Kinetic Equation Approach to Graphene in Strong Fields

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Uniwersytet Wrocławski

mpipks

Introduction

Motivation

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$$\Gamma \approx \frac{(eE)^2}{4\pi^3\hbar^2 c} \exp\left(-\frac{\pi m^2 c^3}{e\hbar E}\right)$$

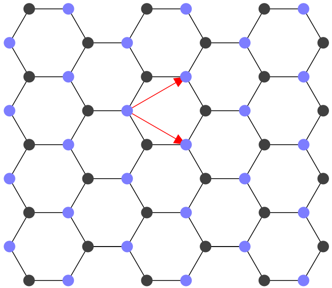
- Experimental realisation of this novel phenomenon is still afar.

$$E_c = \frac{m_e^2 c^3}{e\hbar} \approx 10^{18}\, Vm^{-1}$$

Introduction

Why Graphene?

- Single layer of Carbon atoms arranged in a hexagonal lattice. Two sublattices A and B.



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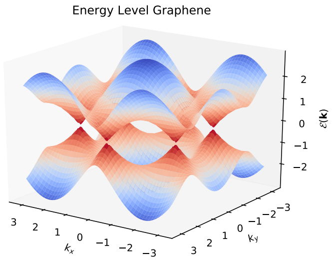

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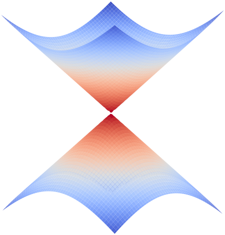

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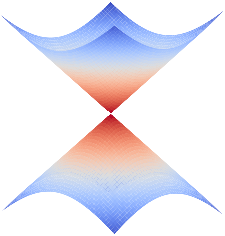
- Theory around Dirac points look exactly like the theory of massless Dirac particles[Novoselov et al. Nature (2005)].



$$\mathcal{E} = v_F|\mathbf{q}|$$

Effect of External Electric Field

- Parallels between Schwinger process in QED vacuum and Graphene

QED	Graphene
Dirac Sea	Fermi Sea
Electron-Positron pairs	Electron-Hole pairs

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$$n(\vec{p}, t) = \Theta(p_x)\Theta(eEt - p_x)\exp\left(-\frac{\pi v_F p_y^2}{\hbar eE}\right)$$

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- Different stages

Classical	Kubo	Schwinger/Kibble-Zurek
$t \ll \frac{h}{W}$	$\frac{h}{W} \ll t \ll \sqrt{\frac{\hbar}{v_F eE}}$	$\sqrt{\frac{\hbar}{v_F eE}} \ll t \ll \frac{\hbar}{eaE}$
$j \approx Et$	$j \approx E$	$j \approx tE^{3/2}$

[1]Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation Approach

Formalism[1]

- Effective hamiltonian near Dirac points

$$H(t) = v_F \frac{1}{L^2} \sum_{\vec{p}} \Psi^\dagger(\vec{p}, t)(\vec{\sigma} \cdot \vec{P})\Psi(\vec{p}, t)$$

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$$\Psi \to U\Psi = \Phi = \begin{pmatrix} a(\vec{p}, t) \\ b^\dagger(-\vec{p}, t) \end{pmatrix}$$

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- Distribution functions

$$f_e(\vec{p}, t) = \langle a^\dagger(\vec{p}, t) a(\vec{p}, t)\rangle \qquad f_h(\vec{p}, t) = \langle b^\dagger(-\vec{p}, t) b(-\vec{p}, t)\rangle$$

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- From the equation of motion we get the following integral equation for the distribution functions $f = f_e = f_h$

$$\dot{f}(\vec{p}, t) = \frac{1}{2}\lambda(\vec{p}, t) \int_{t_0}^{t} dt' \lambda(\vec{p}, t')(1 - 2f(\vec{p}, t')) \cos\theta(t, t')$$

where $\varepsilon(\vec{p}, t) = v_F \sqrt{P_1^2 + P_2^2}$, $\lambda(\vec{p}, t) = e v_F^2 \dfrac{E_1 P_2 - E_2 P_1}{\varepsilon(\vec{p}, t)^2}$ and $\theta(t, t') = \frac{2}{\hbar} \int_{t'}^{t} dt'' \varepsilon(\vec{p}, t'')$

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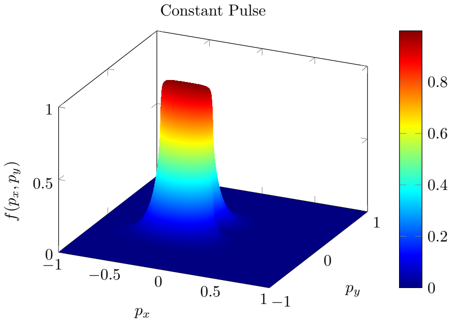
- The above Integral equation is equivalent to the following set of Ordinary Differential Equations

$$\dot{f} = \frac{1}{2}\lambda u \qquad \dot{u} = \lambda(1 - 2f) - \frac{2\varepsilon}{\hbar} v \qquad \dot{v} = \frac{2\varepsilon}{\hbar} u$$

- These can be solved numerically for any given external electric field E.

- Units: Two parameters from graphene Fermi velocity v_F and lattice spacing a.
- Timescale a/v_F and momentum in terms $\hbar/a$.
- Constant electric field, figure shows distribution funciton at $t = 5\frac{a}{v_F}$.
- Number density $N = \int f(\vec{p}, t) d^2 p$



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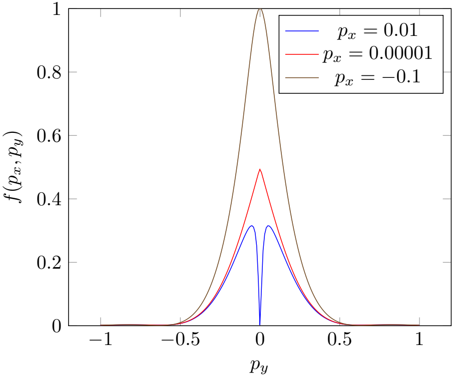
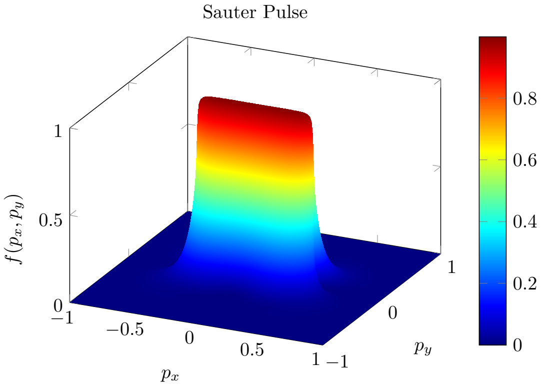


Results

Sauter Pulse

- Sauter Pulse

$$E(t) = E_0 \cosh^{-2}(\kappa t)$$



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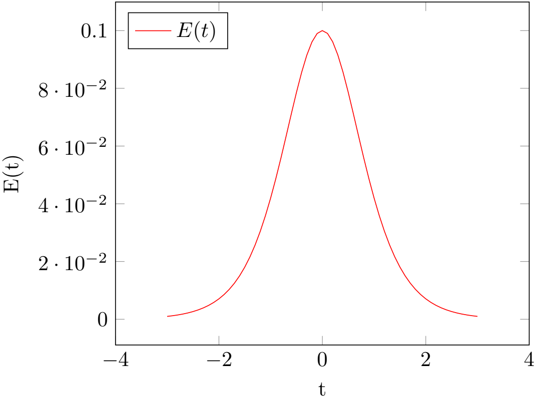

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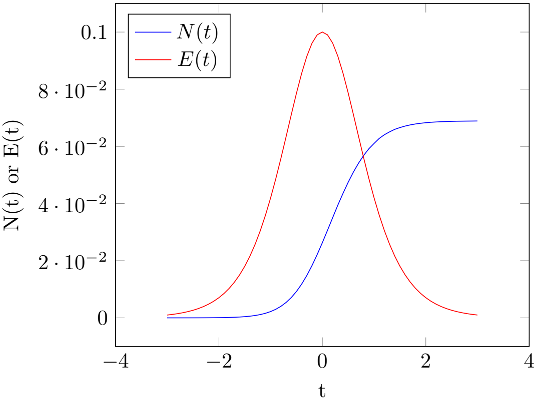

Approximate Solution[2]

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- Low Density Approximation $f \ll 1$

$$f(t) = \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\cos\theta(t',-\infty)\right)^2 + \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\sin\theta(t',-\infty)\right)^2$$

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- Effective mass Approximation
  - Replace momentum with its time average

$$P^2 \rightarrow < P(t)^2 >= p^2 + e^2 < A(t)^2 >= p^2 + e^2 m_*^2 v_F^2$$

  - $\lambda_*(\vec{p}, t) = -\frac{e v_F^2}{\varepsilon_*^2(\vec{p})} E(t) := \Lambda(p) E(t)$

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  - $\lambda_*(\vec{p},t) = -\frac{ev_F^2}{\varepsilon_*^2(\vec{p})}E(t) := \Lambda(p)E(t)$
- Distribution functions can directly be computed via

$$f(\vec{p},t) = \Lambda^2(\vec{p})\left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda_*(t')\cos\theta_*(t',-\infty)\right)^2$$

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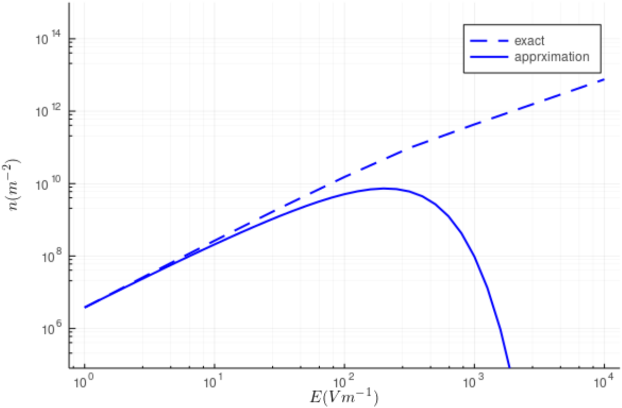
Comparison

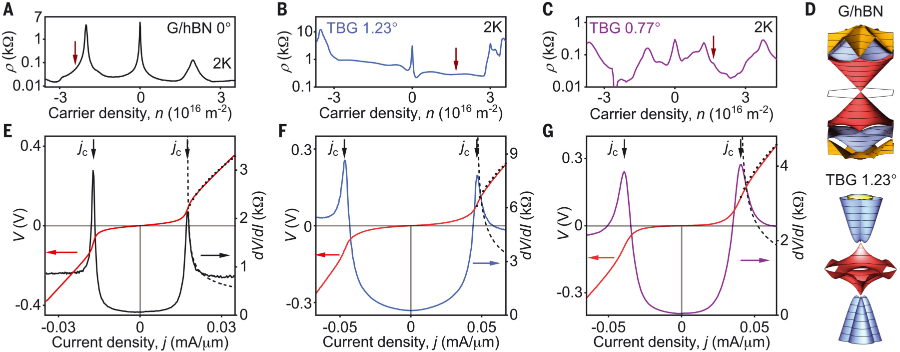


Slope $= 1.52 \approx \frac{3}{2}$

The Approximate solution deviates from the exact solution for large external fields. $\frac{2\pi m_* v_F^2}{\hbar\kappa} < 1$ or $E_0 < \frac{\hbar\kappa^2}{2\pi e v_F}$

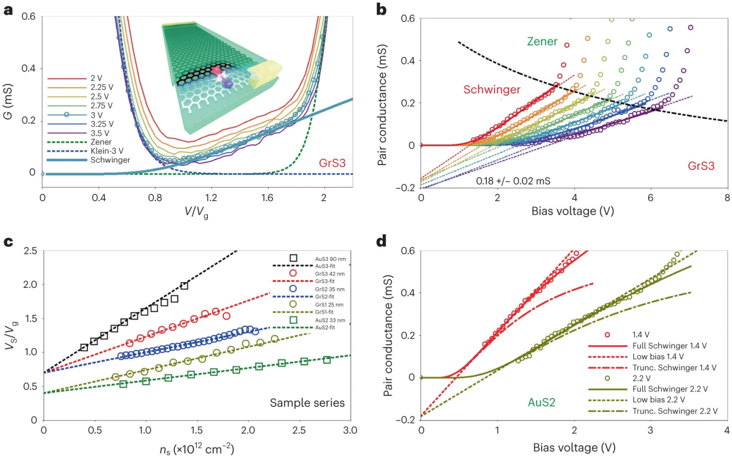
Experiment 1

[Berdyugin et. al., Science, 2022]



Experiment 2

[Schmitt A. et. al. Nature Phys. 2023]



Summary and Outlook

- Graphene can be a testing ground to gain insight about the structure of QED vacuum.
- Kinetic equation approach to it can produce the momentum profile of the created electron hole pairs.

Future Works

- Include back-reaction and collision terms.
- Gapped system [S. P. Gavrilov, D. M. Gitman, Phys. Rev. D,1996]

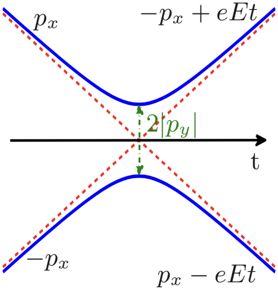
$$\langle j(t)\rangle \approx tE^{\frac{d+1}{2}} \exp\left(\frac{-\pi\Delta^2}{ev_F E\hbar}\right)$$

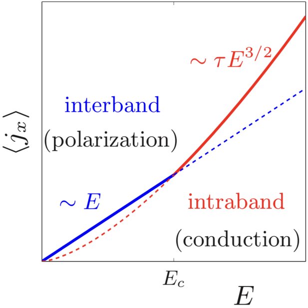
Thank You

Supplimentary slide

1[Dora B., Moessner R.,Phys Rev B., 2010]

Constant Electric field E





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Comparison with other methods

[Panferov A., EPJ WoC 204,06008 (2019); 1901.01395]