Hydrodynamic simulations of superdense fluids

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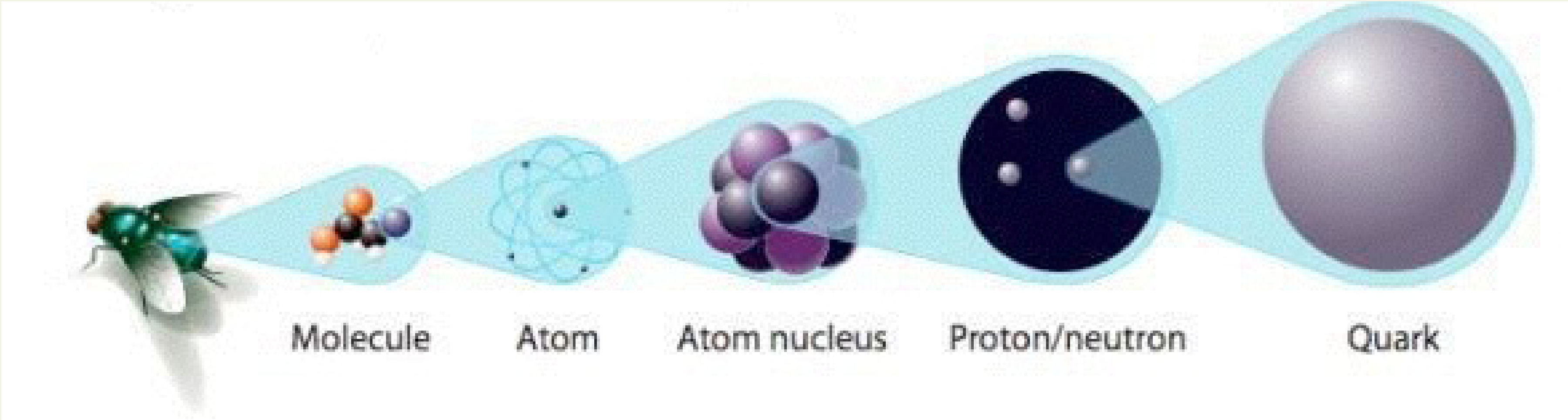
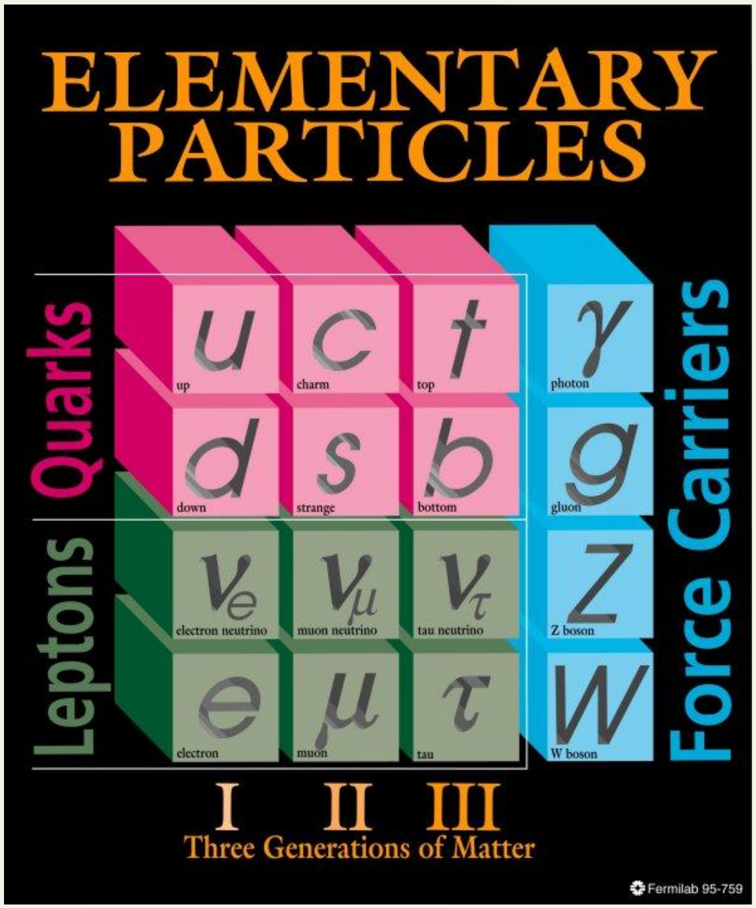
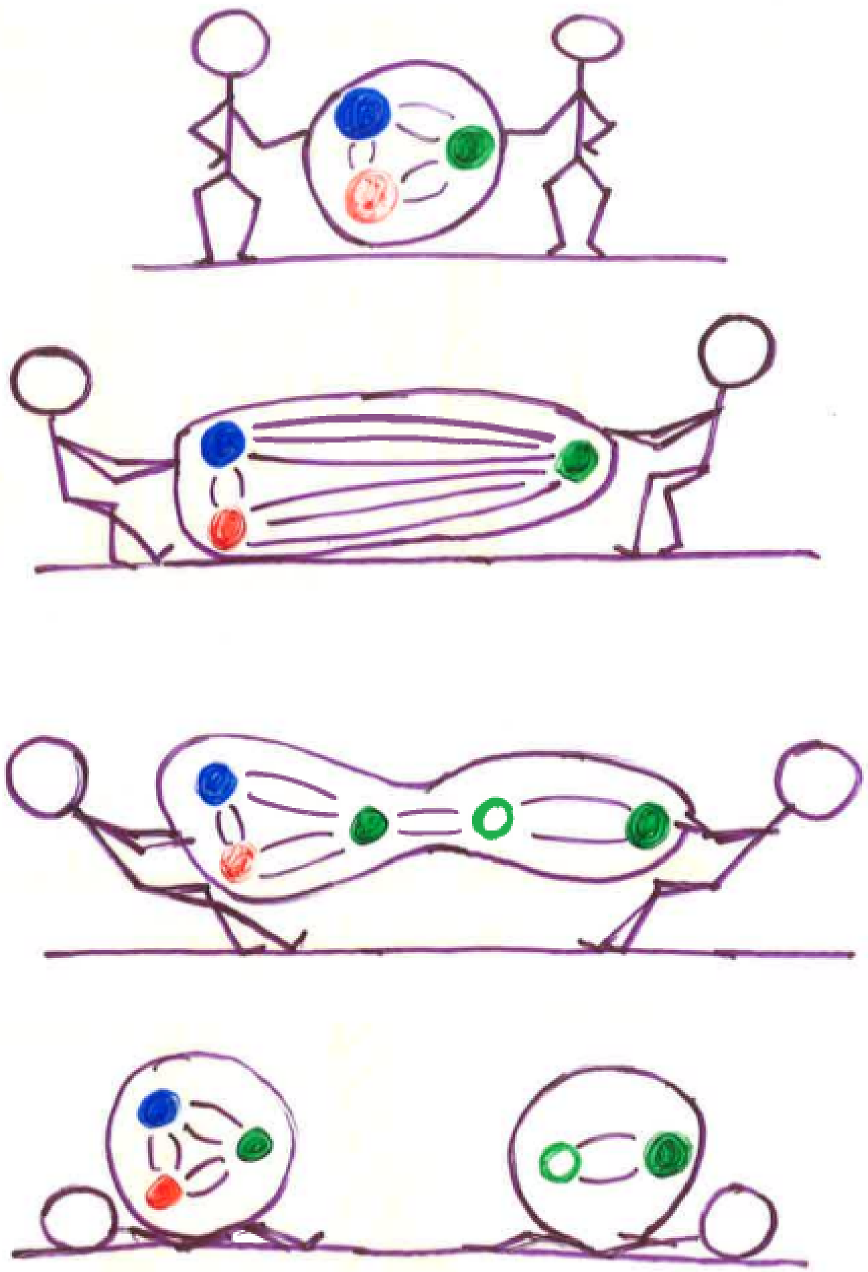
Many particle systems under extreme conditions

December 4, 2023, Görlitz, Germany

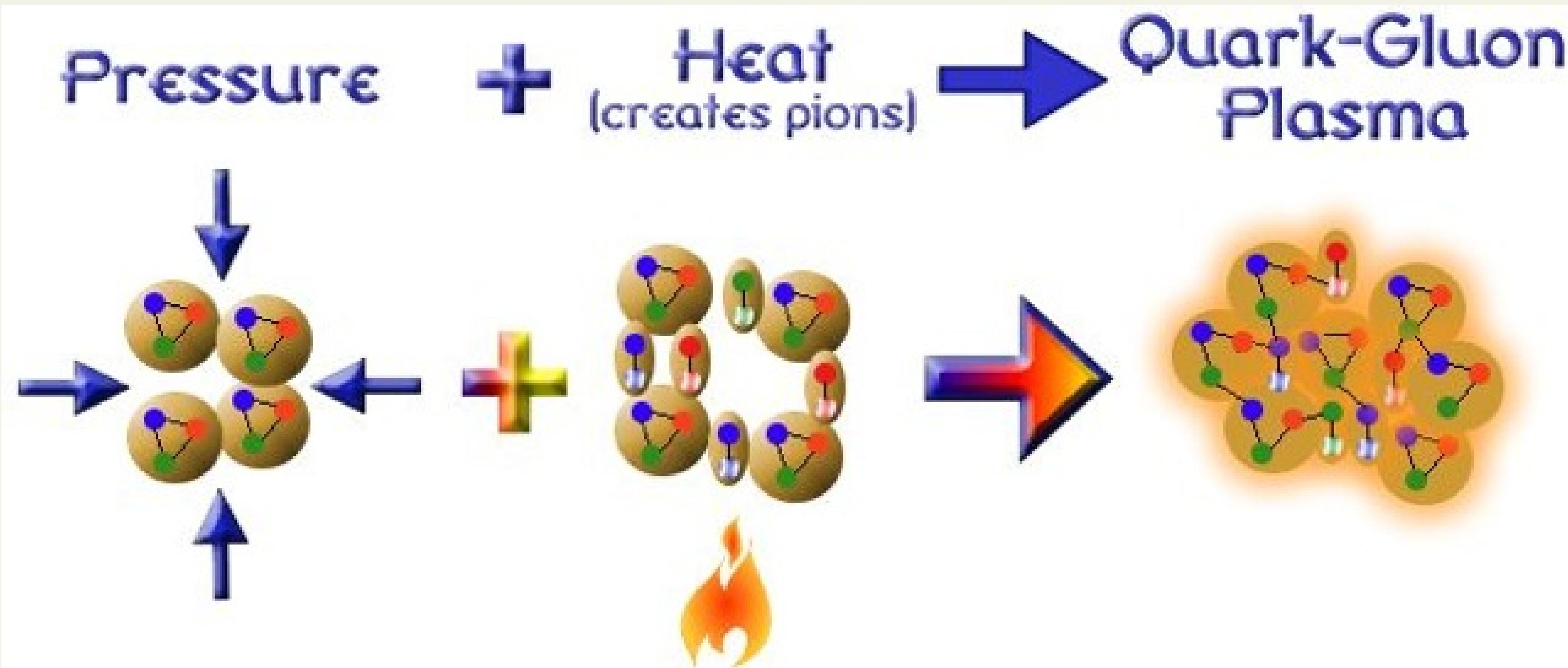




- **12 building blocks of matter (+antiparticles)**
- **observed matter consists only of three of these (u, d, e)**

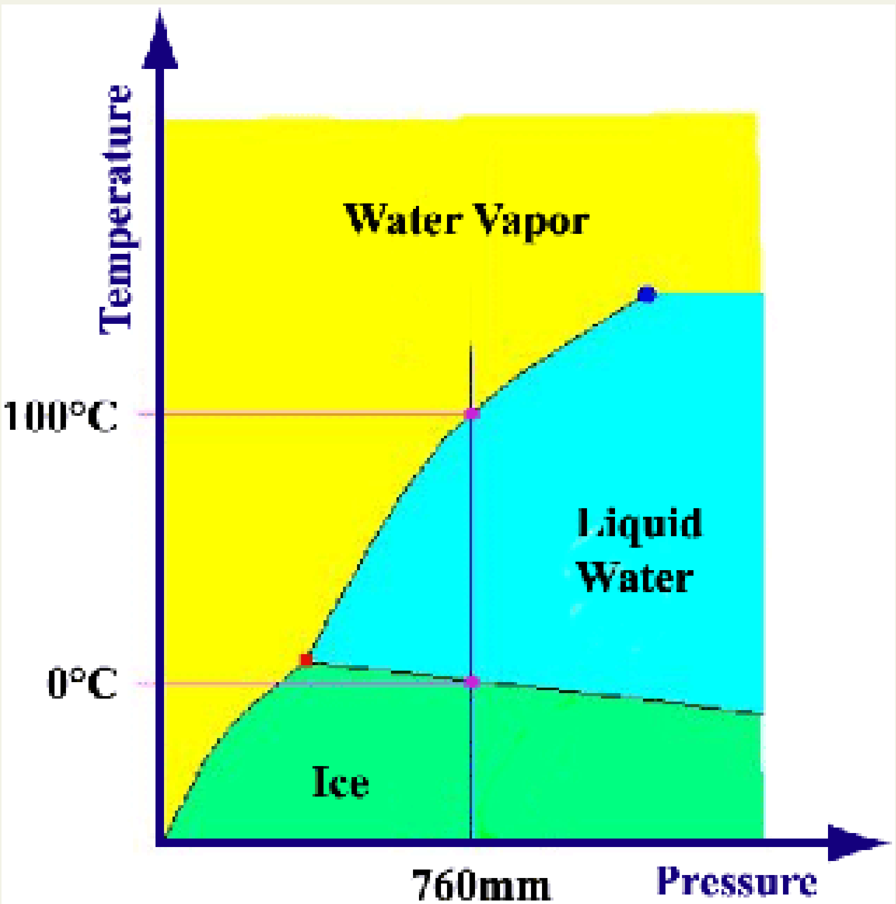


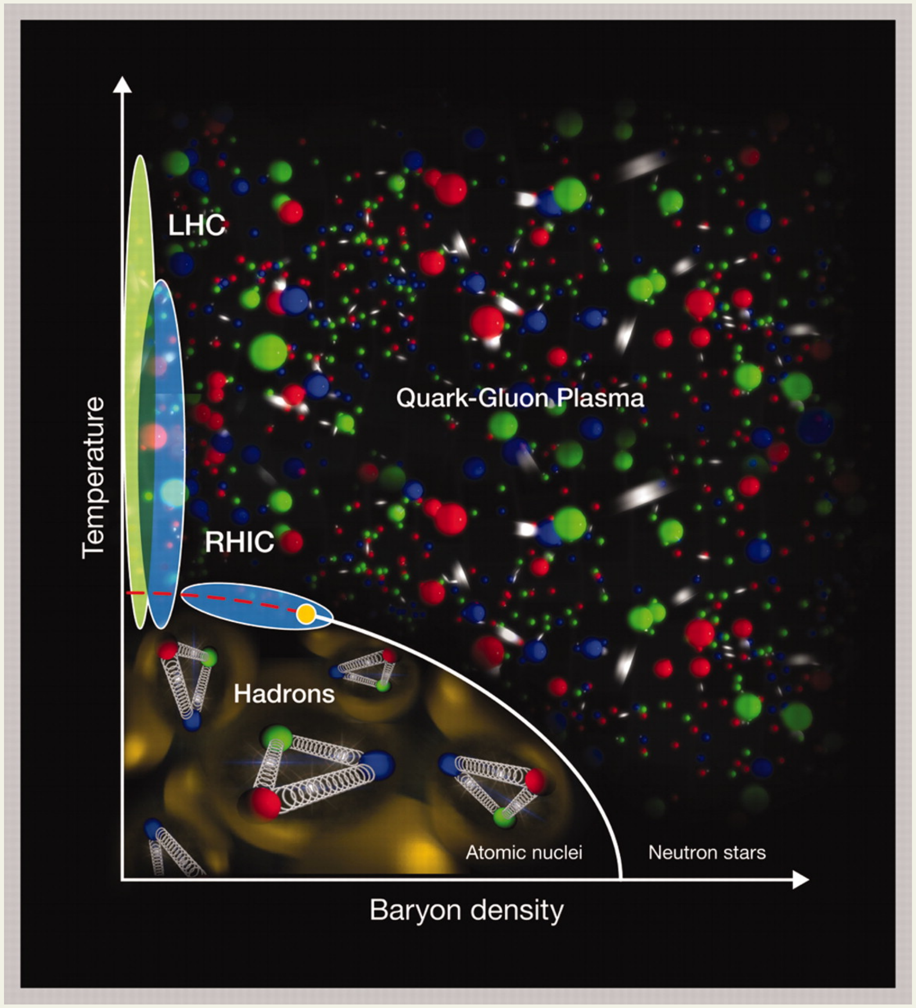
What happens when you compress nuclear matter to very high temperatures and densities?



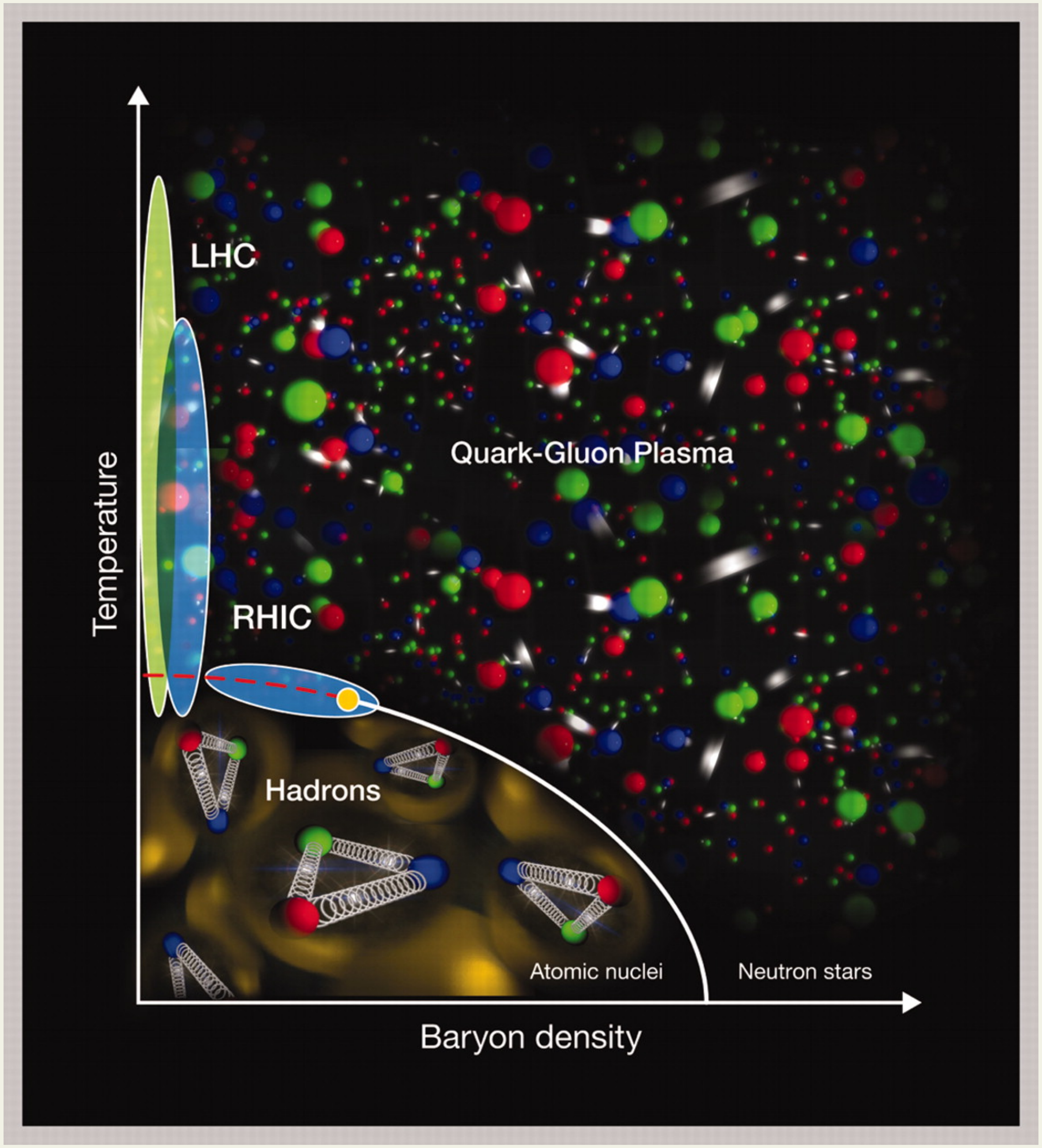
- **phase transition to quark-gluon plasma, QGP**

Phase diagram

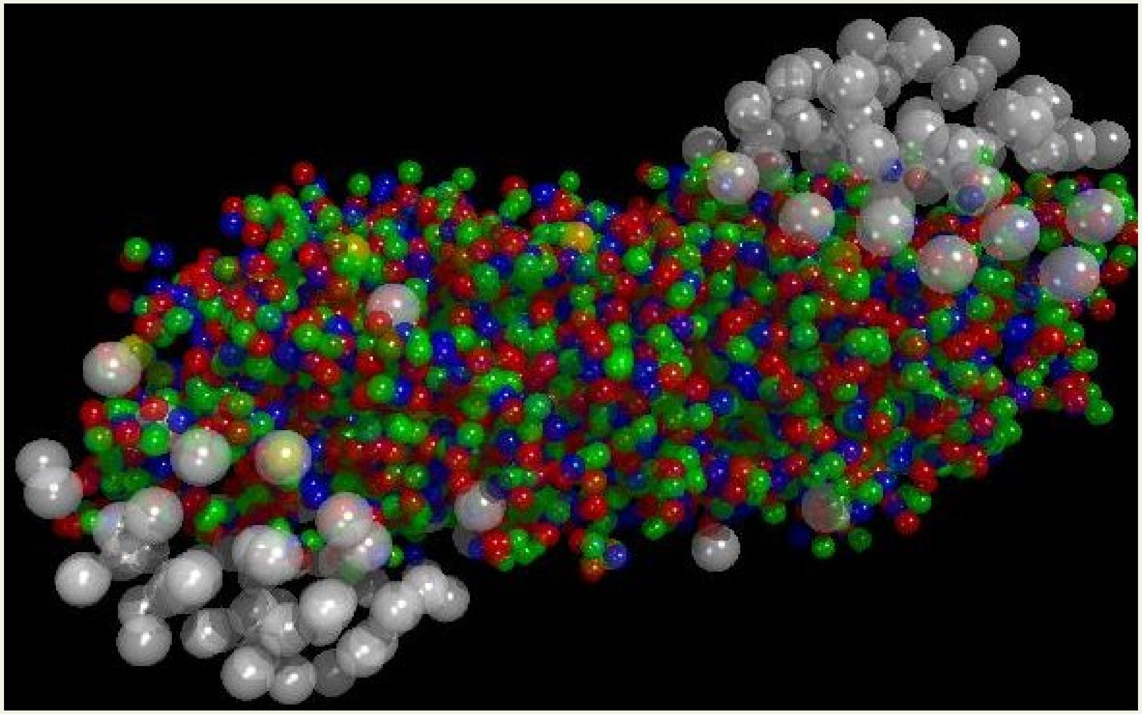




Phase diagram



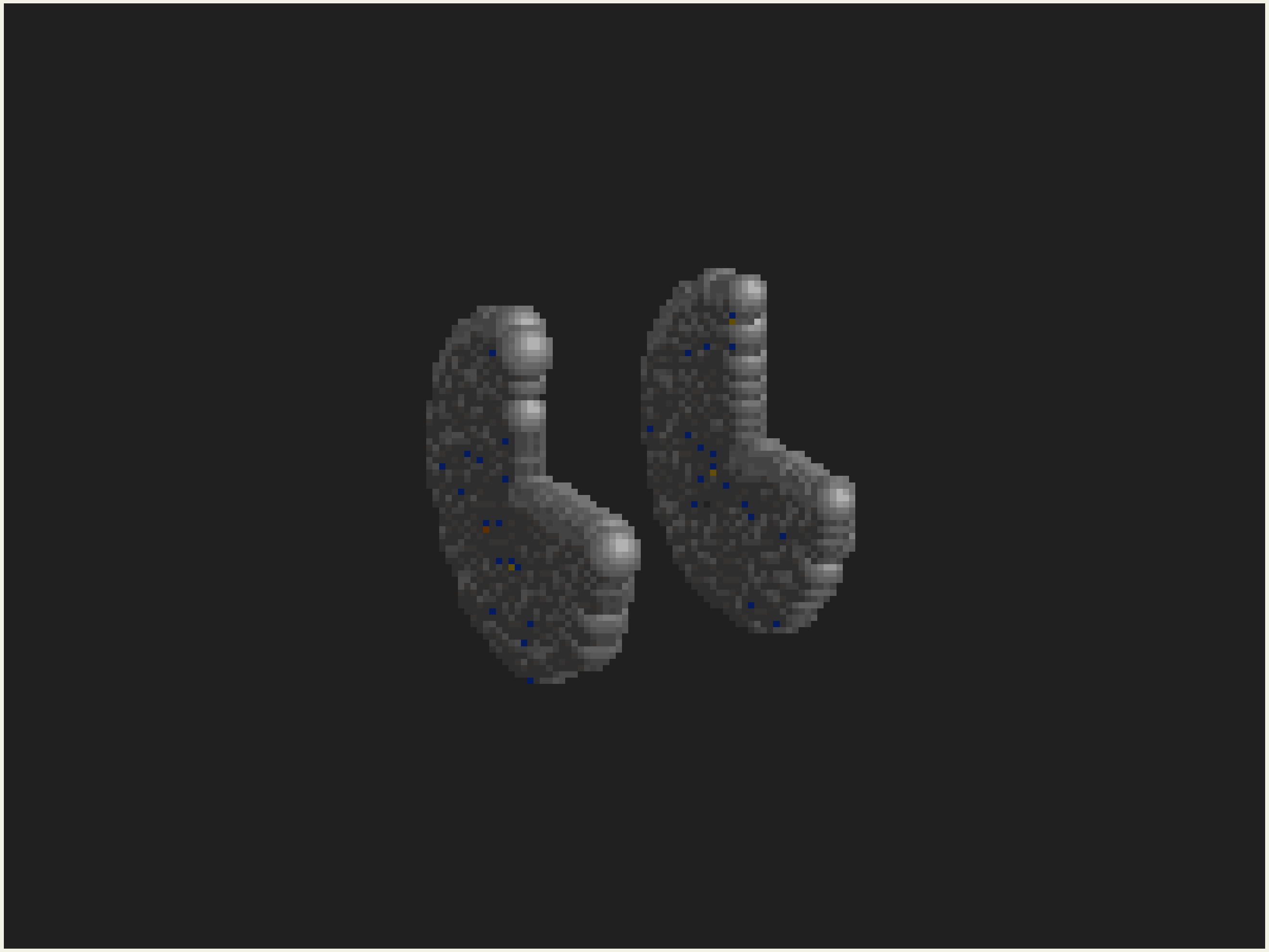
Collisions of heavy nuclei: Little Bang



- **Brookhaven National Laboratory (BNL): RHIC,**
  $Au + Au$ **at** $\sqrt{s_{NN}} = 200$ **GeV**

- **CERN LHC,**
  $Pb + Pb$ **at** $\sqrt{s_{NN}} = 2.76$ **and** 5.02 **TeV**

Heavy-ion collision



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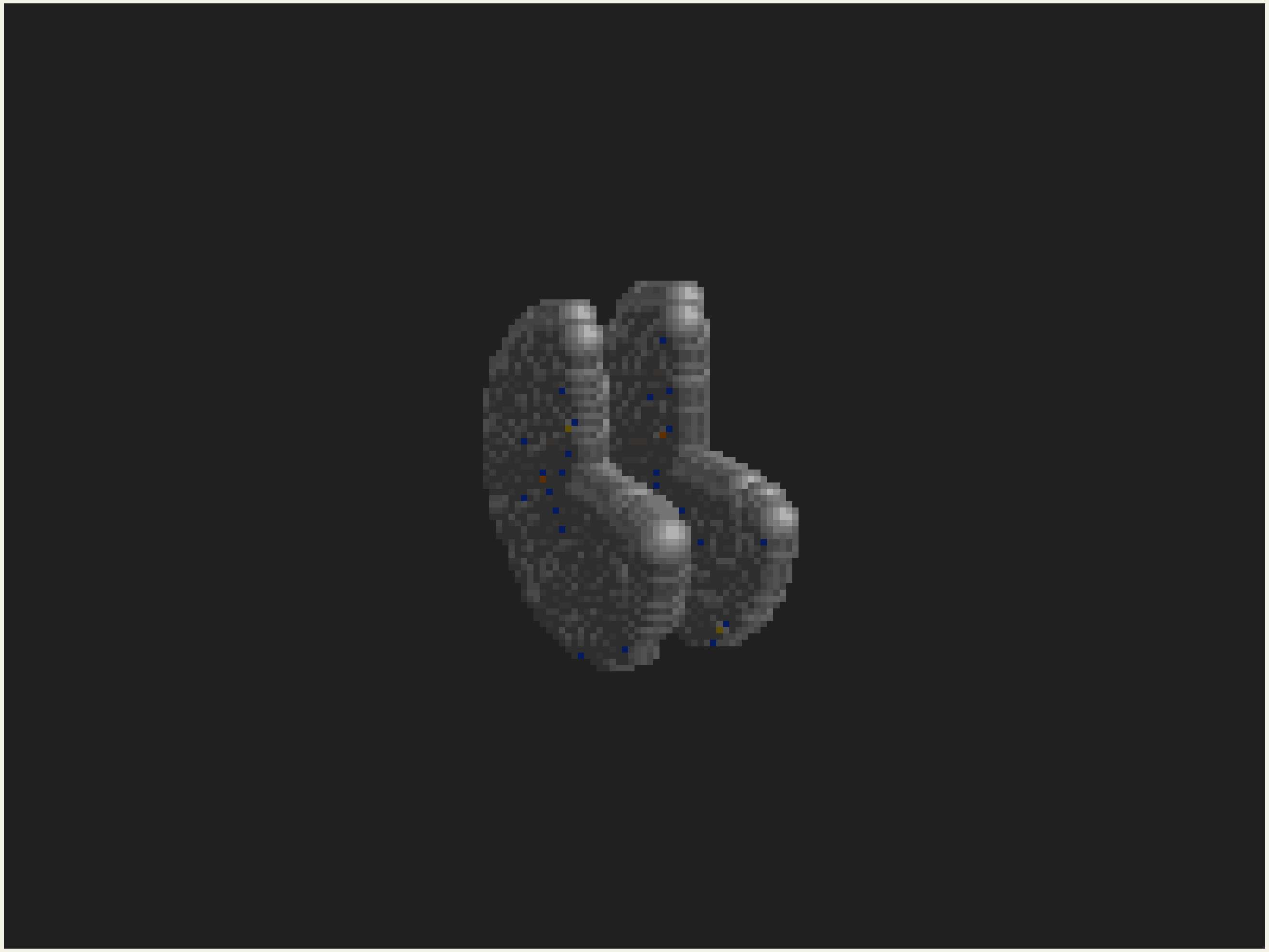
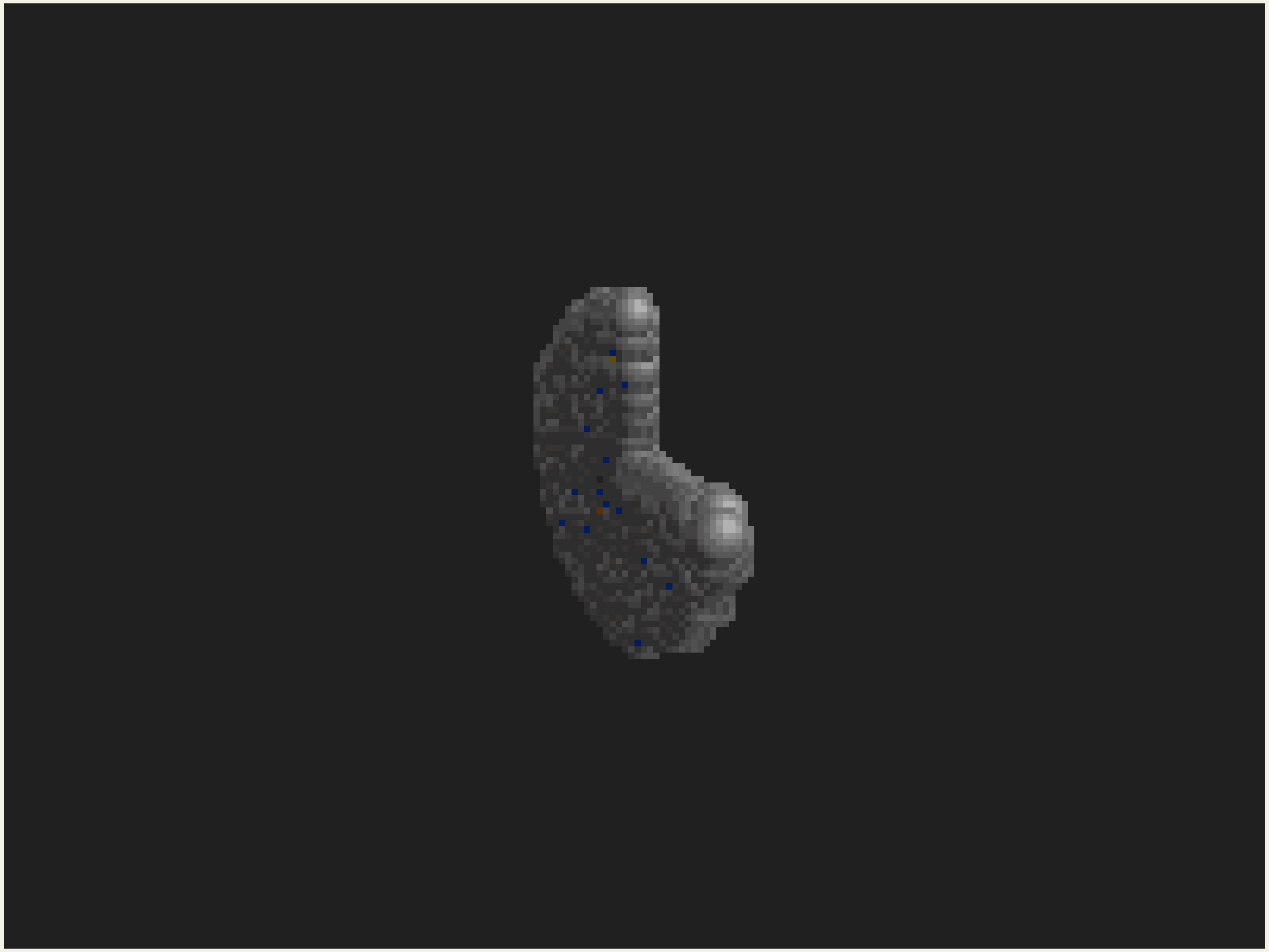
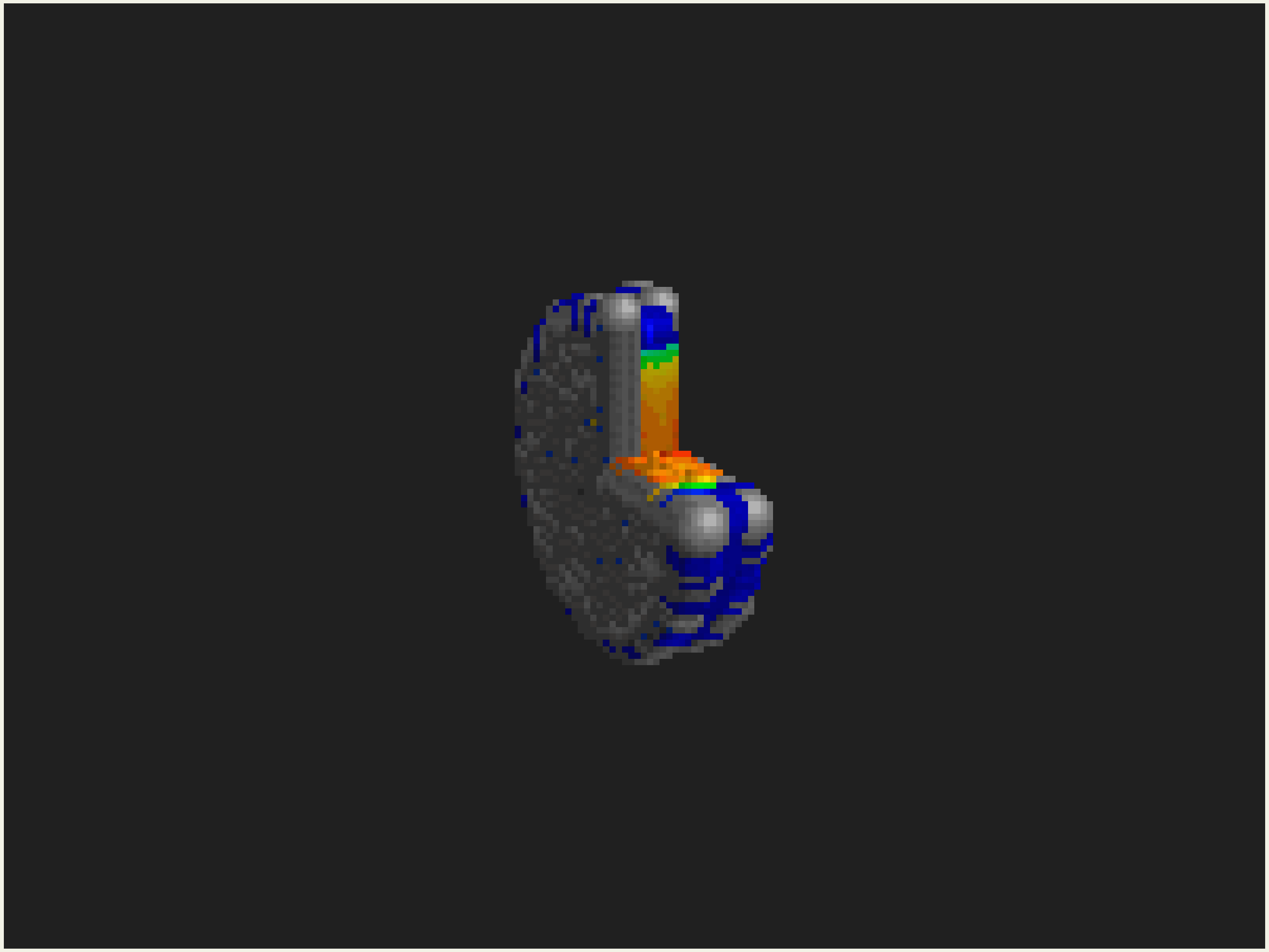
Heavy-ion collision



Heavy-ion collision



Heavy-ion collision



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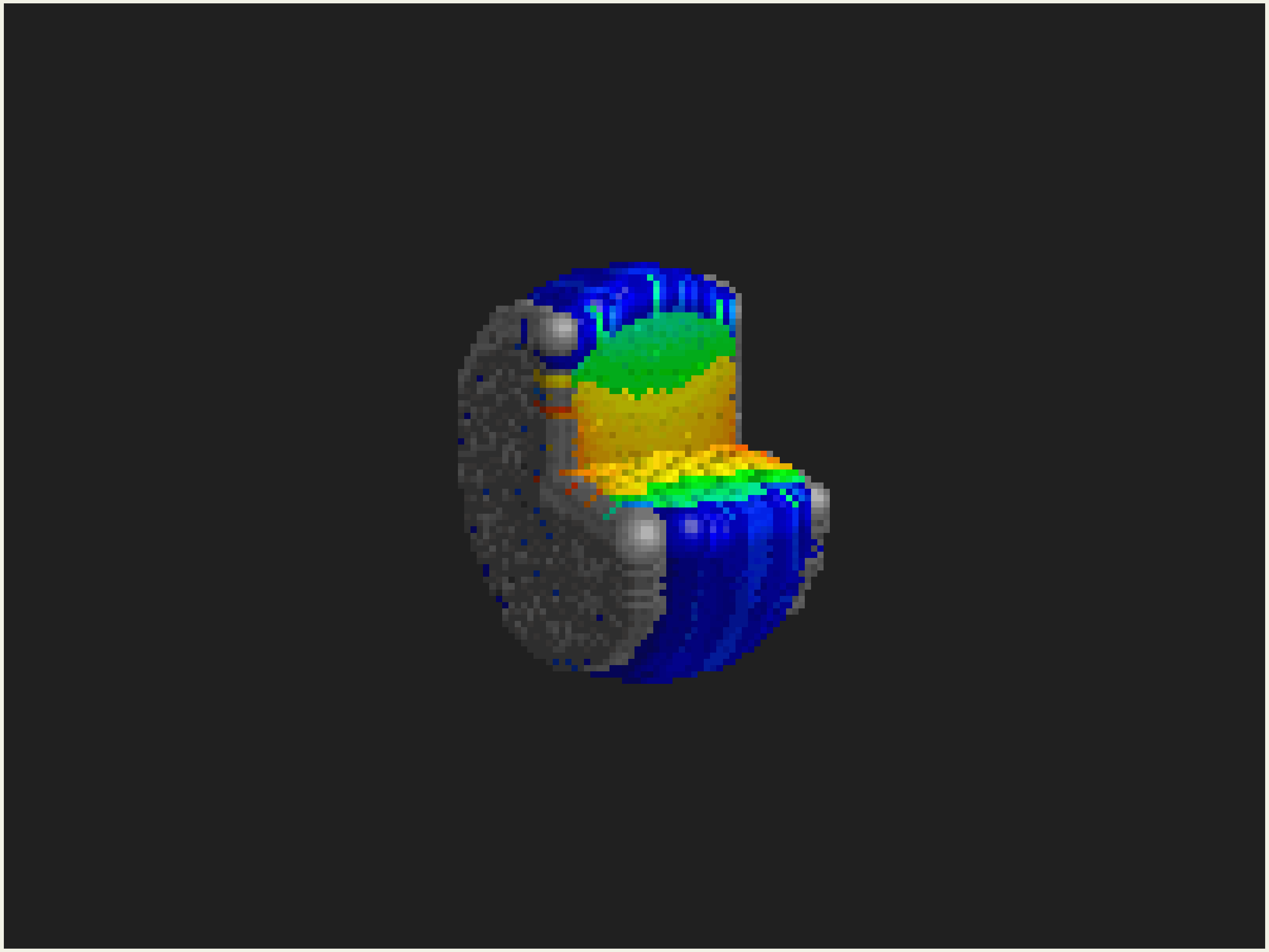
Heavy-ion collision



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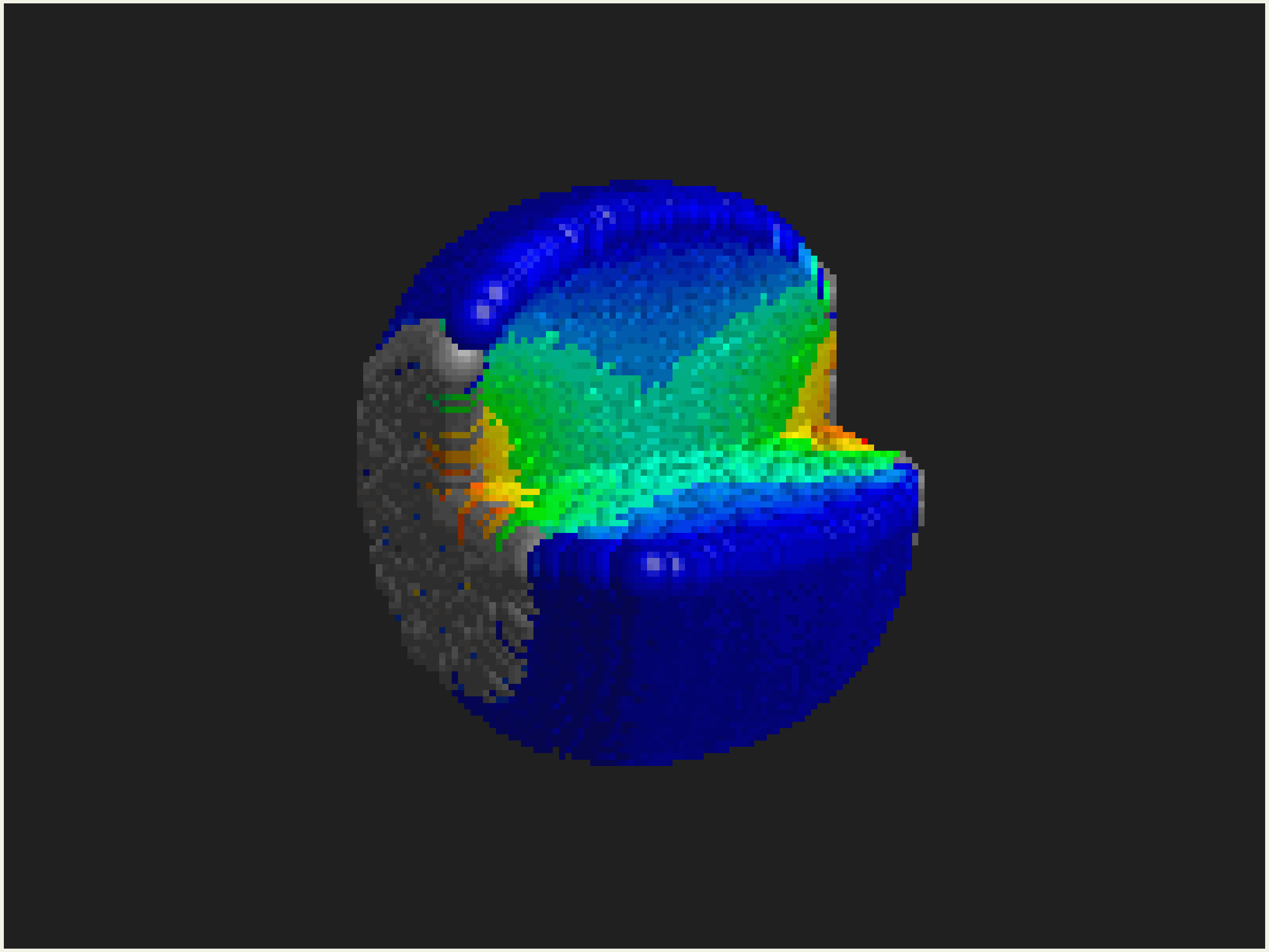
Heavy-ion collision



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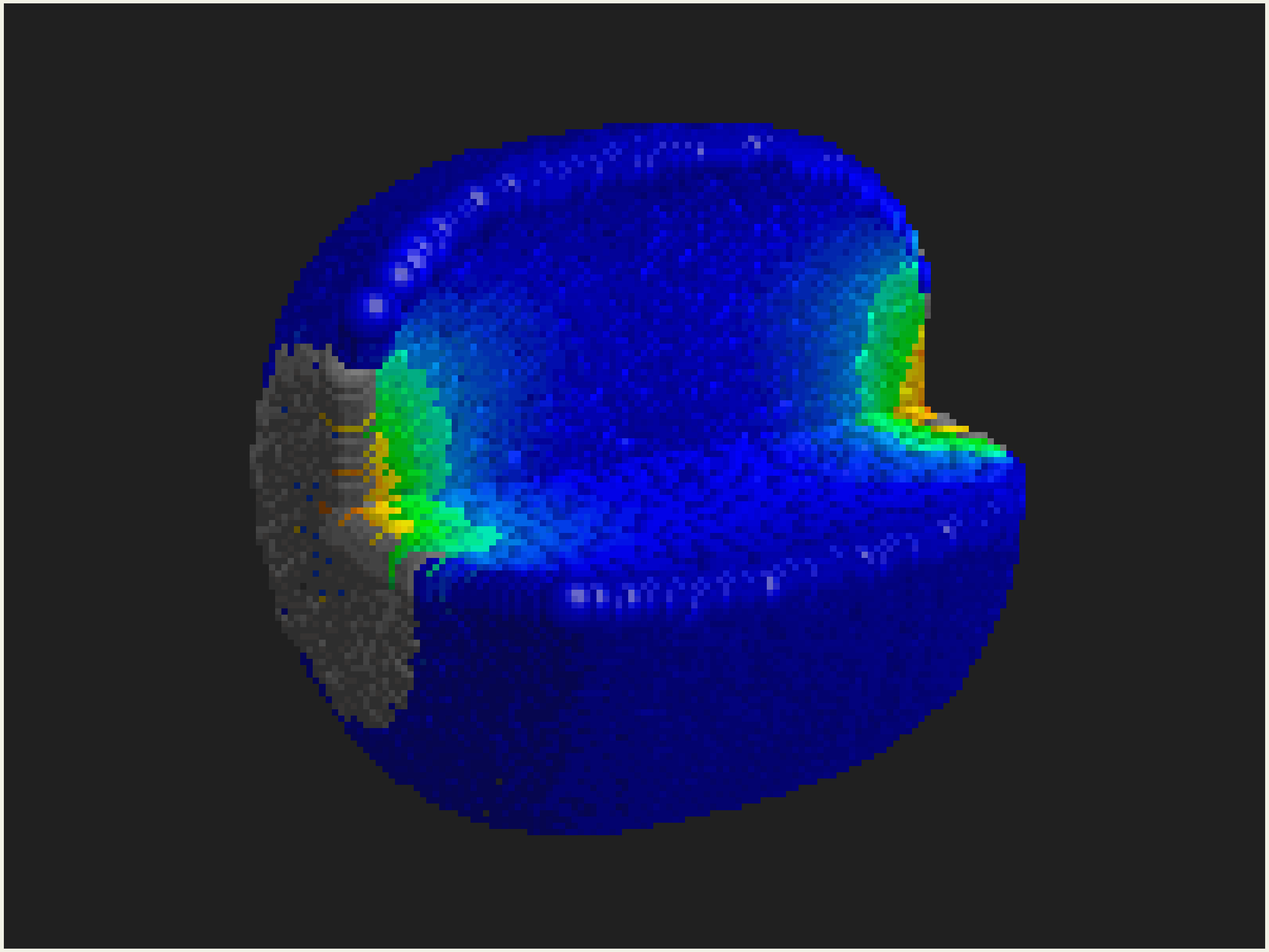
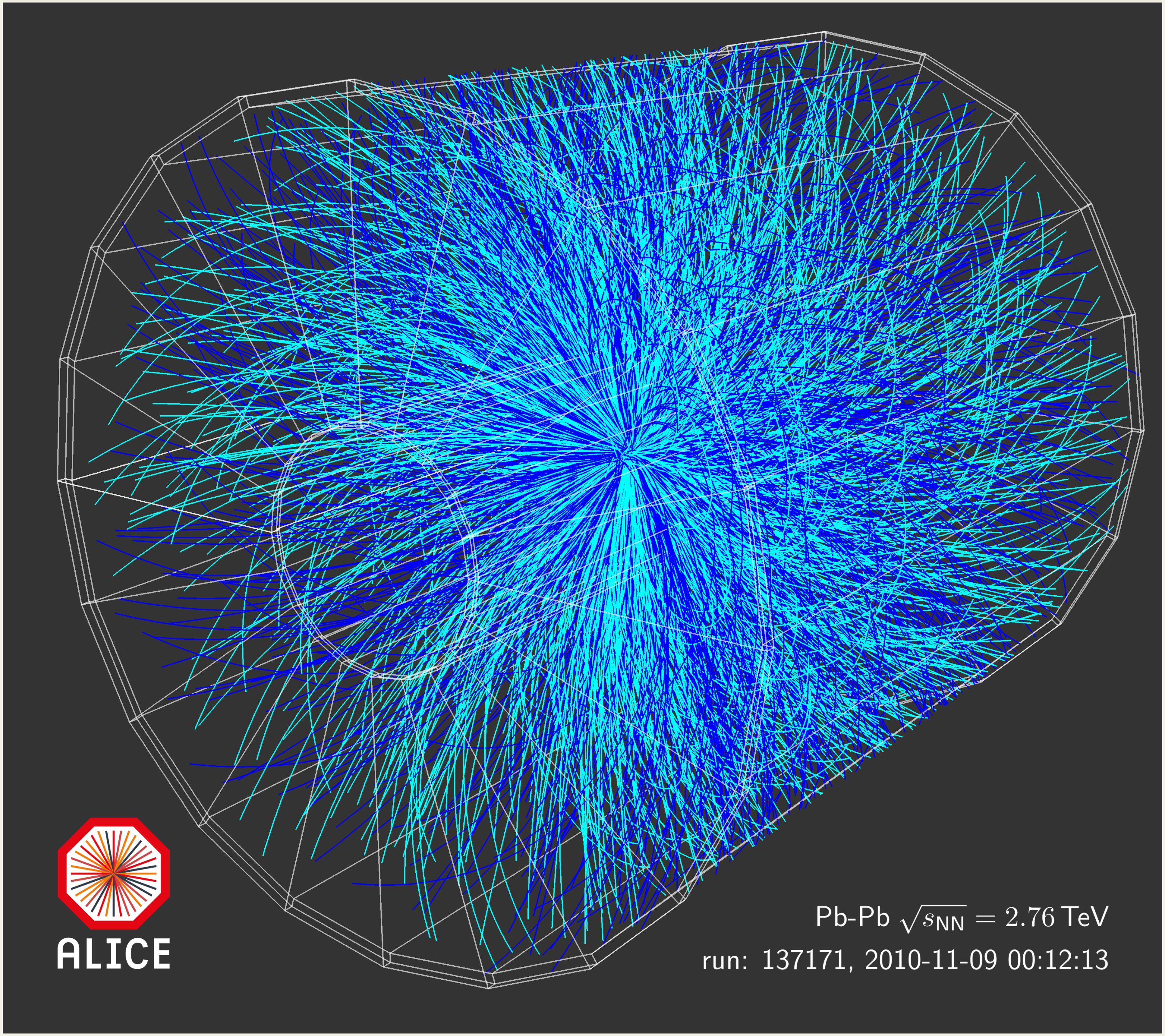
Heavy-ion collision




ALICE
Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV
run: 137171, 2010-11-09 00:12:13

Fluid dynamics

local conservation of energy, momentum and baryon number:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \textbf{and} \quad \partial_\mu N^\mu(x) = 0$$

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n u^\mu + \nu^\mu$$

local, macroscopic variables: **energy density** $\epsilon(x)$
pressure $P(x)$
flow velocity $u^\mu(x)$

matter characterized by: **equation of state** $P = P(T, \{\mu_i\})$
transport coefficients $\eta = \eta(T, \{\mu_i\})$
$\zeta = \zeta(T, \{\mu_i\})$
$\kappa = \kappa(T, \{\mu_i\})$

Evolution equations

- **four coupled differential equations, $\mu \in \{0,1,2,3\}$:**

$$\frac{\partial}{\partial t}T^{0\mu} = -\frac{\partial}{\partial x}T^{1\mu} - \frac{\partial}{\partial y}T^{2\mu} - \frac{\partial}{\partial z}T^{3\mu}$$

which can be cast in the form

$$\frac{\partial}{\partial t}\rho_0 + \nabla \cdot (\vec{v}\rho) = X$$

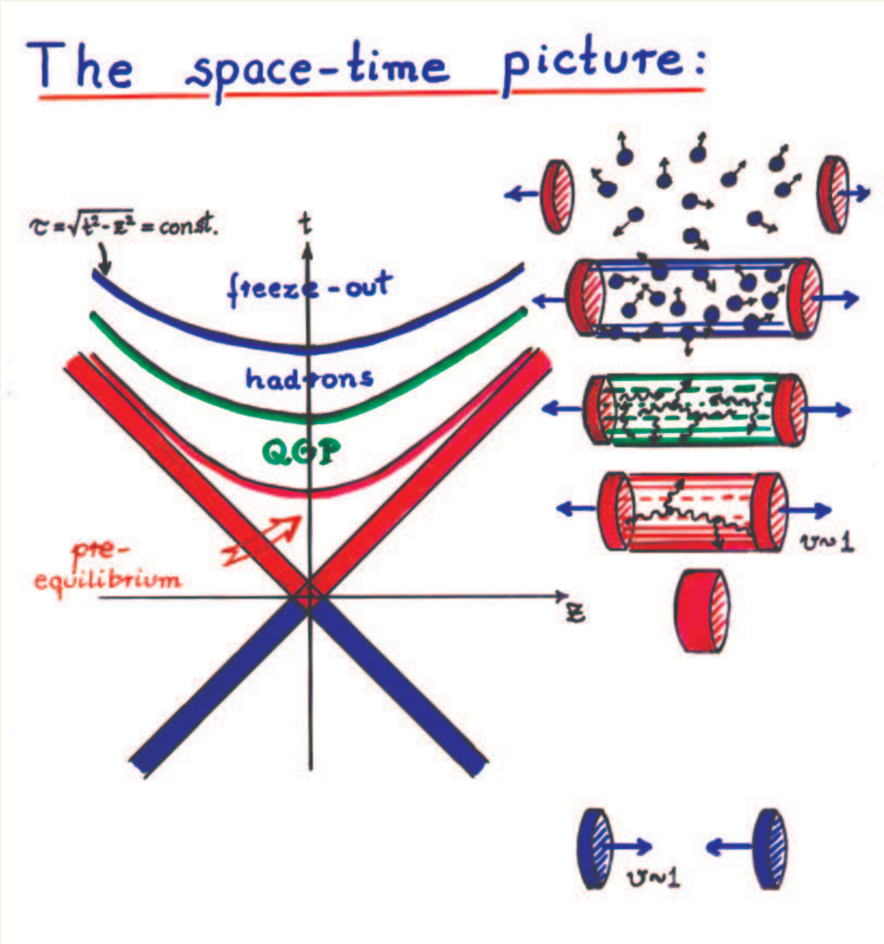
- **must be solved numerically**
- **can be solved using conventional algorithms of fluid dynamics**

usually

- **fixed grid**
- **finite volume with flux correction**

Boundary conditions

- spatial: expansion to vacuum ⇒ vacuum "far away"
- temporal, early: <u>after</u> primary collisions!
- temporal, late: once the system has cooled



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Dissipative relativistic fluid dynamics

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

$\pi^{\mu\nu}$: shear-stress tensor, Π: bulk pressure

relativistic Navier-Stokes: $\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$

- **parabolic equations of motion!**
- **unstable and acausal**

Dissipative relativistic fluid dynamics

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

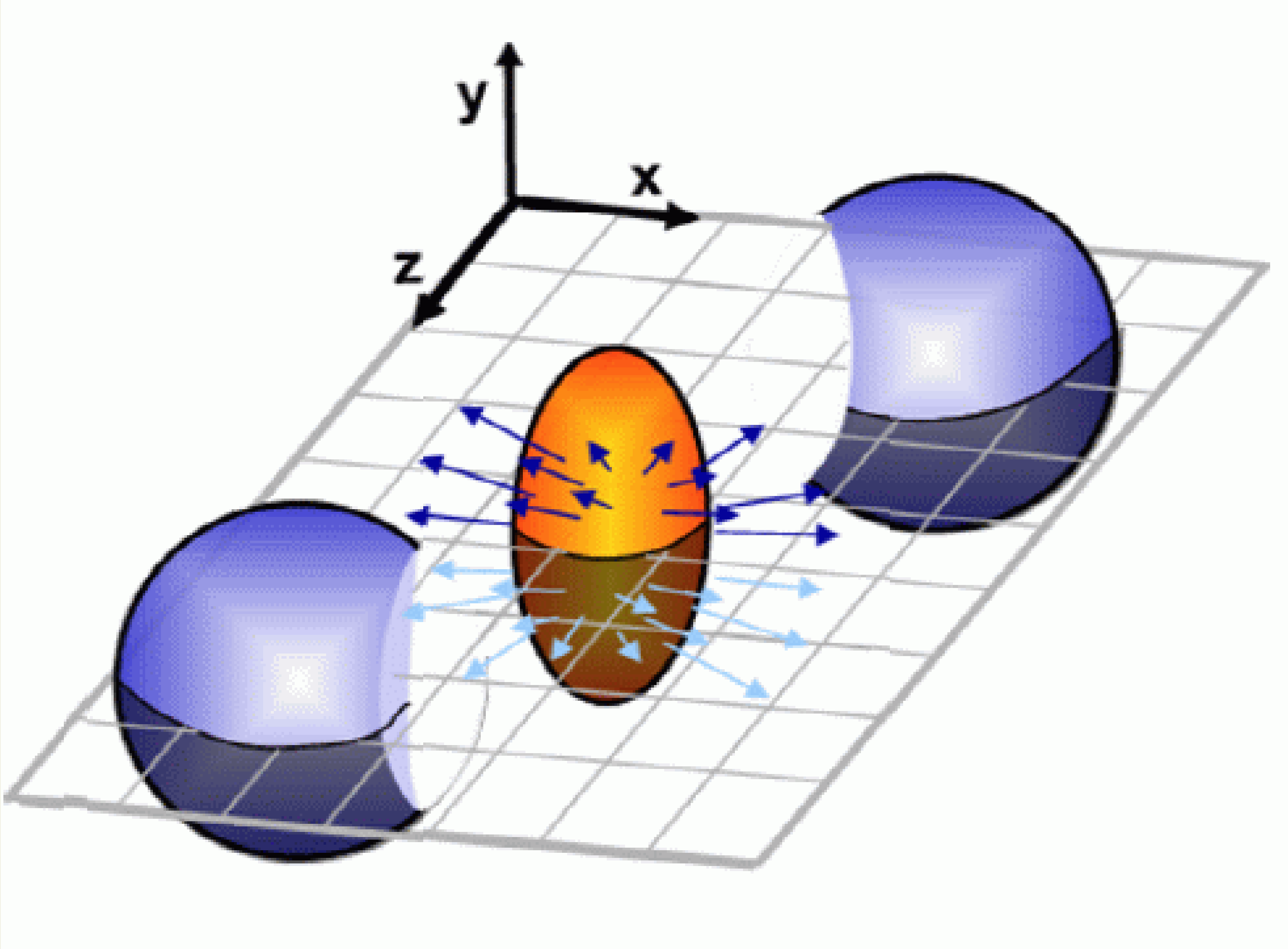
$\pi^{\mu\nu}$: shear-stress tensor, Π: bulk pressure

Israel-Stewart: $\pi^{\mu\nu}$ independent dynamical variable

$$\langle u^\lambda \partial_\lambda \pi^{\mu\nu} \rangle = \frac{\pi^{\mu\nu}_{\rm NS} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\lambda u^\lambda + \cdots$$

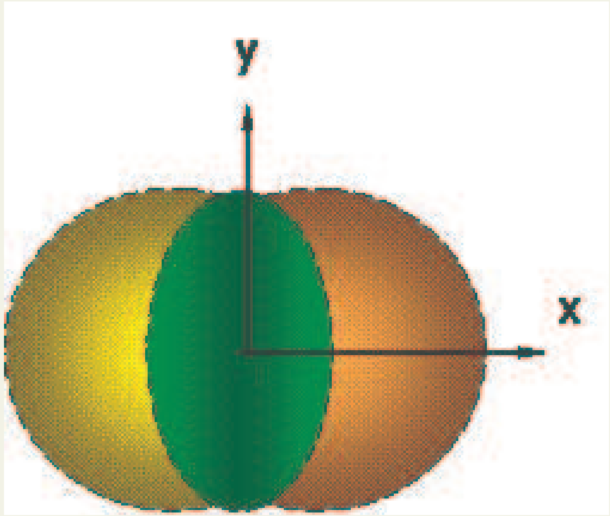
- η: shear viscosity coefficient
- τ_π: shear relaxation time, known for massless particles
- solved numerically
- finite differencing, same grid than for equations of motion

Elliptic flow v_2

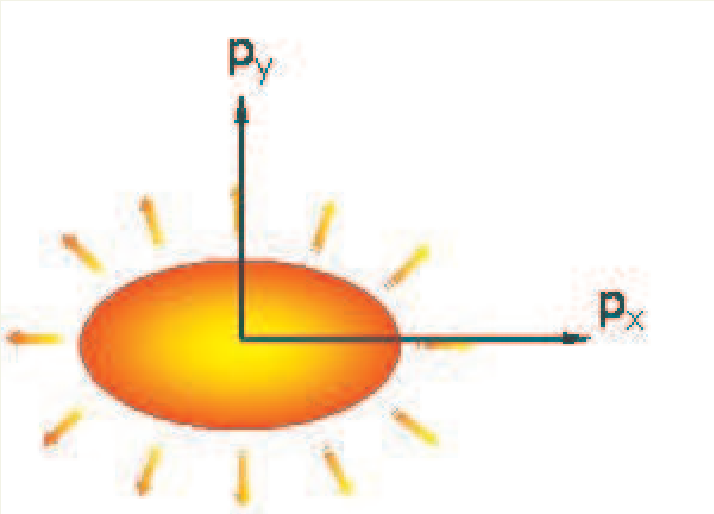


Elliptic flow v_2

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy



$\Rightarrow$



- Anisotropy in coordinate space + rescattering

$\Rightarrow$ Anisotropy in momentum space

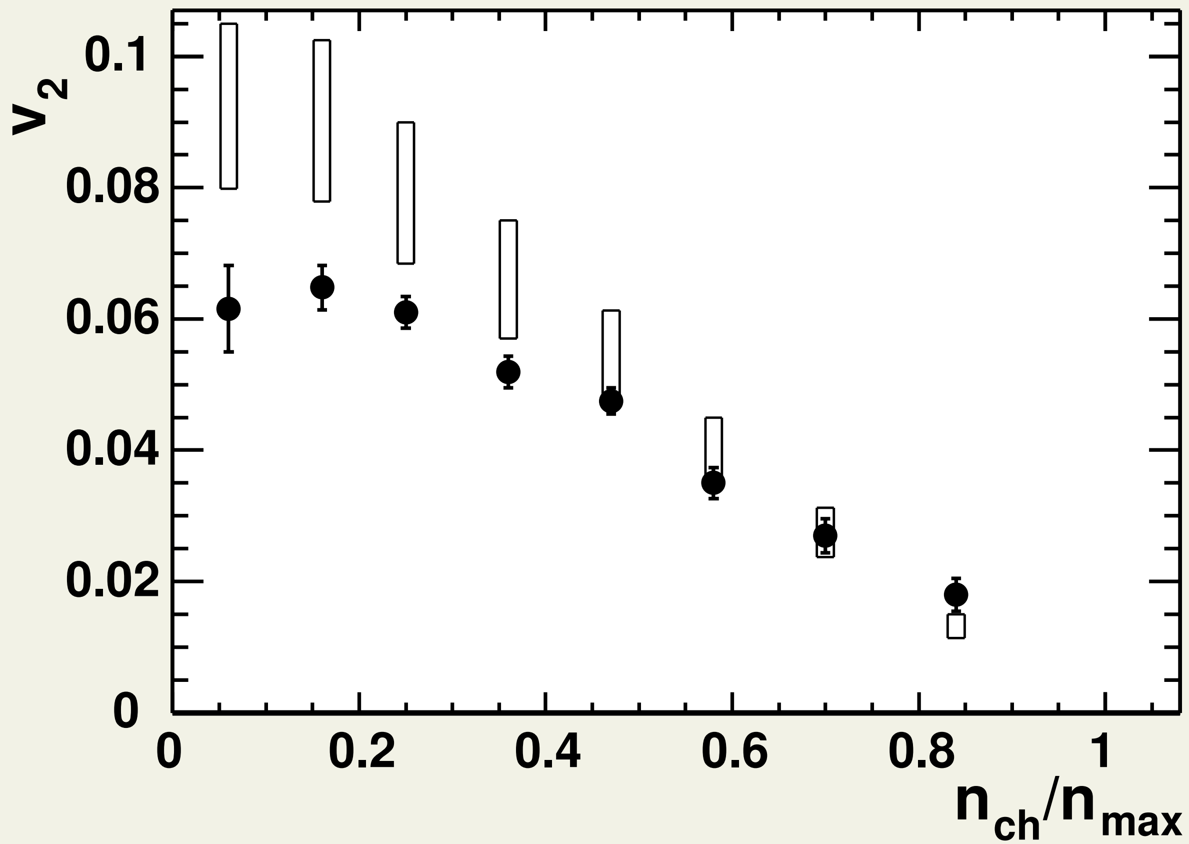
sensitive to speed of sound $c_s^2 = \partial p / \partial e$ and shear viscosity η

- Quantified with Fourier expansion of momentum distribution:

$$\frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T\, \mathrm{d}\phi} = \frac{1}{2\pi} \frac{\mathrm{d}N}{\mathrm{d}y\, p_T \mathrm{d}p_T} (1 + 2v_1(y, p_T) \cos\phi + 2v_2(y, p_T) \cos 2\phi + \cdots)$$

Success of ideal hydrodynamics

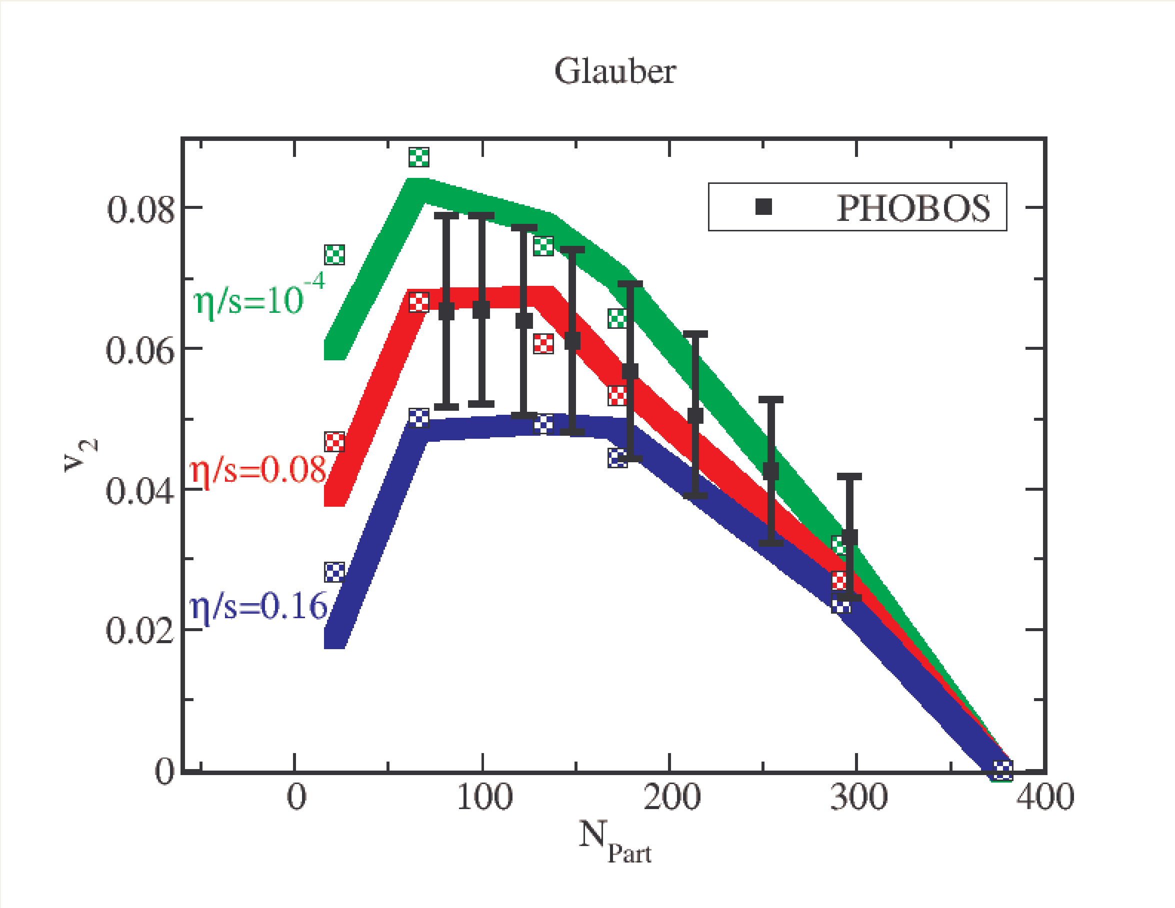
- p_T-averaged v_2 of charged hadrons:
- points: data, boxes: hydrodynamical calculations



- works beautifully in central and semi-central collisions

Dissipative hydrodynamics

- **Luzum & Romatschke, Phys.Rev.C78:034915,2008**



- $\eta/s = 0.08$ **favoured**

Modeling problem

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

$(\tau_0, \epsilon_{\mathrm{init}}, \eta/s, T_{\mathrm{dec}}\ldots)$

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\,\mathrm{exp}}$

$(\mathrm{d}N/\mathrm{d}y, \langle p_T \rangle, v_n, \ldots)$

- **Which values of input parameters $\vec{x}$ give the best reproduction of experimental output $\vec{y}^{\,\mathrm{exp}}$?**
- **What is the level of uncertainty of these values?**

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

(τ_0**,** $\epsilon_{\rm init}$**,** η/s**,** $T_{\rm dec}$**. . .)**

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\,\rm exp}$

($\mathrm{d}N/\mathrm{d}y$**,** $\langle p_T \rangle$**,** v_n**,. . .)**

Bayes' theorem:

Posterior probability $\propto$ **Likelihood** $\cdot$ **Prior knowledge**

Bayesian analysis

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- **Prior knowledge: Range of parameter values**

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

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$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\,\rm exp}$

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Bayes' theorem:

Posterior probability $\propto$ **Likelihood** $\cdot$ **Prior knowledge**

- **Likelihood:** $\mathcal{L}(\vec{x}) \propto \exp\left(-\frac{1}{2}(\vec{y}(\vec{x}) - \vec{y}^{\,\rm exp})\Sigma^{-1}(\vec{y}(\vec{x}) - \vec{y}^{\,\rm exp})^T\right)$,
  where Σ is the covariance matrix

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

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$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\,\rm exp}$

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- evaluation of the likelihood function $\mathcal{O}(10^8)$ runs. . .
- use Gaussian emulator instead
  = stochastic, non-parametric interpolation of the model

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

$(\tau_0,\ \epsilon_{\rm init},\ \eta/s,\ T_{\rm dec}\ldots)$

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ **Experimental values** $\vec{y}^{\,\rm exp}$

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Bayes' theorem:

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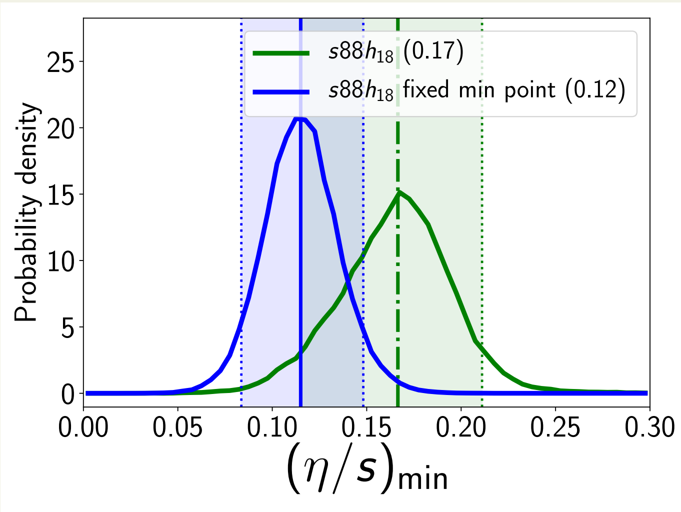
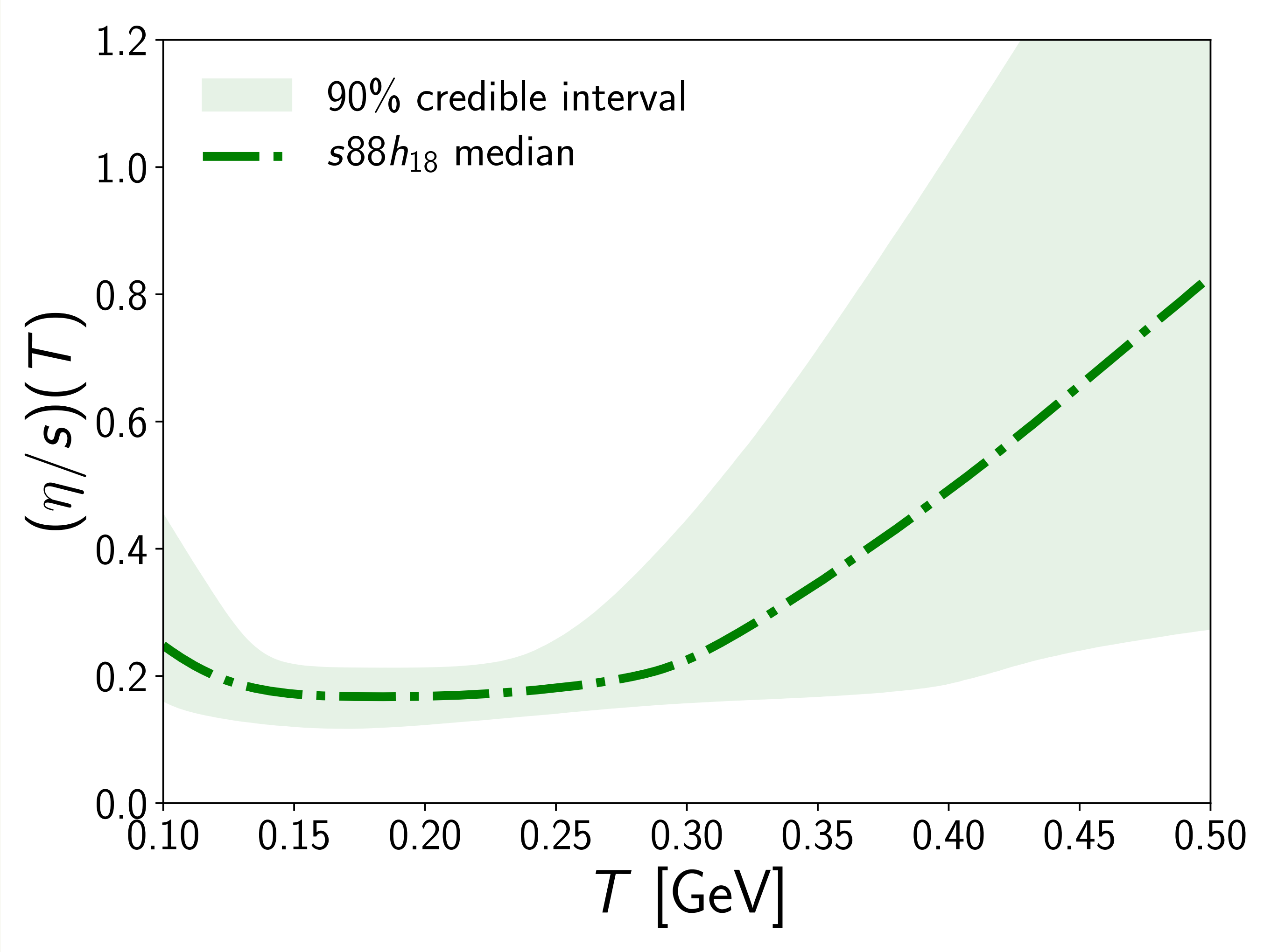
  where Σ is the covariance matrix
- evaluation of the likelihood function $\mathcal{O}(10^8)$ runs. . .
- use Gaussian emulator instead
  = stochastic, non-parametric interpolation of the model
- Sample the likelihood function using Markov chain Monte Carlo
  = random walk in parameter space constrained to favour high likelihood

$\rightarrow$ distribution of Markov chain steps $\equiv$ probability distribution

$(\eta/s)(T)$

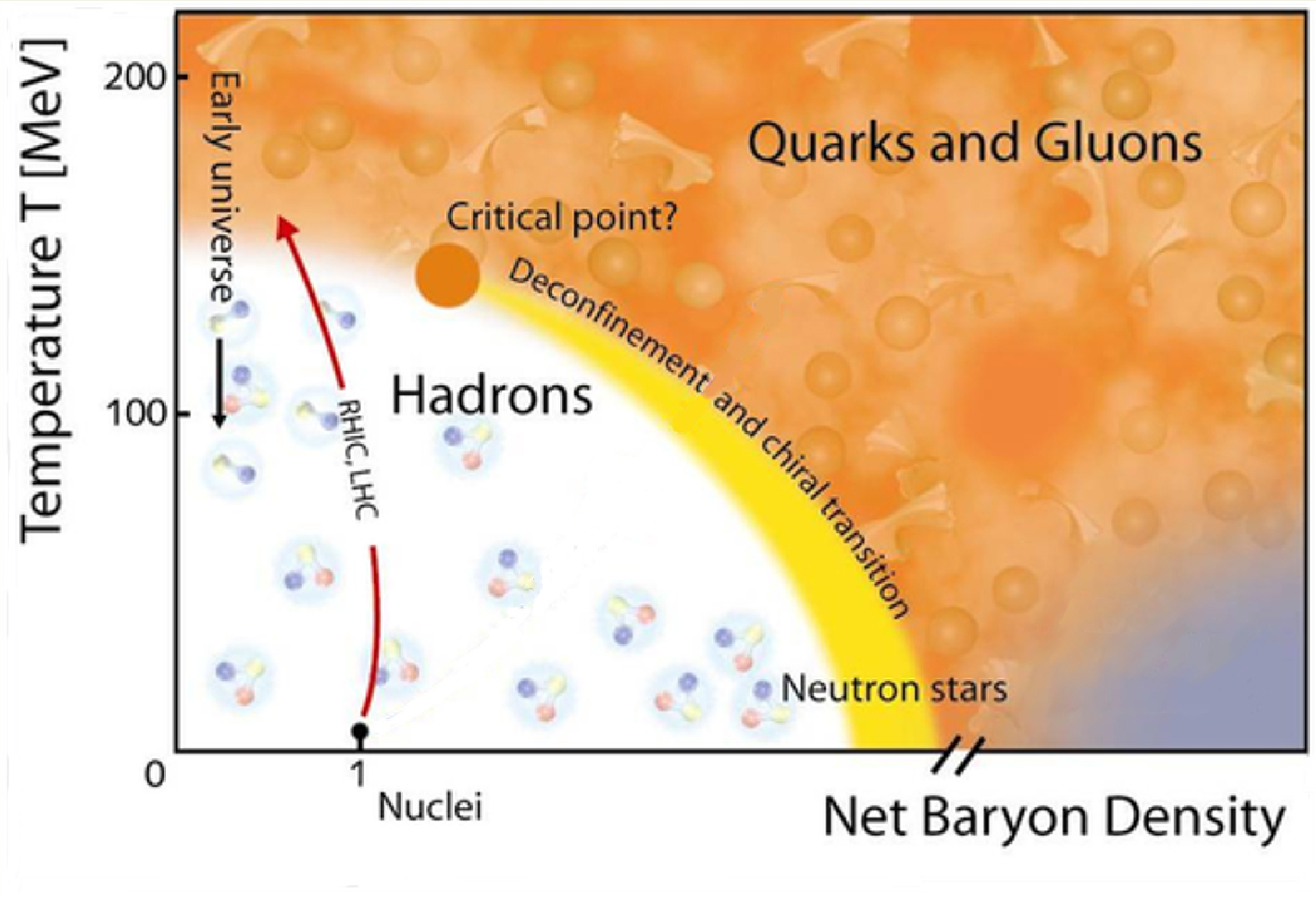




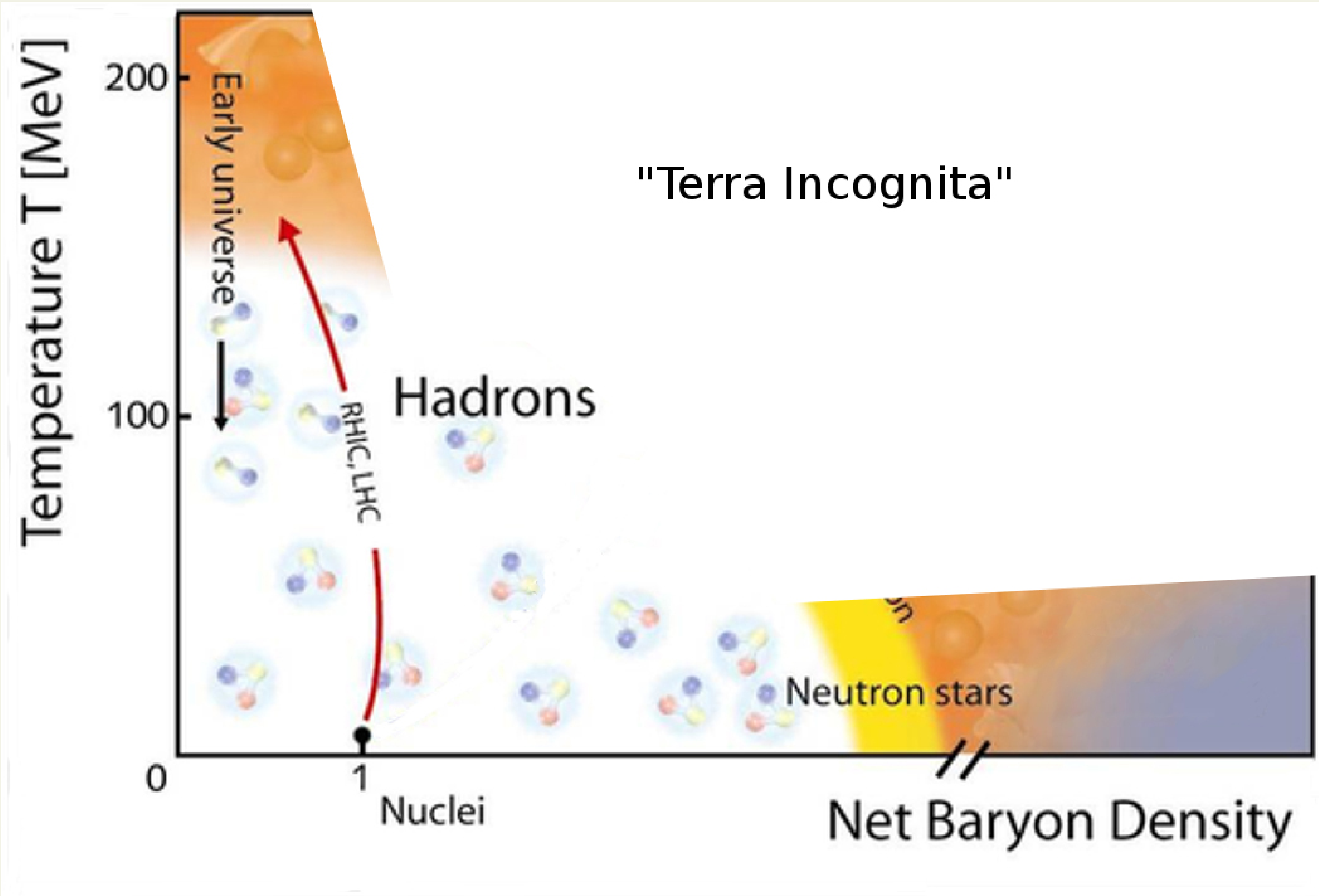
Auvinen *et al.*, PRC 102, 044911 (2020)

- $0.12 < \eta/s < 0.23$ **when** $150 \lesssim T/\mathrm{MeV} \lesssim 220$

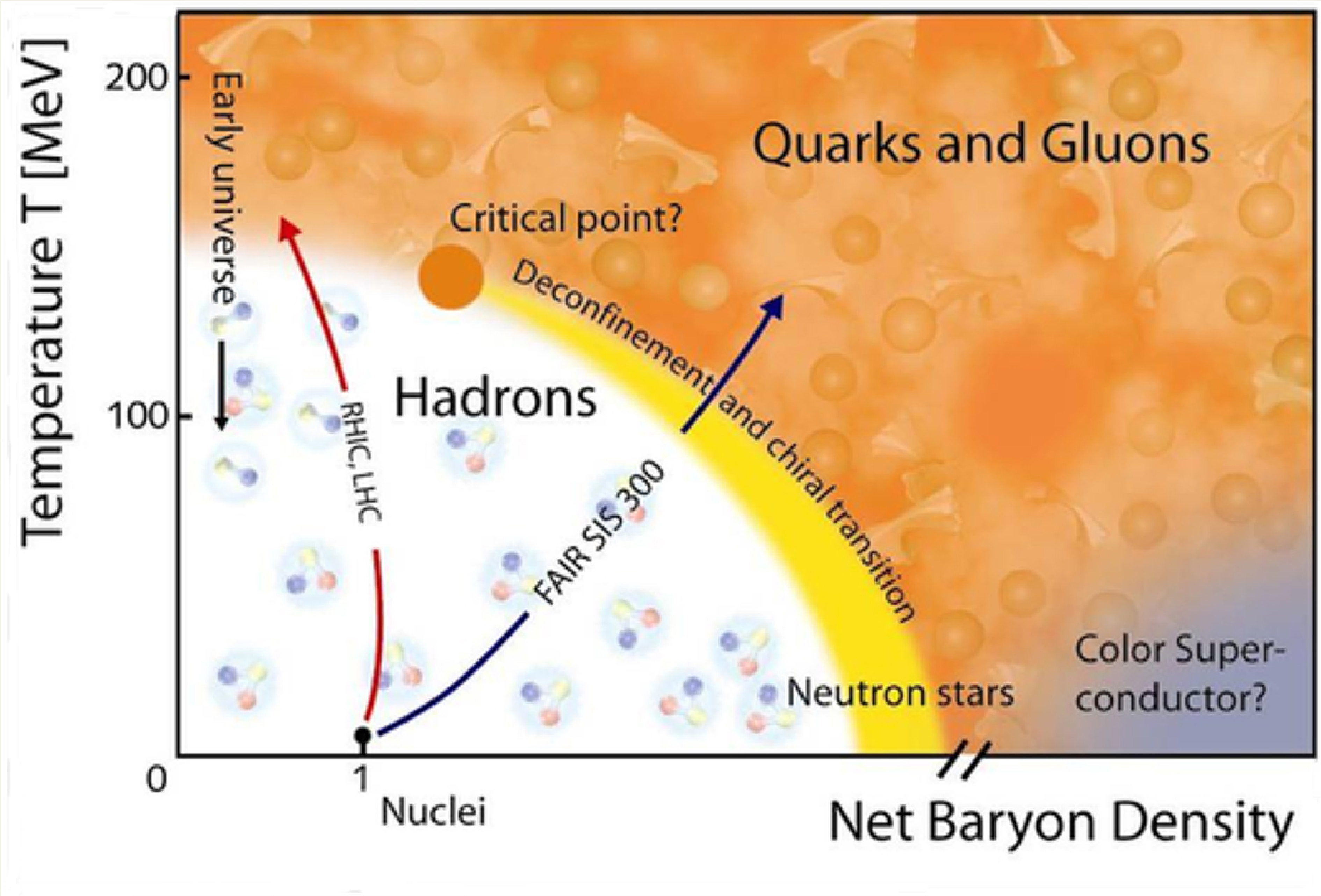
Phase diagram



Phase diagram

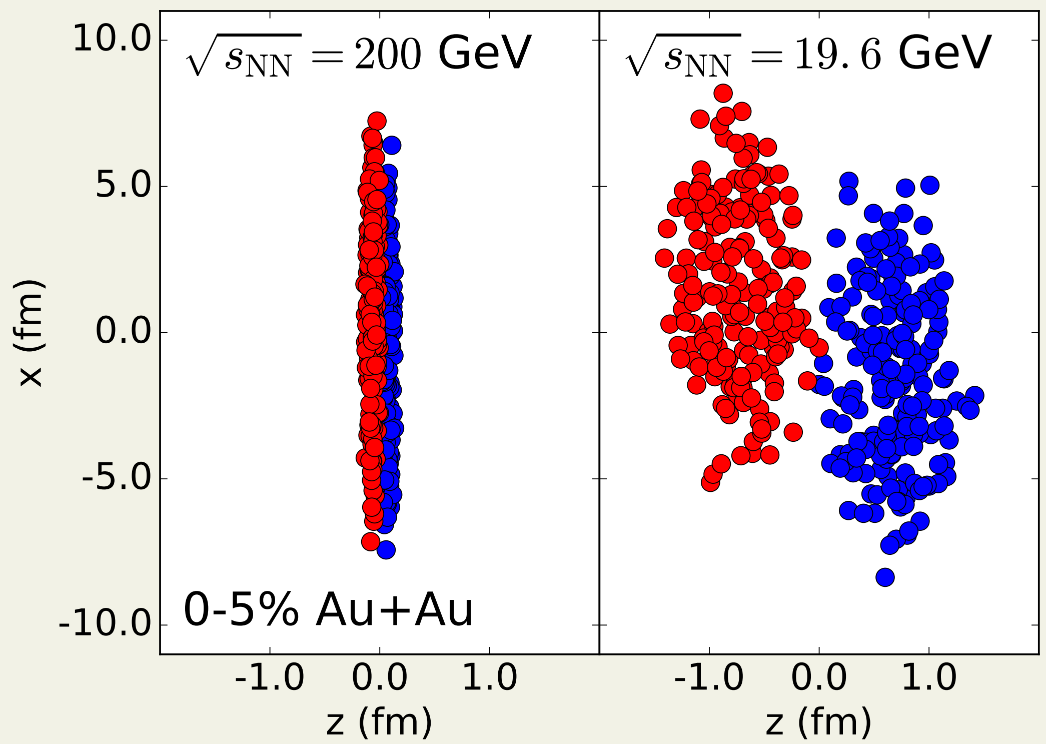


Phase diagram



Challenges

1. primary collisions overlap with secondary collisions

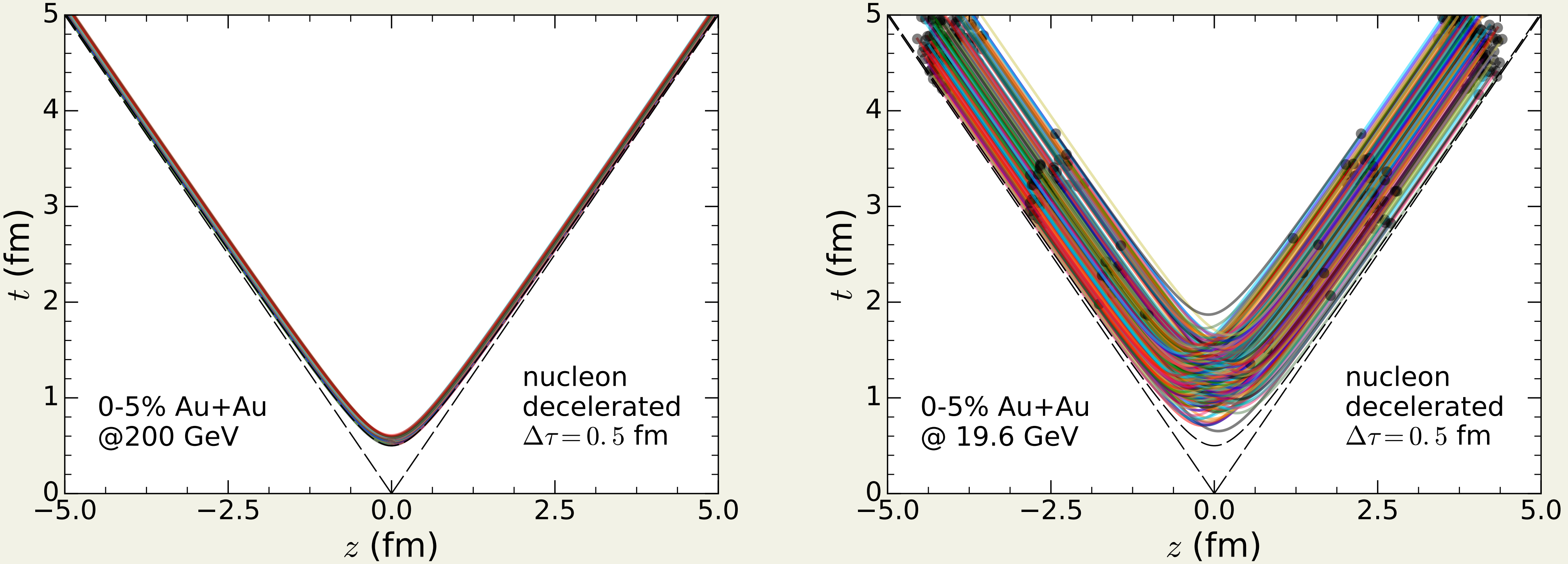


Shen & Schenke, PRC97, 024907 (2018)

Challenges

1. lower multiplicity $\Longrightarrow$ smaller system
   $\Longrightarrow$ larger deviations from equilibrium?

2. primary collisions overlap with secondary collisions



Shen & Schenke, PRC97, 024907 (2018)

3-fluid dynamics

$$0 = \partial_\mu T^{\mu\nu}$$

3-fluid dynamics

$$
\begin{aligned}
0 &= \partial_\mu T^{\mu\nu} \\
&= \partial_\mu T^{\mu\nu}_{\mathrm{t}} + \partial_\mu T^{\mu\nu}_{\mathrm{p}} + \partial_\mu T^{\mu\nu}_{\mathrm{fb}}
\end{aligned}
$$

$T^{\mu\nu}_{\mathrm{t}}$ = **target fluid**
$T^{\mu\nu}_{\mathrm{p}}$ = **projectile fluid**
$T^{\mu\nu}_{\mathrm{fb}}$ = **fireball fluid**

- **target and projectile represent colliding nucleons**
- **fireball (loosely) represents produced particles**

3-fluid dynamics

$$\partial_\mu T^{\mu\nu}_{\mathrm{t}}(x) = -F^\nu_{\mathrm{t}}(x) + F^\nu_{\mathrm{ft}}(x)$$
$$\partial_\mu T^{\mu\nu}_{\mathrm{p}}(x) = -F^\nu_{\mathrm{p}}(x) + F^\nu_{\mathrm{fp}}(x)$$
$$\partial_\mu T^{\mu\nu}_{\mathrm{fb}}(x) = F^\nu_{\mathrm{p}}(x) + F^\nu_{\mathrm{t}}(x) - F^\nu_{\mathrm{fp}}(x) - F^\nu_{\mathrm{ft}}(x)$$

- **interaction between target and projectile:**
  friction terms $-F^\nu_{\mathrm{t}}(x)$ **and** $-F^\nu_{\mathrm{p}}(x)$
- **interaction between fireball and target/projectile:**
  friction terms $F^\nu_{\mathrm{fp}}(x)$ **and** $F^\nu_{\mathrm{ft}}(x)$

Summary

- at (extremely) large temperatures and densities hadrons "melt" to quark-gluon plasma where quarks and gluons are the basic degrees of freedom
- quark-gluon plasma behaves like fluid with very low specific shear viscosity
- models for lower collision energies/higher baryon densities under construction