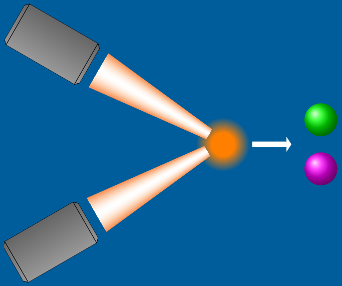
DRESDEN concept

HZDR
HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

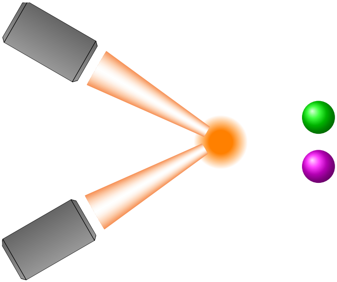
Sauter-Schwinger effect for colliding laser pulses

Christian Kohlfürst, Naser Ahmadiniaz, Ralf Schützhold



c.kohlfuerst@hzdr.de

Particle Systems under Extreme Condition



Transforming energy into matter

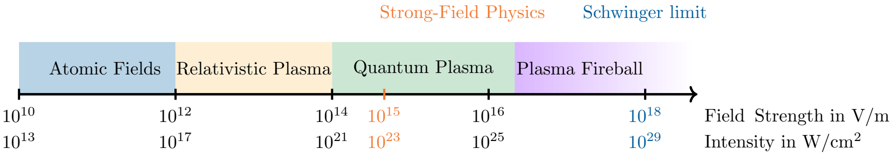
- Convert light into electrons and positrons
- Driving quantum system out-of equilibrium
- Probe non-perturbative aspects of quantum field theory
- Interacting, non-linear vacuum as background
- Search for Emergence

Characteristic Scales

Quantum and relativistic physics

- System of units: $c = \hbar = 1$
- Electron mass $m \sim 9 \times 10^{-31}$kg
- Compton time $\tau_{\text{Compton}} \sim 10^{-21}$ s
- Compton length $\lambda_{\text{Compton}} \sim 10^{-12}$ m
- Strong fields $E \geq 10^{15}$ V/m
- Schwinger limit $E_{\text{cr}} \simeq 10^{18}$ V/m

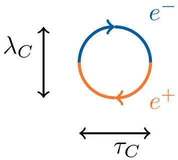
High-intensity fields as the pathway to new physics



Theoretical Concepts

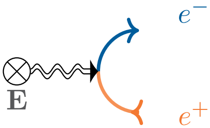
Quantum vacuum fluctuations

- Virtual electron-positron loop
- Scale set by electron mass $m \simeq 511$ keV



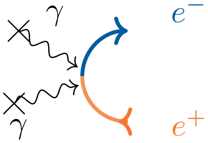
Quantum tunneling pair production

- Non-perturbative pair production:
  work done by field $eE\lambda_C \geq 2m$
- Scaling via $\exp(-\pi E_{\text{cr}} \,/\, E)$

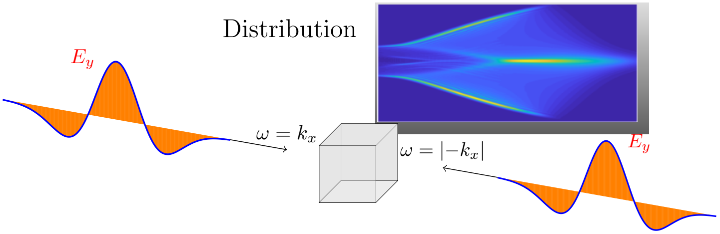


Multiphoton absorption & Breit-Wheeler

- Interaction of n photons
- Absorb energy $\mathcal{E}$ such that $n\mathcal{E}_\gamma \geq 2m$
- In perturbative regime scales as E^{2n}



The Setup



Simulation: A priori

- $\boldsymbol{A}(t, \boldsymbol{r}) = [f(t-x) + f(t+x)]\boldsymbol{e}_y$

  Pulses $f(t) = tE_{\mathrm{cr}}/6\ e^{-\omega^2 t^2}$
- One free parameter:

  Photon energy & Volume: ω

  Peak in frequency spectrum at $E_\gamma \sim \Omega = 2\omega$

Theory: Step-by-step

- Electric field: One dominant peak
- Magnetic field: Quadrupol

- LO: Field at $\mathbf{E}(t, x = 0)$
- NLO: Curvature of $\mathbf{E}(t, x = 0)$

Non-equilibrium Quantum Electrodynamics

Matrix-valued Wigner distribution function in quantum field theory

$$\mathbb{W}_{\alpha\beta}\left(\mathrm{x},\mathrm{p}\right) = \left\langle \Omega \left| \frac{1}{2} \int \mathrm{d}^4\mathrm{s}\ \mathrm{e}^{\mathrm{i}\mathrm{p}_\mu \mathrm{s}^\mu}\ U\left(A_\mu, \mathrm{x}, \mathrm{s}\right) \left[\hat{\bar{\psi}}_\beta \left(\mathrm{x} - \frac{\mathrm{s}}{2}\right), \hat{\psi}_\alpha \left(\mathrm{x} + \frac{\mathrm{s}}{2}\right)\right] \right| \Omega \right\rangle$$

Gauge transporter $U\left(A_\mu, \mathrm{x}, \mathrm{s}\right) = \exp\left(\mathrm{ie} \int_{-1/2}^{1/2} \mathrm{d}\xi\ s^\mu A_\mu\left(x + \xi s\right)\right)$ fixes gauge invariance

- End product of non-equilibrium Green's function in *in-in* formalism
- Quantized spinors $\hat{\bar{\psi}}$ & $\hat{\psi}$ and classical electromagnetic field $A_\mu(x)$
- Relative s, center-of-mass x & kinetic momentum coordinates p

D. Vasak, M. Gyulassy, H. Elze, Annals of Physics, **173**, 462 (1987); I. Bialynicki-Birula, P. Górnicki, and J. Rafelski, Phys. Rev. D **44**, 6 (1991)

- Time-evolution determined by Dirac equation (initial value problem)
- Non-perturbative relativistic quantum transport
- Extension of quantum Vlasov equation

Theory: WKB Approach

Simpler toy model: Riccati equation

- Klein-Gordon equation $$\left[\left(\partial_\mu + iqA_\mu\right)\left(\partial^\mu + iqA^\mu\right) - m^2\right]\phi = 0$$

  Generalized WKB ansatz $$\boldsymbol{\varphi} = (\phi, \dot{\phi}) = \alpha \mathbf{u}_+ e^{+is} + \beta \mathbf{u}_- e^{-is}$$

- Phase function $$s(t,x) = s_0(t) + \frac{x^2}{2} s_2(t) + \mathcal{O}(x^4)$$

  Solve order-by-order in $s_0,\ s_2, \ldots$

- Pair creation through $$\dot{R} = \partial_t\left(\alpha/\beta\right) = \Box s (e^{+2is} - R^2 e^{-2is})/(2\dot{s})$$

Take-away: Only the pre-factor changes

- Up to $\ddot{s} \to \Box s = \ddot{s} - s''$ same as for purely time-dependent field

  Contribution from electric field: $\ddot{s} \to$ pair creation

  Contribution from magnetic field: $s'' \to$ focusing/de-focusing effects

Theory: Effective Models and Approximations

One-loop Euler-Heisenberg effective Lagrangian

- Non-linear in electromagnetic fields

$$\mathcal{L}_{\text{eff}} \propto \frac{1}{m^4}\left(c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu}F^{*\,\mu\nu})^2\right), \qquad c_1,\ c_2 \text{ coefficients}$$

- Onset of pair production: Instability

Locally constant field approximation (LCFA)

- Source term adapted from Sauter-Schwinger effect

$$\Gamma = \frac{1}{2\pi^3}\sum_i \int \mathrm{d}^3p\ \exp\left(-\frac{\pi\left(m^2 + \mathbf{p}_\perp^2\right)}{eE}\right)$$

Spatially homogeneous approximation (SHA)

- Only consider effects along symmetry axis $\boldsymbol{A}(t) = tE_{\text{cr}}/3\ e^{-\omega^2 t^2}\boldsymbol{e}_y$
- Temporal structure fully resolved - one fit parameter for spatial volume

Sauter-Schwinger effect: Regimes of Production

Colliding pulses

- $\boldsymbol{A}(t, \boldsymbol{r}) = [a(t-z) + a(t+z)]\boldsymbol{e}_x$ with $a(t) = tE_{\mathrm{cr}}/6\ e^{-\omega^2 t^2}$
- Photon energy (Laser frequency) & Volume ω

Christian Kohlfürst, Naser Ahmadiniaz, Johannes Oertel, and Ralf Schützhold, Phys. Rev. Lett. 129, 241801 (2022)

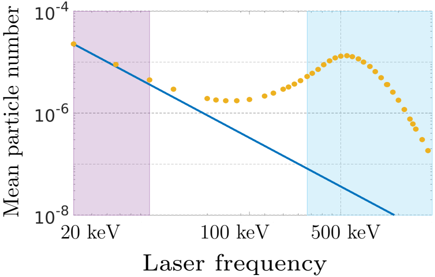
Quantum Tunneling

- Low-energy limit $\omega \leq 0.1m$
- Volume scaling $V \sim 1/\omega^2$

Multiphoton, Breit-Wheeler

- High-energy limit $\omega \approx 500$ keV
- Peak at 2-photon absorption
- Polynomial scaling (perturbative)

Asymptotic particle number



Heisenberg-Wigner formalism

Locally constant field approximation

Intermediate Regime

Downsides of Simulation

- Development and maintainance
- Numerically costly

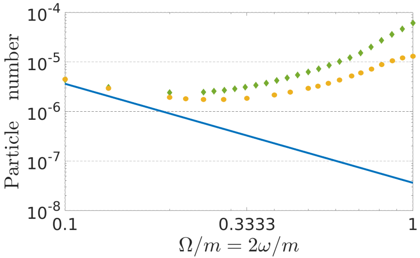
Downsides of LCFA

- Only valid if $\Omega \leq 0.1m$
- Local

Solution: Step-by-step models

- Step 1: SHA
  Right qualitative behaviour
  Quantitative overestimation
- Step 2: Riccati equation in $\Box s$
  Suppressive element
  Improved quantitative accuracy

Asymptotic particle number



Spatially homogeneous approximation

Dirac-Heisenberg-Wigner formalism

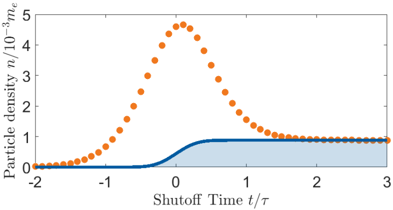
Locally constant field approximation

Time Evolution in Quantum Field Theory

Fundamental problem in quantum field theory: Non-asymptotic physics?

Heisenberg-Wigner formalism

- Real-time formalism
- Particle number as observable



- Full control at all times
- Only creation, no annihilation
- Shutoff t_0 in electric field

  $E(t) = E_{\mathrm{cr.}}/3\,\mathrm{sech}^2(t/\tau)\,\Theta(t_0 - t)$

Heisenberg-Wigner formalism

Locally constant field approximation

Matthias Diez, Reinhard Alkofer, Christian Kohlfürst, Phys. Lett. B 844, 138063 (2023)

Interpretation

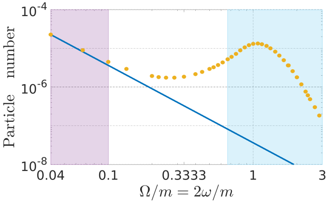
- Schwinger particle creation at all times $\sim \exp\Big(-\pi E_{\mathrm{cr}} \,/\, E(t)\Big)$
- Peak in particle number $n(t)$ due to Uncertainty principle

  Evaluation at specific point in time requires arbitrarily high energy which, in turn, induces perturbative pair production

Summary & Takeaway

Quantum field theory at the extreme

- Non-equilibrium physics at all energies
- Dynamics of fully relativistic systems
- Extract information of a quantum system even at intermediate times



Head-on collision of pulsed fields

$$\boldsymbol{A}\,(x,t) = [f(t-x) + f(t+x)]\,\boldsymbol{e}_y$$

- Regimes of pair production:
  Momentum spectra & particle yield
  (Non)-perturbative aspects
- Beyond dipole field approximation
- Improved schemes

Experimental scenarios

- X-ray free electron lasers
- “Ultraperipheral” collisions
- Solid state analogues

Literature

CK *et al.*, Phys. Rev. Lett. 129, 241801 (2022)

Matthias Diez *et al.*, Phys. Lett. B 844, 138063 (2023)


Thank you!

How to probe pair production?

Experiments

- SLAC Experiment 144
- Probing Strong-field QED at FACET-II, Stanford
- HIBEF, Hamburg

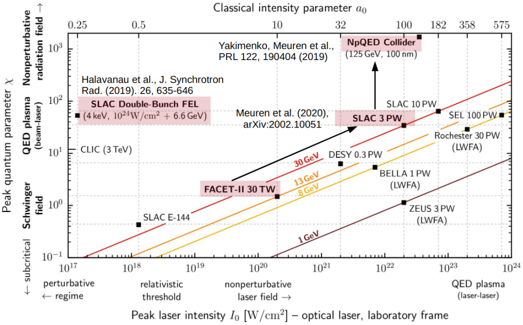
Analytical methods

- World-line instanton technique
- WKB approach

Simulations

- Heisenberg-Wigner formalism
- Quantum Kinetic Theory

Overview over experiments



Sebastian Meuren (for the E-320 collaboration)

WKB Approach

Ansatz for wave function

- Klein-Gordon equation $$\left[\left(\partial_\mu + iqA_\mu\right)\left(\partial^\mu + iqA^\mu\right) - m^2\right]\phi = 0$$
- WKB ansatz $$\phi(t,x,y) = \alpha(t,x)e^{iS(t,x,y,z=0)}$$
- Eikonal equation $$(\partial_\mu S + qA_\mu)(\partial^\mu S + qA^\mu) = m^2$$
- Translational invariance $$S(t,x,y) = k_y y \pm s(t,x)$$
- General phase function $$\partial_t s = \chi(t,x) = \sqrt{m^2 + \left(\partial_x s\right)^2 + \left(k_y + qA_y\right)^2}$$

Parity

- Because of symmetry in $\mathbf{A}$ $$s(t,x) = s_0(t) + \frac{x^2}{2}\,s_2(t) + \mathcal{O}(x^4)$$
- Zeroth order $s_0(t)$ $$\partial_t s_0 = \chi(t)$$
- Curvature contribution $$\partial_t s_2 = \left.\frac{s_2^2 + \left[k_y + qA_y\right] q\partial_x^2 A_y}{\sqrt{m^2 + \left[k_y + qA_y\right]^2}}\right|_{x=0}$$

DRESDEN concept HZDR

Transport Equation

Equal-time formalism

$$\mathbb{W}(\mathbf{x}, \mathbf{p}, t) = \int \frac{\mathrm{d}p_0}{2\pi} \mathbb{W}(\mathrm{x}, \mathrm{p}) = \frac{1}{4}\left(\mathbb{s} + \mathrm{i}\gamma_5 \mathbb{p} + \gamma^\mu \mathbb{v}_\mu + \gamma^\mu \gamma_5 \mathbb{a}_\mu + \sigma^{\mu\nu} \mathbb{t}_{\mu\nu}\right)$$

- Wigner components are transport quantities: mass density $\mathbb{s}$, charge density $\mathbb{v}_0$,...
- Project on $\int \frac{\mathrm{d}p_0}{2\pi}$ for initial-value problem, first order in time
- Time-evolution determined by (adjoint) Dirac equation
- Obtain regular probability densities when integrating out either $\mathbf{x}$ or $\mathbf{p}$
- Integro-differential operators D_t, $\mathbf{\Pi}$ & $\boldsymbol{D}$

$$\begin{aligned}
&D_t \mathbb{s} && && - 2\mathbf{\Pi} \cdot \mathbb{t}_1 &&= 0 \\
&D_t \mathbb{p} && && + 2\mathbf{\Pi} \cdot \mathbb{t}_2 &&= -2m\mathbb{a}_0 \\
&D_t \mathbb{v}_0 &&+ \boldsymbol{D} \cdot \mathbb{v} && &&= 0 \\
&D_t \mathbb{a}_0 &&+ \boldsymbol{D} \cdot \mathbb{a} && &&= 2m\mathbb{p} \\
&D_t \mathbb{v} &&+ \boldsymbol{D}\, \mathbb{v}_0 && + 2\mathbf{\Pi} \times \mathbb{a} &&= -2m\mathbb{t}_1 \\
&D_t \mathbb{a} &&+ \boldsymbol{D}\, \mathbb{a}_0 && + 2\mathbf{\Pi} \times \mathbb{v} &&= 0 \\
&D_t \mathbb{t}_1 &&+ \boldsymbol{D} \times \mathbb{t}_2 && + 2\mathbf{\Pi}\, \mathbb{s} &&= 2m\mathbb{v} \\
&D_t \mathbb{t}_2 &&- \boldsymbol{D} \times \mathbb{t}_1 && - 2\mathbf{\Pi}\, \mathbb{p} &&= 0
\end{aligned}$$

Colliding Pulses within DHW formalism

- Transverse fields, vanishing longitudinal field component
  $\hat{\mathbf{e}}_E \cdot \hat{\mathbf{e}}_B = \hat{\mathbf{e}}_E \cdot \hat{\mathbf{e}}_\kappa = \hat{\mathbf{e}}_B \cdot \hat{\mathbf{e}}_\kappa = 0,$
  $\hat{\mathbf{e}}_E \times \hat{\mathbf{e}}_B = \hat{\mathbf{e}}_\kappa$
- Quasi-1 + 1-dimensional system: Photon propagation $\boldsymbol{\kappa}_\perp^2 \ll \boldsymbol{\kappa}^2$
- Time evolution $D_t = \partial_t$
- Spatial derivative and 'magnetic' Wigner potential $\boldsymbol{D} = \boldsymbol{\nabla}_x + \boldsymbol{\mathcal{B}}$
- Canonical momenta and 'electric' Wigner potential $\boldsymbol{\Pi} = \boldsymbol{k} + \boldsymbol{\mathcal{A}}$

$$
\begin{aligned}
&D_t \mathbb{s} && -2\boldsymbol{\Pi}\cdot\mathbb{t}_1 &&= 0\\
&D_t \mathbb{p} && +2\boldsymbol{\Pi}\cdot\mathbb{t}_2 &&= -2m\mathbb{a}_0\\
&D_t \mathbb{v}_0 + \boldsymbol{D}\cdot\mathbb{v} && &&= 0\\
&D_t \mathbb{a}_0 + \boldsymbol{D}\cdot\mathbb{a} && &&= 2m\mathbb{p}\\
&D_t \mathbb{v} + \boldsymbol{D}\,\mathbb{v}_0 && +2\boldsymbol{\Pi}\times\mathbb{a} &&= -2m\mathbb{t}_1\\
&D_t \mathbb{a} + \boldsymbol{D}\,\mathbb{a}_0 && +2\boldsymbol{\Pi}\times\mathbb{v} &&= 0\\
&D_t \mathbb{t}_1 + \boldsymbol{D}\times\mathbb{t}_2 && +2\boldsymbol{\Pi}\,\mathbb{s} &&= 2m\mathbb{v}\\
&D_t \mathbb{t}_2 - \boldsymbol{D}\times\mathbb{t}_1 && -2\boldsymbol{\Pi}\,\mathbb{p} &&= 0
\end{aligned}
$$

Electromagnetic Wigner potentials

$$
\boldsymbol{\mathcal{B}}\,\mathbb{w}(x,k,t) = -\mathrm{i}q \int \mathrm{d}s \int \mathrm{d}k'\ \mathrm{e}^{\mathrm{i}(\mathrm{k}-\mathrm{k}')\mathrm{s}} \left[\boldsymbol{A}\left(x+\frac{s}{2},t\right) - \boldsymbol{A}\left(x-\frac{s}{2},t\right)\right] \mathbb{w}(x,k',t)
$$

$$
\boldsymbol{\mathcal{A}}\,\mathbb{w}(x,k,t) = -\frac{q}{2} \int \mathrm{d}s \int \mathrm{d}k'\ \mathrm{e}^{\mathrm{i}(\mathrm{k}-\mathrm{k}')\mathrm{s}} \left[\boldsymbol{A}\left(x+\frac{s}{2},t\right) + \boldsymbol{A}\left(x-\frac{s}{2},t\right)\right] \mathbb{w}(x,k',t)
$$

Relation to Plasma Physics

- Collisionless Boltzmann equation (Vlasov equation)
- Coupling particles to fields
- Electrons and Ions
- Lorentz force

$$\frac{\mathrm{d}\, f(\mathbf{r},\mathbf{p},t)}{\mathrm{d}\, t} = \frac{\partial f}{\partial t} + \frac{\mathrm{d}\,\mathbf{r}}{\mathrm{d}\, t}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{\mathrm{d}\,\mathbf{p}}{\mathrm{d}\, t}\cdot\frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e\cdot\nabla f_e - q\left(\mathbf{E} + \frac{\mathbf{v_e}}{c}\times\mathbf{B}\right)\cdot\frac{\partial f_e}{\partial \mathbf{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i\cdot\nabla f_i + Z_i q\left(\mathbf{E} + \frac{\mathbf{v_i}}{c}\times\mathbf{B}\right)\cdot\frac{\partial f_i}{\partial \mathbf{p}} = 0$$

$$\hbar \to 0 \quad \& \quad \delta(m^2 - p^2)$$

$$\mathbf{p}\mathbb{s} = \mathbb{v},\ \mathbb{s} = \mathbb{v}_0/\sqrt{m^2+\mathbf{p}^2}\ldots$$

$$\boldsymbol{A}\left(x \pm \frac{s}{2}, t\right) \approx \boldsymbol{A}\left(x, t\right)$$

- Vlasov equation as classical limit of Wigner transport equations
- On-shell particles and slowly varying fields $\partial A \ll m$

Wigner formalism as extension of Vlasov equation

+ fully relativistic
+ quantum mechanical component
  (spin-field coupling,...)

+ spinorial decomposition
+ quantum field theoretical effects
  (pair production,...)

Particle Momentum Spectrum

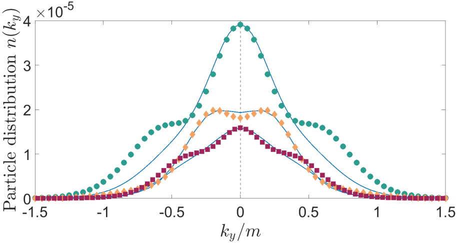
Qualitative Analysis

- Symmetry in k_y
- Distribution is reproduced: non-perturbative elements and photon absorption
- Distribution maxima not necessarily at $k_y = 0$

Quantitative Analysis

- Regime $\Omega/m = [0.1, 1.0]$
- Overestimate by $[1, 4.7]$
- Correction $[1, 4.3]$

Comparison

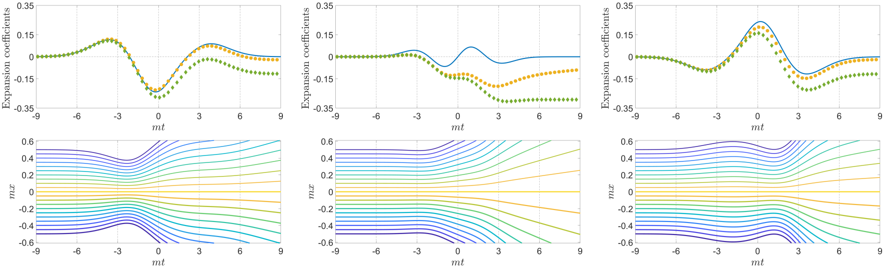


Spatially homogeneous approximation

Dirac-Heisenberg-Wigner formalism

$\omega = m/3$, $\omega = m/4$, $\omega = m/5$

Results: Focusing & De-Focusing



- WKB expansion coefficients $\ddot{s}_0$, s_2(linearized), s_2
- Characteristic curves of $(\partial_t s)^2 = m^2 + (\partial_x s)^2 + (k_y + qA_y)^2$
- Strong correlation between s_2 and dispersion of curves

- Pre-factor in pair production: $\ddot{s}_0$ - s_2
- Configuration:

  $\omega = m/3$ at $k_y = \{-m,\, 0,\, m\}$

Results: Curvature Contribution

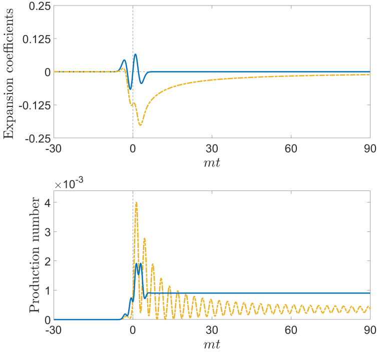
Expansion coefficients

- $\ddot{s}_0$ in comparison to s_2
- Stable in s_2 despite $\dot{s}_2 \sim s_2^2$
- Converge for $t \to \infty$

Production number

- Approximately given by $|R|^2$
- Homogeneous: $\ddot{s}_0$
- Inhomogeneous: $\ddot{s}_0$ - s_2
- Oscillations dampened for $t \to \infty$
- Reduction by factor of roughly two

Computation



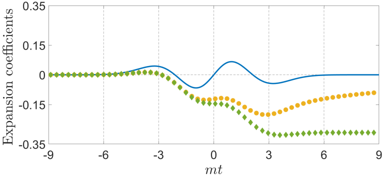
- Configuration:
  $\omega = m/3$ at $k_y = 0$

DRESDEN concept HZDR

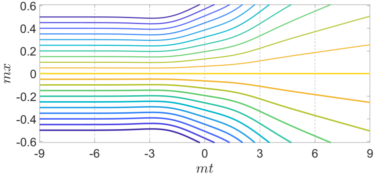
Results: Focusing & De-Focusing

Overview

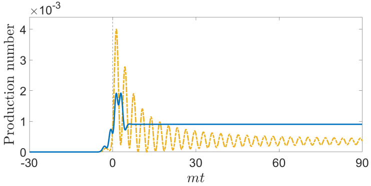
- WKB expansion coefficients $\ddot{s}_0$, s_2(linearized), s_2



- Characteristic curves of $(\partial_t s)^2 = m^2 + (\partial_x s)^2 + (k_y + qA_y)^2$
- Strong correlation between s_2 and dispersion of curves



- Pre-factor in pair production: $\ddot{s}_0$ - s_2
- Production number approximately given by $|R|^2$
- Pulses with frequency $\omega = m/3$ studied at $k_y = 0$



DRESDEN concept HZDR