Correlations and Semi-Universal Relations Connecting Nuclear Matter and Neutron Stars

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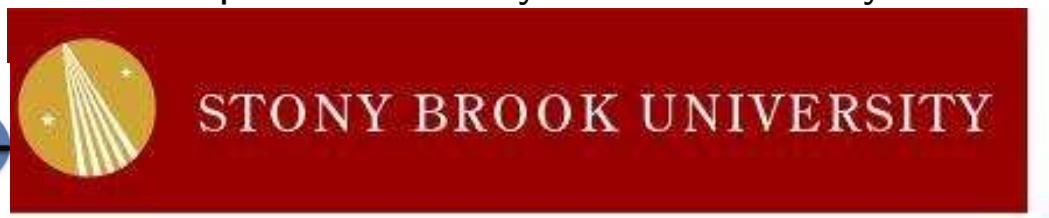

Many-Particle Systems Under Extreme Conditions
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DOE - Nuclear Physics

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Recent Collaborators:

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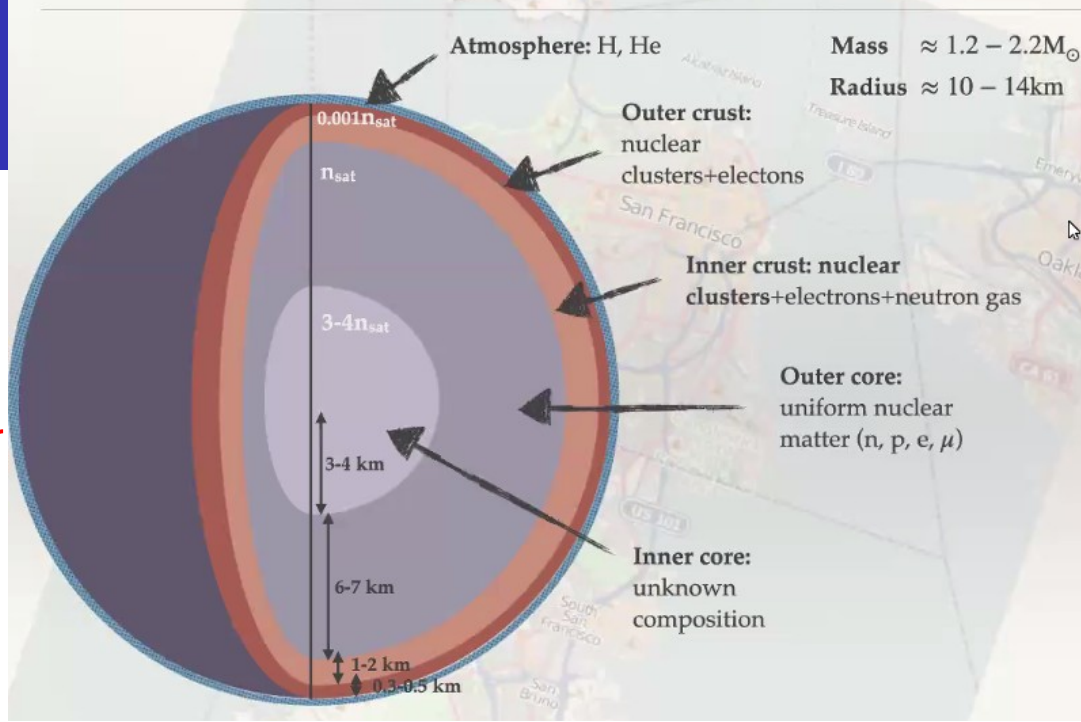
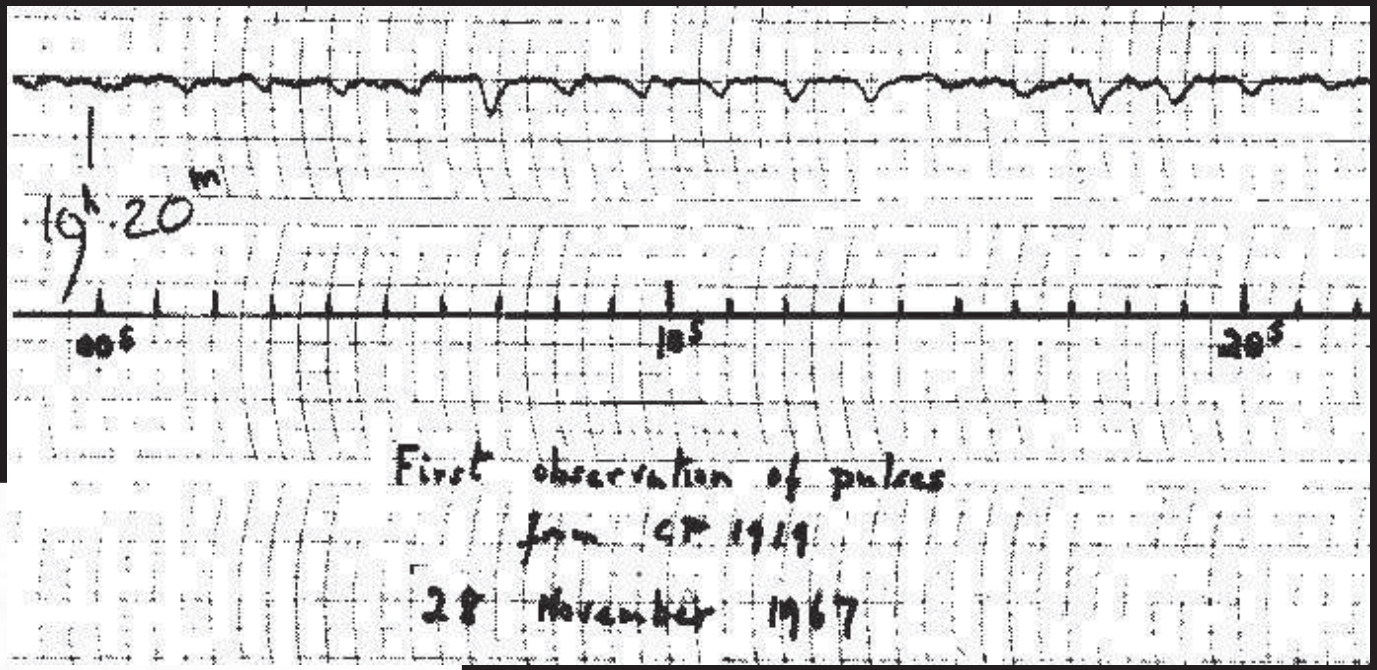
Neutron Stars: Basics

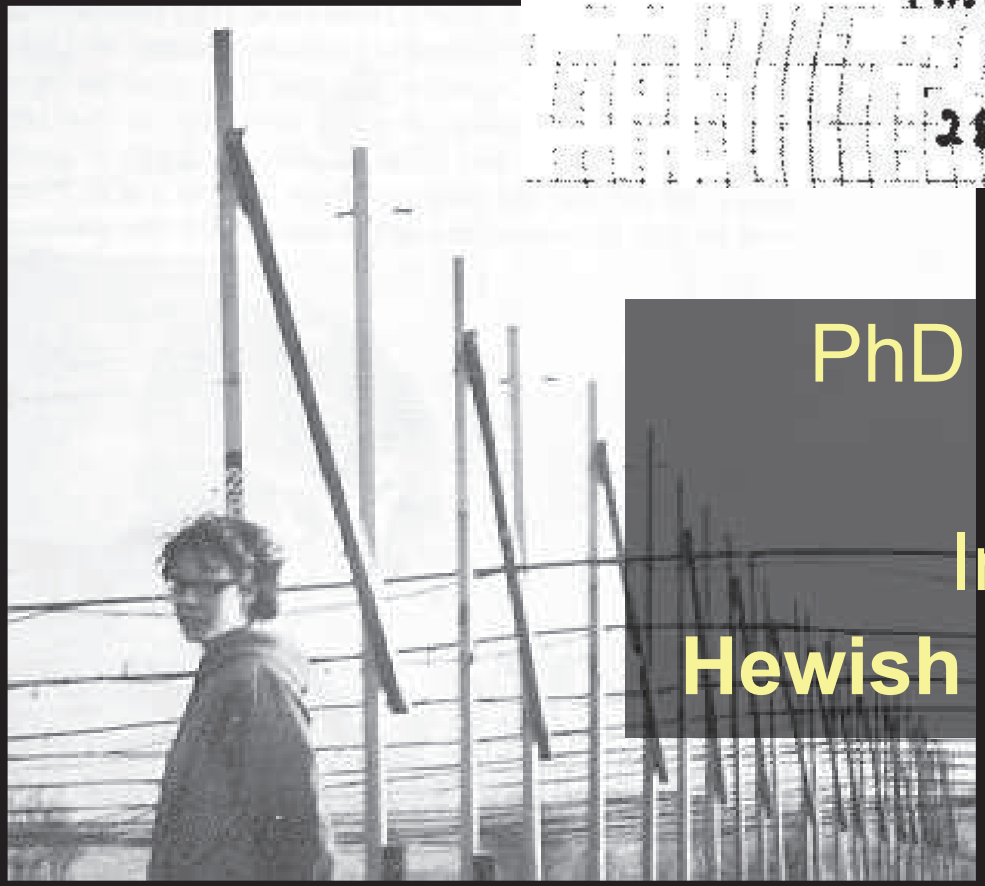
- Nearly all known NSs are pulsars (rapidly rotating and highly magnetized) that emit X-ray, optical or radio beams from their poles, like a lighthouse.



- The radii of most NSs are about 12 km.
- Most, if not all, NSs are formed in the gravitational collapse of massive stars at the ends of their lives; some of those collapses produce black holes instead. Some massive NSs may be formed in the aftermath of a binary merger of two lower-massed neutron stars.
- The minimum possible NS mass is $0.1M_\odot$, but none are observed to be less massive than $1M_\odot$.

The Discovery of Pulsars

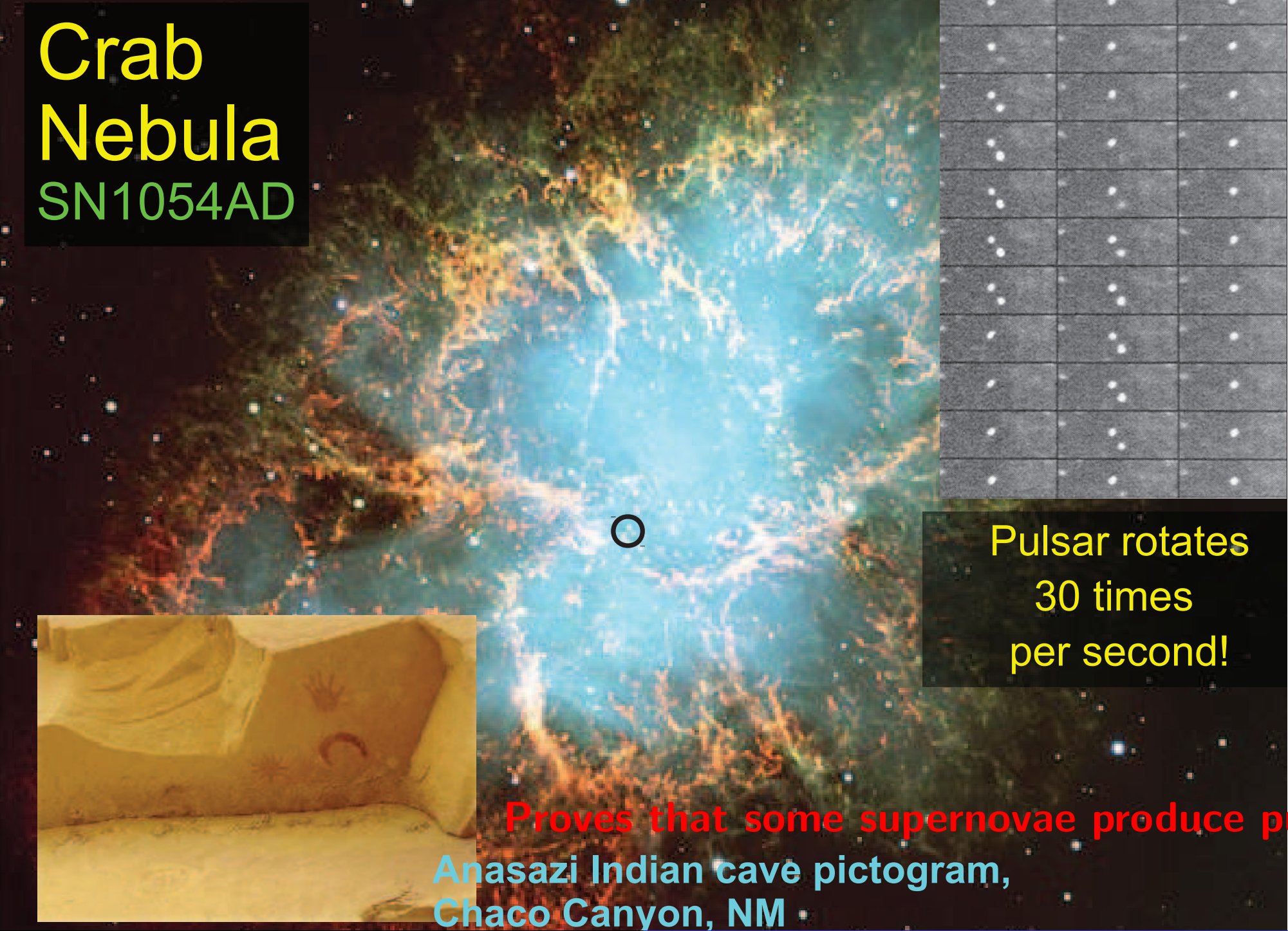




PhD student **Jocelyn Bell** and
Prof. **Antony Hewish**
Initially "**Little Green Men**"
Hewish won Nobel Prize in 1974

Crab Nebula

SN1054AD



Pulsar rotates
30 times
per second!

Proves that some supernovae produce p

Anasazi Indian cave pictogram,
Chaco Canyon, NM

Pulsars are Precise Clocks

PSR J0437-4715

At 00:00 UT Jan 18 2011:

P = 5.7574519420243 ms

+/- 0.000000000001ms

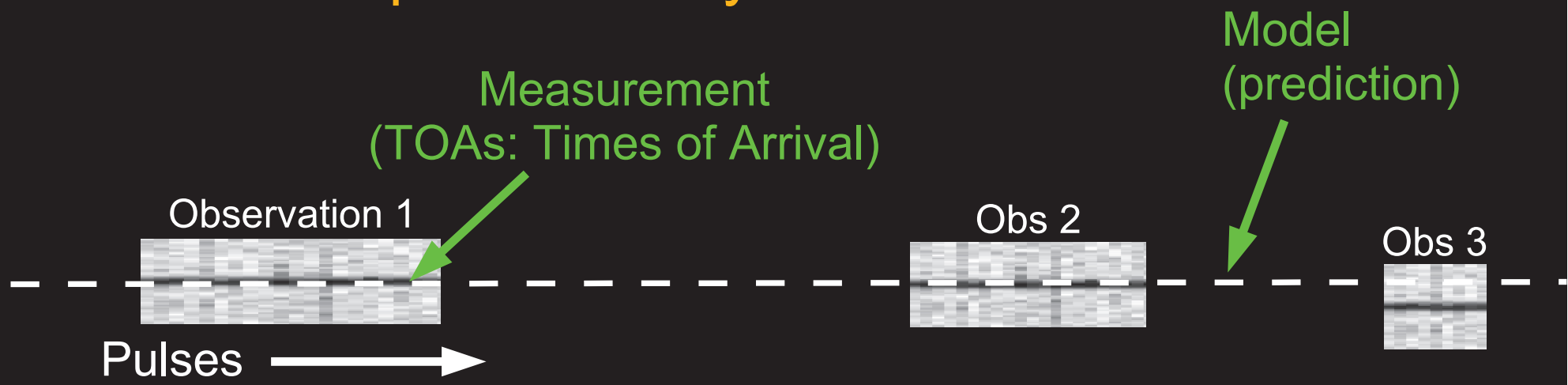
The last digit changes by 1 every half hour!

This digit changes by 1 every 500 years!

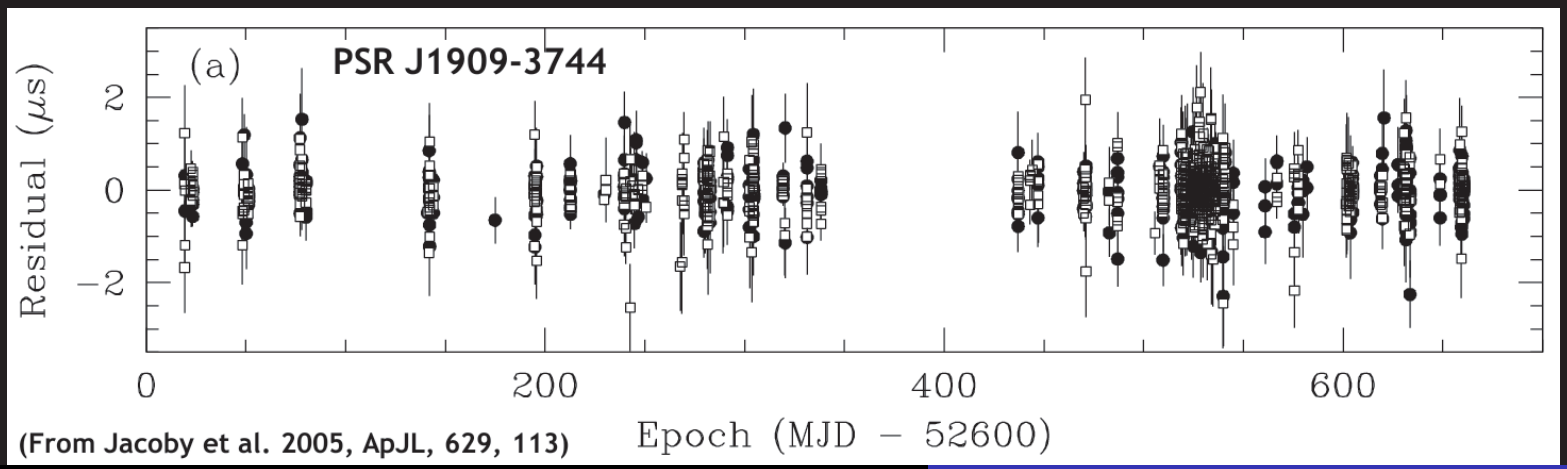
This extreme precision is what allows us to ***use pulsars as tools*** to do unique physics!

Pulsar Timing: *Pulse Phase Tracking*

Unambiguously account for every rotation of a pulsar over years



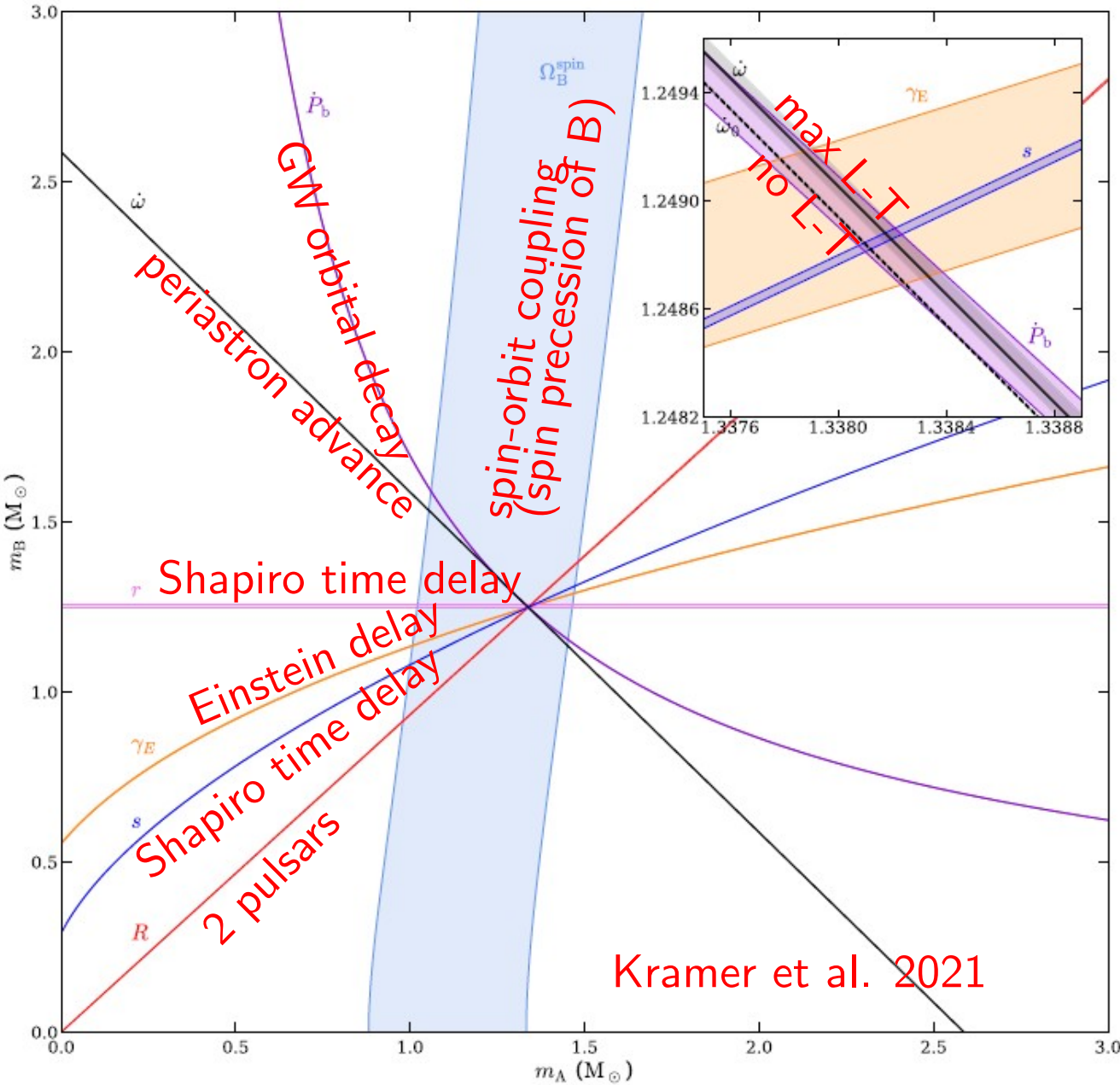
Measurement - Model = Timing Residuals



(From Jacoby et al. 2005, ApJL, 629, 113)

200ns RMS over 2 yrs

Pulsar Timing for PSR J0737-3039

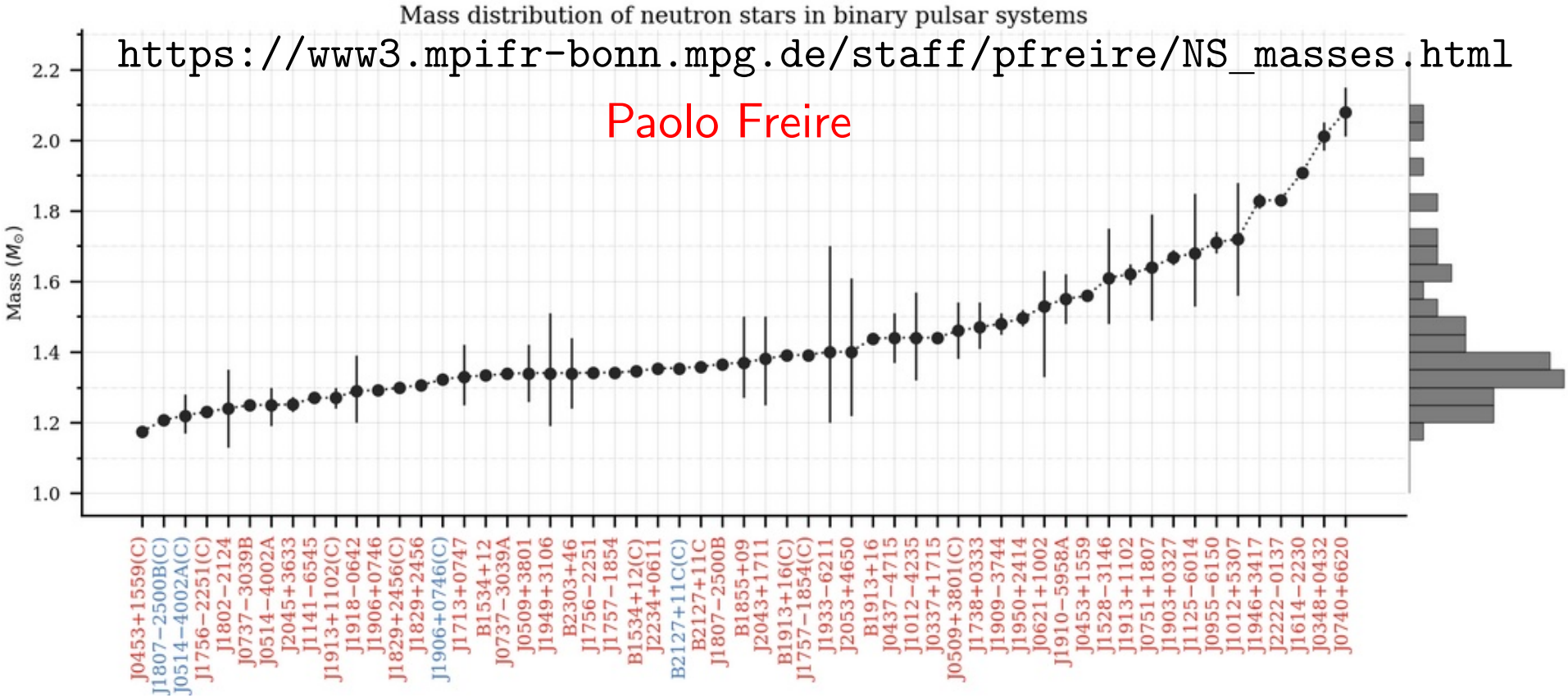


$m_A = 1.338185^{+12}_{-14}\ M_\odot$

$m_B = 1.248868^{+13}_{-11}\ M_\odot$

These are the most precisely known masses of any astronomical object.

Masses of Pulsars in Binaries from Pulsar Timing



Largest: 2.08 ± 0.07 $M_\odot$

Smallest: 1.174 ± 0.004 $M_\odot$

Several other NS masses have been measured by other means, including some of more than $2M_\odot$ (e.g., black widow pulsars), but their mass uncertainties are generally large.

How Can a Neutron Star's Radius Be Measured?

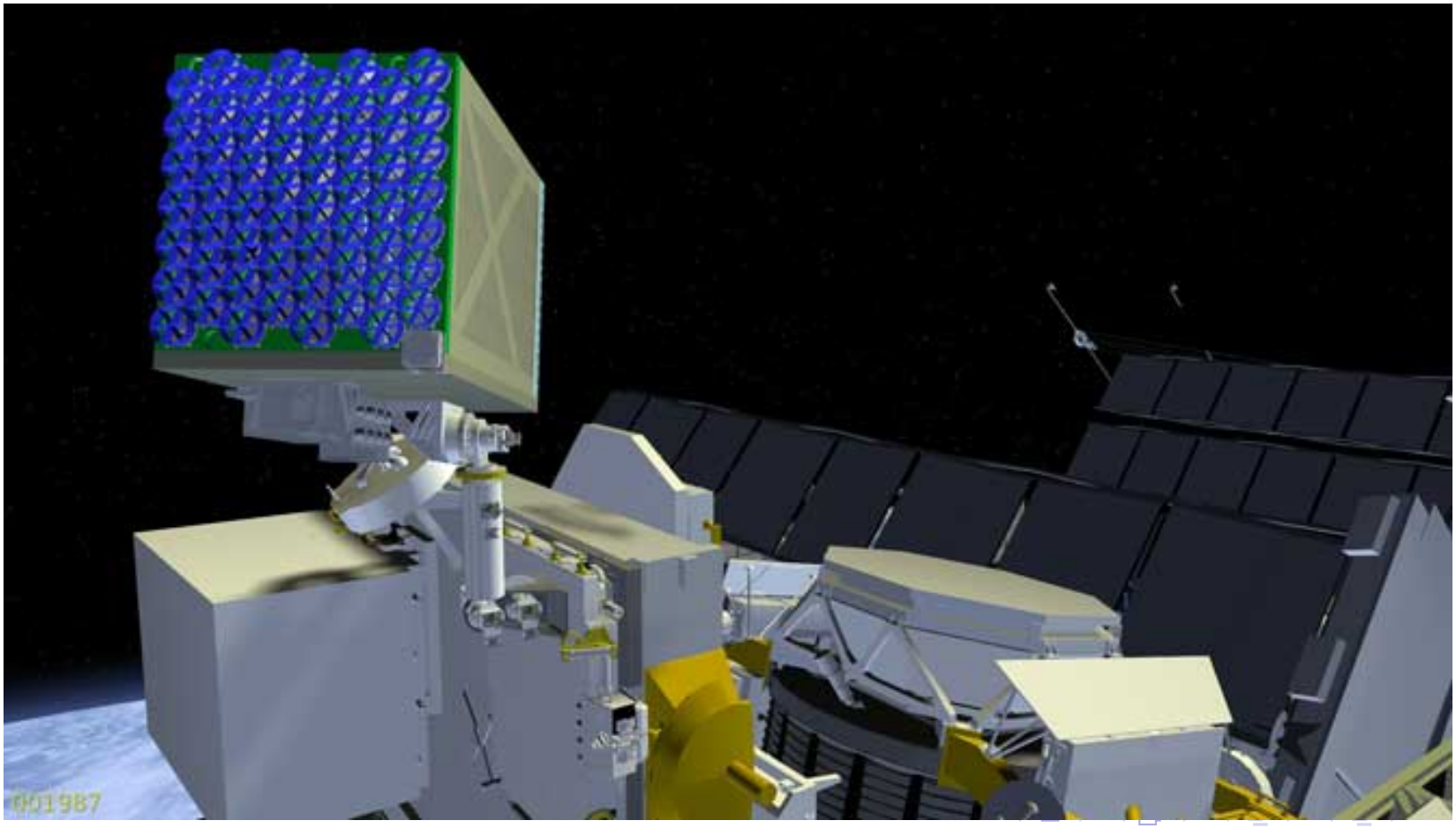
- Flux $= \frac{\text{Luminosity}}{4\pi D^2} = \frac{4\pi R^2 \sigma_B T_s^4}{4\pi D^2} = \left(\frac{R}{D}\right)^2 \sigma_B T_s^4$
  X-ray observations of quiescent neutron stars in low-mass X-ray binaries to measure distance D, Flux and surface temperature T_s. GR effects introduce an R dependence.
- $F_{Edd} = \frac{GMc}{\kappa D^2}$ X-ray observations of bursting neutron stars in accreting systems to measure the Eddington flux F_{Edd}. κ is the poorly-known opacity. GR effects introduce an R dependence.
- Phase-resolved spectroscopy of millisecond pulsars with nonuniform surface emissions. NICER: PSR J0030+0451, PSR J0437-4715 (closest and brightest millisecond pulsar) and PSR J0740+6620 (most massive pulsar).
- $R_{1.4} = (11.5 \pm 0.3) \frac{\mathcal{M}}{M_\odot} \left(\frac{\tilde{\Lambda}}{800}\right)^{1/6} \text{km}, \qquad \mathcal{M} = \frac{(M_A M_B)^{3/5}}{(M_A + M_B)^{1/5}}$
  GW observations of neutron star mergers to measure the chirp mass $\mathcal{M}$ and the binary tidal deformability $\tilde{\Lambda}$ (GW170817).
- $I \propto M_A R_A^2$ Radio observations of spin-orbit coupling in extremely relativistic binary pulsars to measure masses M_A, M_B and moment of inertia I_A [PSR J0737-3039 ($P_b = 0.102$d), PSR J1757-1854 (0.164 d), PSR J1946+2052 (0.078 d)].

Neutron Star Interior Composition ExploreR (NICER)

Launched aboard a SpaceX Falcon 9 rocket on June 3, 2017.

It is installed aboard the International Space Station.

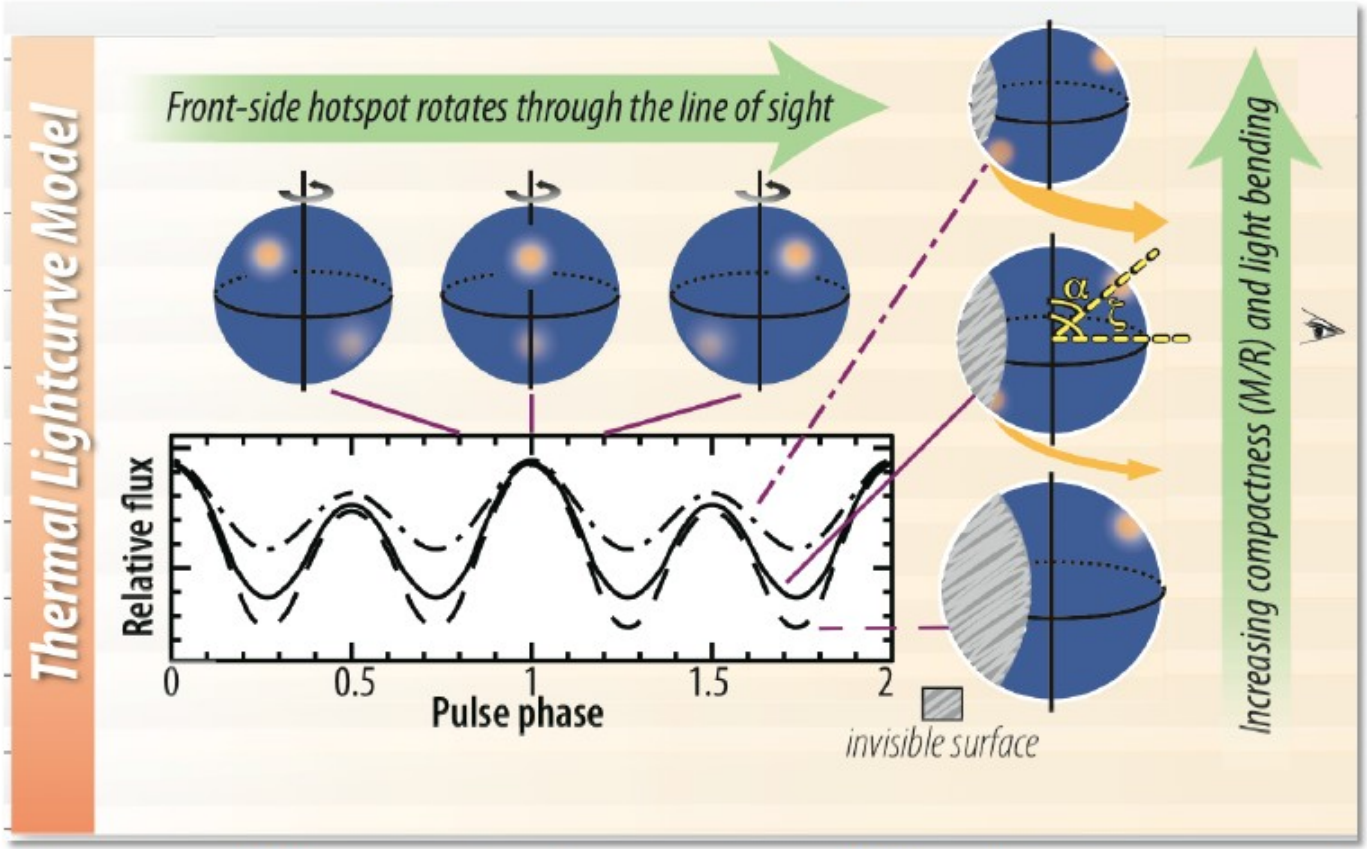
Dedicated to the study of neutron stars through soft X-ray timing.



Science Measurements

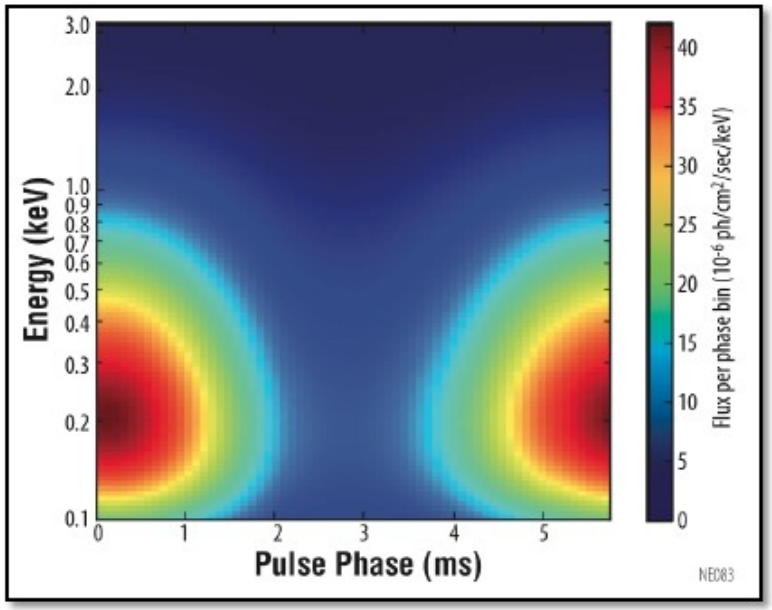
NICER

Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches

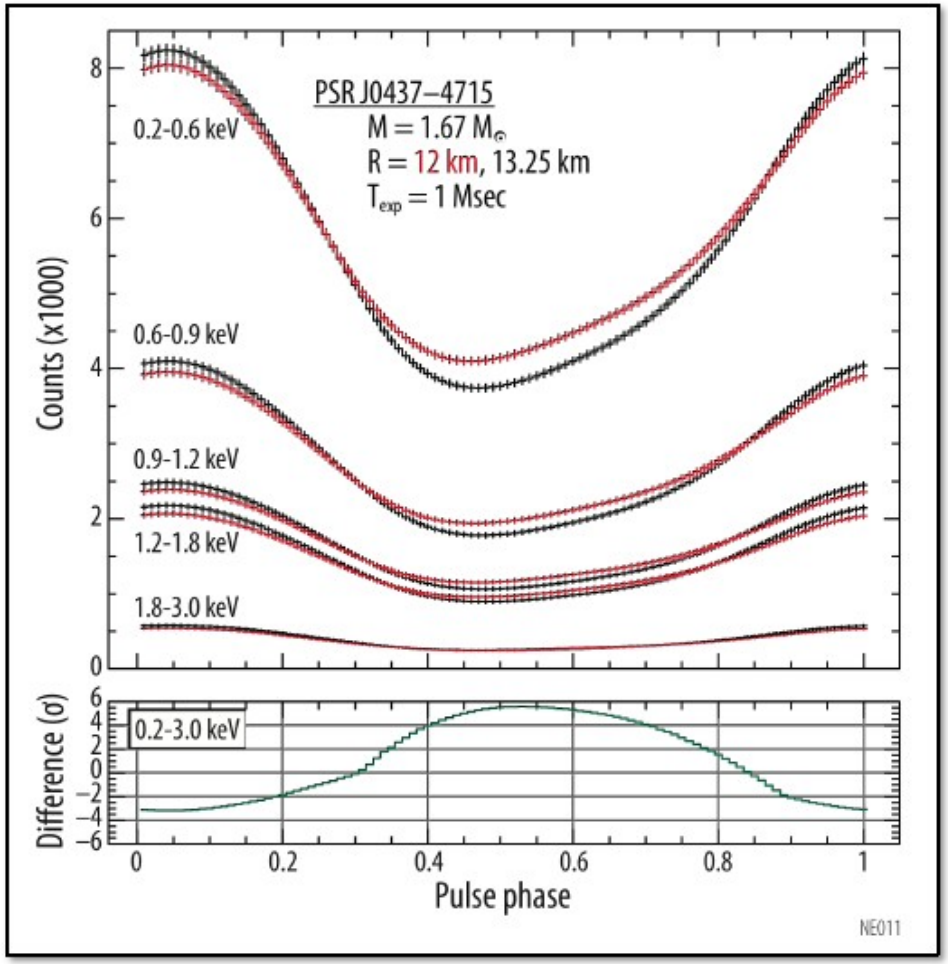


Lightcurve modeling constrains the compactness (M/R) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending…

Science Measurements (cont.)

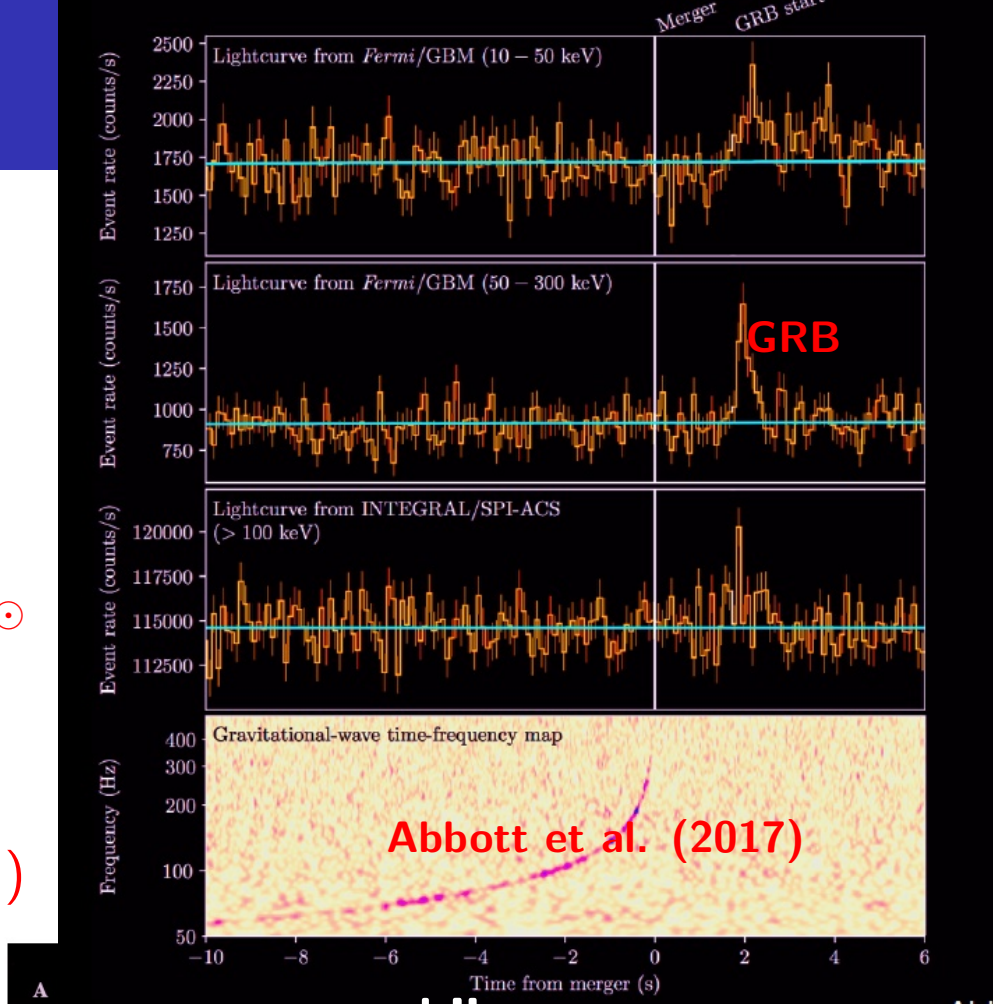


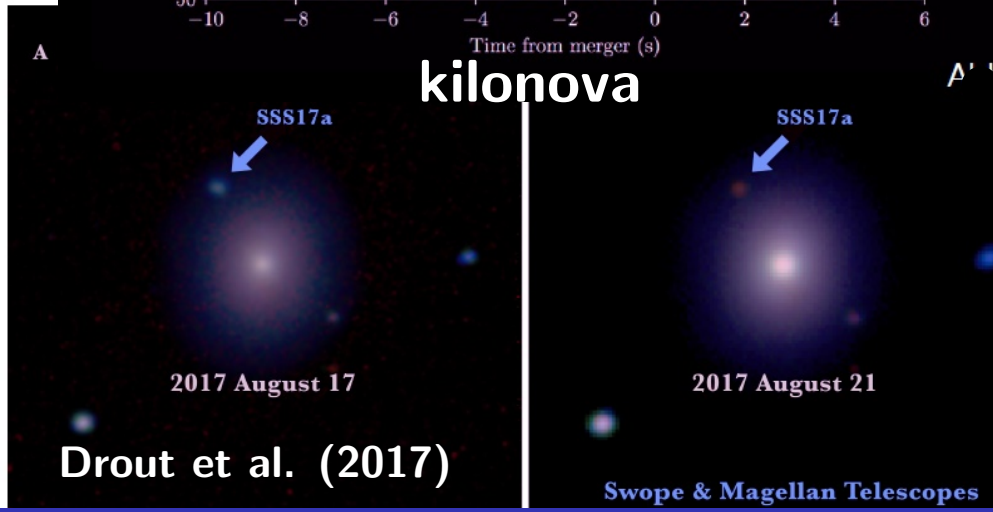
… while phase-resolved spectroscopy promises a direct constraint of radius *R*.



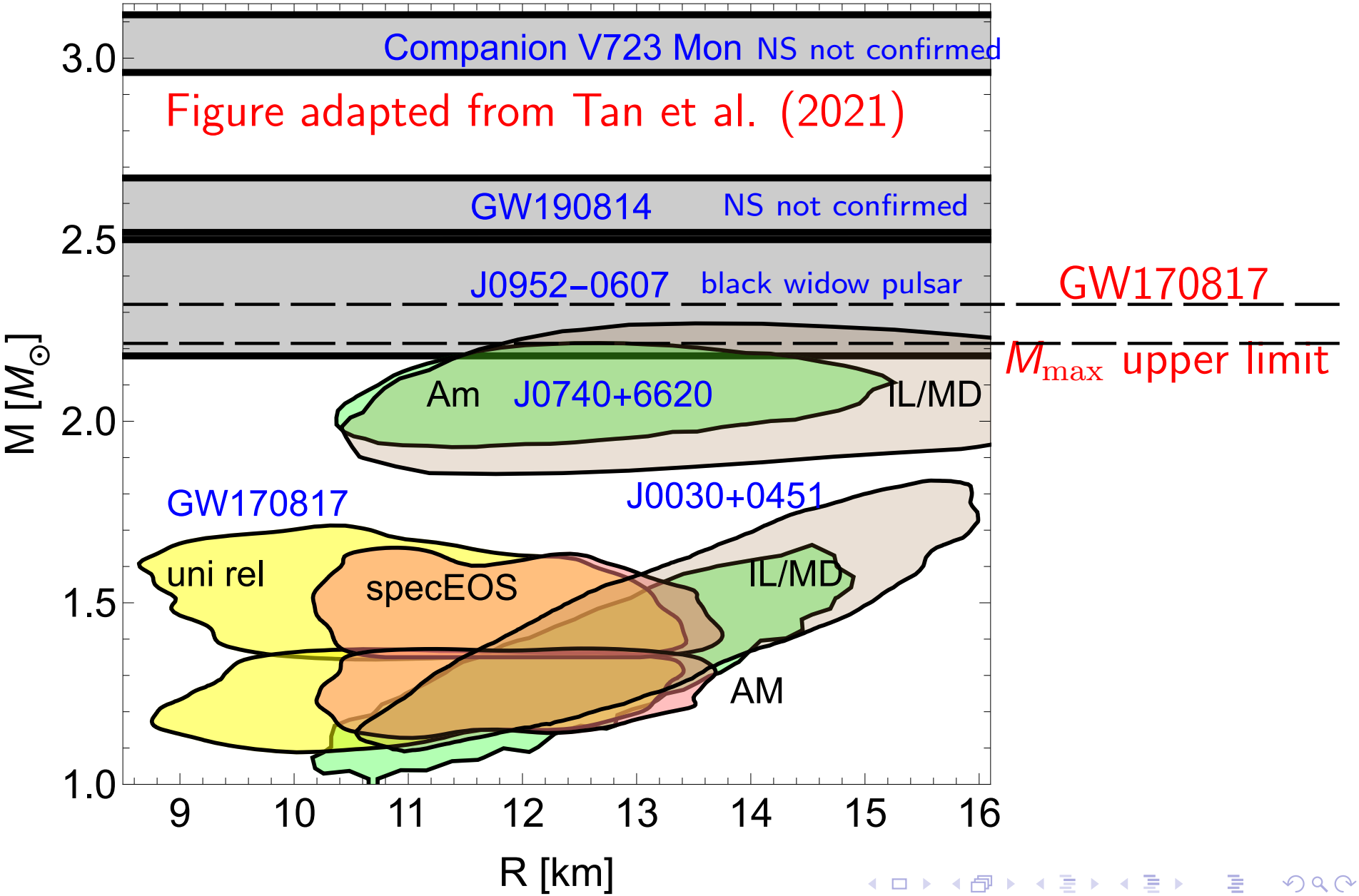
GW170817

- LVC detected a signal consistent with a BNS merger, followed 1.7 s later by a weak gamma-ray burst.
- $\sim$ 10100 orbits observed over 317 s.
- $\mathcal{M} = 1.186 \pm 0.001\ M_\odot$
- $M_{\rm T} = M_A + M_B \gtrsim 2^{6/5}\mathcal{M} = 2.725 M_\odot$
- $E_{\rm GW} > 0.025 M_\odot c^2$
- $D_L = 40^{+8}_{-14}$ Mpc
- $75 < \tilde{\Lambda} < 560$ (10.9 km$< \bar{R} <$13.3 km)
- $M_{\rm ejecta} \sim 0.06 \pm 0.02\ M_\odot$
- Blue ejecta: $\sim 0.01 M_\odot$
- Red ejecta: $\sim 0.05 M_\odot$
- Highly opaque ejecta implies substantial r-process production
- M_T+Ejecta+GRB: $M_{max} \lesssim 2.22 M_\odot$





Summary of Astrophysical Observations

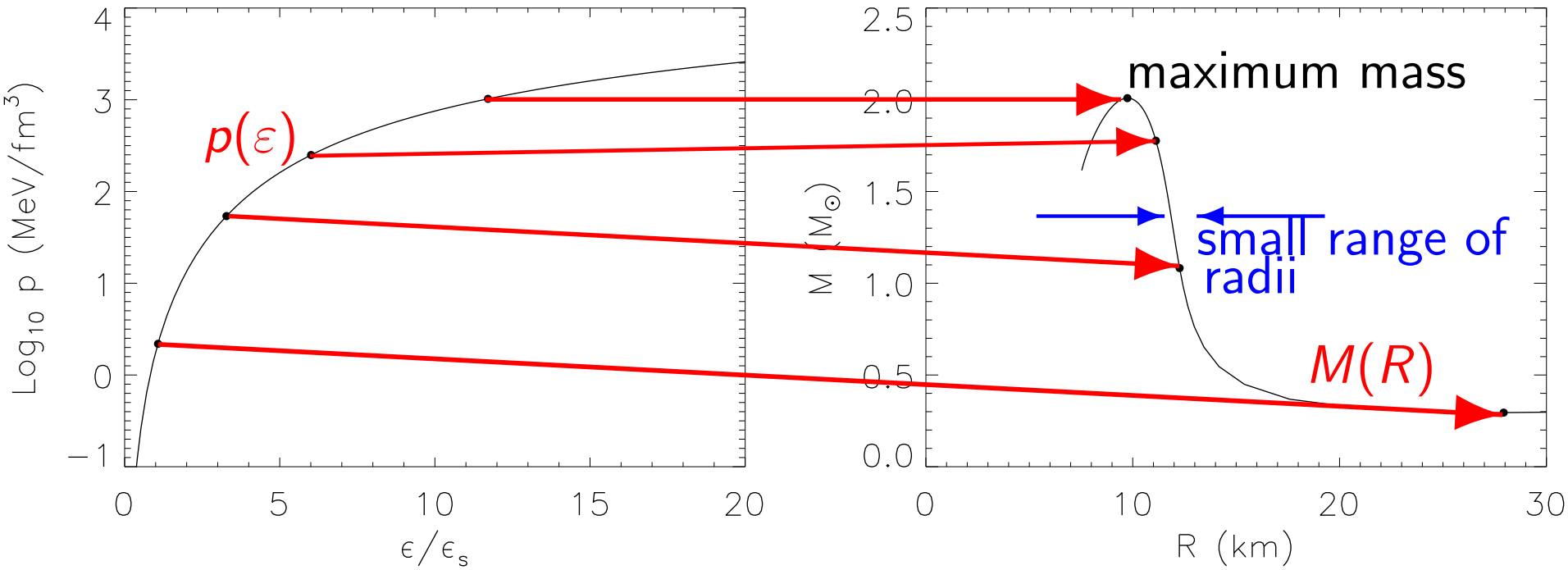


Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

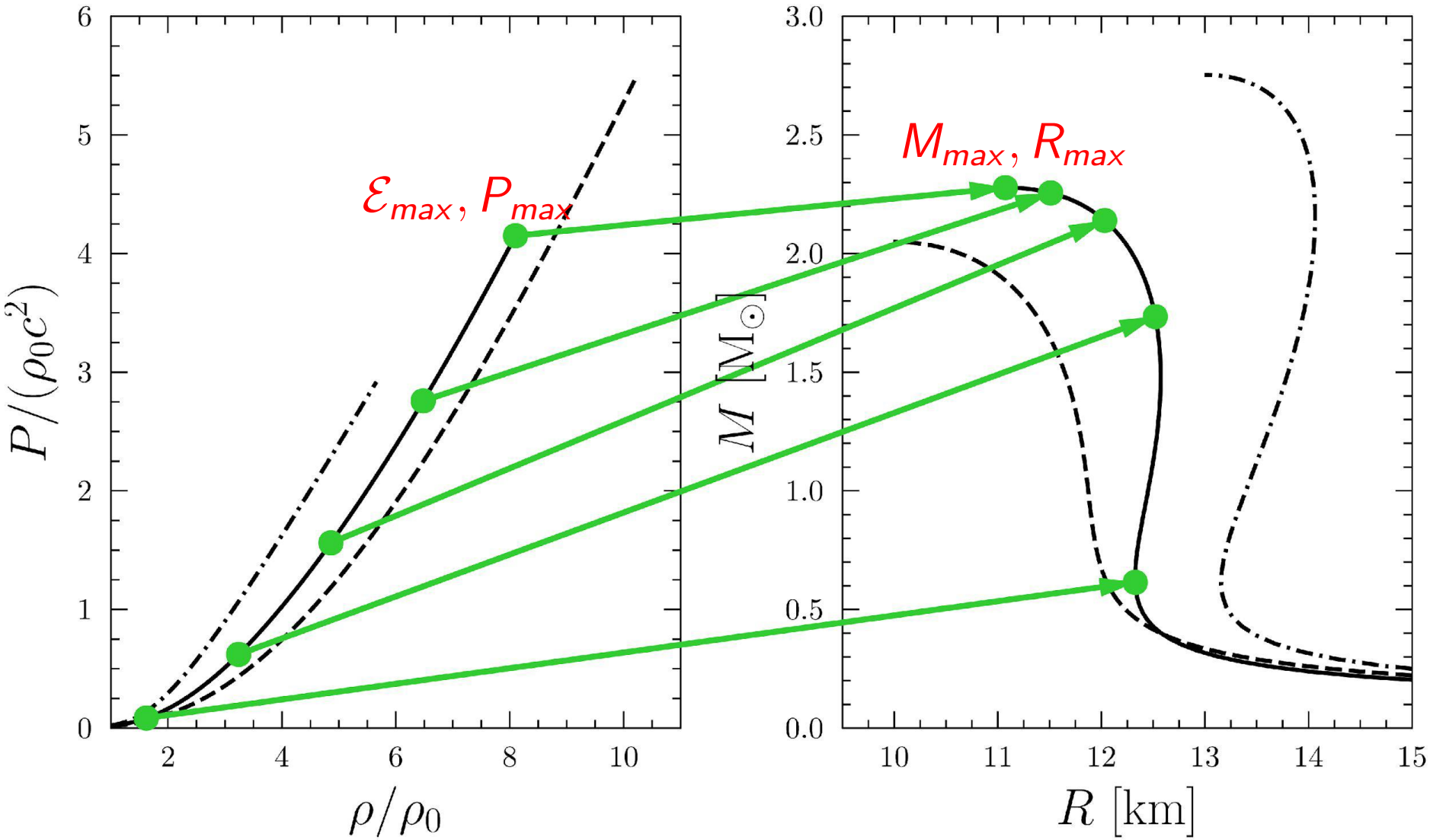
$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$

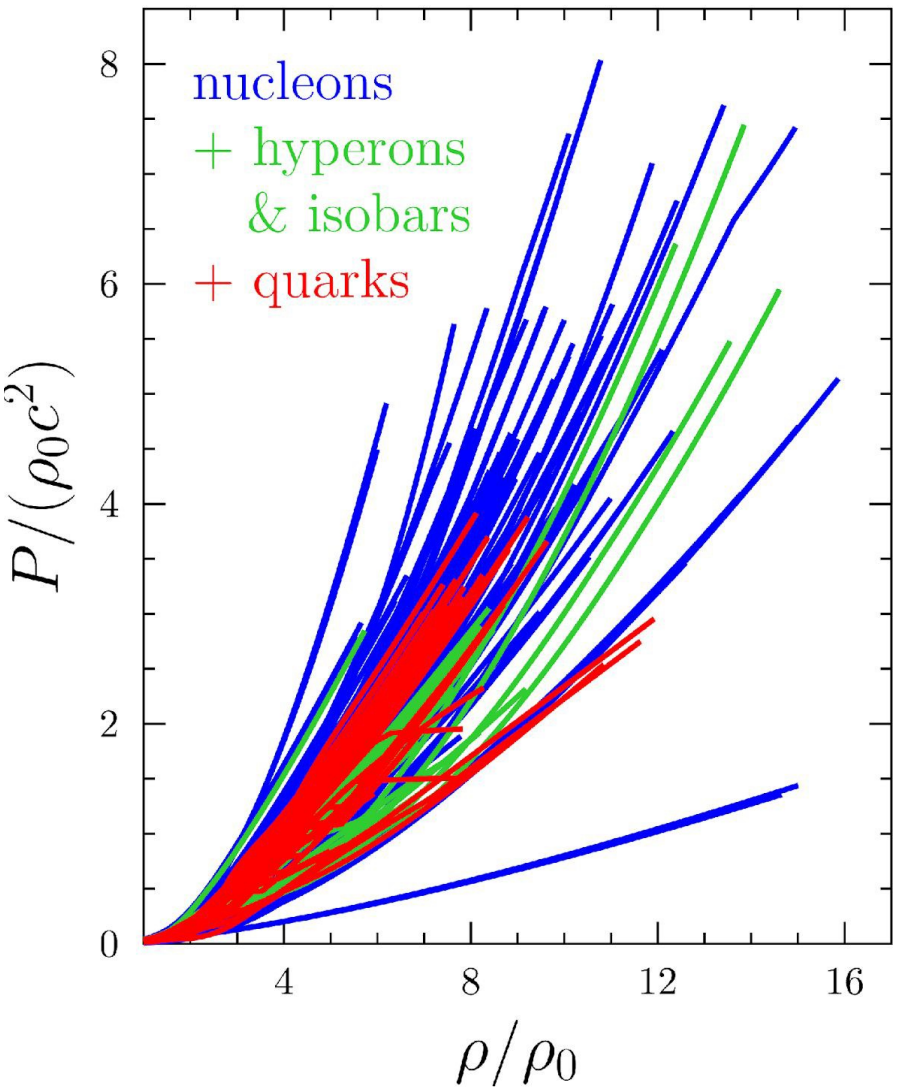


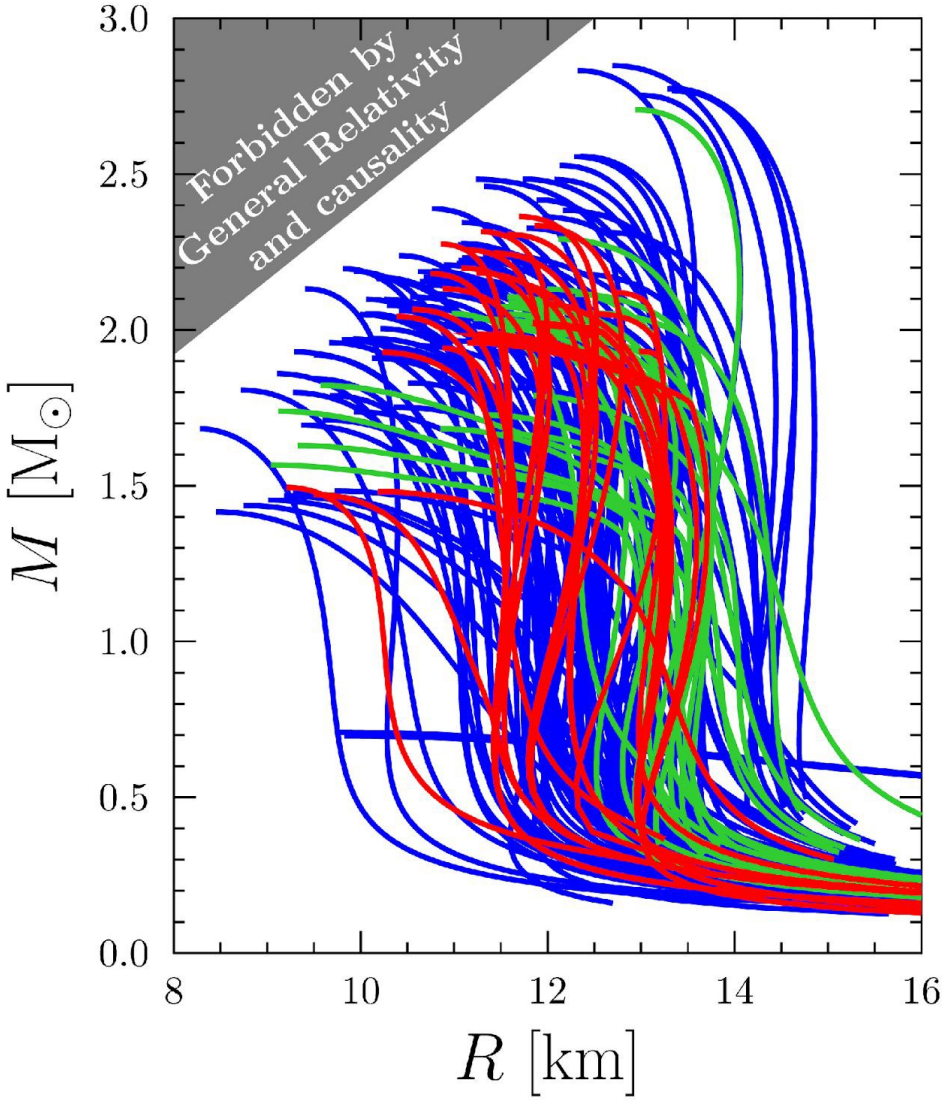
Equation of State ⟵ Observations

Varying the EOS

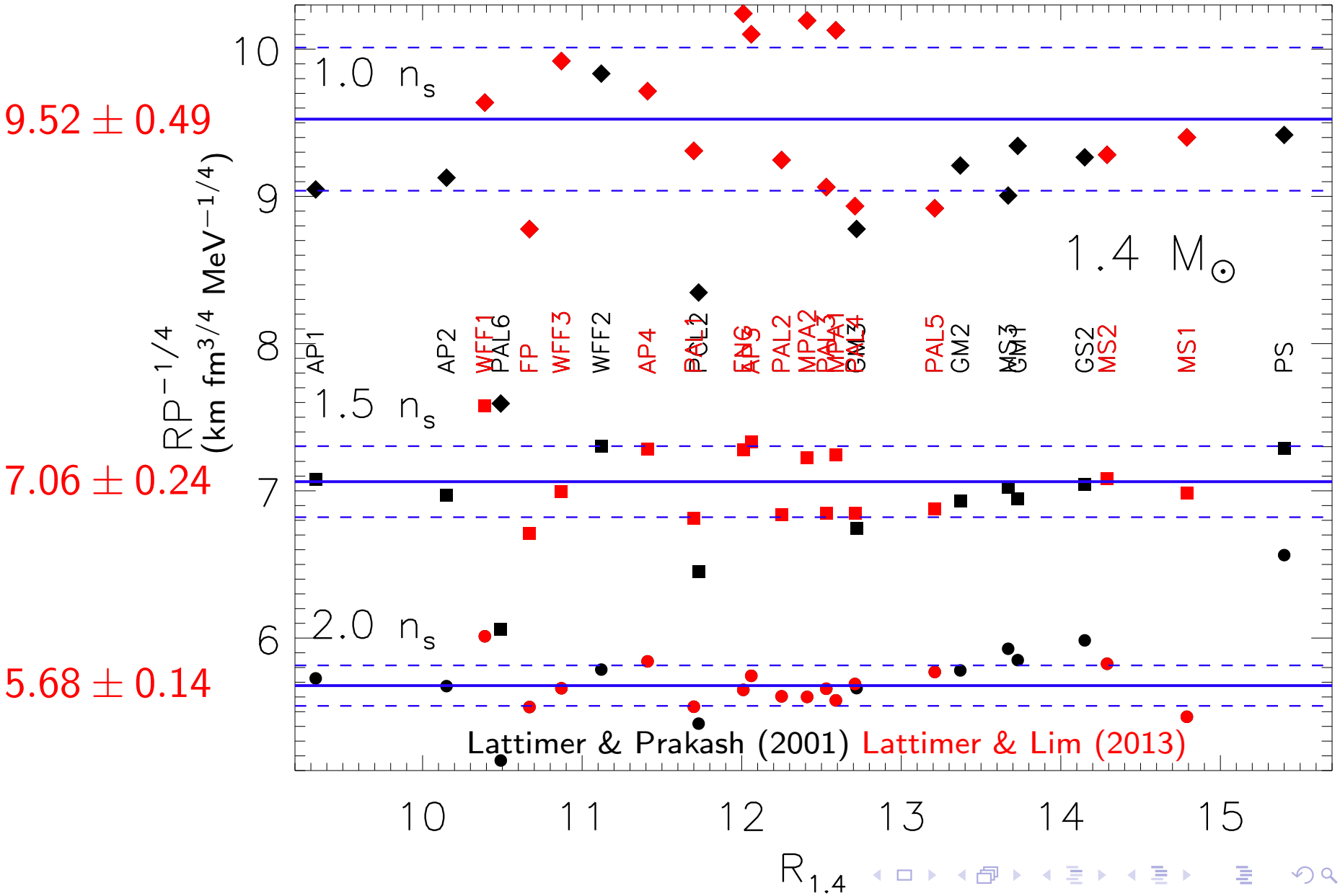


Varying the EOS

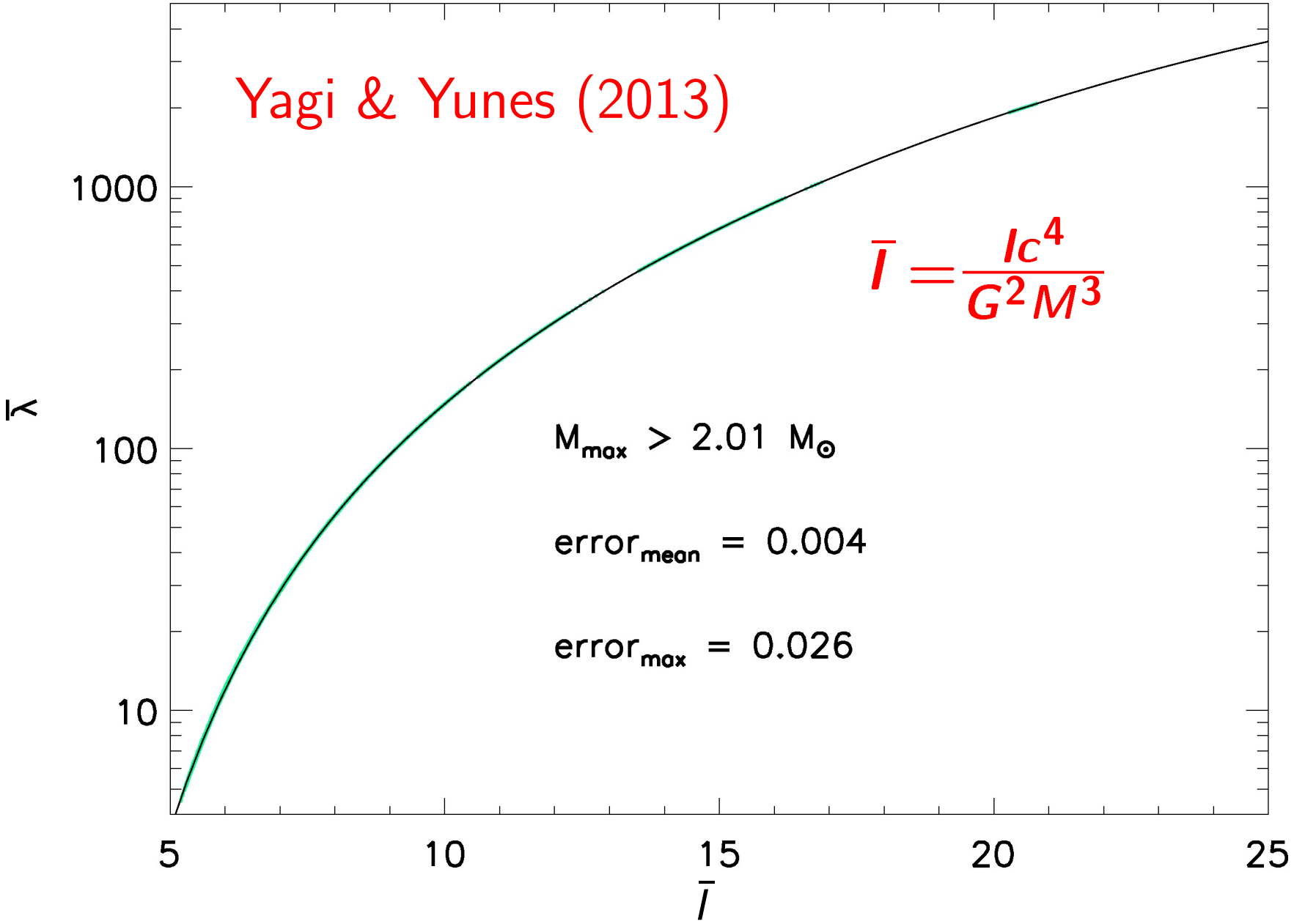




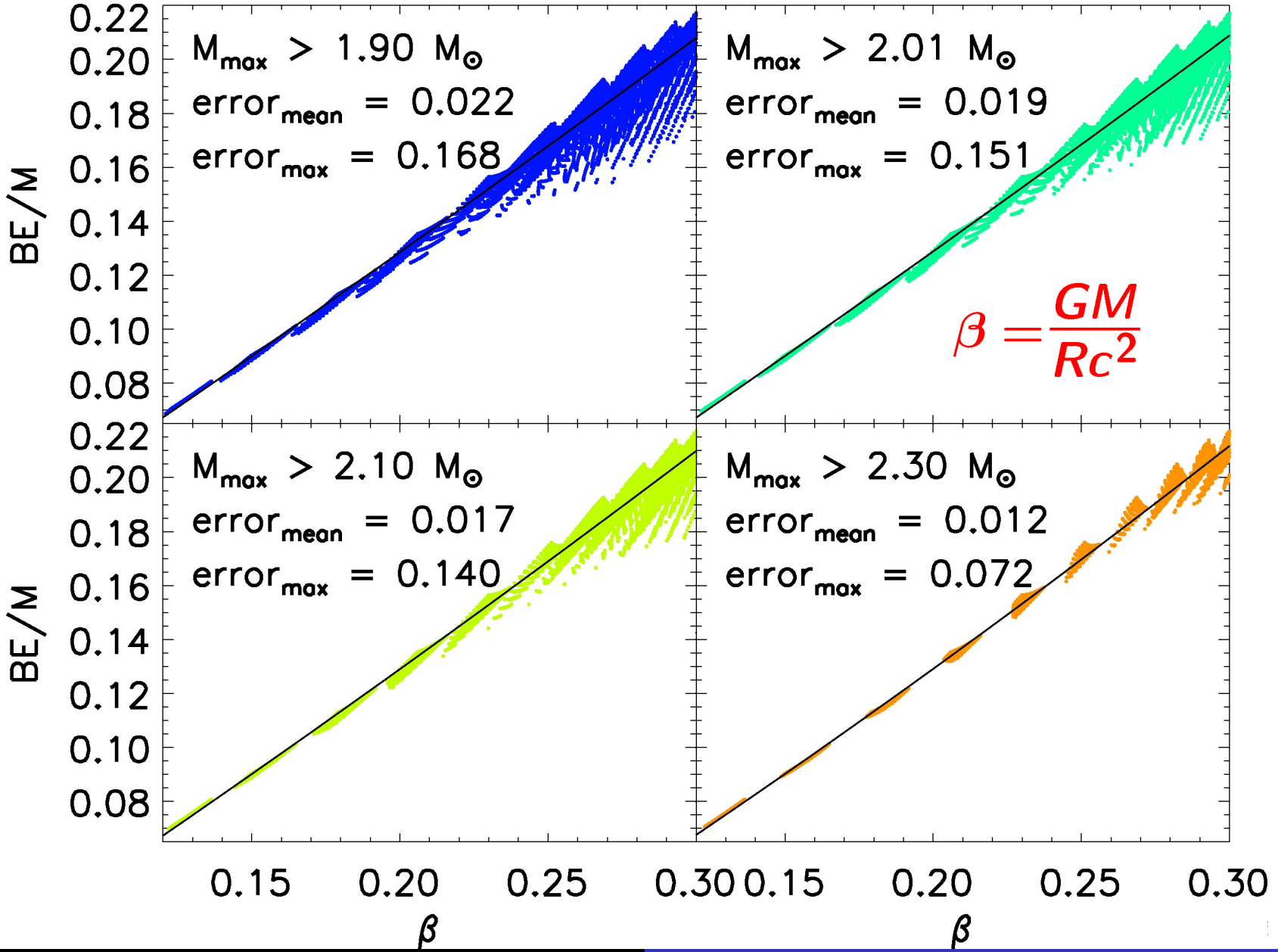
The Radius – Pressure Correlation



Tidal Deformatibility - Moment of Inertia



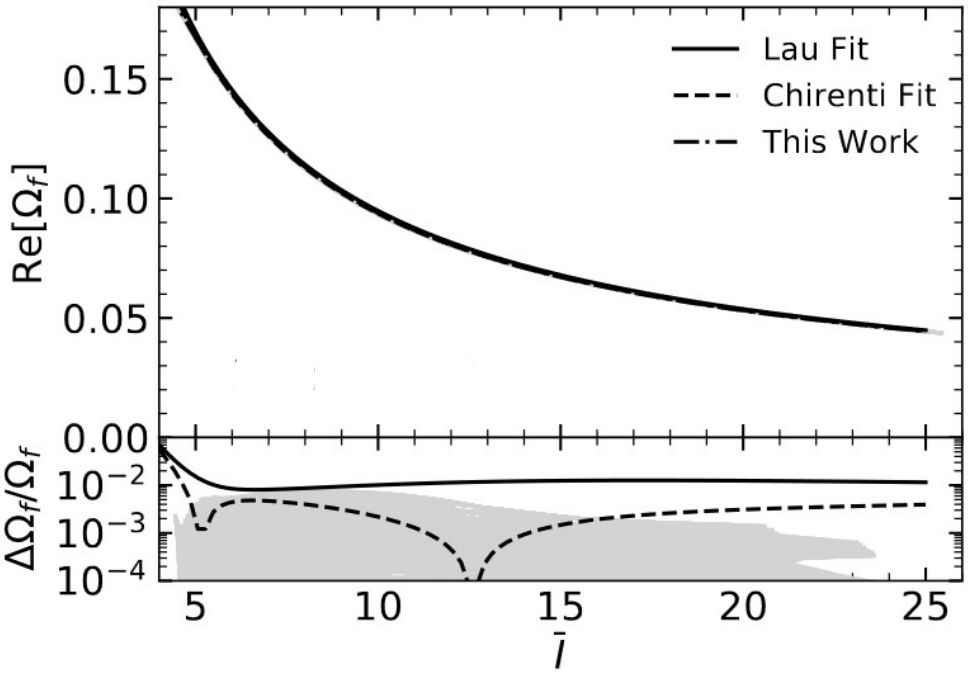
Binding Energy - Compactness

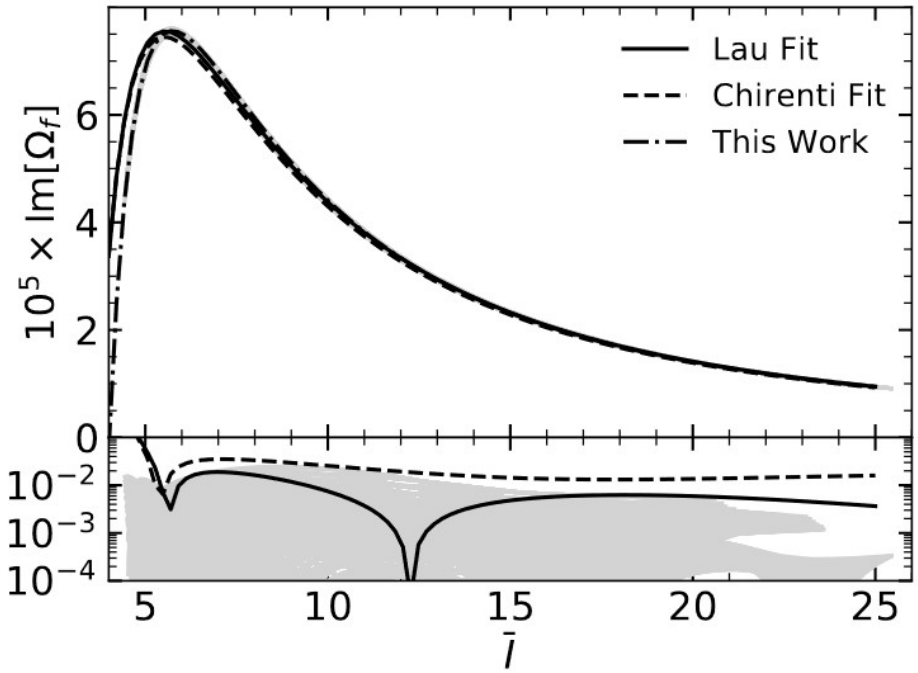


F-Mode Properties - Moment of Inertia

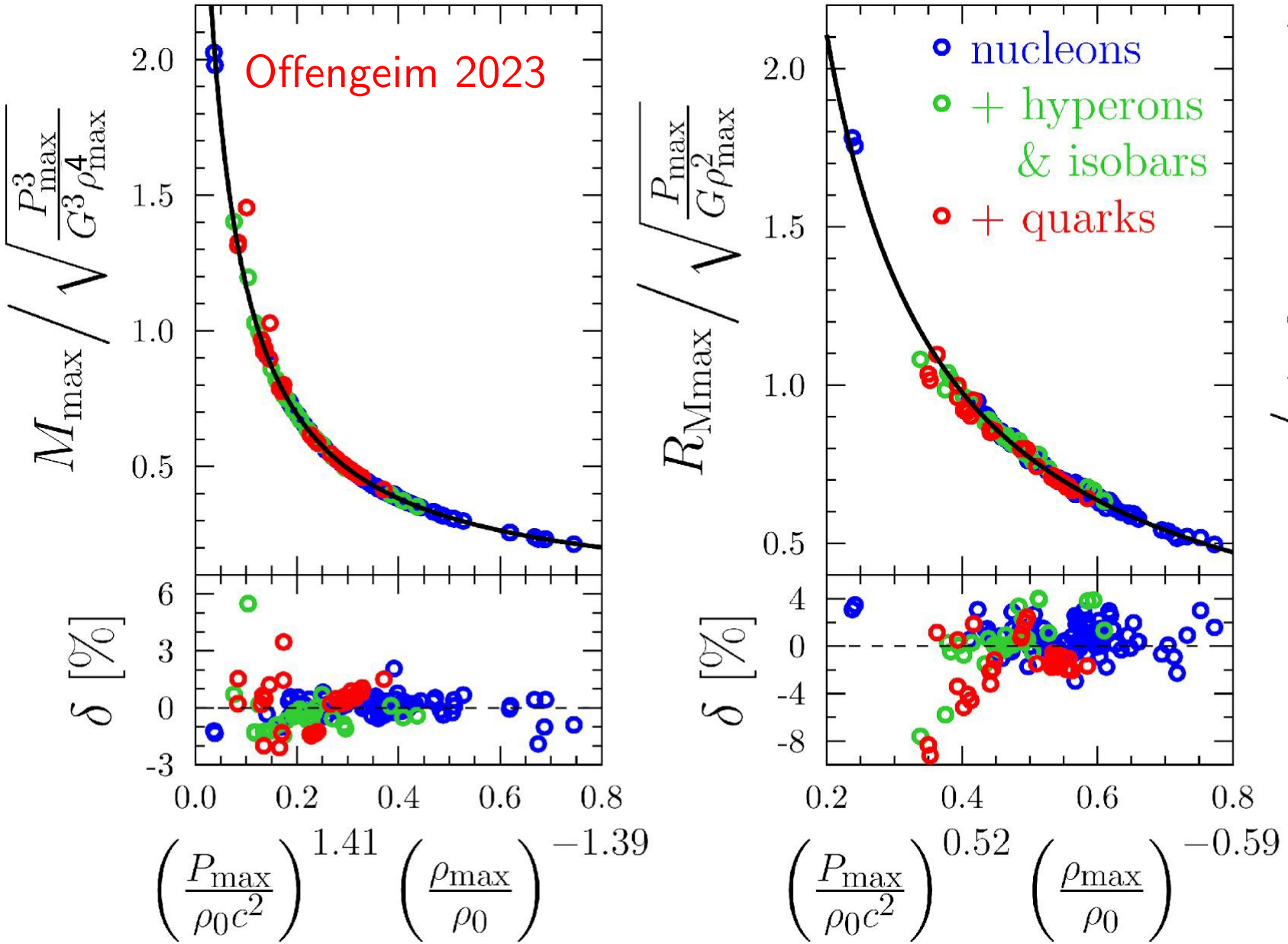
$$\Omega_f = \frac{GM\omega_f}{c^3}$$

Zhao & Lattimer 2022





$M_{\rm max}, R_{\rm max}, \mathcal{E}_{\rm max}, P_{\rm max}$ Correlation



$M_{\rm max}, R_{\rm max}, \mathcal{E}_{\rm max}, P_{\rm max}$ Correlation

Offengeim's finding suggest the power-law relations

$$\mathcal{E}_{\rm max} = (1.809 \pm 0.36) \left(\frac{R_{\rm max}}{10{\rm km}}\right)^{-1.98} \left(\frac{M_{\rm max}}{M_\odot}\right)^{-0.171} \ {\rm GeV\ fm^{-3}},$$

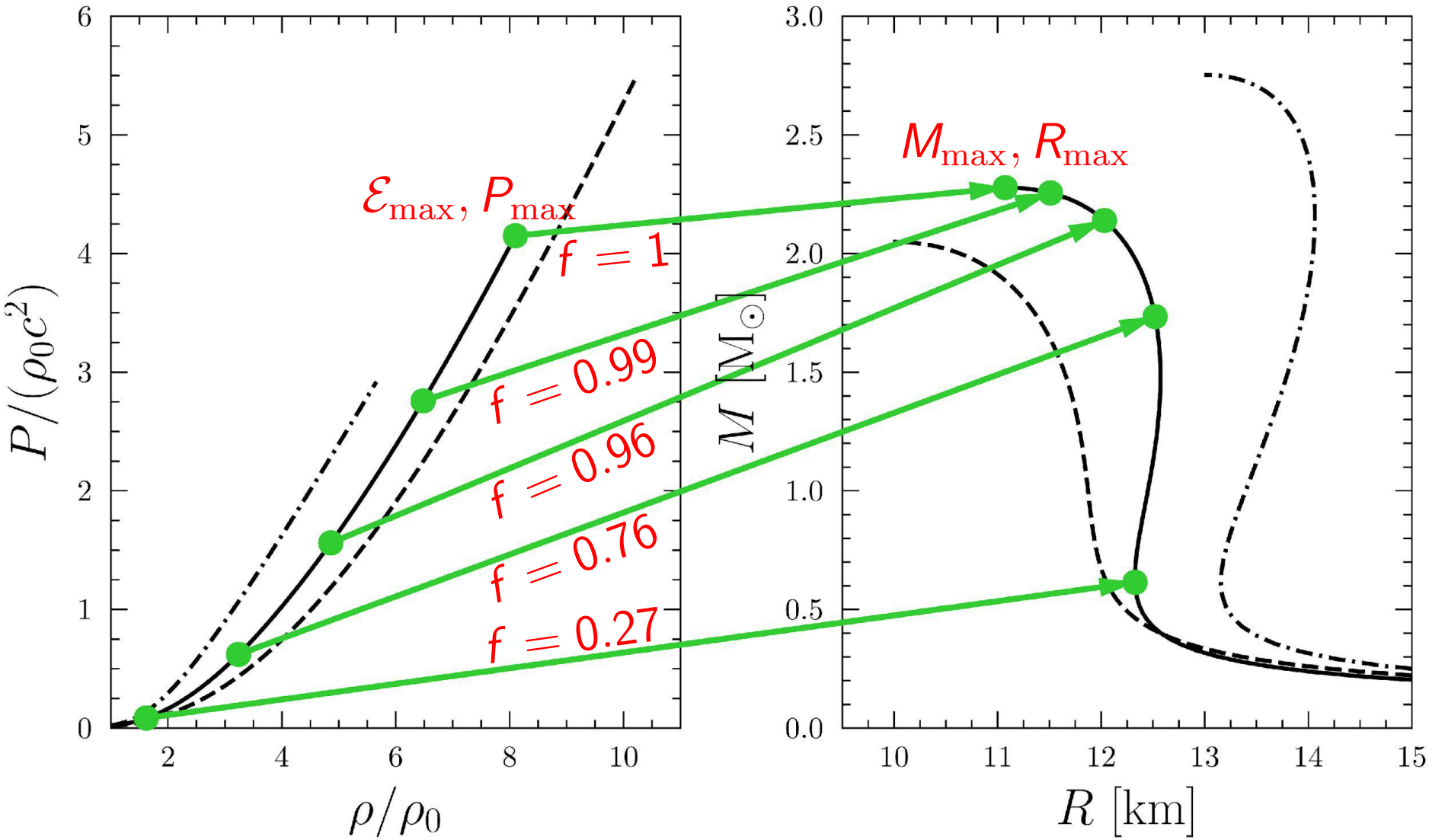
$$P_{\rm max} = (118.5 \pm 6.2) \left(\frac{R_{\rm max}}{10{\rm km}}\right)^{-5.24} \left(\frac{M_{\rm max}}{M_\odot}\right)^{2.73} \ {\rm MeV\ fm^{-3}},$$

We found, in addition, that additional points along the $M - R$ curve, at $M = fM_{\rm max}$, have similarly accurate correlations:

$$\mathcal{E}_f = a_{\mathcal{E},f} \left(\frac{R_{fM_{max}}}{10{\rm km}}\right)^{b_{\mathcal{E},f}} \left(\frac{M_{max}}{M_\odot}\right)^{c_{\mathcal{E},f}},$$

$$P_f = a_{P,f} \left(\frac{R_{fM_{max}}}{10{\rm km}}\right)^{b_{P,f}} \left(\frac{M_{max}}{M_\odot}\right)^{c_{P,f}},$$

Correlations at $M = fM_{\rm max}$



Correlations With $fM_{\rm max}$ and 2 R Values

$$\mathcal{E}_f = a_{\mathcal{E},f} \left(\frac{R_{M_1}}{10{\rm km}}\right)^{b_{\mathcal{E},f}} \left(\frac{R_{M_2}}{10{\rm km}}\right)^{c_{\mathcal{E},f}} \left(\frac{M_{max}}{M_\odot}\right)^{d},$$

$$P_f = a_{P,f} \left(\frac{R_{M_1}}{10{\rm km}}\right)^{b_{P,f}} \left(\frac{R_{M_2}}{10{\rm km}}\right)^{c_{P,f}} \left(\frac{M_{max}}{M_\odot}\right)^{d_{P,f}},$$

$f = M/M_{\rm max}$	$M_1/M_{\rm max}$	$M_2/M_{\rm max}$	$\Delta(ln\mathcal{E}_f)$
1	1	1/2	0.0046047928
4/5	2/3	1/2	0.0035713650
2/3	2/3	3/5	0.0050640415
3/5	4/5	1/2	0.0024633916
1/2	2/3	1/2	0.0047856724
2/5	3/5	1/2	0.0047133738

$f = M/M_{\rm max}$	$M_1/M_{\rm max}$	$M_2/M_{\rm max}$	$\Delta(\ln P_f)$
1	1	1/2	0.019032030
4/5	4/5	1/2	0.0096117432
2/3	2/3	3/5	0.014041491
3/5	2/3	1/2	0.00068963633
1/2	3/5	1/2	0.020100887
2/5	3/5	1/2	0.032359011

greatly reduced uncertainties!

Applications

- An analytic method of directly inverting the TOV equations, accuracy can be made arbitrarily high (number of f and R values). Existing techniques use parameterized EOS models in probabilistic (Bayesian) approaches having unquantified systematic uncertainties stemming from the model choice and parameter ranges (prior distributions).
- Correlations of $c_{s,\max}(M_{\max}, R_{\max})$ also exist (Offengeim 2023 found $\sim 10\%$ uncertainty), but accuracies can be improved using ≥ 2 R values. Accurate values for specific f values would be useful for interpolating within the $\mathcal{E} - P$ grid. They could also allow probing the composition of the neutron star interior (phase transitions, etc.).
- Correlations of $\tilde{\Lambda}, \bar{I}$ and BE$/M$ with M_{max} and R_{max} remain to be explored.