Thermodynamics of QCD with quarks and multi-quark clusters

Oleksii Ivanytskyi, David Blaschke and Gerd Röpke
(credits to Mateusz Cierniak)

arxiv:2308.07950 [nucl-th]

Many Particle Systems Under Extreme Conditions
Görlitz, 3-6 December 2023

Outline

- **Introduction**
- **Density functional approach to quark matter**
- **Multiquark clusters**
- **QCD thermodynamics**

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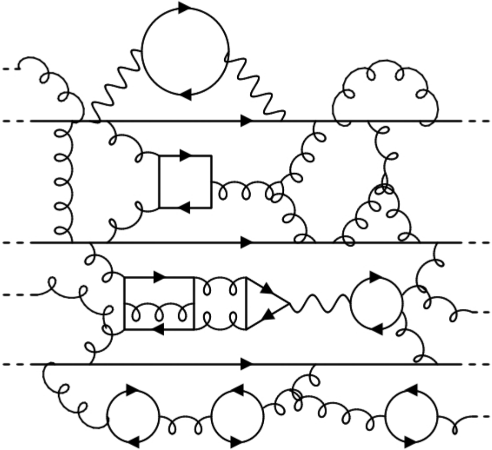
QCD: elegant Lagrangian, ...

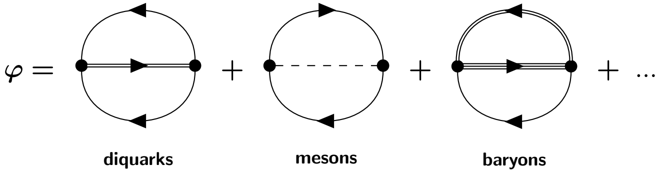
$$\mathcal{L}_{\color{red}{Q}\color{green}{C}\color{blue}{D}} = \overline{q}\left(i\hat{\not{D}} - \hat{m}\right)q - \frac{1}{4}G^2$$

QCD: ..., sophisticated nonperturbative dynamics



This is just a three quark bound state, e.g. nucleon

Cluster decomposition in two-loop approximation

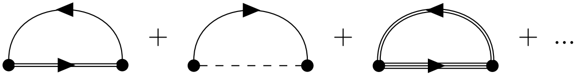


G. Baym, L.P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, Phys. Rev. 127, 1391 (1962)

- **Two-loop self energies & Dyson-Schwinger propagators**

$$\Pi_n = \frac{\delta\varphi}{\delta S_n}, \quad (S_n)^{-1} = (S_n^{free})^{-1} - \Pi_n, \quad n = 1, 2, 3, \ldots$$

quarks: $\Pi_1 =$



Dyson-Schwinger problem requires solving all S_n simultaneously

- **Mean-field approximation for quark propagators**

The Dyson-Schwinger problem reduces to a subsequent solving S_n using $S_{m<n}$

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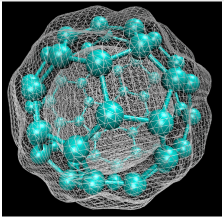
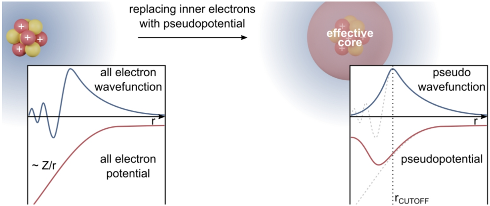
Context: density functional theory





(Dirac)Brueckner-Hartree-Fock T-, G-matrix based theories

$\Updownarrow$

Density functional theory

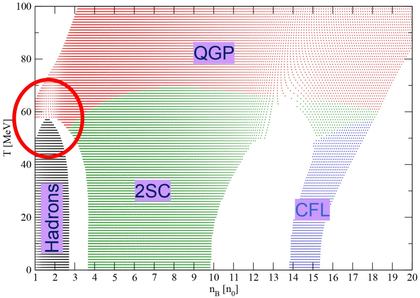
- **Many body problems**
- **Quantum chemistry**
- **Skyrme-type models for nuclear physics**
- **String Flip model for quark matter**
- **...**

Why? False quark dominance in hybrid quark-hadron EoS

- **Hadronic EoS consistent with astro (DDf4) + NJL model**

$\Downarrow$

False quark onset already @ T $\simeq$ 60 MeV



- **Hadron decays are energetically favorable**

$$\begin{array}{lcc} M_q \simeq 330 MeV & & M_{meson} > 2M_q \\ M_\omega = 783 MeV & \Rightarrow & quarks\ are\ too\ light \\ M_\rho = 775 MeV & & to\ be\ confined \end{array}$$

Effective quark "confinement" is needed

Confining density functional @ $N_f = 2$

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m)q - \mathcal{U}, \quad \mathcal{U} = D_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\varkappa}$$

O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

- **Parameters**

D_0 - coupling, controls interaction strength

α - dimensionless constant, controls vacuum quark mass

$\langle\overline{q}q\rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

$\varkappa = 1/3$

$\Downarrow$

motivated by String Flip model, $\mathcal{U}_{SFM} \propto \langle q^+q\rangle^{2/3}$

$$\Sigma_{SFM} = \frac{\partial\mathcal{U}_{SFM}}{\partial\langle q^+q\rangle} \propto \langle q^+q\rangle^{-1/3} \propto separation$$

$\varkappa = 1$

$\Downarrow$

Nambu–Jona-Lasinio model

C.J.Horowitz, E.J. Moniz, J. W. Negele, PRD 31, 1689 (1985)
G. Röpke, D. Blaschke, H. Schultz, PRD 34, 3499 (1986)

- **Dimensionality**

$$[\mathcal{U}] = energy^4 \qquad [\overline{q}q] = energy^3 \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = energy^2 = [string\ \ tension]$$

self energy = string tension × separation $\Longrightarrow$ confinement

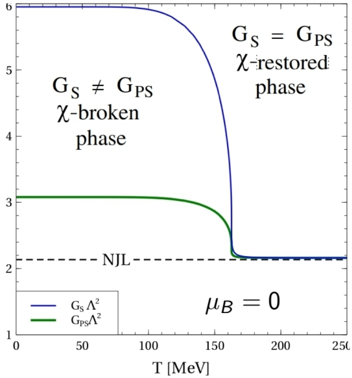
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{\left(\overline{q}q - \langle\overline{q}q\rangle\right)\Sigma_S}_{1^{\text{st}}\ \text{order}} - \underbrace{G_S\left(\overline{q}q - \langle\overline{q}q\rangle\right)^2 - G_{PS}\left(\overline{q}i\vec{\tau}\gamma_5 q\right)^2}_{2^{\text{nd}}\ \text{order}} + \ldots$$

- **Mean-field scalar self-energy**

$$\Sigma_S = \frac{\partial \mathcal{U}_{MF}}{\partial\langle\overline{q}q\rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\overline{q}q\rangle^2}, \quad G_{PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{\left(\bar{q}q - \langle \bar{q}q \rangle\right) \Sigma_{MF}}_{1^{\text{st}}\ \text{order}} - \underbrace{G_S \left(\bar{q}q - \langle \bar{q}q \rangle\right)^2 - G_{PS} \left(\bar{q} i \vec{\tau} \gamma_5 q\right)^2}_{2^{\text{nd}}\ \text{order}} + \dots$$

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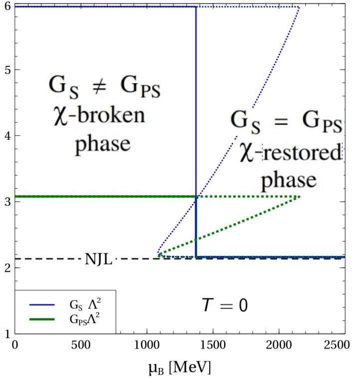


Comparison to NJL model @ $N_f = 2$

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \underbrace{(m + \Sigma_S)}_{\textbf{effective mass } \mathbf{m}^*})q + G_S(\bar{q}q)^2 + G_{PS}\,(\bar{q}i\vec{\tau}\gamma_5 q)^2 + \cdots + \mathcal{L}_V + \mathcal{L}_D$$

- **Similarities:**
  - current-current interaction
  - (pseudo)scalar, vector, diquark, ... channels

- **Differences:**
  - high m^* at low T, $\mu \Rightarrow$ **"confinement"**

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2G_0}{3\alpha^{2/3}\langle\bar{q}q\rangle_0^{1/3}}$$

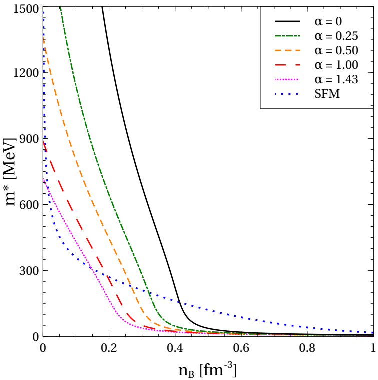
$$\Downarrow$$

$$m^* \to \infty \; at \; \alpha \to 0$$

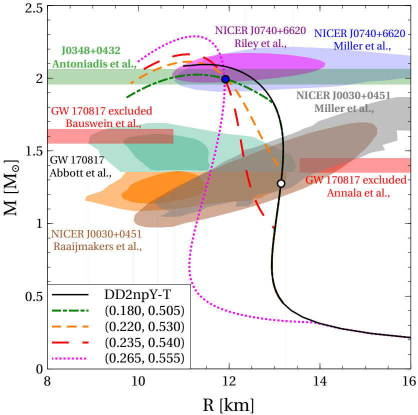
  - medium dependent couplings:

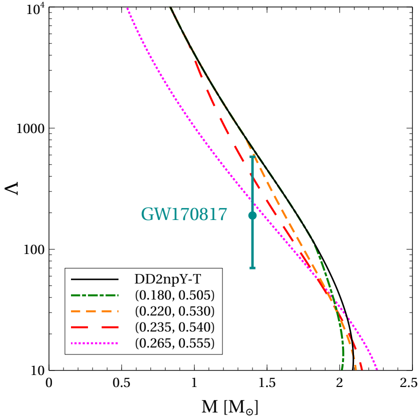
$$\begin{array}{llllll} \text{low} & T,\ \mu, & \Rightarrow & G_S \neq G_{PS} & \Rightarrow & \chi\text{-broken} \\ \text{high} & T,\ \mu, & \Rightarrow & G_S = G_{PS} & \Rightarrow & \chi\text{-symmetric} \end{array}$$

T = 0



Modeling neutron stars with quark cores @ $N_f = 2$





O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

Agreement with the observational constraints

on mass-radius relation and tidal deformability of neutron stars

Confining density functional with Polyakov loop @ $N_f = 3$

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ + gA\!\!\!/ - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = diag(m_u, m_d, m_s)$$

A_μ - homogeneous static gluon field in the Polyakov gauge

- **Density functional**

$$\mathcal{U}_\chi = D_0 \left[(1+\alpha)\langle\hat{\mathcal{O}}\rangle_0 - \hat{\mathcal{O}}\right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2}\sum_{a=0,8}\left[(\overline{q}\tau_a q)^2 + (\overline{q}i\gamma_5\tau_a q)^2\right]$$

D. Blaschke, O. Ivanytskyi, M. Shahrbaf, 2202.05061 [nucl-th]

- **Polyakov loop potential**

$$\Phi = \frac{1}{N_c} Tr_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b\log M_H + \frac{1}{2}c\left(\Phi^3 + \bar{\Phi}^3\right) + d\left(\bar{\Phi}\Phi\right)^2$$

T-dependence of $a, b, c,\ d$ is fitted to the pure SU(3) gauge lattice data

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

Expansion around mean-field solution @ $N_f = 3$

$$\mathcal{U}_\chi = \underbrace{\mathcal{U}_\chi^{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{\overline{q}\hat{\Sigma}q - \langle\overline{q}\hat{\Sigma}q\rangle}_{1^{\text{st}}\ \text{order}}$$

$$\underbrace{-\sum_{f,f'}\left(\overline{f}f - \langle\overline{f}f\rangle\right) G_S^{ff'}\left(\overline{f'}f' - \langle\overline{f'}f'\rangle\right) - G_{PS}\sum_f\left(\overline{f}i\gamma_5 f\right)^2}_{2^{\text{nd}}\ \text{order}} + \dots$$

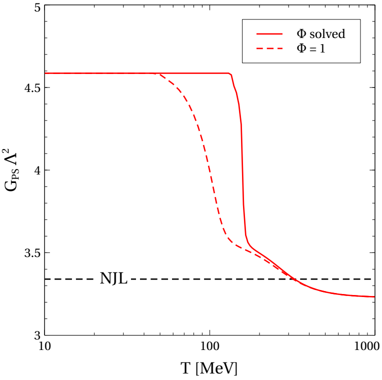
- **Mean-field scalar self-energy**

$$\hat{\Sigma} = diag(\Sigma_u, \Sigma_d, \Sigma_s), \quad \Sigma_f = \frac{\partial\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle}$$

- **Effective medium dependent couplings**

$$G_S^{ff'} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle\partial\langle\overline{f'}f\rangle}$$

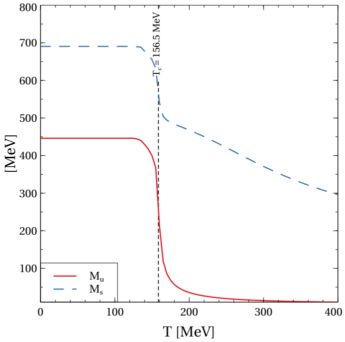
$$G_{PS} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}i\gamma_5 f\rangle^2}$$

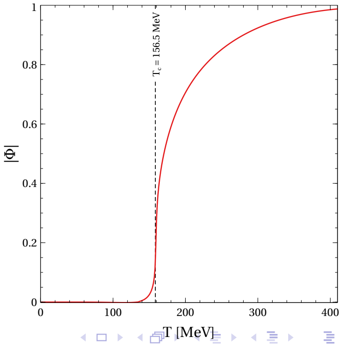


- **Fitting vacuum phenomenology**

$$\begin{cases} M_\pi = 140\ MeV \\ F_\pi = 93\ MeV \\ M_K = 494\ MeV \\ F_K = 112\ MeV \\ T_c = 156.5\ MeV \end{cases} \Rightarrow \quad \begin{array}{l} m_u = m_d = 4.4\ MeV \\ m_s = 134.8\ MeV \\ \Lambda = 636.1\ MeV \\ \sqrt{D_0} = 729.6\ MeV \\ \alpha = 1.44 \end{array}$$

- **Effective masses and Polyakov loop**





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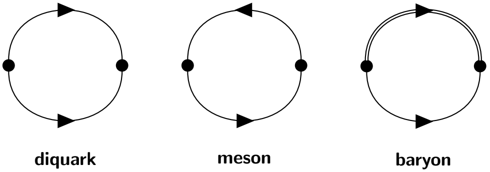
Generalized Beth-Uhlenbeck approach

- **Large size clusters as correlations of the smaller size ones $\Rightarrow$ propagators**

polarization loops :



- **Phase shift of multiquark clusters**

$$S_n = |S_n| e^{i\delta_n} \quad \Rightarrow \quad \delta_n = \Im \ln S_n$$

- **Generalized Beth-Uhlenbeck formula**

$$\Omega_n = \frac{d_n}{\kappa_n} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} (tr_D)^{2-\kappa_n} \ln(\beta^{\kappa_n} S_n^{-1}) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}$$

$\kappa_n = 1$ - fermions, $\kappa_n = 2$ - bosons

G. Röpke, N.U. Bastian, D. Blaschke, T. Klähn, S. Typel, H. Wolter, NPA 897 (2013)

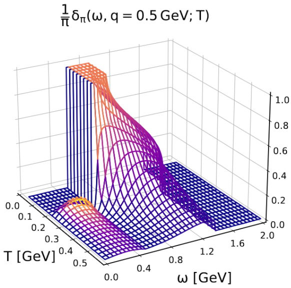
Phase shifts of multiquark states

- **Microscopic calculations for pions**

  K. Maslov, D. Blaschke, PRD 107, 094010 (2023)

  1. discontinuous jump at the on-shell energy below the dissociation temperature
  2. continuous growth at small energies above the dissociation temperature
  3. continuous fall above the decay threshold
  4. vanishing at high energies (Levonson's theorem)

  M. Wellner, Am. J. Phys. 32, 787–789 (1964)

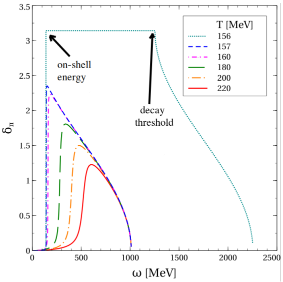


- **Parametric model of $\delta = \delta(\mathbf{T}, \omega)$**

  D. Blaschke, M. Cierniak, O. Ivanytskyi, G. Röpke, arxiv:2308.07950 [nucl-th]

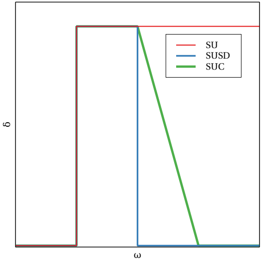
  1. parametric expression reproduces all the properties of the microscopic calculations
  2. T-dependence of the hadron masses & widths agree with the microscopic calculations
  3. requires hadron decay threshold given by quark masses $M_{u,d,s}$

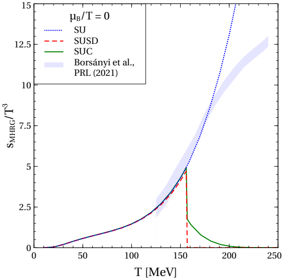
$$M_h^{\mathrm{Th}} = N_h^u M_u + N_h^d M_d + N_h^s M_s$$



Beth-Uhlenbeck vs Hadron resonance gas

- **Step-up (SU)**
  1. is generated by the pole of $S_{n>1}$
  2. corresponds to a bound multiquark state
  3. is present only below the dissociation temperature
  4. generates a HRG-like term in Ω
- **Step-down (SUSD)**
  1. rough account of the decay threshold
  2. partially/totally compensates HRG-like term in Ω
- **Continuum (SUC)**
  1. corresponds to a scattering multiquark state
  2. partially compensates HRG-like term in Ω





Mass-spectrum

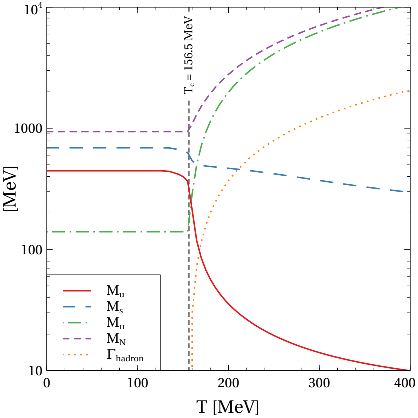
$$M_{n>1} = M_{n>1}^{vacuum} + A(T - T_c)\theta(T - T_c)$$

$$\Gamma_{n>1} = B\theta(T - T_c)$$

$T_c = 156.6$ MeV, A, B - fitted to lQCD

- **Low T (χ-broken matter)**
  1. heavy quarks
  2. stable multiquark clusters
  3. constant mass of multiquark states
  4. zero width of multiquark states

- **High T (χ-symmetric matter)**
  1. light quarks
  2. unstable multiquark clusters
  3. growing mass of multiquark states
  4. growing width of multiquark states



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Thermodynamic potential

$$\Omega = \Omega_{\text{quarks}} + \mathcal{U}_\chi - \langle \bar{\mathbf{q}} \hat{\boldsymbol{\Sigma}} \mathbf{q} \rangle + \mathcal{U}_\phi + \underbrace{\Omega_{\text{hadrons}} + \Omega_{\text{colored clusters}}}_{\text{multiquark clusters}}$$

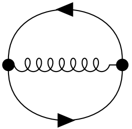
- **Quarks**
  - Non-perturbative states at low momenta $k < \Lambda$

$$\Omega^{k<\Lambda}_{quarks} = -\frac{1}{\beta V} Tr \ln(\beta S^{-1}_{quarks})$$

S_{quarks} - quark propagator @ mean-field

  - Perturbative states at high momenta $k > \Lambda$

$$\Omega^{k>\Lambda}_{quarks} = \frac{1}{2\beta V}$$



J.I. Kapusta, Finite Temperature Field Theory, Cambridge (1989)

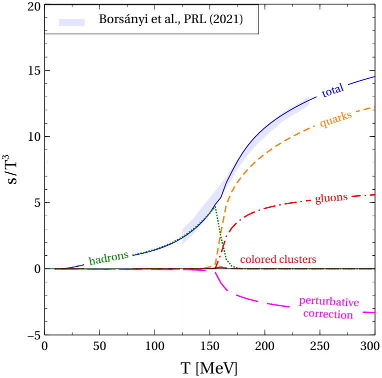
- **Hadrons - 62 meson, 60+60 baryon-antibaryon states with $M < 2.6$ GeV**
- **Colored multiquark states** - diquarks, tetraquarks, pentaquarks coupled to Φ

$$M^{vacuum} = \sum_f M^{vacuum}_f N_f - B \left(\sum_f N_f - 1 \right)$$

Entropy density

$$s = -\frac{\partial \Omega}{\partial T}$$

- **Low T**
  1. hadron dominance
- **High T**
  1. quark-gluon dominance
  2. negative perturbative contribution
- **Colored multiquark states ($\mu_B = 0$ only)**
  1. suppressed by the Polyakov loop at high T
  2. suppressed by high mass at high T



Chiral condensate

$$\langle \bar{f} f \rangle = -\frac{\partial \Omega}{\partial m_f} = \underbrace{2N_c \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{M_f}{E_f} (f_f^+ + f_f^- - 1)}_{\textbf{quarks}}$$

$$+ \underbrace{\sum_{n>1} \frac{d_n \sigma_n^f}{m_f} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{M_n}{\omega} (f_n^+ + f_n^-) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}}_{\textbf{multiquark clusters}}$$

- **σ-factor**
  1. σ_π^f, σ_K^f – defined from the GMOR relations
  2. other multiquark clusters

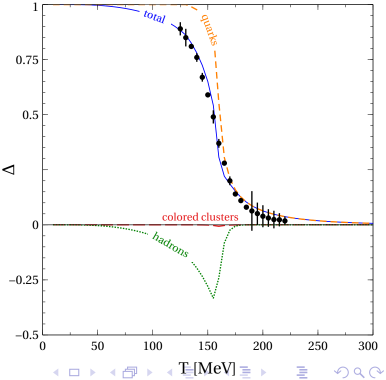
$$\sigma_n^f = m_f \frac{\partial M_n}{\partial m_f} = m_f N_n^f$$

J. Jankowski, D. Blaschke, M. Spalinski, PRD 87, 10 (2013)

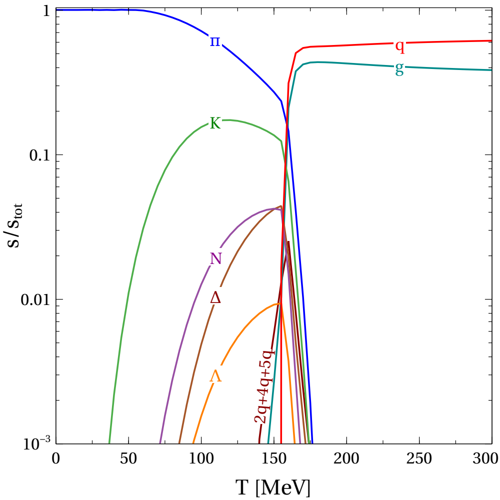
- **Scaled chiral condensate**

$$\Delta = \frac{m_s \langle \bar{l} l \rangle - m_l \langle \bar{s} s \rangle}{m_s \langle \bar{l} l \rangle_0 - m_l \langle \bar{s} s \rangle_0}$$

Almost constant quark term below T_c
Hadrons are necessary to reproduce the lQCD data



Composition



Sharp switching between partonic & hadronic degrees of freedom

Conclusions

- A unified EoS of strongly interacting matter based on a cluster decomposition approach
- Agreement with the lattice QCD data on entropy density and chiral condensate

  (see arxiv:2308.07950 [nucl-th] for baryon density and stay tuned for more)
- Sudden switching between partonic and hadronic degrees of freedom

Speculation

1. **Sharp switching between hadrons and partons**
2. **Absence of colored mediators of interaction below T_c**

$$\Downarrow$$

Narrow range of the freeze-out temperatures for all hadrons?