T-Matrix Theory of Strongly Coupled QGP

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Many-Particle Systems under Extreme Conditions

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Outline

1.) Introduction

- Nonperturbative QCD Force + sQGP

2.) Thermodynamic T-Matrix

- Selfconsistent Quantum Many-Body Theory
- Input Potential + Constraints from Lattice QCD

3.) Hamiltonian Approach to sQGP Structure

- Thermodynamic Potential + EoS
- Spectral + Transport Properties

4.) Heavy-Flavor Transport in Heavy-Ion Collisions

- QCD Matter in the Lab
- Open HF Phenomenology

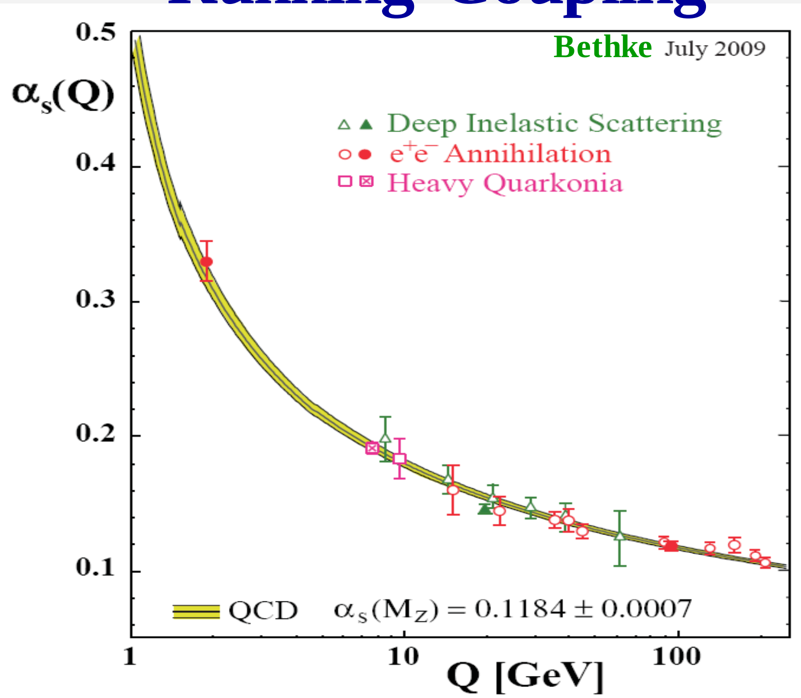
5.) Conclusions

1.1 Quantum Chromodynamics in Vacuum

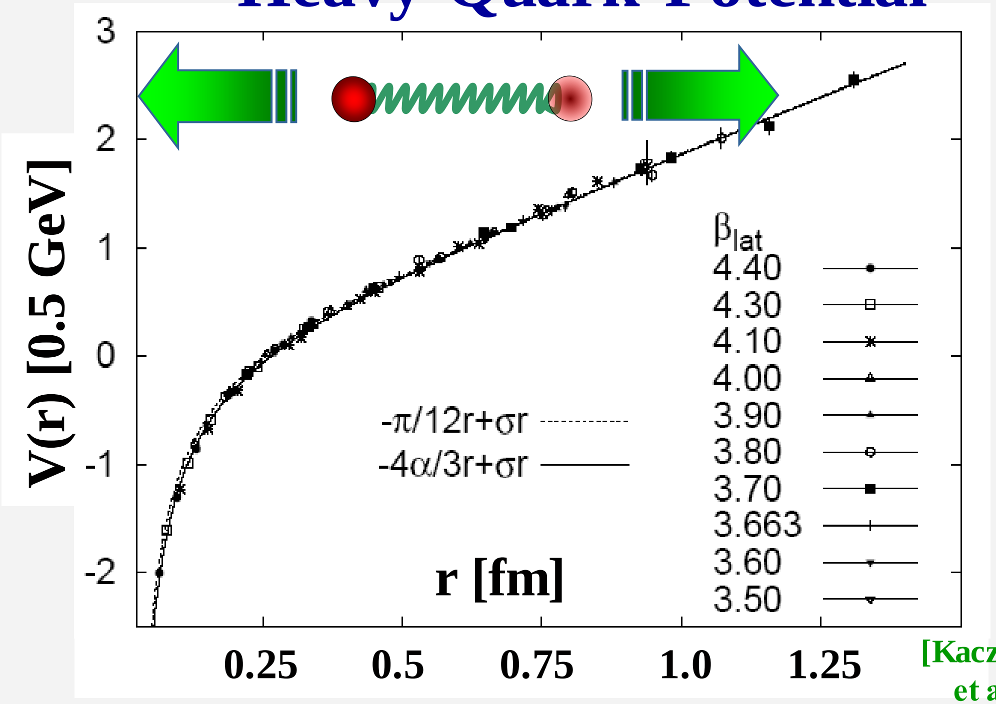
$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^{\mu} D_{\mu} - \mathcal{M}_q)q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

[Gross,Politzer +Wilczek '73]

Running Coupling



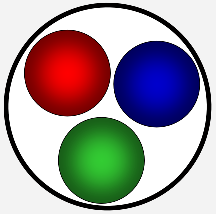
Heavy-Quark Potential



[Kaczmarek et al '05]

$Q^2 \leq 1$ GeV^2 → **"strong QCD"**

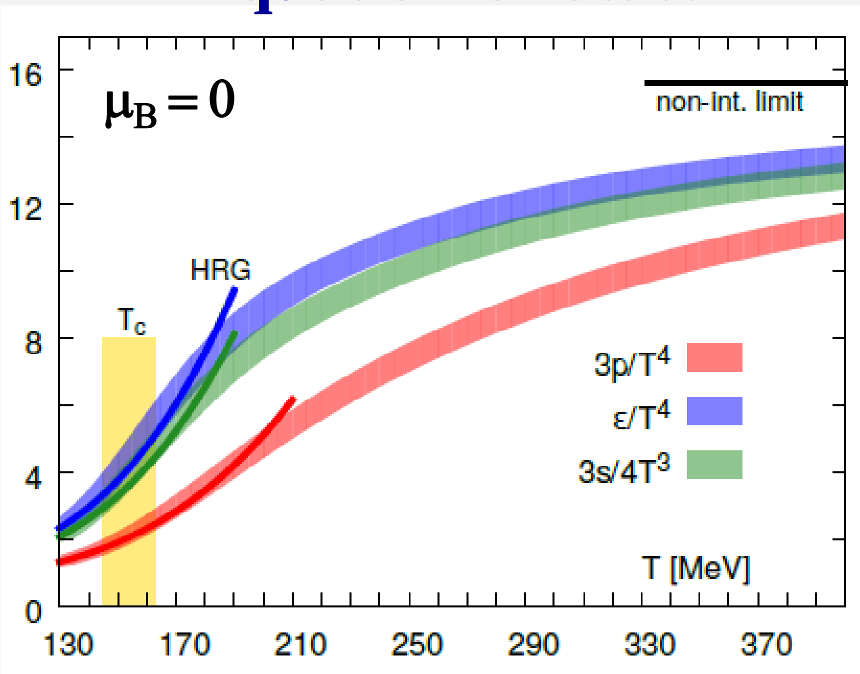
- **Confinement**
- mass generation (*$M_p \approx 1$ GeV*)



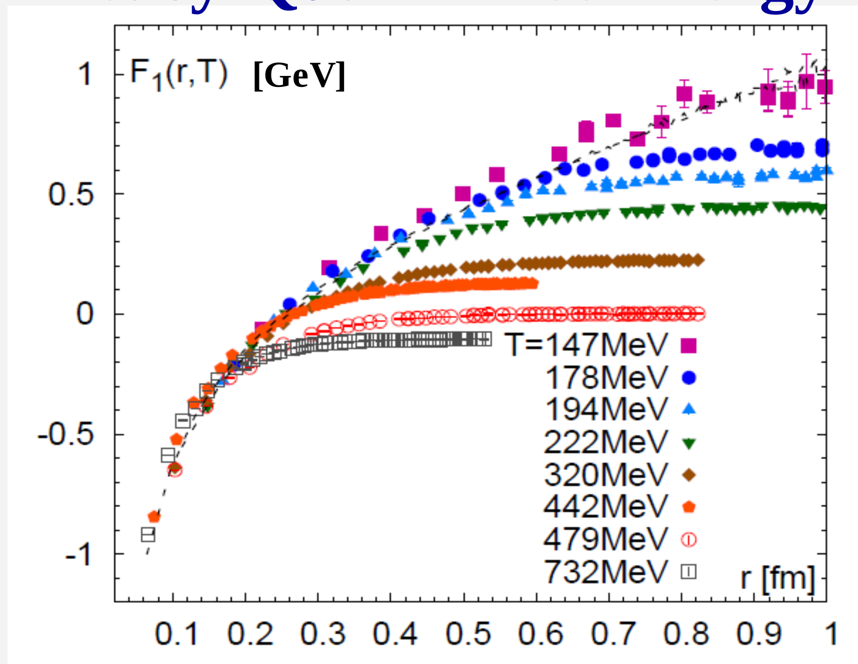
- Non-perturbative for *$r \geq 0.25$ fm*
- Operational definition of confinement
- Well-calibrated force describes charm-/bottom-onium spectroscopy

1.2 QCD Matter and In-Medium Force

Equation of State



Heavy-Quark Free Energy



[Bazavov et al '13,'14]

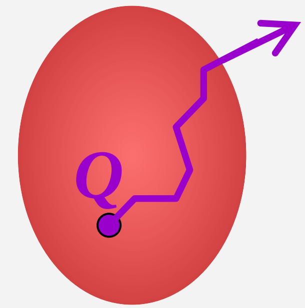
- "Change" in degrees of freedom above **T~160 MeV**

- $F_{Q\bar{Q}} = U_{Q\bar{Q}} - T\, S_{Q\bar{Q}}$
- Non-perturbative above T_c

- **Connection between QCD Matter and in-medium forces?**
- **Heavy-quark transport as a microscopic probe**
  ⇒ Deduce structure of strongly coupled QGP

<u>1.3 Strongly Coupled Quark-Gluon Plasma</u>

⇔ Quantum liquid with transport coefficients near conjectured lower bounds



- Microscopic calculation of transport coefficients should **obey** quantum-lower bounds

- **Bulk medium** should encode strong-coupling properties

  → universality? $\mathcal{D}_s(2\pi T) \sim (4\pi)\eta/s \sim \sigma_{el}/T$

- **Large scattering rates** → large collisional widths

  → broad spectral functions → **off-shell (quantum) effects**

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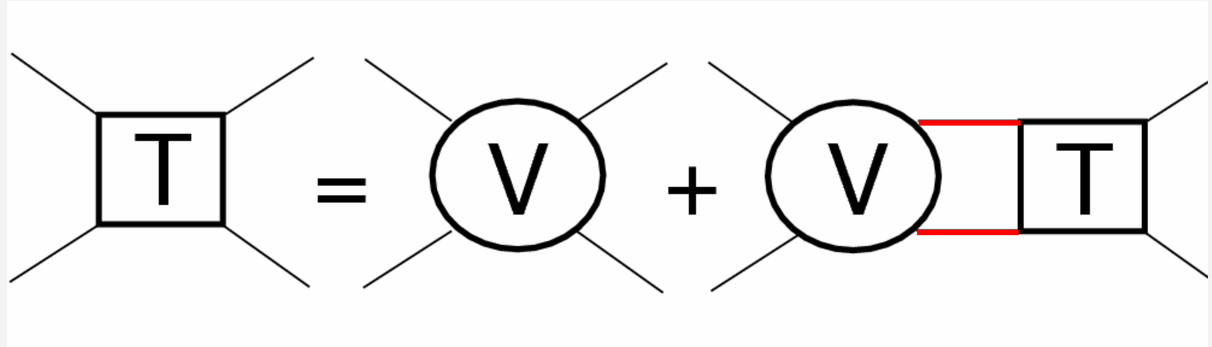
5.) **Conclusions**

2.1 Thermodynamic T-Matrix

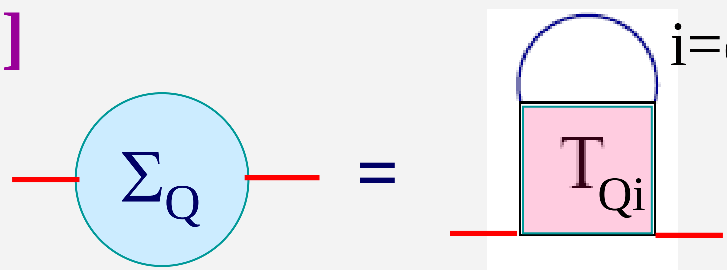
- **Scattering equation**

j=Q

i=Q,q,g



- Born approximation (**weak** coupling): $\mathbf{T_{Qi} \approx V_{Qi}}$
- **Strong** coupling → resummation: $\boxed{\mathbf{T_{ji} = V_{ji} + \int V_{ji}\, G_j\, G_i\, T_{ji}}}$
- Dyson equation: $\mathbf{G_i = 1 / [\omega - \omega_k - \Sigma_i(\omega,k)]}$
- Parton **self-energies** → **self-consistency**:



- **Vacuum**: Cornell potential → HQ spectroscopy
- **In-medium**: $V_{\mathcal{C}} + V_{\mathcal{S}} = -\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r} - \frac{\sigma e^{-m_s r - (c_b m_s r)^2}}{m_s}$

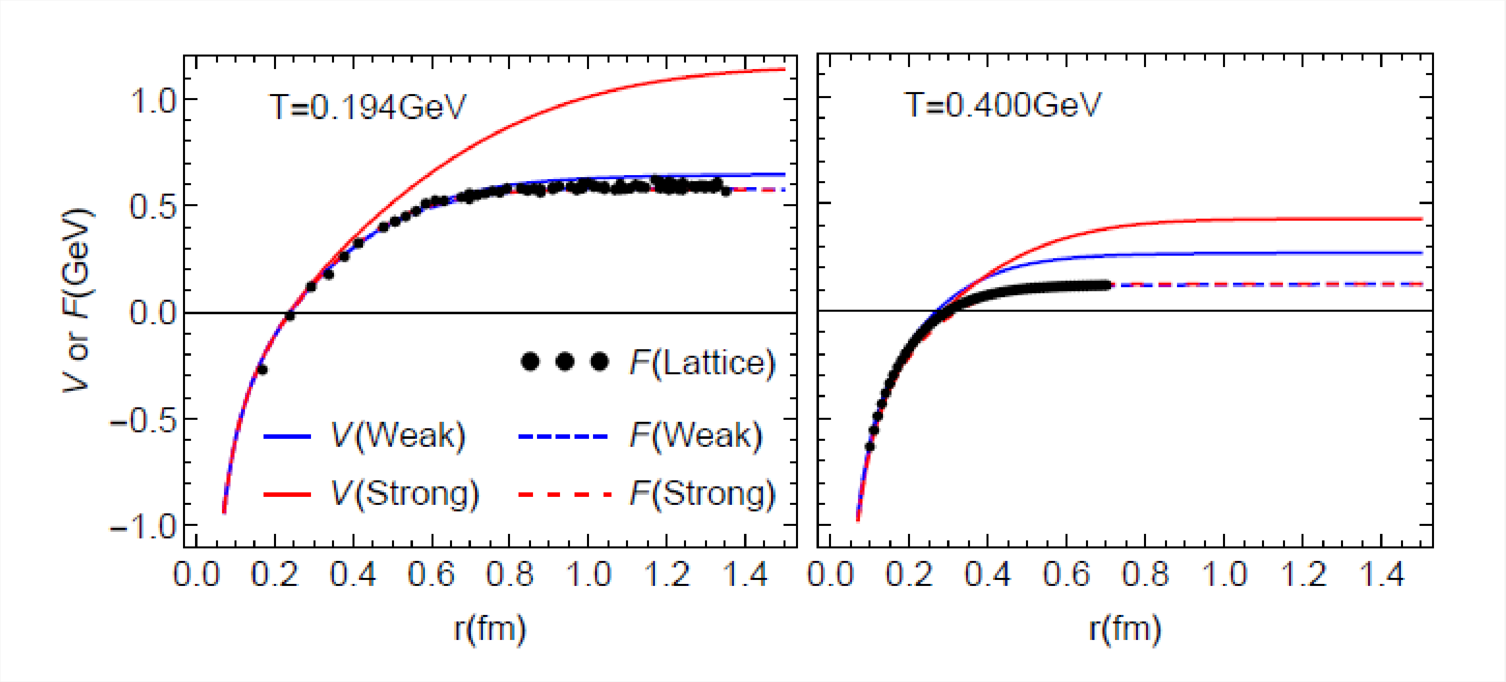
[TAMU '04-'23]

→ **constrain screening masses $m_{d,s}$ using lattice-QCD data**

2.2 Potential Extraction from Lattice QCD

- **Free energy:** $$F_{Q\bar{Q}}(r_1 - r_2) = -\frac{1}{\beta}\ln\left(\int_{-\infty}^{\infty} d\omega\sigma(\omega, r_1 - r_2)\, e^{-\beta\omega}\right)$$

- **Quarkonium Spectral Function:** $$\sigma(\omega, r) = \frac{1}{\pi}\frac{(V+\Sigma)_I(\omega)}{(\omega-(V+\Sigma)_R)^2 + (V+\Sigma)_I^2(\omega)}$$



- **Weakly Coupled Scenario: potential close to F, $\Gamma_Q \sim 0.1$ GeV**
- **SCS: remnants of confining force, $\Gamma_Q \sim 0.6$ GeV;**
  Quantum effects + self-consistency limit coupling strength

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3.) Hamiltonian Approach to QGP

- **In-Medium Parton Hamiltonian** with 2-body interactions

$$H = \sum \varepsilon_i(\mathbf{p})\psi_i^\dagger(\mathbf{p})\psi_i(\mathbf{p}) + \psi_i^\dagger(\frac{\mathbf{P}}{2} - \mathbf{p})\psi_j^\dagger(\frac{\mathbf{P}}{2} + \mathbf{p})V_{ij}^a\psi_j(\frac{\mathbf{P}}{2} + \mathbf{p}')\psi_i(\frac{\mathbf{P}}{2} - \mathbf{p}')$$

  - effective parton masses $\varepsilon_i(\mathbf{p}) = \sqrt{M_i^2 + \mathbf{p}^2}$

- Interaction: **Cornell potential** with relativistic corrections

$$V_{ij}^a(\mathbf{p}, \mathbf{p}') = \mathcal{R}_{ij}^{\mathcal{C}}\mathcal{F}_a^{\mathcal{C}}V_{\mathcal{C}}(\mathbf{p} - \mathbf{p}') + \mathcal{R}_{ij}^{\mathcal{S}}\mathcal{F}_a^{\mathcal{S}}V_{\mathcal{S}}(\mathbf{p} - \mathbf{p}')$$

  - **as used in heavy-quark sector**

- **Implement into Brueckner / Luttinger-Ward-Baym approach**

3.2 QGP Equation of State + Spectral Functions

Thermodynamic Potential (2-PI)

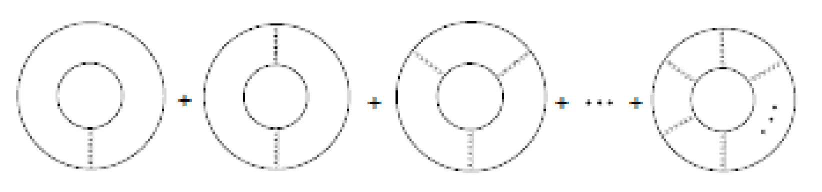
$$\Omega = \mp \frac{-1}{\beta} \sum_n \mathrm{Tr}\{\ln(-G^{-1}) + \Sigma\, G\} \pm \Phi$$

Selfconsistent SFs

$$G = G_0 + G_0 \Sigma G \qquad \Sigma = GT \qquad T = V + VGGT$$

Lutttinger-Ward Functional

$$\Phi = \frac{-1}{\beta} \sum_{n,\nu} \mathrm{Tr}\{\frac{1}{2\nu}(\frac{-1}{\beta})^{\nu}[(-\beta)^{\nu}\Sigma_{\nu}(G)]G\}$$



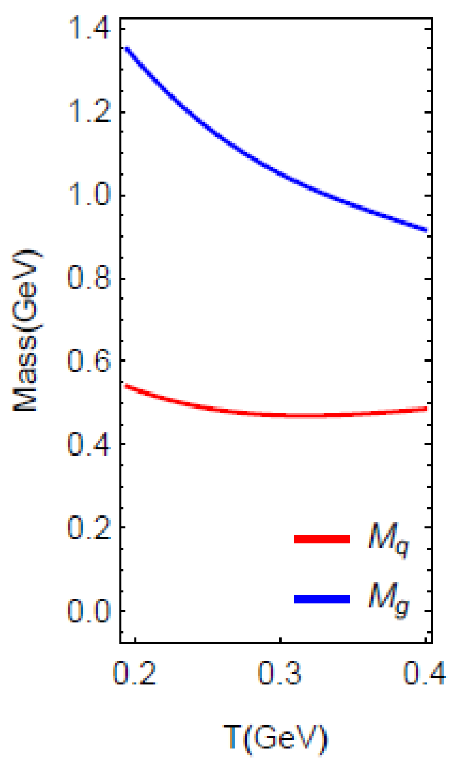
- **Fully off-shell resummed Luttinger-Ward functional (!)**

  $\rightarrow$ allows for dynamically emerging **hadronic resonances**

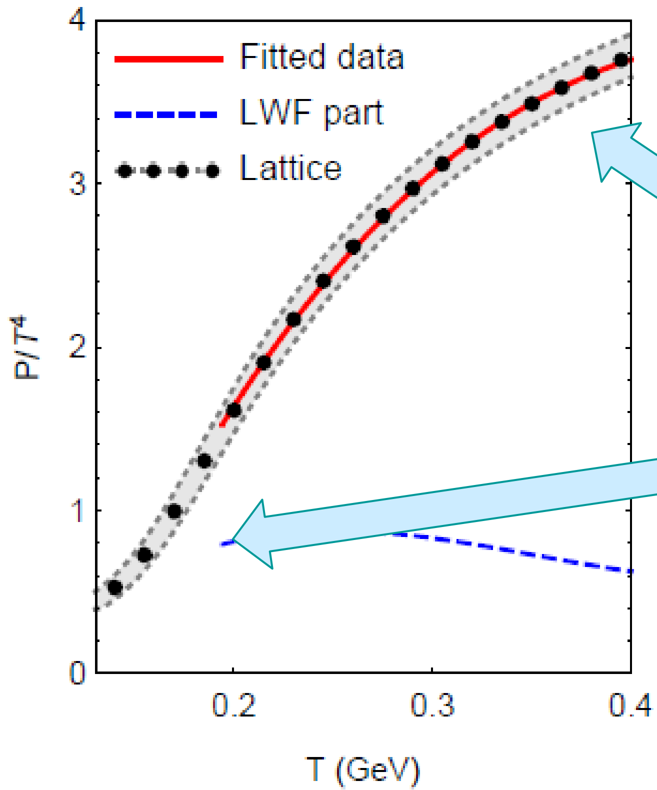
- Fit "bare" quark + gluon masses

3.3 QGP Equation of State + Degrees of Freedom

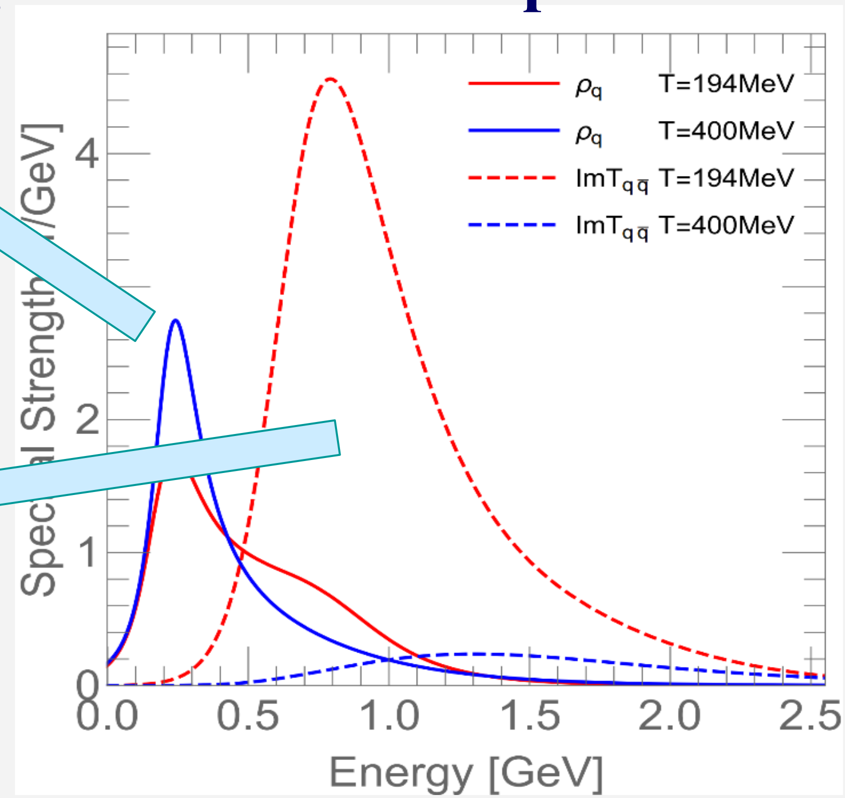
Input Masses



Pressure



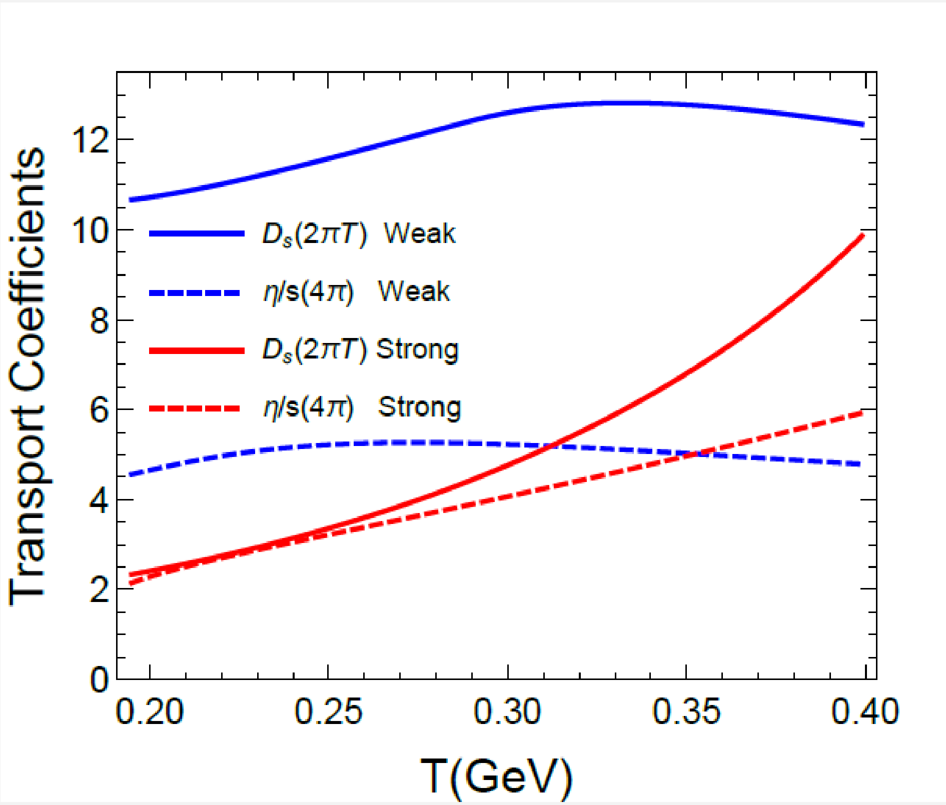
Quark + "Meson" Spectral Fcts

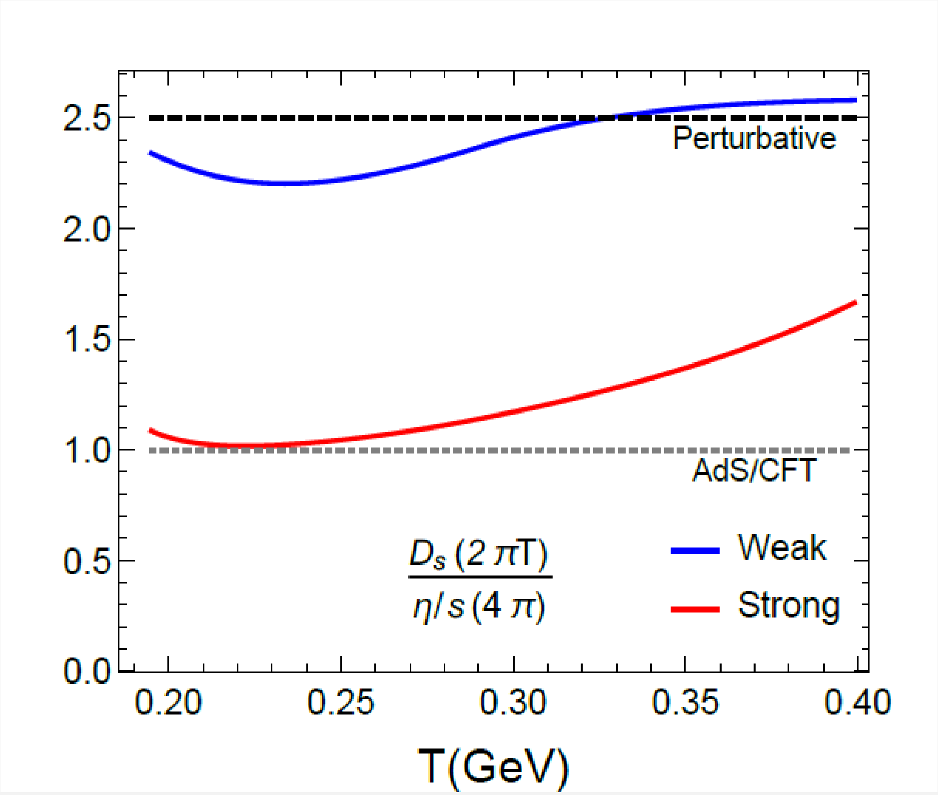


- **Confining force near T_c : $\Rightarrow$ soft partons melt, $\Gamma_q \geq m_q$**
  $\Rightarrow$ broad hadronic resonances emerge

3.4 Transport Coefficients + Strong-Coupling Limit

Shear Viscosity and Heavy-Quark Diffusion





- Strongly coupled: $(2\pi T)\,\mathcal{D}_s \sim (4\pi)\,\eta/s$
- Perturbative: $(2\pi T)\,\mathcal{D}_s \sim 5/2\,(4\pi)\,\eta/s$
- Transition as **T** increases

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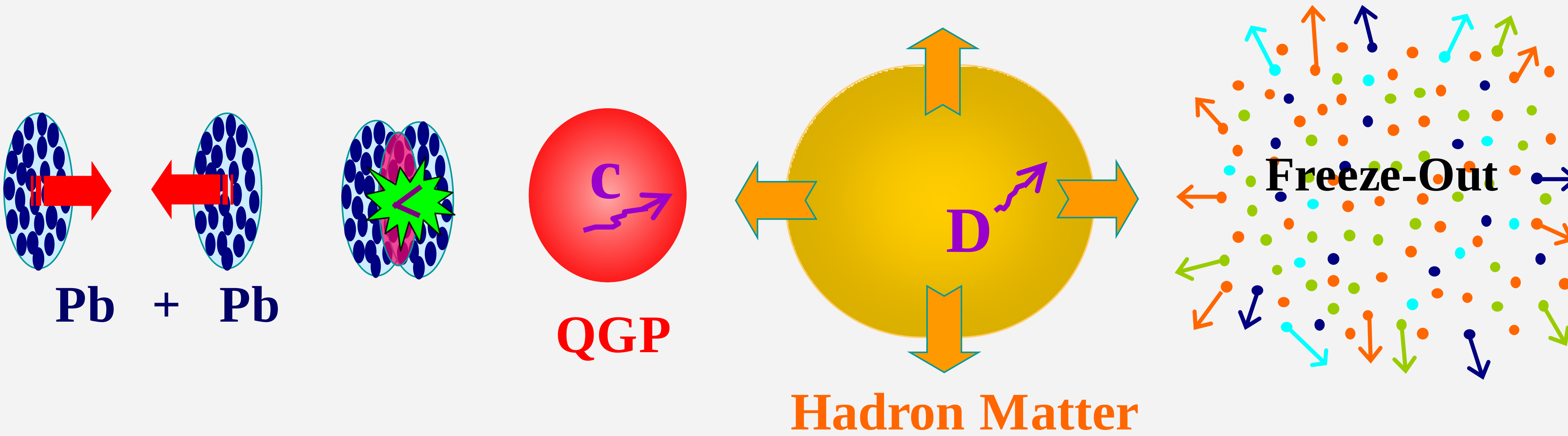
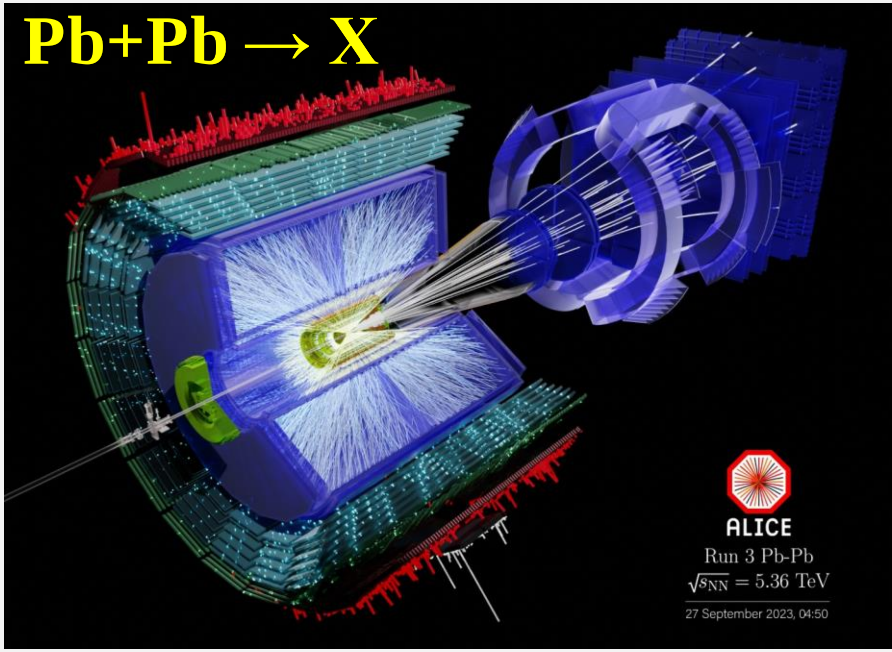
4.1 QCD Matter in the Lab: Heavy-Ion Collisions

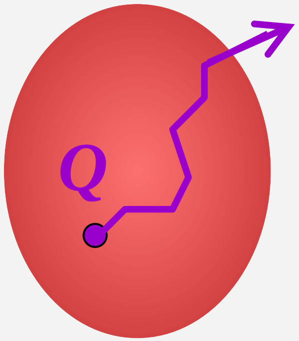


Heavy-Quark Probe of QCD Medium

- "Brownian markers" of coupling strength: $m_Q >> T$
- Requires transport approach



4.2 Heavy-Flavor Transport in QGP



$p^2 \sim m_Q\, T >> q^2 \sim T^2 \;\Rightarrow$ **Fokker-Planck Equation:**

$$\boxed{\frac{\partial}{\partial t} f_Q(t,p) = \gamma \frac{\partial}{\partial p_i}[p_i f_Q(t,p)] + D_p \Delta_{\vec{p}} f_Q(t,p)}$$

thermalization rate

$$\gamma p = \int d^3q\, w_Q(q,p)\, q$$

$$\sim \int |T_{Qi}|^2\, (1-\cos\theta)\, f^i$$

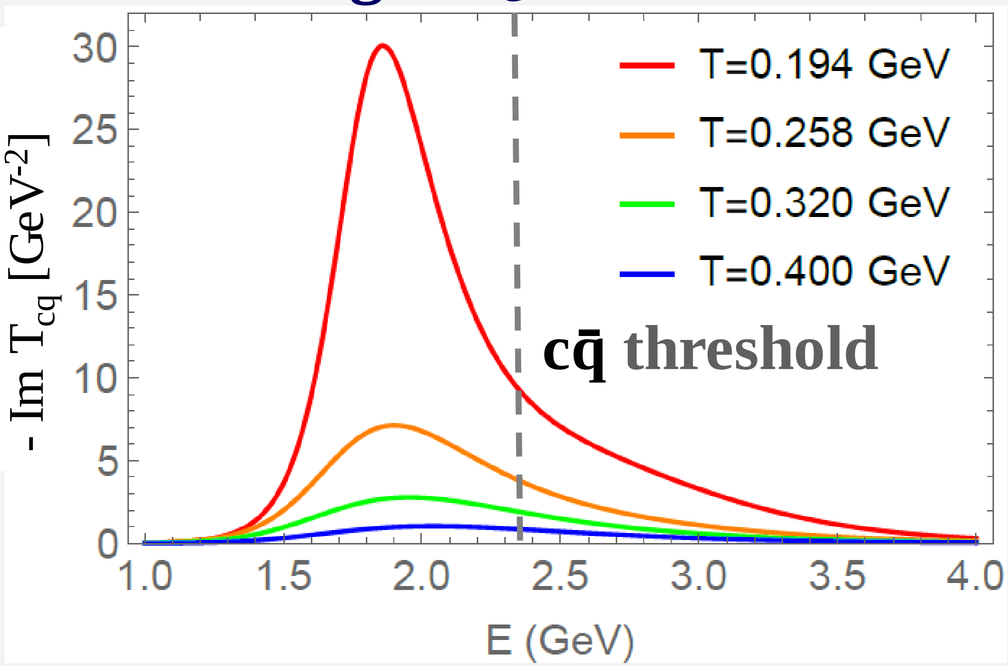
momentum diffusion coefficient

$$D_p = \int d^3q\, w_Q(q,p)\, q^2$$

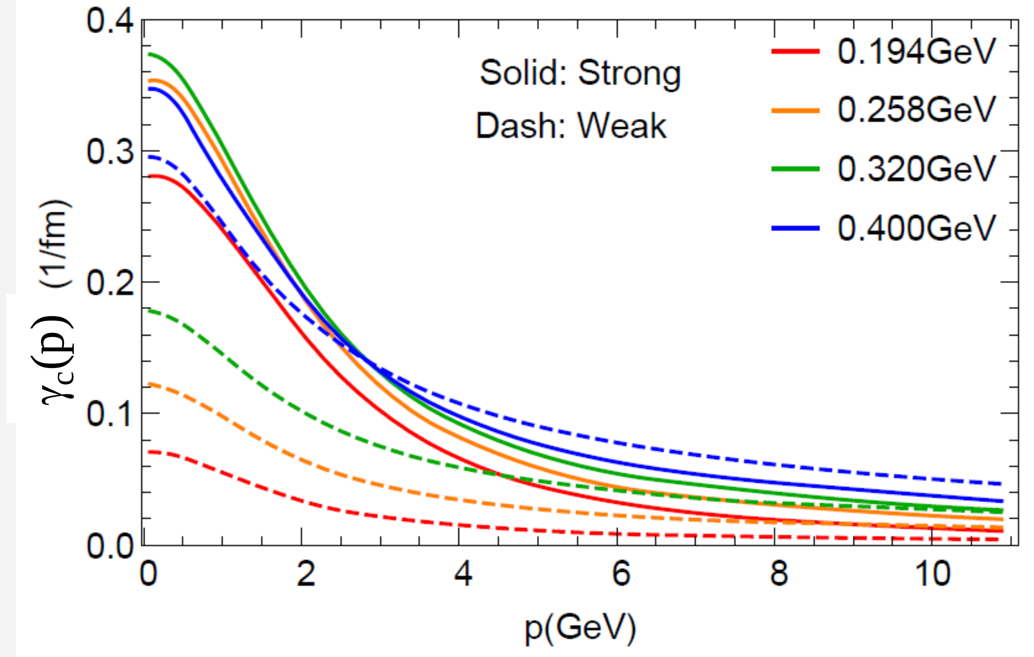
- Thermal relaxation time $\tau = 1/\gamma$
- **Spatial diffusion constant**: $\boxed{\mathcal{D}_s = T / \gamma(p=0)\, m_Q}$
- Key ingredient: **heavy-light scattering amplitude** T_{Qi}

4.3 Charm-Quark Scattering in QGP

Charm-Light Quark T-Matrix



Thermal Relaxation Rate

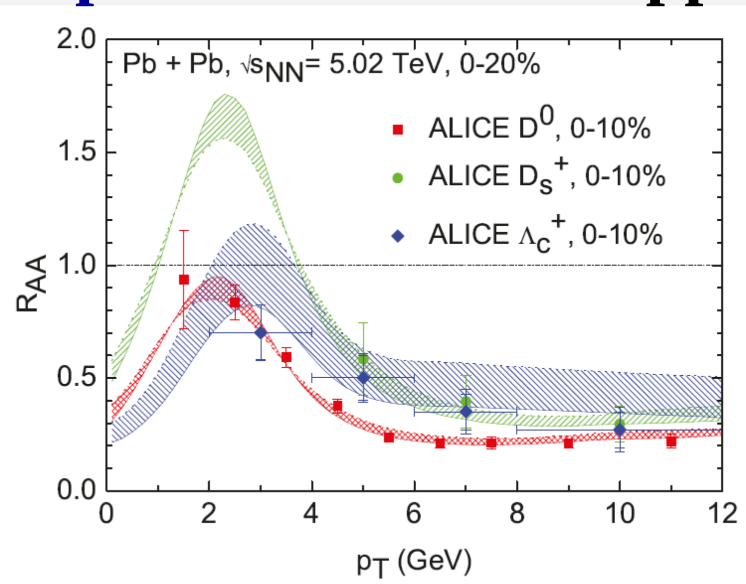


- Broad **D**-mesons emerge near T_c → connects **diffusion** + **hadronization**!
- $\tau_c = 1/\gamma_c \approx 3$ fm/c , $\mathcal{D}_s\,(2\pi T) \approx 2$
- **Off-shell (quantum) effects critical!**

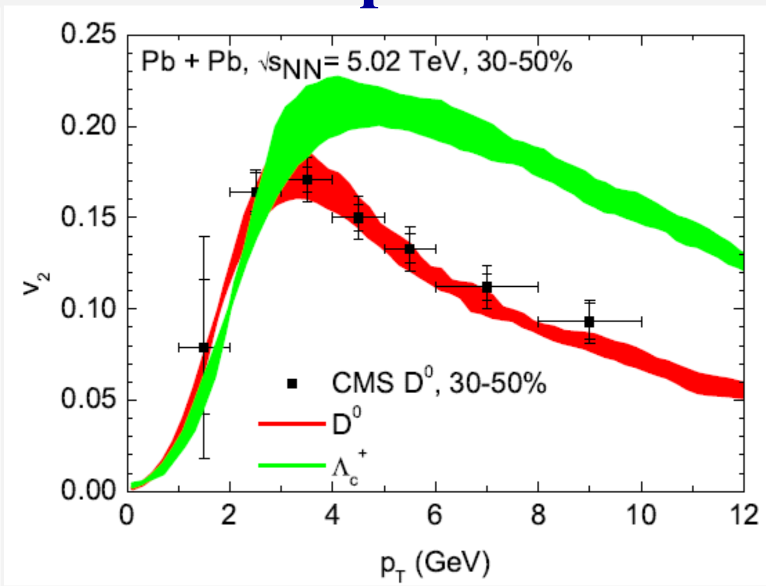
[Liu,He+RR '19]

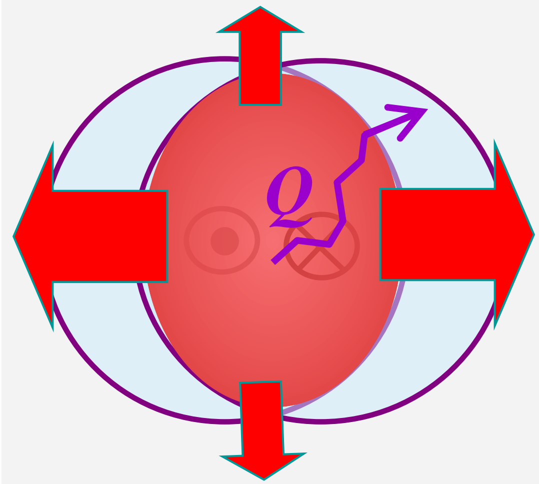
4.4 Charm-Hadron Spectra in Heavy-Ion Collisions

Spectra Ratio PbPb/pp



``Elliptic" Flow

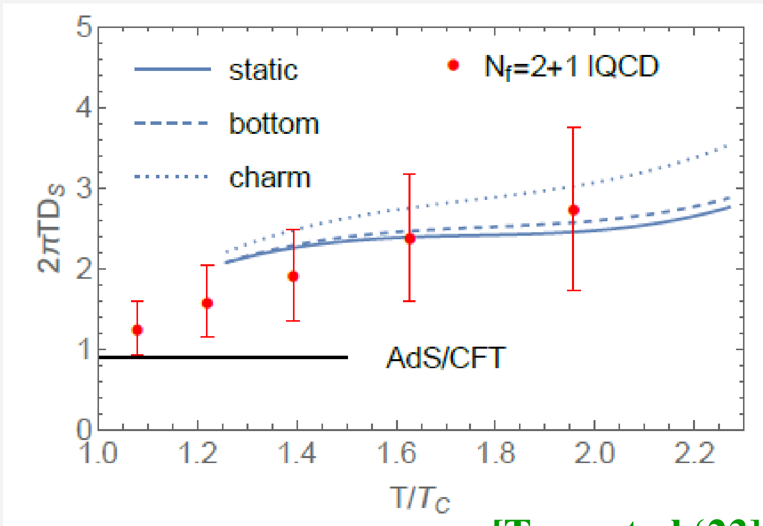




- Charm quarks dragged along with fireball

  ⇒ "**flow bump**" in $\mathbf{R_{AA}}$ + **large** $\mathbf{v_2}$

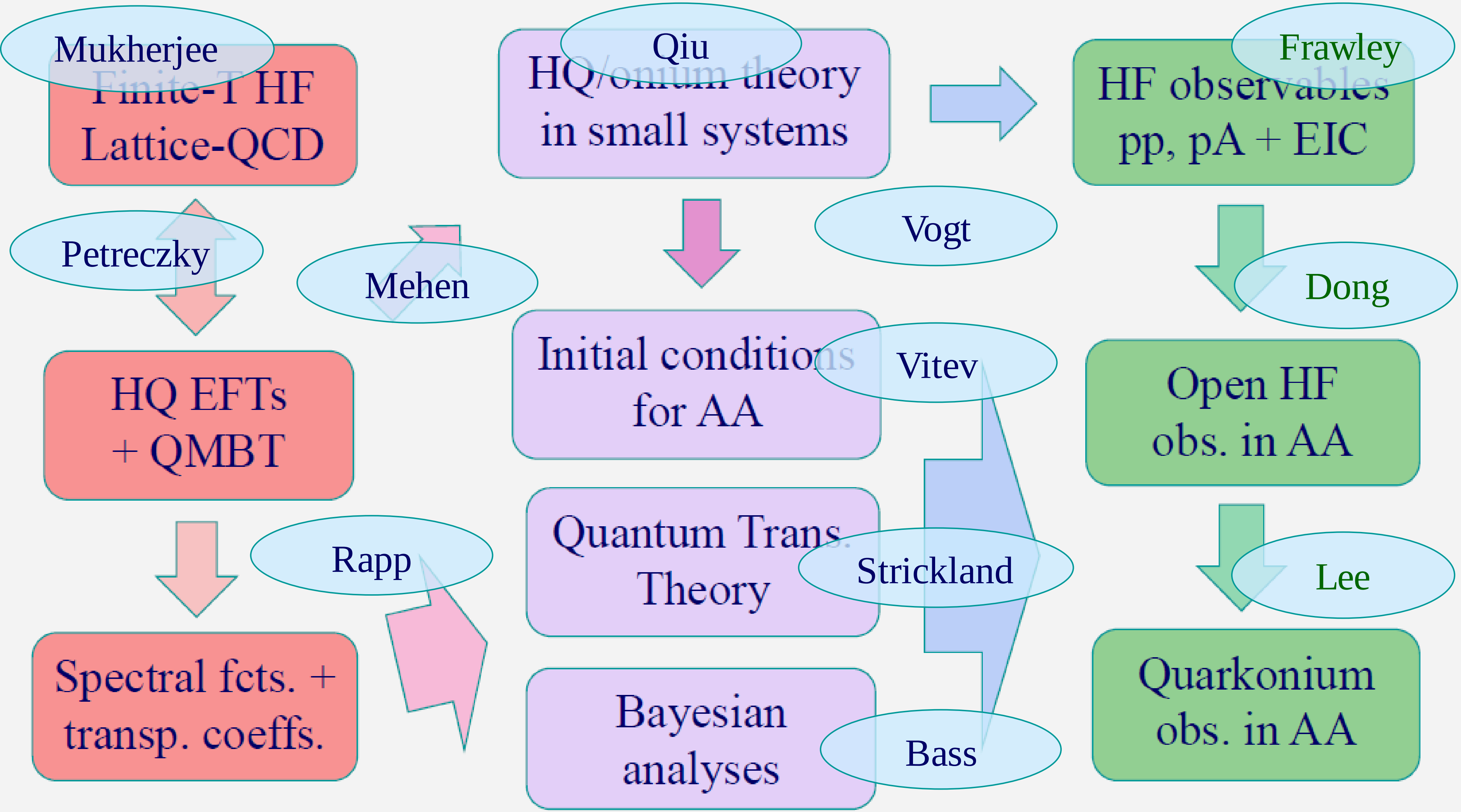
  ⇒ **strong coupling** near $\mathbf{T_c}$ **[M.He+RR '19]**



- Diffusion coefficient $\mathcal{D}_s(2\pi T)$ ~ 2 near T_c

[Tang et al '23]

Topical Collaboration in Nuclear Theory: HEavy Flavor TheorY (HEFTY) for QCD Matter



5.) Summary

- Quantum Many-Body Theory of **s**QGP:
  - Accounts for key aspects of **strong coupling**:
    nonperturbative forces, resummation, quantum effects (large widths)
  - Input parameters constrained by lattice QCD, recover vacuum limit
- QGP structure driven by remnants of **long-range confining force**
  - $\rightarrow$ - **melts** long-wavelength quasi-partons
  - forms **hadronic bound states** near $\mathbf{T_c}$
  - generates **liquid properties** (small $\mathcal{D}_s$, η/s, …)
  - **quantum effects** ubiquitous
- Heavy quarks serve as **Brownian Markers** of **s**QGP in HICs
- Self-consistency essential for quantum bounds!?