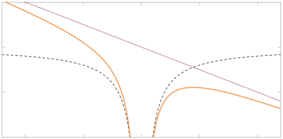
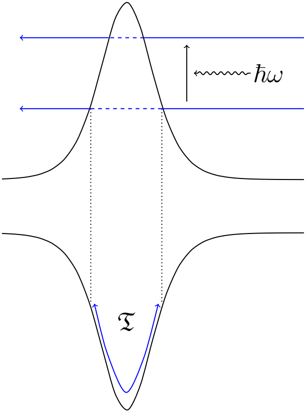
Adiabatic and non-adiabatic effects







- Pre-acceleration
  ("classical acceleration")
- Energy mixing (Floquet states)
- Deformation of the potential $V_0(x)$
- Pushing out of the potential $V_0(x)$

Kramers-Henneberger Map

- Schrödinger equation for time-dependent electrical field

$$i\partial_t \psi(t,x) = -\frac{(\partial_x - iqA(t))^2}{2m}\psi(t,x) + V_0(x)\psi(t,x)$$

- Vector potential can be translated to displacement
  $\psi(t,x) \to \psi(t, x - \chi(t))$
- Point particle in electric field: $m\dot{\chi}(t) = -q\mathfrak{A}(t)$
- Equivalent to Schrödinger equation with quivering potential
  $V_0(x) \to V_0(x + \chi(t))$

W. C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968)

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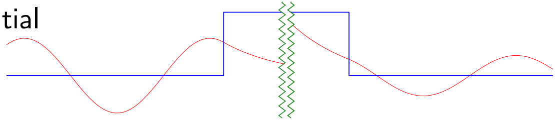
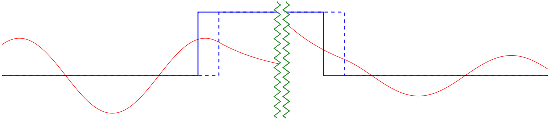
Dynamically Assisted Tunneling: **Box potential**

- Static potential



- Kramers-Henneberger displacement $m\ddot{\chi}(t) = q\mathfrak{E}(t)$



- Energy mixing: $\psi(t, x < 0) = e^{-iE_{\rm in}t + i\sqrt{2mE_{\rm in}}[x - \chi(t)]}$

$$+ \int dE \psi_{\rm ref}(E) e^{-iEt - i\sqrt{2mE}[x - \chi(t)]}$$

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

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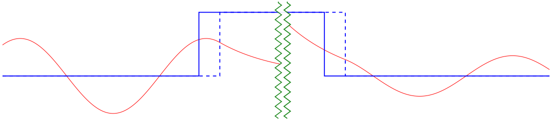
Opaque Barrier approximation

- Transmitted wave for small energies $E, E_{\rm in} \ll V_0$ and $\chi \ll L$

$$\psi_{\rm tra}(E) \approx \psi_E^0 \int \frac{dt}{2\pi}\, e^{i(E-E_{\rm in})t-\sqrt{2mV_0}[\chi(t+i\mathfrak{T})-\chi(t)]}$$

- Energy mixing $\chi(t+i\mathfrak{T})$ & Displacement ("pushing out") $\chi(t)$



- Analytical continuation implies exponential increase of amplitude (cf. $\chi(t) = \chi_0 \cos(\omega t) \to \chi_0 \exp(\omega \mathfrak{T})$)

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

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Instanton picture

- Exponent: Change of instanton action

$$\sqrt{2mV_0}[\chi(t+i\mathfrak{T})-\chi(t)] = -\int_t^{t+i\mathfrak{T}} dt' \frac{dx}{dt'} qA(t')$$

$$= -\sqrt{\frac{2V_0}{m}} \int_t^{t+i\mathfrak{T}} dt' qA(t')$$

- Leading order in $\mathfrak{T}$: Energy shift: $\triangle E = \sqrt{2mV_0}\mathfrak{T}\ddot{\chi}(t) = mL\ddot{\chi}(t)$
- Second order in $\mathfrak{T}$: Real contribution (quasistatic deformation):

$$\sqrt{\frac{mV_0}{2}}\mathfrak{T}^2\ddot{\chi}(t) = \frac{\mathfrak{T}\triangle E}{2}$$

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

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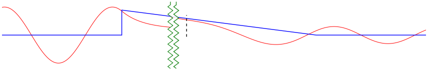
Dynamically Assisted Tunneling: **Triangular Barrier**

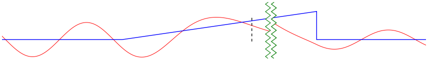
Steep incidence



- Energy mixing at front end

$$\psi_{\mathrm{tra}}(E) \approx \psi_E^0 \int \frac{dt}{2\pi} \times e^{i(E-E_{\mathrm{in}})t-\sqrt{2mV_0}\chi(t+i\mathfrak{T})}$$

Gradual incidence

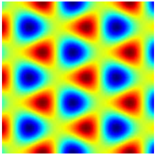


- Displacement at rear end

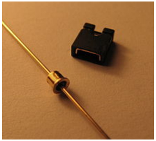
$$\psi_{\mathrm{tra}}(E) \approx \psi_E^0 \int \frac{dt}{2\pi} \times e^{i(E-E_{\mathrm{in}})t+\sqrt{2mV_0}\chi(t)}$$

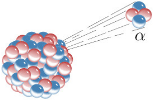
Energy mixing different from “pushing out” effect $\rightarrow$ Quantum Ratchets

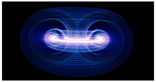
C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

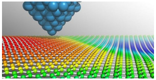


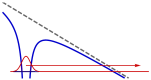
- Onset of non-adiabatic effects: $\omega\mathfrak{T} \sim 1$
- Estimate: $\mathfrak{T} \sim \mathcal{O}(L^2 m)$







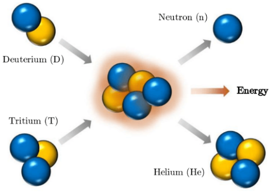


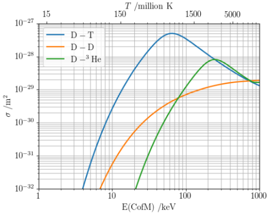


Length	System	Energy	Field Strength
μm	optical lattices	peV	n.a.
nm	solids	meV	10^5 V/m
	atoms	eV	10^{10} V/m
pm	nuclear fusion	keV	10^{16} V/m
fm	α-decay	MeV	10^{18} V/m

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

Nuclear fusion





- Nuclear fusion: Quantum tunneling through Coulomb barrier
- Estimate of tunneling probability:

  Gamow factor $P \sim \exp\left[-\pi\sqrt{\frac{2mc^2}{E}}\alpha_{\mathrm{QED}}\right]$
- Dynamical assistance of nuclear fusion?

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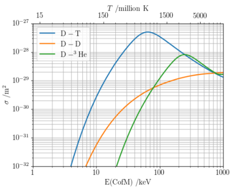
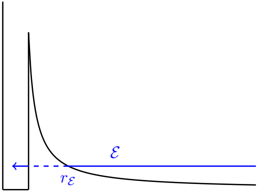
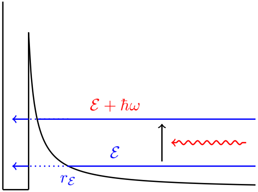
Dynamically Assisted Nuclear Fusion







- ${}^2_1\mathrm{D} + {}^3_1\mathrm{T} \to {}^4_2\mathrm{He} + {}^1_0\mathrm{n} + 17.6\ \mathrm{MeV}$, ${}^1_1\mathrm{p} + {}^{11}_5\mathrm{B} \to 3 \times {}^4_2\mathrm{He} + 8.7\ \mathrm{MeV}$
- Assistance by XFEL pulse $A_x(t) = A_0 / \cosh^2(\omega t)$ or field of α-particles
- Tunneling from increased energy level $\mathcal{E} + \hbar\omega$ through highly asymmetric potential

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)
FQ and R. Schützhold, Phys. Rev. C 100, 041601(R) (2019)

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Analytical Model

- Two-body Lagrangian with Coulomb (+nuclear) field and XFEL
  $\boldsymbol{L}_{12} = \frac{\boldsymbol{m}_1}{2}\,\dot{\boldsymbol{r}}_1^2 + \frac{\boldsymbol{m}_2}{2}\,\dot{\boldsymbol{r}}_2^2 - \boldsymbol{V}(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) + (\boldsymbol{q}_1\dot{\boldsymbol{r}}_1 + \boldsymbol{q}_2\dot{\boldsymbol{r}}_2)\cdot \boldsymbol{A}(\boldsymbol{t})$
- Center-of-mass and relative coordinates with reduced mass
  $\boldsymbol{L} = \frac{\boldsymbol{m}}{2}\,\dot{\boldsymbol{r}}^2 - \boldsymbol{V}(|\boldsymbol{r}|) + \boldsymbol{q}_{\mathrm{eff}}\,\dot{\boldsymbol{r}}\cdot \boldsymbol{A}(\boldsymbol{t})$
- Approximate scaling symmetry: dimension-less parameters
  $\eta = 2mEr_E^2 = \frac{2m}{E}\left(\frac{q_1 q_2}{4\pi\epsilon_0}\right)^2 , \quad \zeta = \frac{q_{\mathrm{eff}}A}{m\omega r_E} = \frac{q_{\mathrm{eff}}A}{mc}\,\frac{E}{\omega}\,\frac{4\pi\epsilon_0 c}{q_1 q_2}$
- WKB tunneling exponent $\mathcal{P} \sim \exp\{-\pi\sqrt{\eta}\}$
- Scaling $E_{\mathrm{p+B}} \leftrightarrow 19E_{\mathrm{D+T}}$

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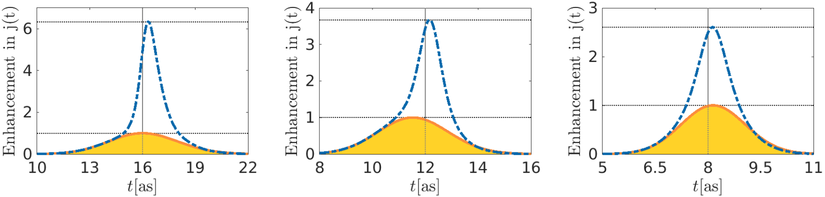
Numerical Simulations I



$E_{\rm in} = 2$ keV | $E_{\rm in} = 4$ keV | $E_{\rm in} = 8$ keV

- Enhancement of Deuterium-tritium fusion rates
- Solution of Schrödinger equation
- Initial kinetic energy: 2 keV, 4 keV and 8 keV
- Illustration of dynamical assistance

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

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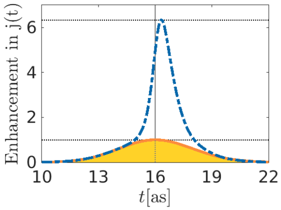
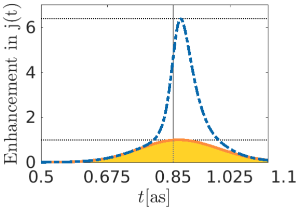
Numerical Simulations II





Deuterium-tritium fusion

$E_{\mathrm{in}} = 2\mathrm{keV}$,
$\omega = 1\mathrm{keV}$, $\mathfrak{E}_0 = 10^{16}\mathrm{V/m}$

Proton-boron fusion

$E_{\mathrm{in}} = 38\mathrm{keV}$,
$\omega = 19\mathrm{keV}$, $\mathfrak{E}_0 = 28 \times 10^{16}\mathrm{V/m}$

Scaling behavior between different fusion reactions

C. Kohlfürst, FQ and R. Schützhold, Phys. Rev. Research 3, 033153 (2021)

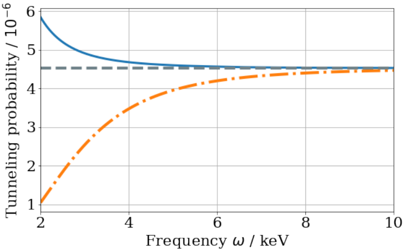
Floquet analysis

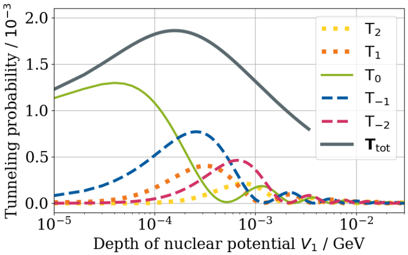
- Periodic driving $\mathfrak{E}(t) = \mathfrak{E}_0 \cos(\omega t)$
- Kramers-Henneberger transformation
- Floquet ansatz for wavefunction $\Psi(x,t) = \sum_n \phi_n(x) e^{i\omega n t}$
- Nonlinear coupled channel equations for amplitudes

$$\frac{dR_i}{dx} = f_i(R_j, T_j, x)$$
$$\frac{dT_i}{dx} = g_i(R_j, T_j, x)$$

- Boundary value problem $\Rightarrow$ initial value problem

Tunneling probabilities





- Averaged potential vs Floquet approach
- $\mathfrak{E}_0 = 2 \cdot 10^{16}$V/m, M=1.13 GeV

- Attractive short-range potential V_1
- $\mathfrak{E}_0 = 2 \cdot 10^{16}$V/m, M=1.13 GeV, $\omega = 6$ keV

D. Ryndyk, C. Kohlfürst, FQ and R. Schützhold, arXiv:2309.12205

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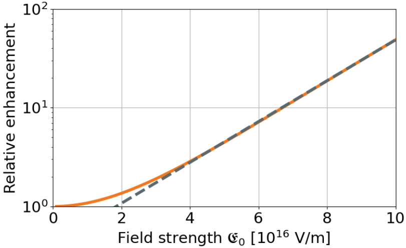
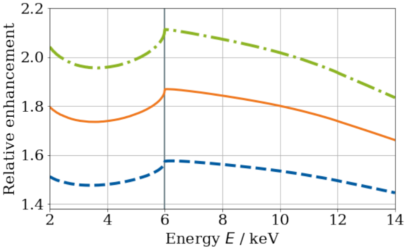
Enhancement and resonances





- Relative enhancement
- $\omega = 2$ keV

- Imaginary channel turns real ($\omega = E$) $\rightarrow$ Resonance

D. Ryndyk, C. Kohlfürst, FQ and R. Schützhold, arXiv:2309.12205

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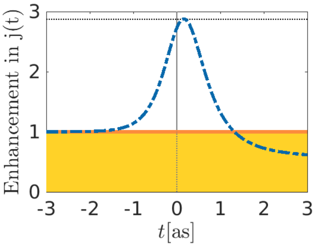
Main takeaway

Dynamically assisted nuclear fusion

- Steep rear end of Coulomb potential: displacement effects
- Scale: $\omega = 1\ \mathrm{keV}$ and $10^{16}\ \mathrm{V/m}$
- Assistance by α-particles?
- Muon-assisted fusion



Dynamically assisted tunneling

- Pre-acceleration
- Energy mixing (front end)
- Deformation of potential
- Displacement (rear end)
- adiabatic vs. non-adiabatic: Landauer-Büttiker time $\mathfrak{T}$