Goerlitz, 04. 12. 2023

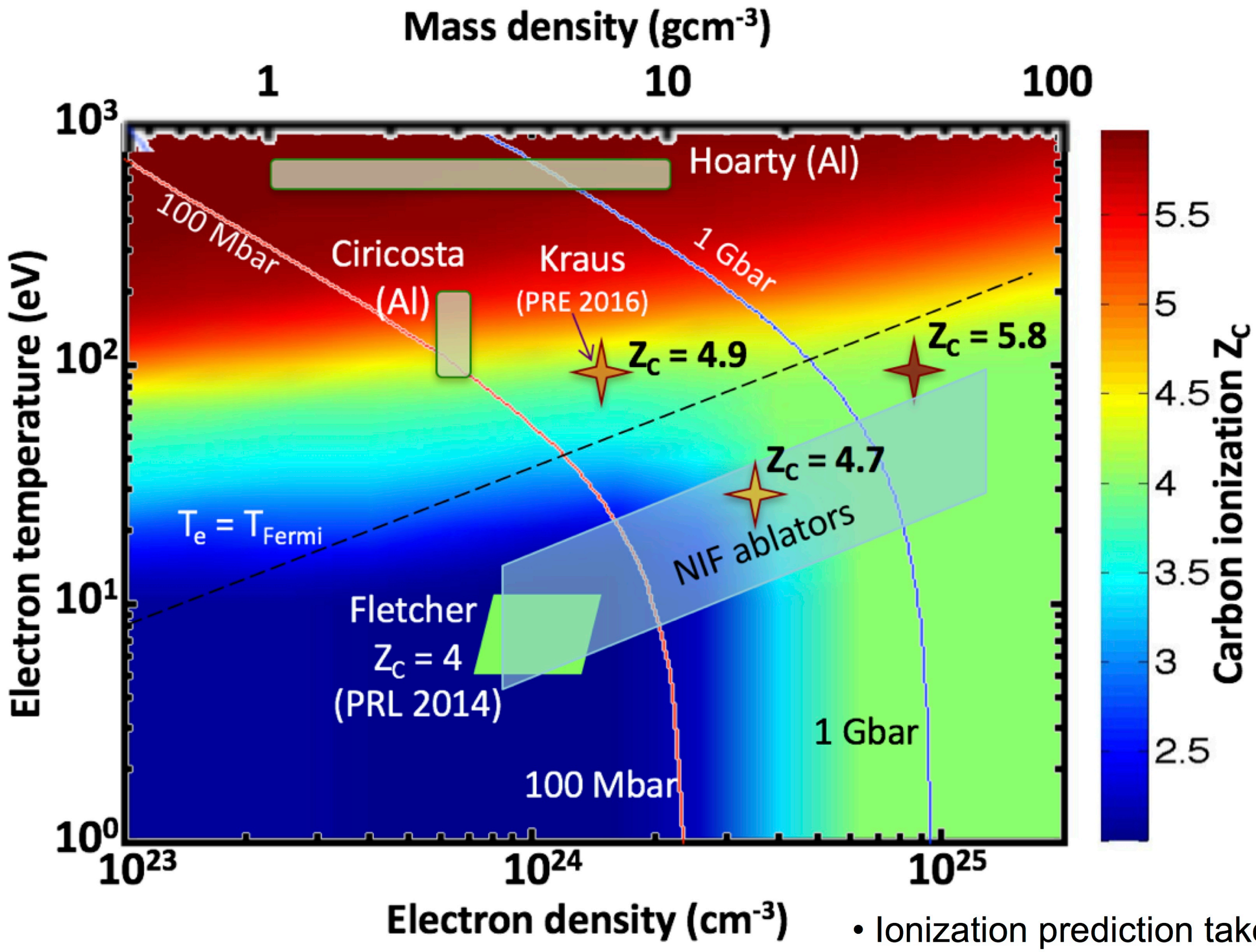
Polish-German WE-Heraeus Seminar & Max Born Symposium

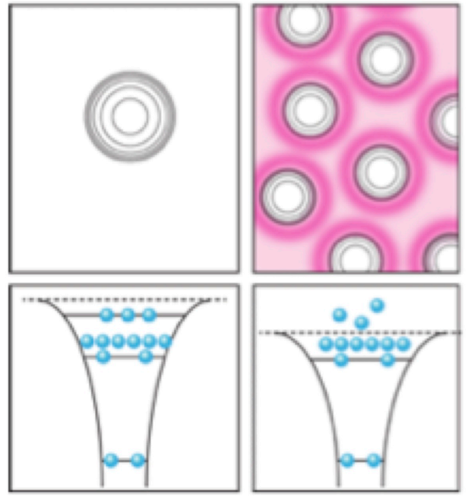
Many-particle systems under extreme conditions

Cluster-virial expansions for correlated matter

G. Röpke, Rostock

NIF XRTS experiments find higher carbon K-shell ionization than predicted by widely used IPD models (Stewart & Pyatt, OPAL)





NIF data point

Hoarty et al., PRL **110**, 265003 (2013)

Ciricosta et al., PRL **109**, 065002 (2012)

Fletcher et al., PRL **112**, 145004 (2014)

Kraus et al., PRE **94**, 011202(R) (2016)
[C. Lin et al., PRE 96, 013202 (2017)]

- Ionization prediction taken from OPAL

Rogers et al., APJ **456**, 902 (1996)

Preliminary, from Tilo Döppner et al., LLNL

Nuclear matter phase diagram

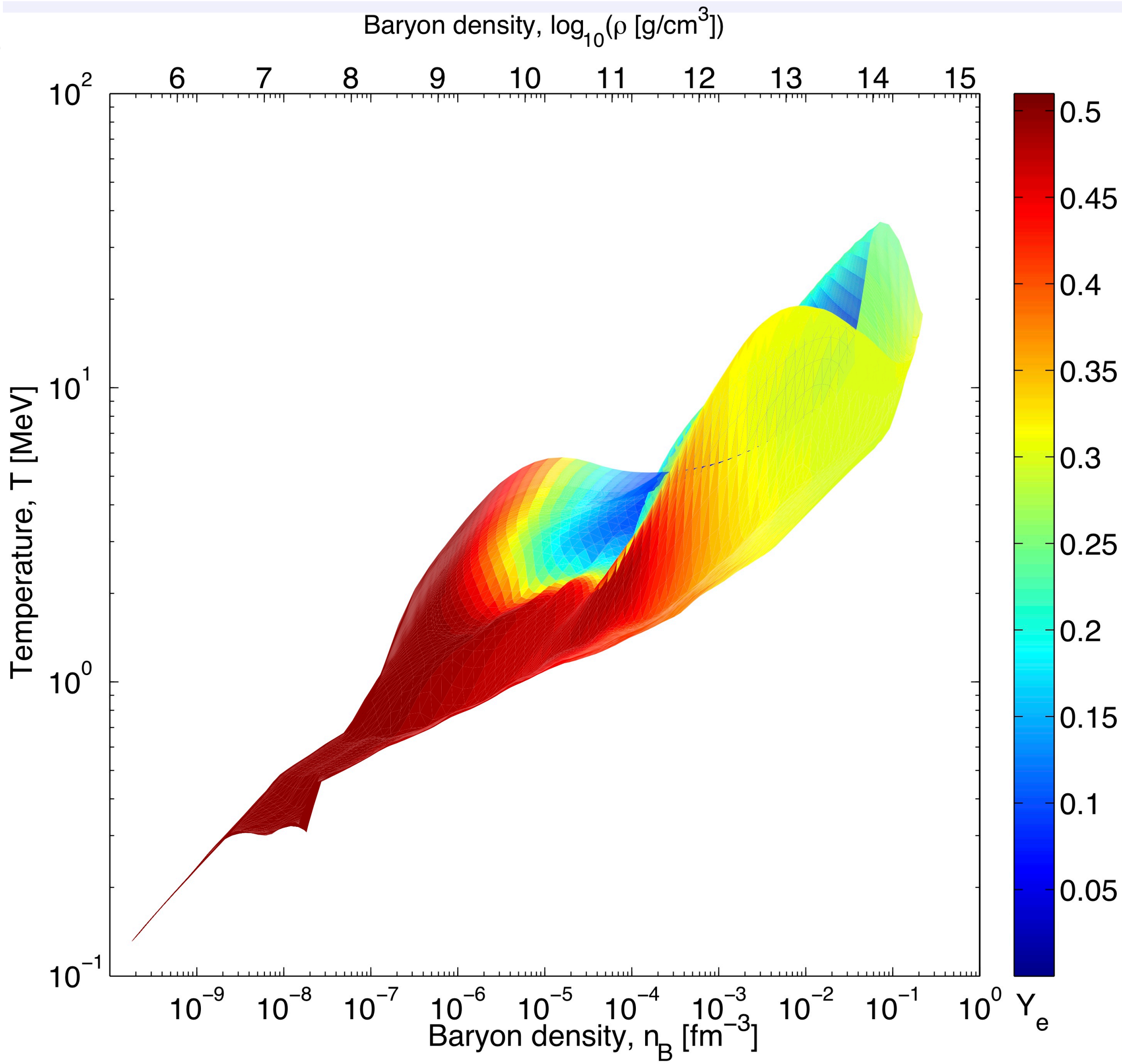
Core collapse supernovae

Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})

- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)

- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer, arXiv 1307.6190

Outline

1. Equation of state and virial expansions in plasmas
   Virial coefficients for the uniform electron gas, interpolation formulas

2. Electrical conductivity: virial expansion for hydrogen plasmas
   Inclusion of electron-electron collisions

3. Beth-Uhlenbeck formula for the second virial coefficient
   In-medium Schroedinger equation for nuclear matter

4. Cross-over Bose Einstein condensate – BCS pairing
   Quartetting in low-density nuclear matter, Hoyle state and α decay

5. Cluster Beth-Uhlenbeck formula, few-body problem, continuum
   Cluster-mean field, composition, freeze-out concept

1. Equations of state

many-particle system, temperature *T*, volume Ω, particle number *N*, density $n=N/\Omega$
thermodynamic potential: Free energy $F(T,\Omega,N)$

pressure $$p(T,n) = \left(\frac{\partial}{\partial \Omega} F(T,\Omega,N) \right) \Big|_{T,N}$$

mean potential energy $$V(T,\Omega,N) = e^2 \frac{\partial}{\partial (e^2)} F(T,\Omega,N)$$

quantum statistical approach: grand canonical ensemble

statistical operator, $T = 1/\beta,\ \mu$ chemical potential/T

$$\rho(\beta,\mu) = \frac{1}{Z_{g.c.}} e^{-\beta H + \mu N} \qquad Z_{g.c.} = \mathrm{Tr}\, e^{-\beta H + \mu N} \qquad p\Omega = -k_B T \ \ln Z_{g.c.}$$

density $n(T,\mu) = \frac{1}{\mathrm{Vol}} \int d^3 r \langle \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \rangle$

Fermi function

$$\mathrm{Tr}\{\rho\, \psi^\dagger(1',t')\, \psi(1,t)\} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{\frac{i}{\hbar}\omega(t'-t)} f(\omega) A(1,1';\omega)$$

spectral function

Correlation function

Green's function method

path integral Monte Carlo (PIMC) simulations

Virial expansions

short-range interaction

$$p^{\rm sr}(T,n) = b_1^{\rm sr}(T)n + b_2^{\rm sr}(T)n^2 + b_3^{\rm sr}(T)n^3 + \dots .$$

second virial coefficient: classical limit $b_2^{\rm sr}(T) = k_BT \int d^3r \left(e^{-V(r)/k_BT} - 1\right)$

Coulomb systems: long-range Coulomb interaction

$$\begin{aligned} F(T,\Omega,N) = \Omega k_BT \{ & n\ln n + [\ln(\Lambda^3) - 1]n \\ & -A_0(T)n^{3/2} - A_1(T)n^2\ln n - A_2(T)n^2 \\ & -A_3(T)n^{5/2}\ln n - A_4(T)n^{5/2} + \mathcal{O}(n^3\ln n)\} \end{aligned}$$

Debye $A_0(T) = \kappa^3/(12\pi n^{3/2})$ screening parameter $\kappa^2 = ne^2/(\epsilon_0 k_BT)$

second virial coefficient $A_2(T) = 2\pi\lambda^3 K(\xi) + \frac{\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 k_BT}\right)^3 \ln(\kappa\lambda/n^{1/2})$

thermal wave length $\lambda^2 = \hbar^2/(mk_BT)$ $\xi = -e^2/(4\pi\epsilon_0 k_BT\lambda) = ({\rm Hartree}/k_BT)^{1/2}$

W.-D. Kraeft, D. Kremp, W. Ebeling, G.R., *Quantum Statistics of Charged Particle Systems,* 1986

Homogeneous (uniform) electron gas

specific mean potential energy $v = V/N$

virial expansion

$$\kappa^2 = \frac{ne^2}{\epsilon_0 k_B T}, \qquad \lambda^2 = \frac{\hbar^2}{mk_B T}, \qquad \tau = \frac{e^2\sqrt{m}}{4\pi\epsilon_0\sqrt{k_B T}\hbar}.$$

$$v(T,n) = v_0(T)n^{1/2} + v_1(T)n\ln\left(\kappa^2\lambda^2\right) + v_2(T)n + v_3(T)n^{3/2}\ln\left(\kappa^2\lambda^2\right) + v_4(T)n^{3/2} + \mathcal{O}(n^2\ln(n))$$

$$\begin{aligned}
v_0(T) &= -\frac{\sqrt{\pi}}{T^{1/2}}, \qquad v_1(T) = -\frac{\pi}{2T^2}, \\
v_2(T) &= -\frac{\pi}{T}\left[\frac{1}{2} - \frac{\sqrt{\pi}}{2}(1+\ln(2))\frac{1}{T^{1/2}} + \left(\frac{C}{2} + \ln(3) - \frac{1}{3} + \frac{\pi^2}{24}\right)\frac{1}{T}\right. \\
&\quad \left. -\sqrt{\pi}\sum_{m=4}^{\infty}\frac{m}{2^m\Gamma(m/2+1)}\left(\frac{-1}{T^{1/2}}\right)^{m-1}\left[2\zeta(m-2) - (1-4/2^m)\zeta(m-1)\right]\right], \\
v_3(T) &= -\frac{3\pi^{3/2}}{2T^{7/2}}.
\end{aligned}$$

(atomic units)

fourth virial coefficient? $v_4(T)$

analytical expressions from perturbation theory

Mean potential energy

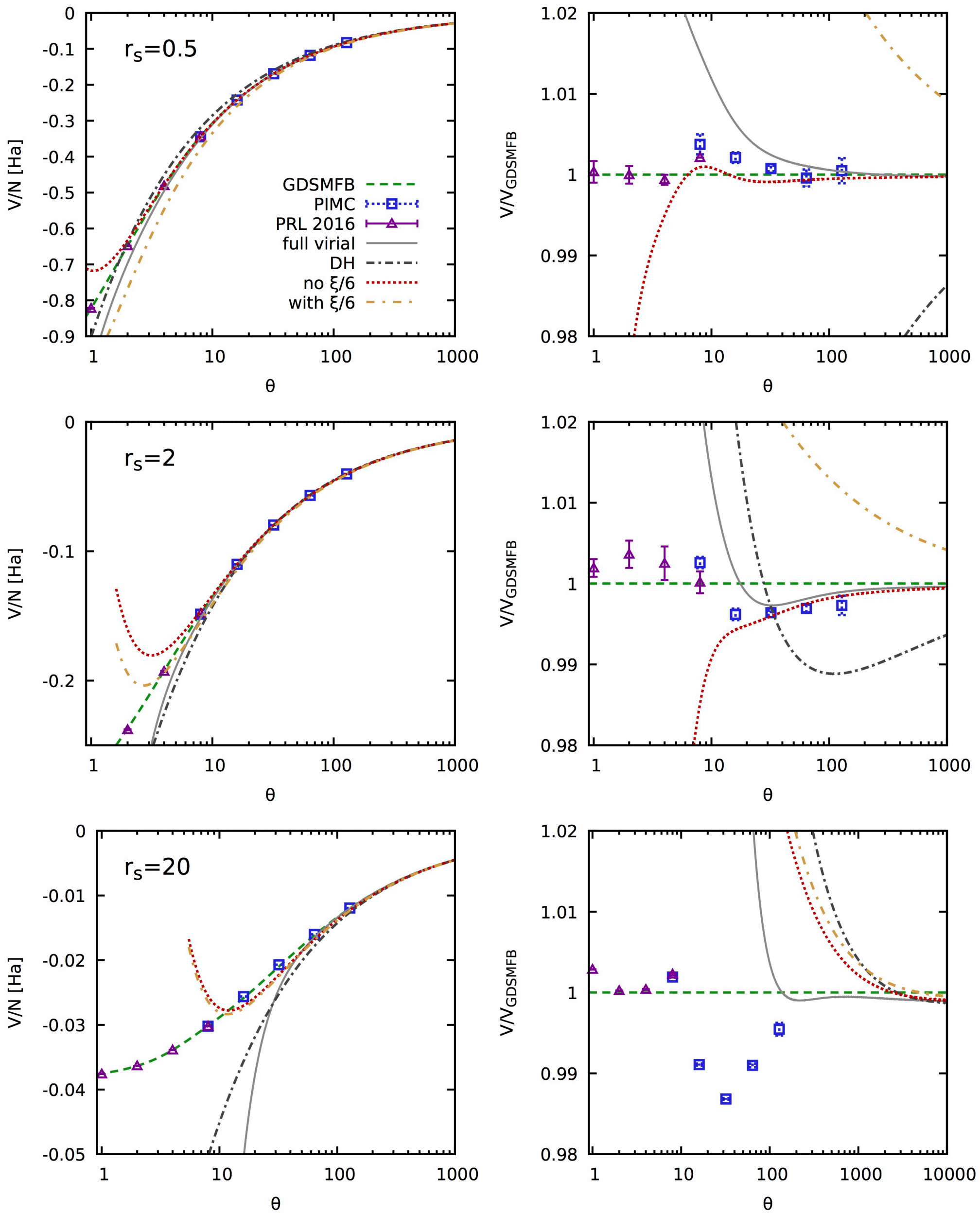
$$\frac{V}{Nk_BT} = -\frac{\kappa^3}{8\pi n} - 2\pi n\lambda^3\left[-\frac{\xi}{4} - \frac{\sqrt{\pi}}{4}\xi^2(1+\ln 2)\right.$$
$$\left. -\frac{\xi^3}{2}\left(\ln\kappa\lambda + \frac{C}{2} + \ln 3 - \frac{1}{3} - \frac{\pi^2}{24}\right)\right].$$

$$\frac{V}{N\,k_BT} = \frac{V_1}{N\,k_BT} + \frac{V_2}{N\,k_BT}$$

Exact results, not debated:
Debye, logarithmic ξ^3-term

$$\frac{V_1}{N\,k_BT} = -\frac{\kappa^3}{8\pi n} + \pi n\lambda^3\xi^3\ln(\kappa\lambda)$$

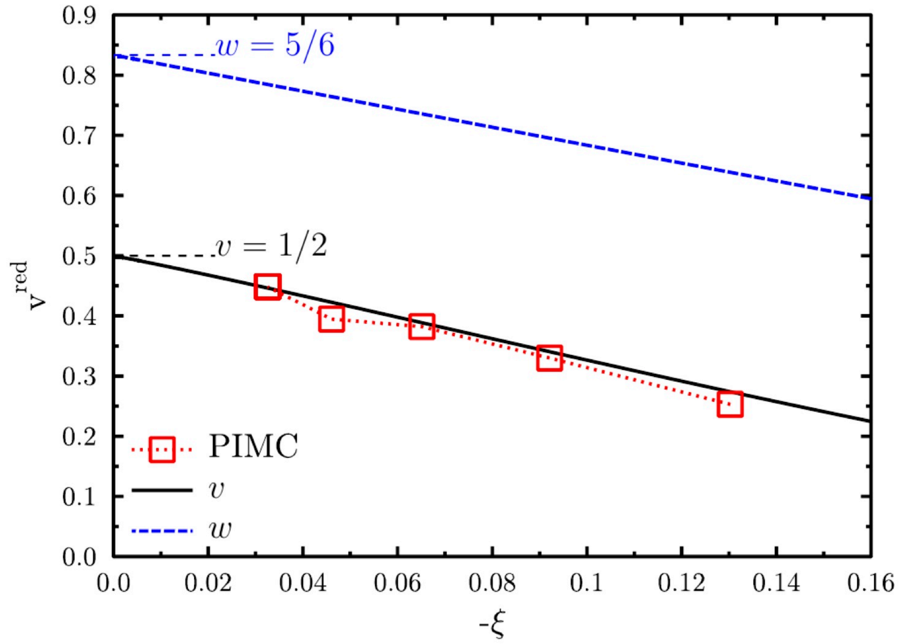
$$v^{\mathrm{red}} = \frac{\Delta v}{\pi n\lambda^3\xi\,k_BT} = \left[\frac{V}{N\,k_BT} - \frac{V_1}{N\,k_BT}\right]\frac{1}{\pi n\lambda^3\xi}$$



T. Dornheim et al., High Energy Density Physics **45**, 101015 (2022)

Equation of state of the uniform electron gas

$$v^{\text{red}} = \frac{\Delta v}{\pi n \lambda^3 \xi \, k_B T} = \left[\frac{V}{N \, k_B T} - \frac{V_1}{N \, k_B T} \right] \frac{1}{\pi n \lambda^3 \xi}$$



$$B_2(T) = A_2(T) - \frac{\pi}{3} \lambda^3 \xi;$$

Figure 10. Dependence of the reduced interaction energy per particle v^{red} (56) on the Born parameter ξ for the density of $r_s = 0.5$. Red squares: PIMC data from this work, solid black line labelled v: virial expansion Eq. (45), dashed blue line: including the debated $\xi/6$ term (denoted by w).

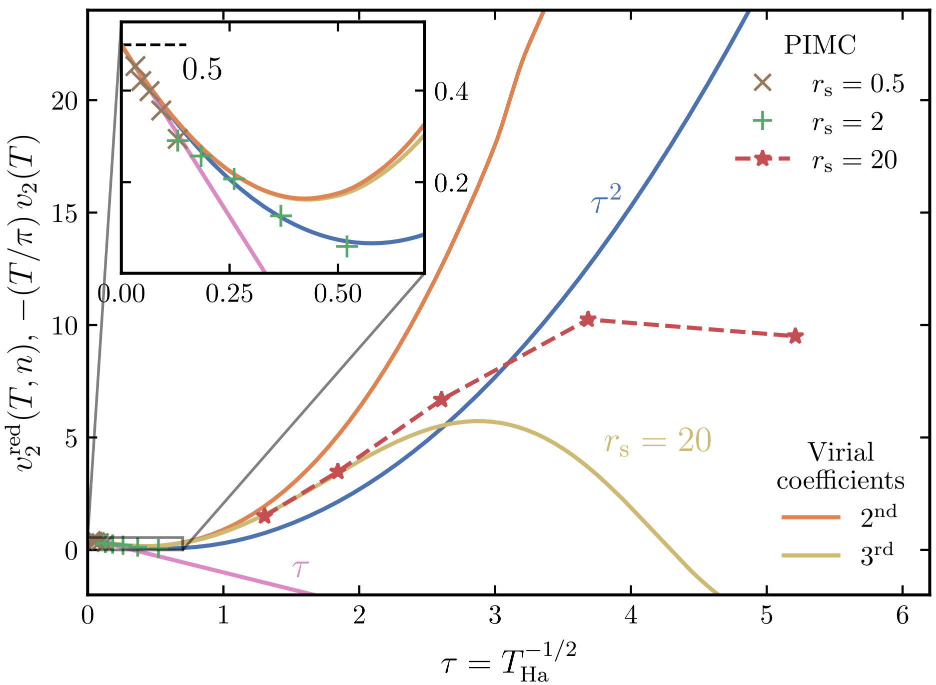
Virial plot

extraction of the second virial coefficient

$$v^{(1)}(T,n) = -\frac{\sqrt{\pi}}{T^{1/2}} n^{1/2} - \frac{\pi}{2T^2} n \ln\left(\frac{4\pi n}{T^2}\right)$$

$$v_2^{\mathrm{red}}(T,n) = \left[v^{\mathrm{PIMC}} - v^{(1)}(T,n)\right] \frac{-T}{\pi n} = \frac{-T}{\pi} v_2(T) + \mathcal{O}(n^{1/2}\ln(n))$$

$$= \frac{1}{2} - \frac{\sqrt{\pi}}{2}(1+\ln(2))\tau + \left(\frac{C}{2} + \ln(3) - \frac{1}{3} + \frac{\pi^2}{24}\right)\tau^2 + \mathcal{O}(\tau^3) + \mathcal{O}(n^{1/2}\ln(n))$$



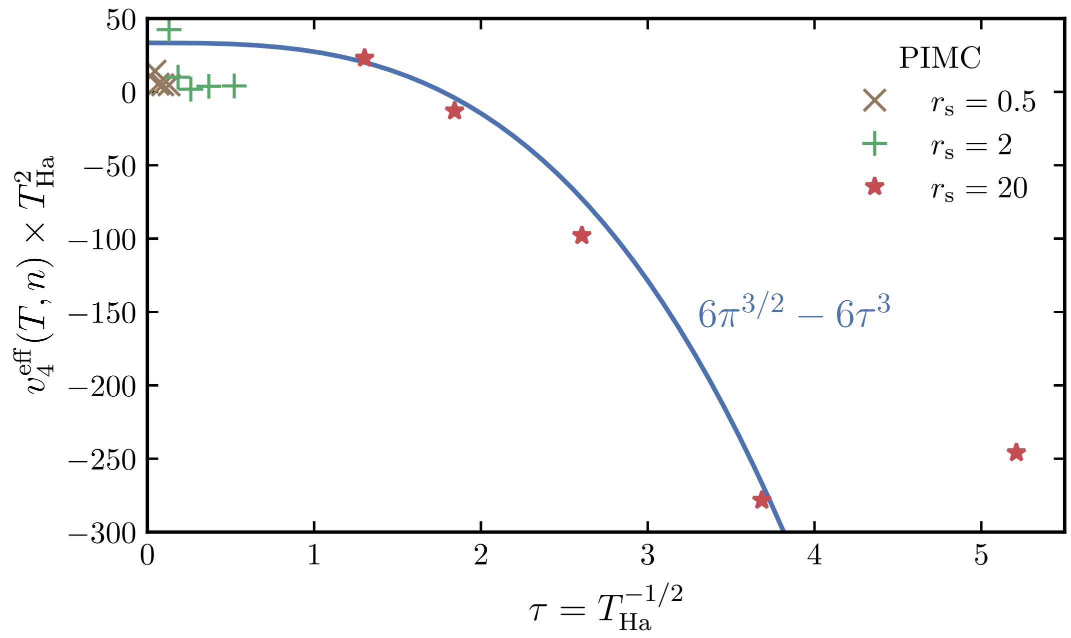
G.R., Contrib. Plasma Phys. **63**, e202300002 (2023)

Fourth virial coefficient

extraction of the fourth virial coefficient

$$\Delta v_3^{\text{red}}(T,n) = \left[v^{\text{PIMC}} - v^{(1)}(T,n) - v_2(T)n - v_3(T)n^{3/2}\ln\left(\frac{4\pi n}{T^2}\right)\right]\frac{T}{\pi n}.$$

$$v_4^{\text{eff}}(T,n) = \Delta v_3^{\text{red}}(T,n)\,\frac{\pi}{T n^{1/2}} = v_4(T) + \mathcal{O}(n^{1/2}\ln(n))$$



Interpolation formulas:

G.R., T. Dornheim, J. Vorberger, D. Blaschke, B. Mahato, submitted, arXiv: 2310.17583

Two-component plasmas: Thermodynamics of atomic and ionized hydrogen: Analytical results versus equation-of-state tables and Monte Carlo data

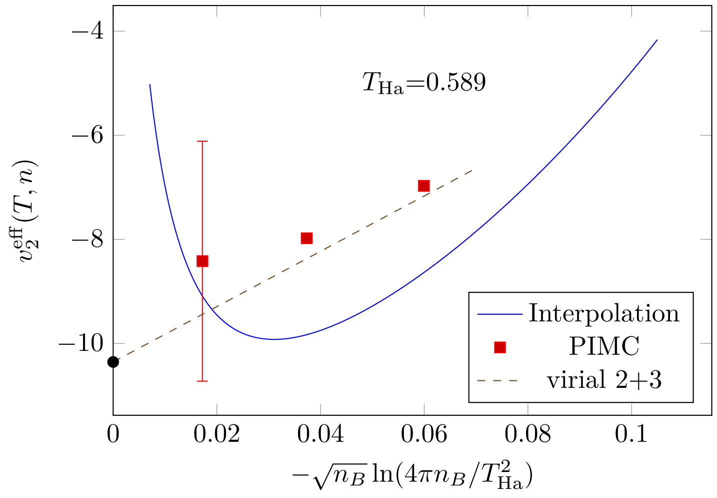
A. Alastuey and V. Ballenegger, Phys. Rev. E **86**, 066402 (2012)

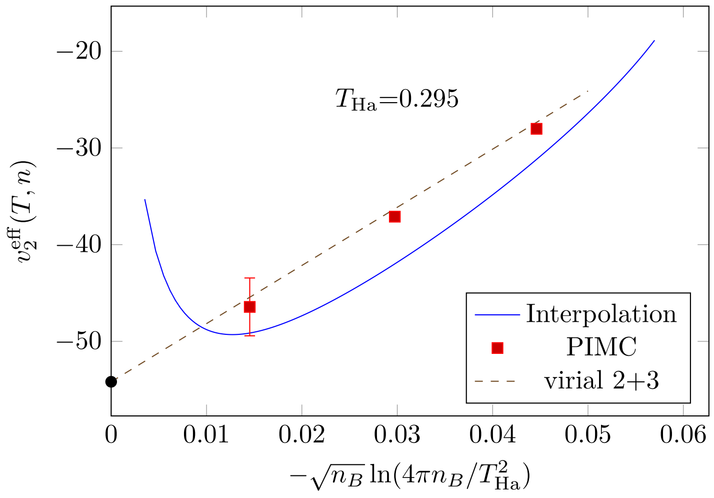
Virial plots for isotherms

$$v_2^{\rm eff}(T,n) = \Big[v(T,n) - v_0(T) n_{\rm B}^{1/2} - v_1(T) n_{\rm B} \ln\left(\frac{4\pi n_{\rm B}}{T_{\rm Ha}^2}\right)\Big] / n_{\rm B}$$

$$v_2^{\rm eff}(T,n) = v_2(T) + v_3(T) n_{\rm B}^{1/2} \ln(4\pi n_{\rm B}/T_{\rm Ha}^2) + \mathcal{O}[n^{1/2}].$$

Isotherms for T_{Ha} = 0.589 and 0.295





Interpolation formula for the free energy (S.Groth et al., Phys. Rev. Lett. **119**, 135001 (2017))

$$f_{\rm XC}^{\rm GDSMFB}(r_S,\Theta) = -\frac{1}{r_s}\frac{a(\Theta)+b(\Theta)\sqrt{r_s}+c(\Theta)r_s}{1+d(\Theta)\sqrt{r_s}+e(\Theta)r_s}$$

$$v(r_s,\Theta) = 2 f_{\rm XC}(r_s,\Theta) + r_s \frac{\partial f_{\rm XC}(r_s,\Theta)}{\partial r_s}\Big|_\Theta$$

G.R., T.Dornheim, J Vorberger, D.Blaschke, B.Mahato, submitted, arXiv: 2310.17583

2. Electrical conductivity of plasmas

- Kinetic theory (Boltzmann equation): Spitzer (low-density limit)
- Linear response theory: Kubo formula (warm dense matter)

$$\sigma(T,\mu) = \frac{e^2\beta}{3m^2\mathrm{Vol}} \int_{-\infty}^{0} dt e^{\epsilon t} \int_0^1 d\lambda \langle \mathbf{P} \cdot \mathbf{P}(t + i\hbar\beta\lambda) \rangle$$

electron total momentum $\mathbf{P} = \sum_k \hbar\, \mathbf{k}\, a_k^\dagger a_k$

- Generalized linear response theory (unified)
- Fluctuation-dissipation theorem: equilibrium correlation functions
- Green functions: perturbation theory, diagram techniques

Kubo-Greenwood formula, DFT-MD simulations: electron-electron collisions included?

(M. P. Desjarlais et al. 2017, N.R. Shaffer and C.E. Starrett 2020)

$$\mathrm{Re}\,[\sigma(\omega)] = \frac{2\pi e^2}{3m_e^2\omega\Omega} \sum_k w_k \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{\alpha=1}^{3} \left[f(\epsilon_{j,k}) - f(\epsilon_{i,k}) \right] |\langle \Psi_{j,k} | \hat{p}_\alpha | \Psi_{i,k} \rangle|^2 \delta(\epsilon_{i,k} - \epsilon_{j,k} - \hbar\omega)$$

Conductivity of warm dense matter including electron-electron collisions:
H. Reinholz, G. R., S. Rosmej, R. Redmer, Phys. Rev. E **91**, 043105 (2015).
DFT-MD contains e-e interaction only in mean-field approximation,
wrong low-density limit of electrical conductivity (Lorentz-model)

Virial Expansion of the Electrical Conductivity of Hydrogen Plasmas

dc conductivity $\sigma(n,T) = \frac{(k_B T)^{3/2}(4\pi\epsilon_0)^2}{m_e^{1/2}e^2}\,\sigma^*(n,T)$

dimensionless resistivity: virial expansion

$$\rho^*(n,T) = 1/\sigma^*(n,T) = \rho_1(T)\ln\frac{1}{n} + \rho_2(T) + \rho_3(T)\,n^{1/2}\,\ln\frac{1}{n} + \dots$$

dimensionless parameters $\Gamma = \frac{e^2}{4\pi\epsilon_0 k_B T}\left(\frac{4\pi}{3}\hat{n}_e\right)^{1/3}$ $\Theta = \frac{2m_e k_B T}{\hbar^2}(3\pi^2\hat{n}_e)^{-2/3}$

$$\rho^*(n,T) = \tilde{\rho}_1(T)\ln\left(\frac{\Theta}{\Gamma}\right) + \tilde{\rho}_2(T) + \dots .$$

$$\tilde{\rho}(x,T) = \frac{\rho^*}{\ln(\Theta/\Gamma)} = \tilde{\rho}_1(T) + \tilde{\rho}_2(T)x + \dots \qquad x = 1/\ln(\Theta/\Gamma)$$ virial plot

exact results $\rho_1^{\mathrm{Spitzer}} = 0.846$ $\lim_{T\to\infty}\tilde{\rho}_2(T) = \tilde{\rho}_2^{\mathrm{QLB}} = 0.4917$

(benchmarks)

V. S. Karakhtanov, Contrib. Plasma Phys. **56**, 343 (2016)

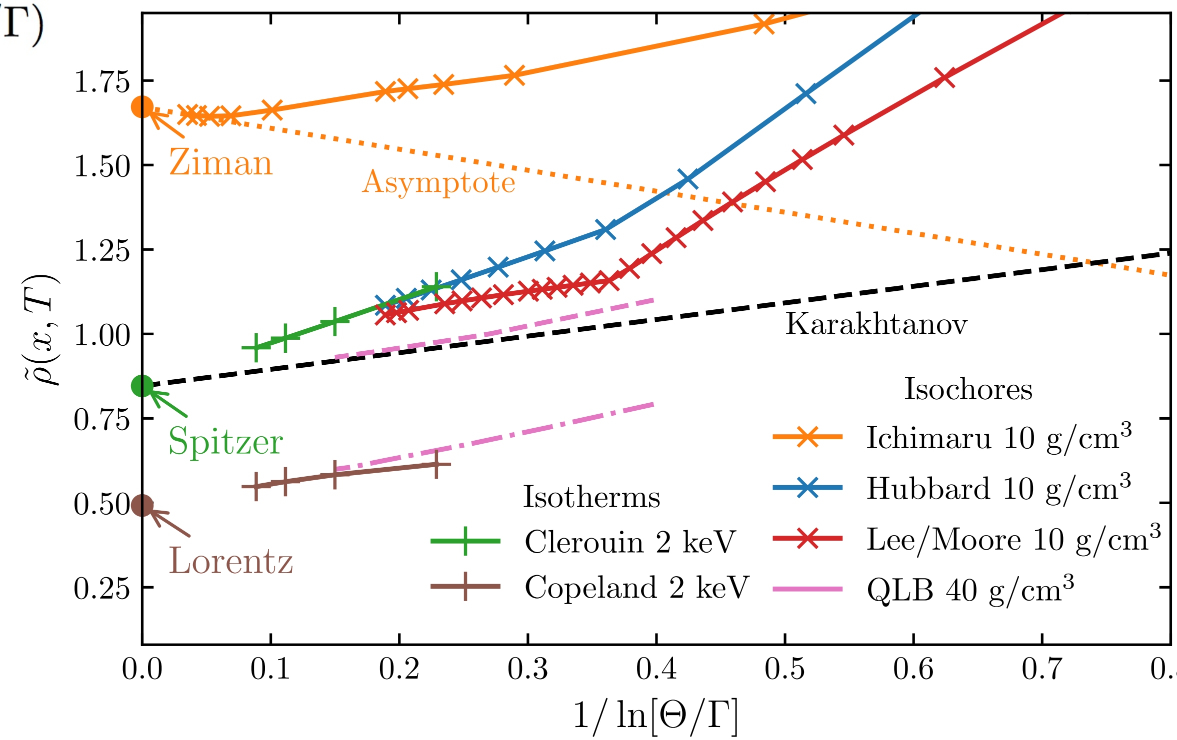
Virial Expansion of the Electrical Conductivity of Hydrogen Plasmas

$$\tilde{\rho}(x,T) = \frac{\rho^*}{\ln(\Theta/\Gamma)}$$

$$\rho_1^{\text{Ziman}} = \frac{2}{3}(2\pi)^{1/2} = 1.67109$$

$$\rho_1^{\text{Spitzer}} = 0.846$$

$$\rho_1^{\text{Lorentz}} = \frac{1}{16}(2\pi^3)^{1/2} = 0.492126$$



Clerouin, Copeland: P. E. Grabowski et al. High Energy Dens. Phys. **37**, 100905 (2020) ,
Review of the first charged-particle transport coefficient comparison Workshop

Ichimaru, Hubbard, Lee/More:
F. Lambert, V. Recoules, A. Decoster, J. Clerouin, M. Desjarlais, Phys. Plasmas **18**, 056306 (2011)

Free electron density

Kubo-Greenwood formula: conductivity, DFT-MD

$$\sigma^{\rm tot}(\omega) = \frac{2\pi e^2}{3\Omega\omega} \sum_{\mathbf{k}\nu\mu} (f_{\mathbf{k}\nu} - f_{\mathbf{k}\mu}) \left|\langle \mathbf{k}\nu | \hat{\mathbf{v}} | \mathbf{k}\mu \rangle\right|^2 \times \delta\left(E_{\mathbf{k}\mu} - E_{\mathbf{k}\nu} - \hbar\omega\right).$$

$$\sigma^{\rm tot}(\omega) = \sigma^{\rm f\text{-}f}(\omega) + \sigma^{\rm b\text{-}f}(\omega) + \sigma^{\rm b\text{-}b}(\omega)$$

Thomas-Reiche-Kuhn sum rule

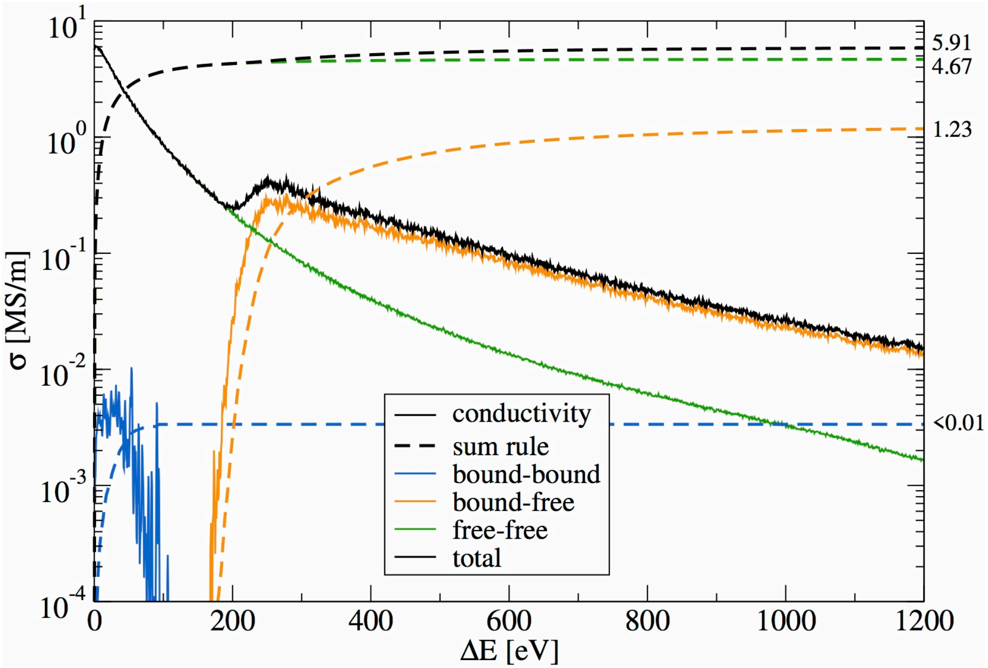
$$Z^{\rm tot} = \frac{2m_e V}{\pi e^2 n_{\rm i}} \int_0^{+\infty} d\omega \; \sigma^{\rm tot}(\omega)$$

Ionization degree $Z^{\rm free}/Z^{\rm tot}$

$$Z^{\rm free} = \frac{n_{\rm e}^{\rm free}}{n_{\rm i}} = \frac{2m_e V}{\pi e^2 n_{\rm i}} \int_0^{+\infty} d\omega \; \sigma^{\rm f\text{-}f}(\omega)$$

Carbon, T = 100 eV, n = 50 g cm^{-3}

final sum rule values



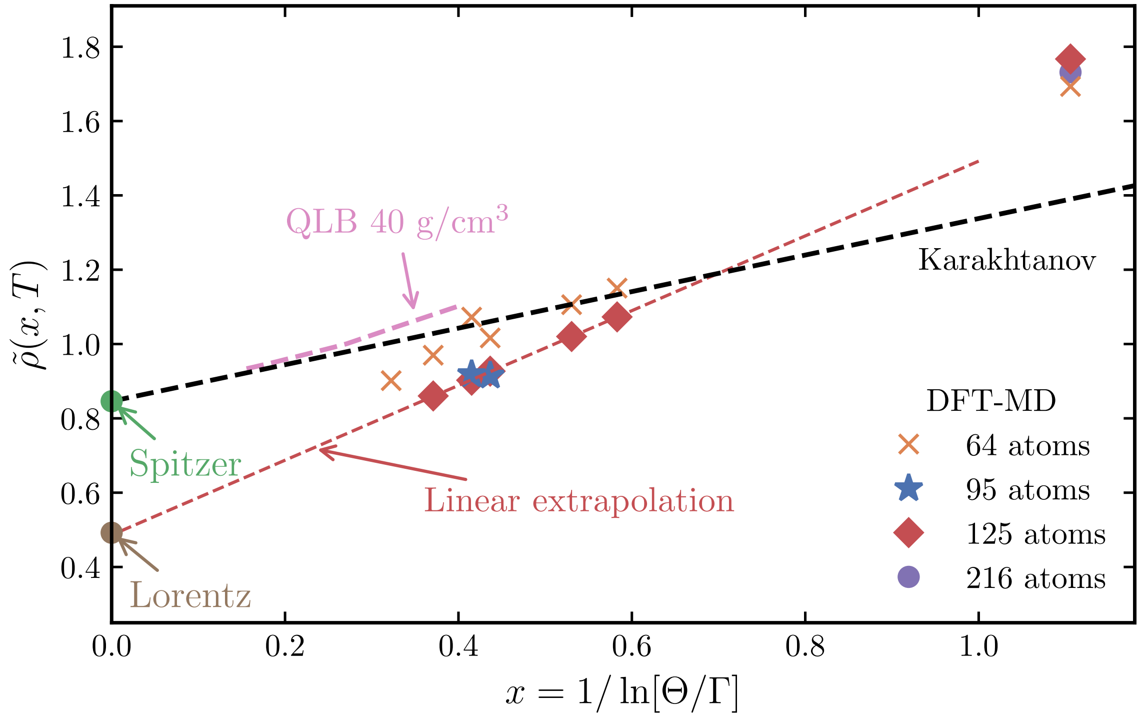
M. Bethkenhagen et al., Phys. Rev. Research 2, 023260 (2020)

Virial Expansion of the Electrical Conductivity of Hydrogen Plasmas

DFT-MD simulations: are electron-electron collisions rigorously included?

Spitzer:
e-e collisions included

Lorentz:
no e-e collisions



F. Lambert, V. Recoules, A. Decoster, J. Clerouin, M. Desjarlais, Phys. Plasmas **18**, 056306 (2011)
M. Desjarlais, C.Scullard, L. Benedict, H. Whitley, R. Redmer, Phys. Rev. E **95,** 033203 (2017)
G. R., M. Schoerner, M. Bethkenhagen, R. Redmer, Phys. Rev. E **104,** 045204 (2021)

PIMC simulations can solve the problem of the contribution of e – e collisions

3. Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\text{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$|1\rangle = |\mathbf{p}_1, \sigma_1\rangle$$

which is related to the Green function and the self-energy as

$$A(1,\omega) = 2\,\mathrm{Im}\,G(1,\omega - i0) = 2\,\mathrm{Im}\frac{1}{\omega - E(1) - \Sigma(1,\omega - i0)}$$

$$E(1) = p_1^2/(2m)$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\text{quasi}}(1))}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma_e(1,z)|_{z=E_e^{\text{quasi}}-\mu_e}} - 2\mathrm{Im}\,\Sigma_e(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega + \mu_e - E_e^{\text{quasi}}(1)}$$

$$E^{\text{quasi}}(1) = p_1^2/(2m) + \mathrm{Re}\Sigma(1,\omega)|_{\omega=E^{\text{quasi}}(1)}$$

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\rm total}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$A(1,\omega) = 2\,{\rm Im}\,G(1,\omega - i0) = 2\,{\rm Im}[\omega - E(1) - \Sigma(1,\omega - i0)]^{-1}$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\rm quasi}(1))}{1 - \frac{d}{dz}{\rm Re}\,\Sigma_e(1,z)|_{z=E_e^{\rm quasi}-\mu_e}} - 2{\rm Im}\,\Sigma_e(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega + \mu_e - E_e^{\rm quasi}(1)}$$

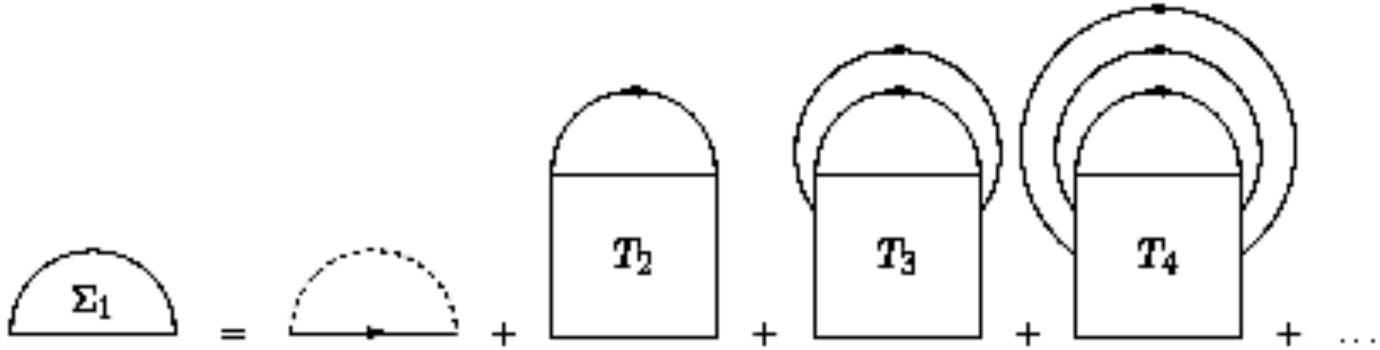
We obtain the generalized Beth-Uhlenbeck formula (quasiparticles)

$$n_e^{\rm total}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 f_e(E^{\rm quasi}(1))$$

$$+\frac{1}{\Lambda^3}\sum_{i,\gamma} Z_i e^{\beta\mu_i}\left[\sum_\nu^{\rm bound}(e^{-\beta E_{i,\gamma,\nu}} - 1) + \frac{\beta}{\pi}\int_0^\infty dE e^{-\beta E}\left\{\delta_{i,\gamma}(E) - \frac{1}{2}\sin[2\delta_{i,\gamma}(E)]\right\}\right]$$

In-medium Schrödinger equation for $E_{i,\gamma,\nu}(T,\mu)$, $\delta_{i,\gamma}(T,\mu)$, channel (spin…) γ

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Effective wave equation for the deuteron in matter

Green functions, spectral function, quasiparticles, self energy, Bethe-Salpeter equation

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

self-energy

Pauli-blocking

phase space occupation:
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

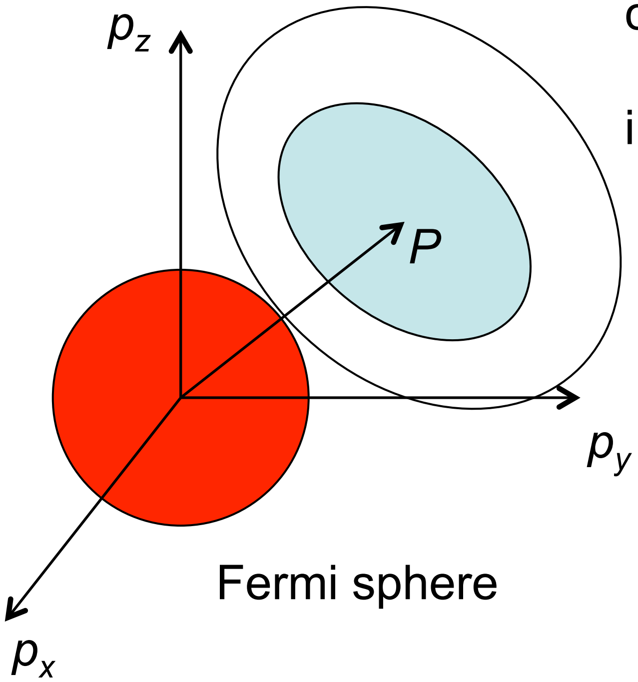
Thouless criterion

$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:

medium:
uncorrelated mean field (-> shell model)
correlated mean field (-> α matter)

Pauli blocking – phase space occupation



momentum space

cluster wave function
(atoms, ions, …deuteron, alpha,…)
in momentum space

P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

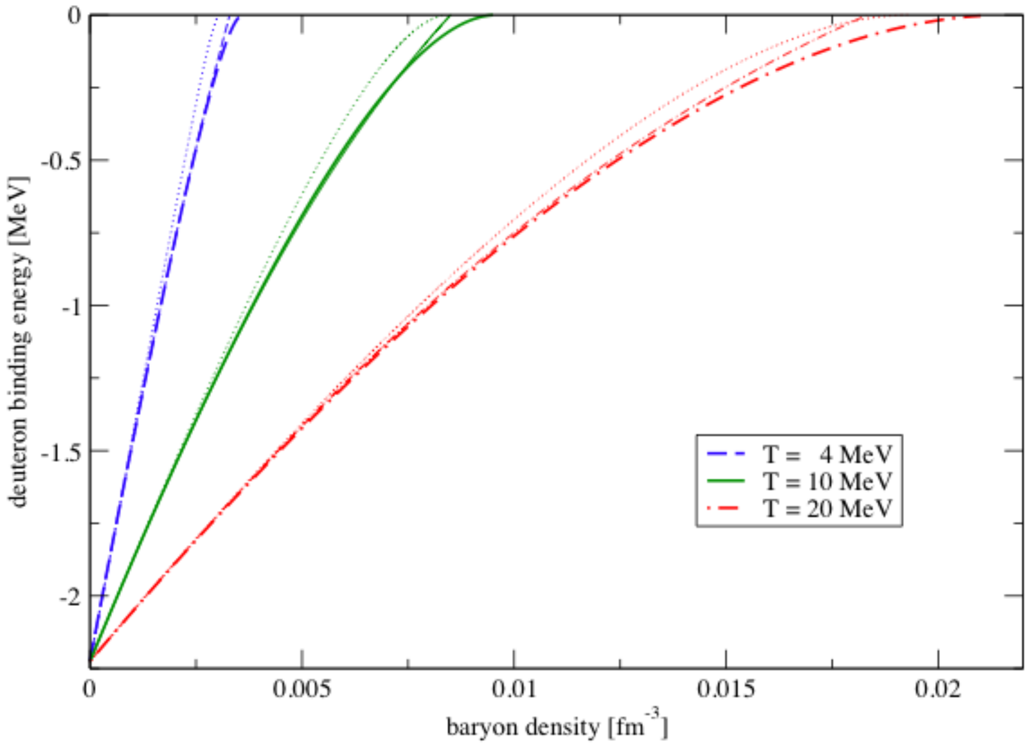
The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

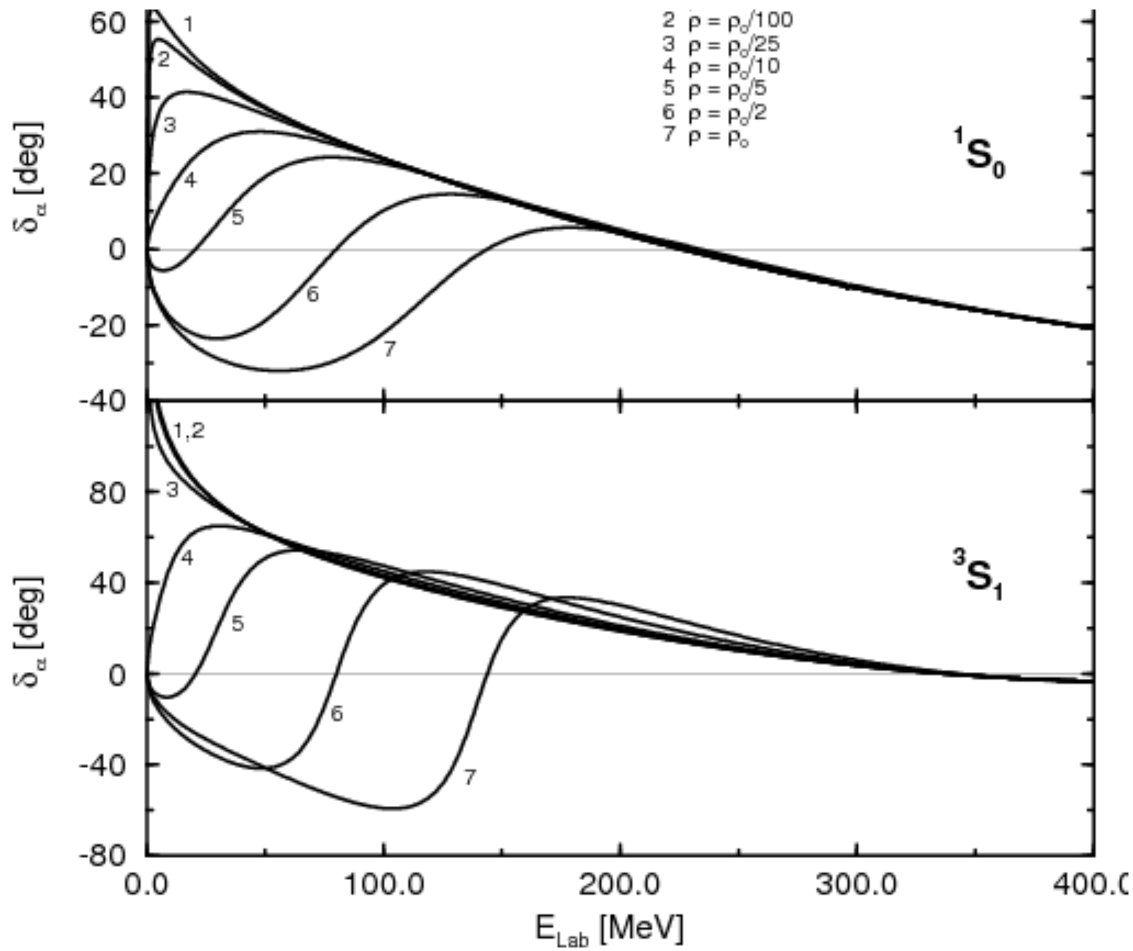
thin lines:

fit formula



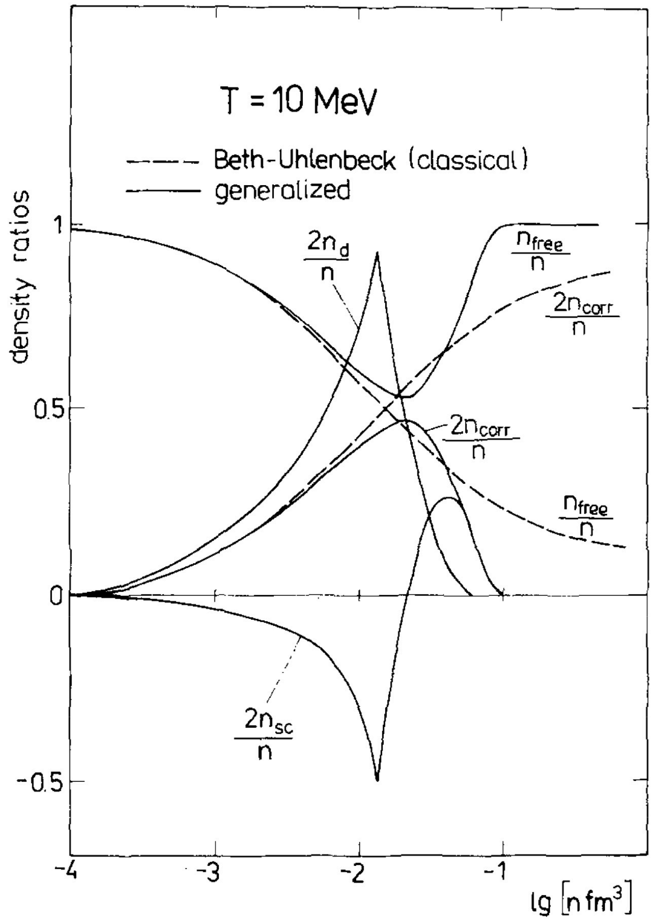
G.R., Nucl. Phys. A **867**, 66 (2011)

Scattering phase shifts in matter



Two-particle correlations

Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter



M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

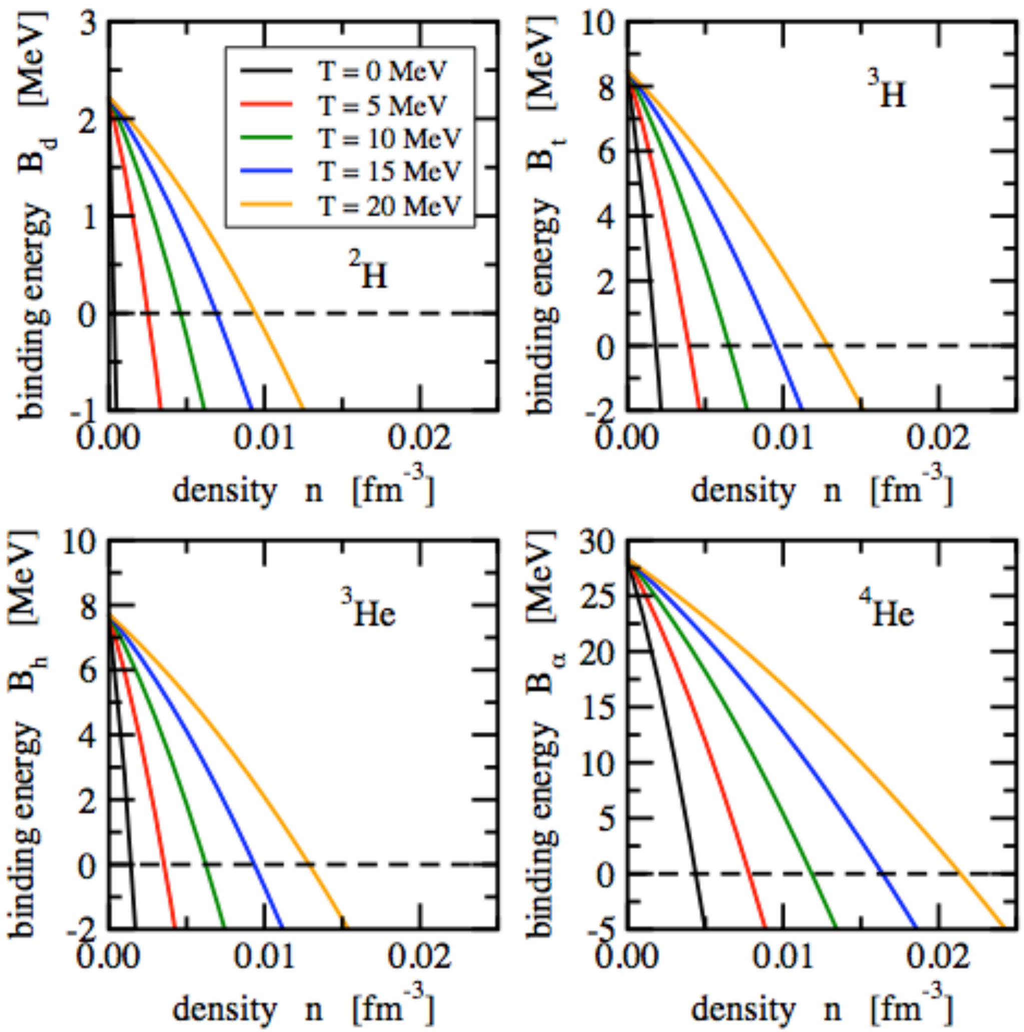
mass number A
charge Z_A
energy $E_{A,\nu,K}$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p) \approx \left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T) = v_{T_I=0}^0(T) \approx 0.30857+0.65327\, e^{-0.102424\; T/{\rm MeV}}$$

G. R., PRC 92,054001 (2015)

Few-body problem and continuum contributions ?

4. Effective wave equation for the deuteron in matter

Green functions, spectral function, quasiparticles, self energy, Bethe-Salpeter equation

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1} + \Delta_1 + \frac{p_2^2}{2m_2} + \Delta_2\right)\Psi_{d,P}(p_1,p_2) + \sum_{p_1',p_2'}(1 - f_{p_1} - f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$= E_{d,P}\Psi_{d,P}(p_1,p_2)$$

self-energy

Pauli-blocking

phase space occupation:
Fermi distribution function

$$f_p = \left[e^{(p^2/2m-\mu)/k_B T} + 1\right]^{-1}$$

Thouless criterion

$$E_d(T,\mu) = 2\mu$$

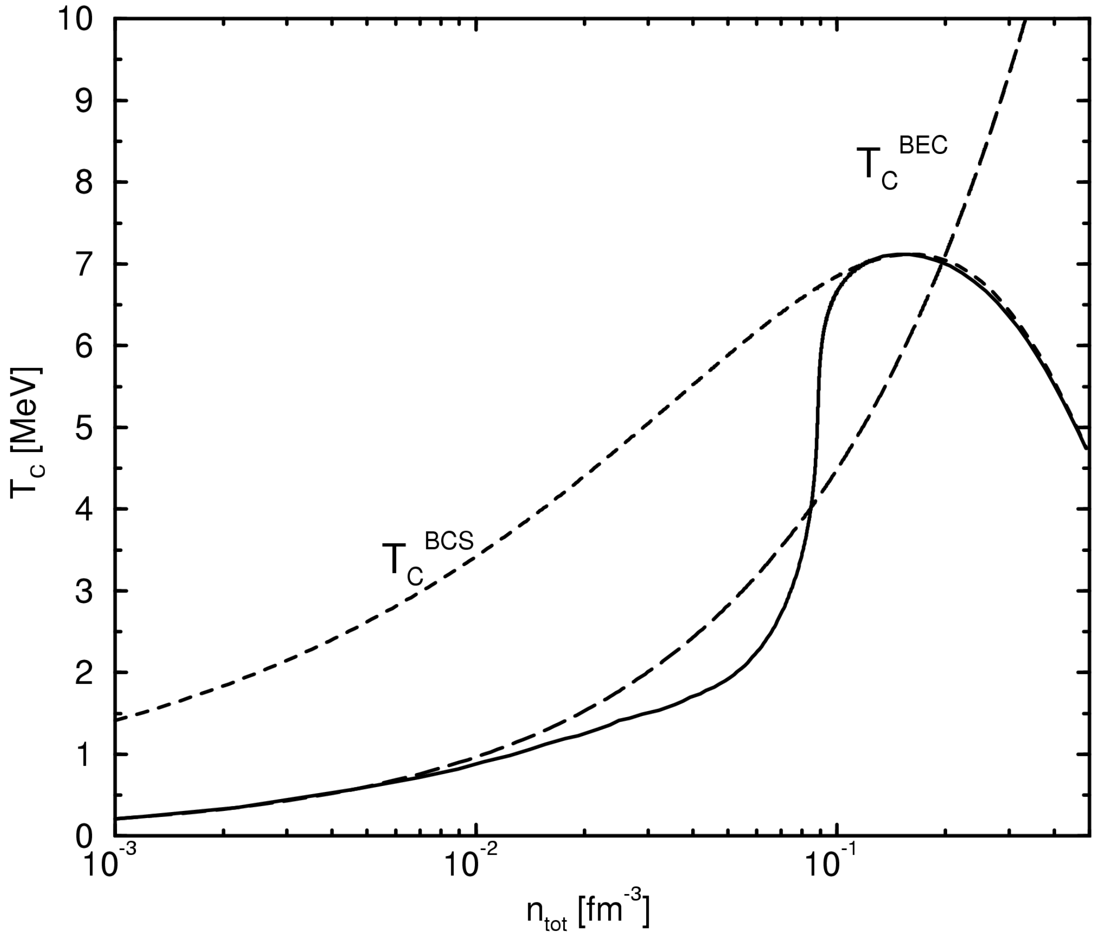
BEC-BCS crossover:

medium:
uncorrelated mean field (-> shell model)
correlated mean field (-> α matter)

2. Quantum condensate

Bose-Einstein-
Condensation
of deuterons
(BEC)

Bardeen-Cooper
Schrieffer
pairing
(BCS)



T. Alm *et al*. Z. Phys. **A337**, 355 (1990)
H. Stein *et al*., Z. Phys. **A351**, 259 (1995)
M. Baldo *et al*., Phys. Rev. C **52**, 975 (1995)
Meng Jin, M. Urban, and P. Schuck, Phys. Rev. C **82**, 024911 (2010)

Few-particle Schrödinger equation in a dense medium

α particles are strongly bound (7.07 MeV/A) compared to deuterons (1.1MeV/A)

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

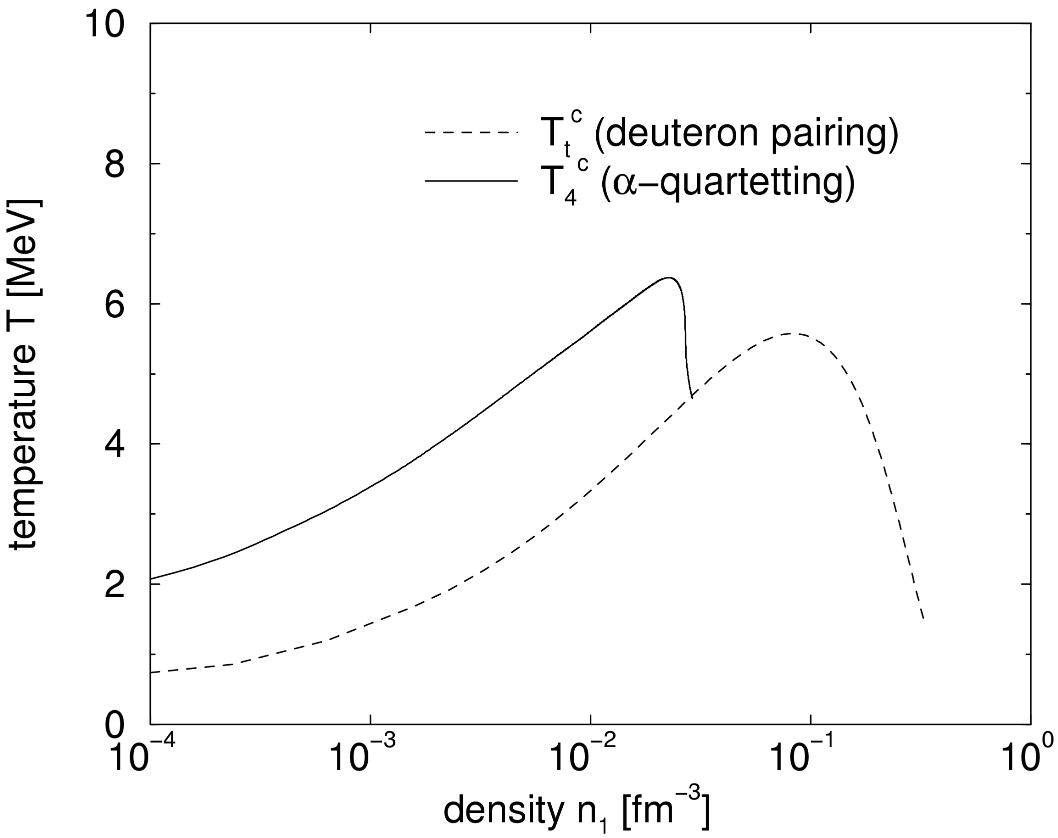
$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Thouless criterion
for quantum condensate:

$E_{n,P=0}(T,\mu) = 4\mu$

α-cluster-condensation

(quartetting)



G.R., A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

Is there a sharp transition from quartetting to pairing ?

Quantum condensate: quartetting

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α-particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r-space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)\Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8)\cdots\Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n})\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \mathrm{BCS} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2)\Phi(\vec{r}_3, \vec{r}_4)\cdots\}$$

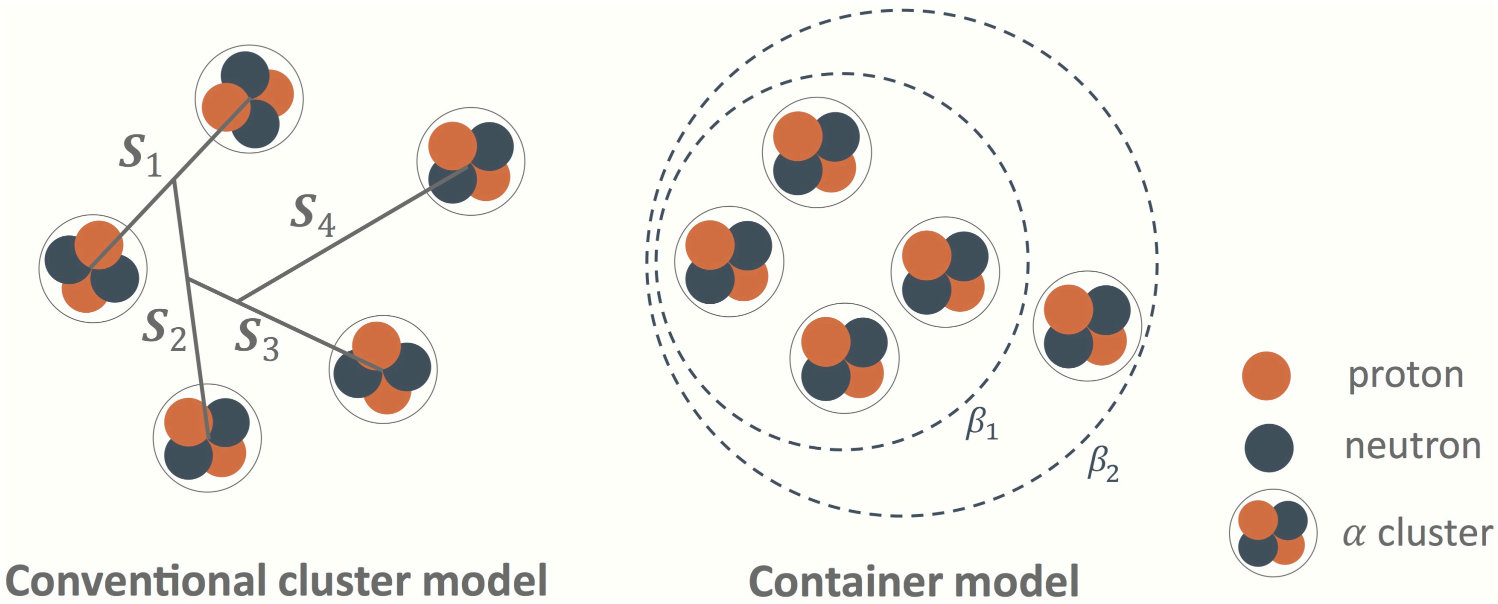
A. Tohsaki et al., PRL **87**, 192501 (2001)

The α condensate state in 4n nuclei

The Hoyle state as Bose-like gas of 3 α clusters, shell model not applicable

α condensation is a general phenomenon: ^{12}C, ^{16}O, ^{20}Ne, …

Container picture: B. Zhou et al., Phys. Rev. Lett. 110, 262501 (2013)



B. Zhou, Y. Funki, H. Horiuchi, Y. Ma, G. R., P. Schuck, A. Tohsaki, T. Yamada, to be published

Is there a sharp transition from quartetting to pairing ?

5. Cluster virial expansion for nuclear matter within a quasiparticle statistical approach

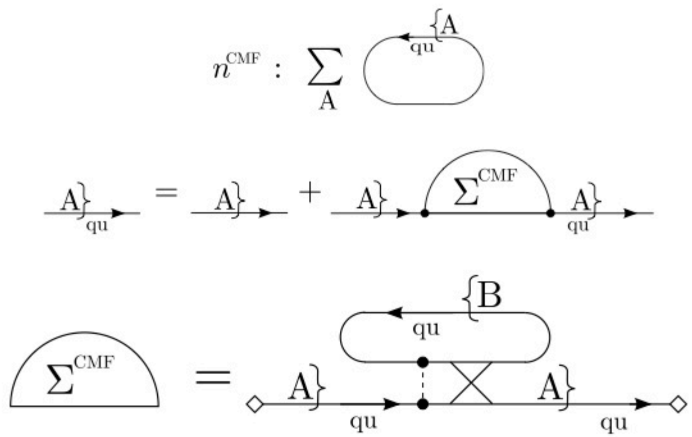
Generalized Beth-Uhlenbeck approach

$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \frac{A}{\Omega} \sum_{\substack{\vec{P} \\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$\begin{aligned} n_2^{\mathrm{qu}}(T,\mu_p,\mu_n) &= \sum_{A,Z,\nu} \sum_{A',Z',\nu'} \frac{A+A'}{\Omega} \sum_{\vec{P}} \sum_{c} g_c \frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi} \\ &\times \int_0^\infty dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right) 2\sin^2(\delta_c)\frac{d\delta_c}{dE} \end{aligned}$$

avoid double counting

generating functional



G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Cluster - mean field approximation

Cluster (A) interacting with a distribution of clusters (B) in the medium,

fully antisymmetrized (correlated medium)

$$\sum_{1'\ldots A'} \{H_A^0(1\ldots A,1'\ldots A') + \sum_i \Delta_i^{A,mf} \delta_{k,k'} + \frac{1}{2}\sum_{i,j} \Delta V_{ij}^{A,mf} \delta_{l,l'} - E_{A\nu P}\delta_{k,k'}\} \psi_{A\nu P}(1'\ldots A') = 0$$

self-energy

$$\Delta_1^{A,mf}(1) = \sum_2 V(12,12)_{ex} f^*(2) + \sum_{B\nu P}\sum_{2\ldots B'} f_B(E_{B\nu P}) \sum_i V_{1i}(1i,1'i') \psi^*_{B\nu P}(1\ldots B) \psi_{B\nu P}(1'\ldots B')$$

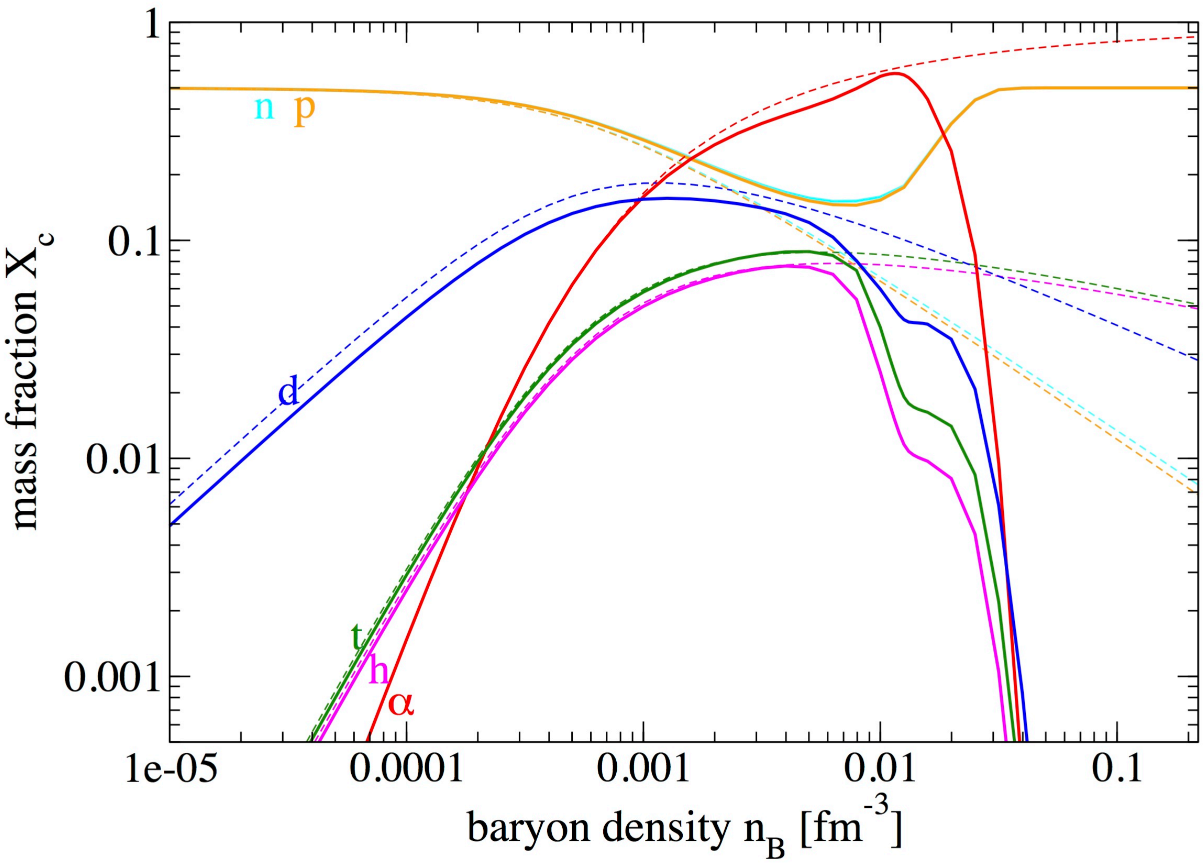
effective interaction

$$\Delta V_{12}^{A,mf} = -\frac{1}{2}[f^*(1) + f^*(2)] V(12,1'2') - \sum_{B\nu P}\sum_{2^*\ldots B''} f_B(E_{B\nu P}) \sum_i V_{1i} \psi^*_{B\nu P}(22^*\ldots B^*) \psi_{B\nu P}(2'2''\ldots B'')$$

phase space occupation $\quad f^*(1) = f_1(1) + \sum_{B\nu P}\sum_{2\ldots B} f_B(E_{B\nu P}) \,|\psi_{B\nu P}(1\ldots B)|^2$

self-consistent solutions for clusters in a clustered medium ?

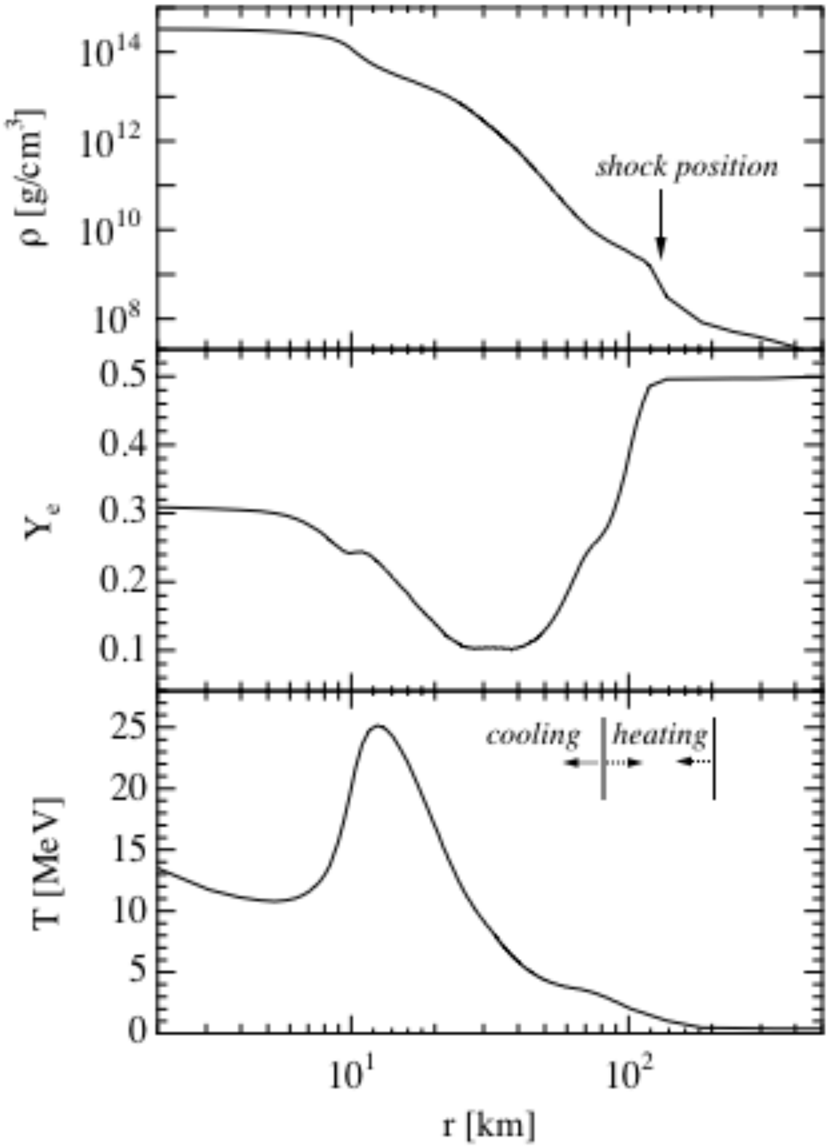
Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

G. R., PRC **92**, 054001 (2015)

Core-collapse supernovae



snapshot

density

proton fraction, and

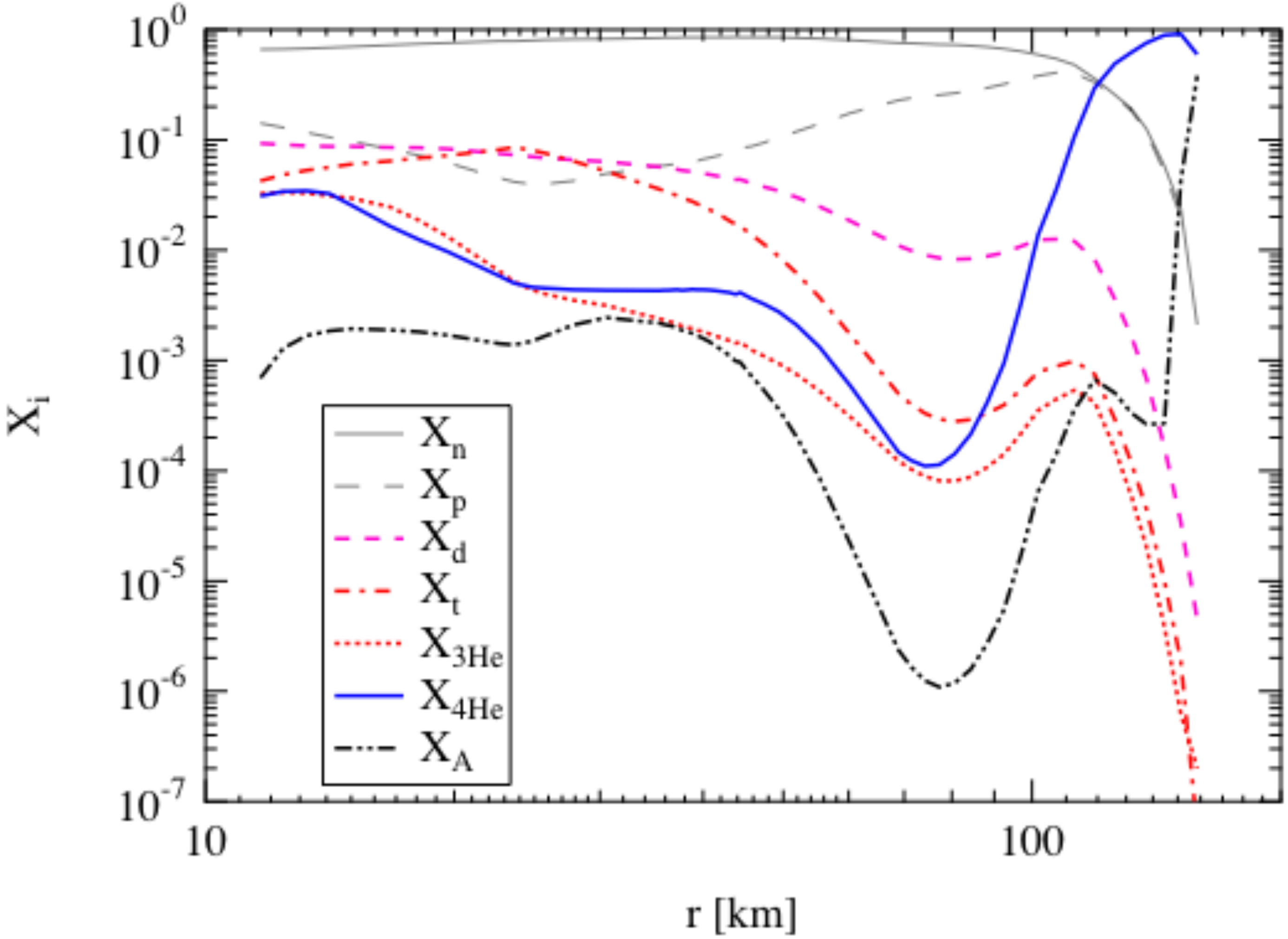
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi, G. R.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core



Mass fraction X
of light clusters
for a post-bounce
supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

EoS at low densities from HIC

PRL 108, 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

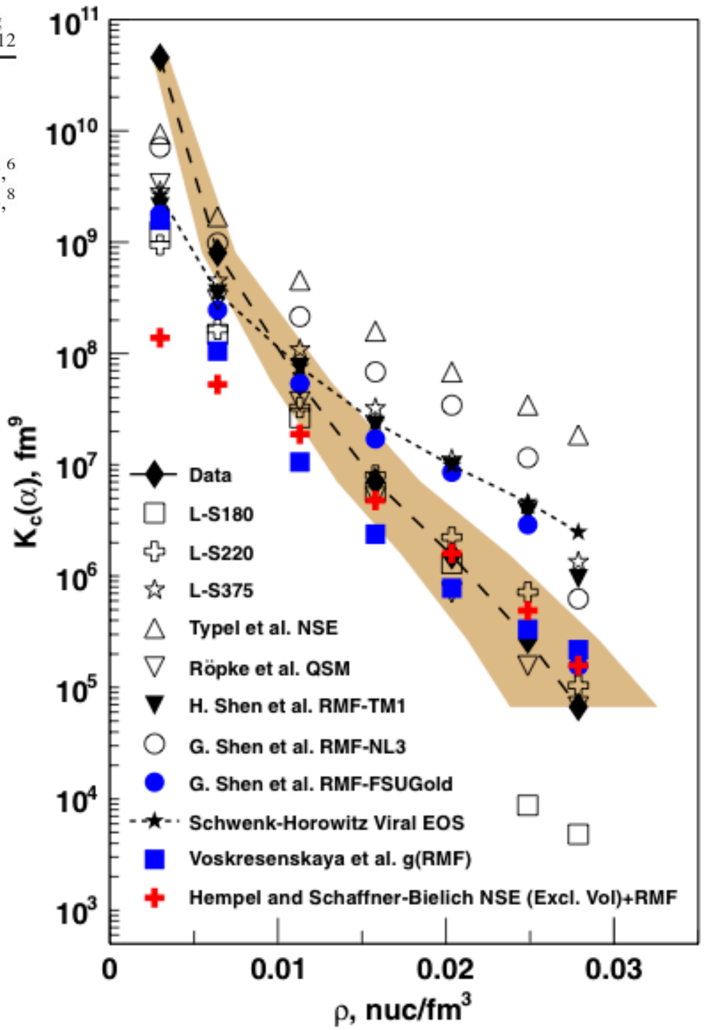
L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

inhomogeneous,
non-equilibrium



Nonequilibrium statistical operator (NSO)

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1 - t)} U(t, t_1) \rho_{\rm rel}(t_1) U^\dagger(t, t_1)\, dt_1$$

time evolution operator $U(t, t_0)$ relevant statistical operator $\rho_{\rm rel}(t)$

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$

maximum of information entropy $S_{\rm rel}(t) = -k_{\rm B}\, \mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}$

generalized Gibbs distribution $\rho_{\rm rel}(t) = \exp\left\{ -\Phi(t) - \sum_n \lambda_n(t) B_n \right\}$

von Neumann equation

$$\frac{\partial}{\partial t} \varrho_\varepsilon(t) + \frac{i}{\hbar} [H, \varrho_\varepsilon(t)] = -\varepsilon \left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t) \right)$$

$$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$$

Expanding nuclear matter: freeze-out and reaction processes (feed-down)

Freeze-out at heavy ion collisions

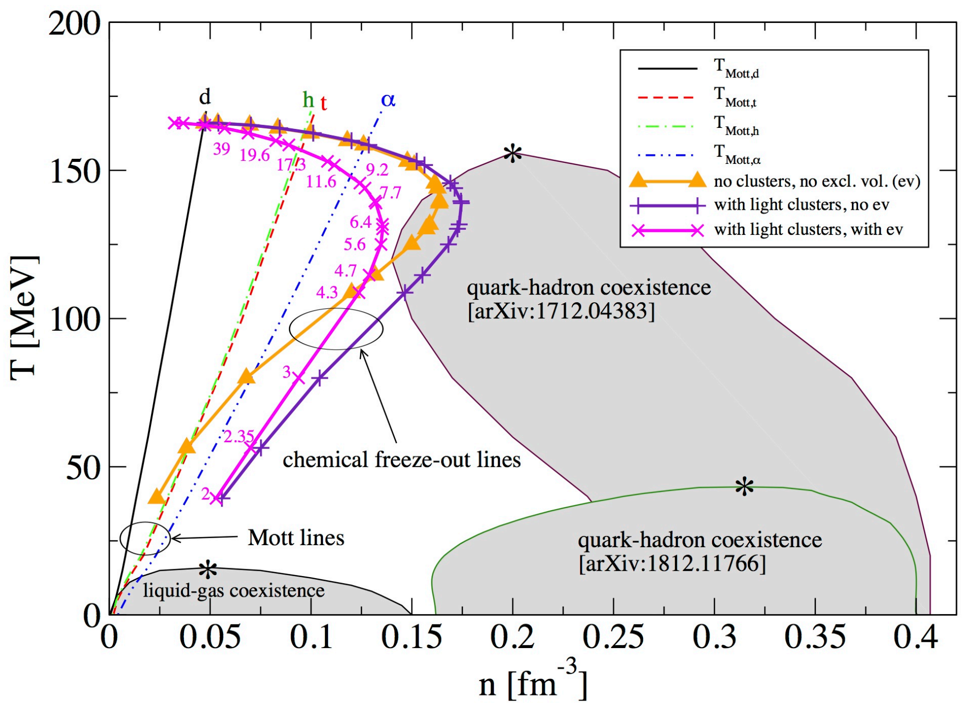
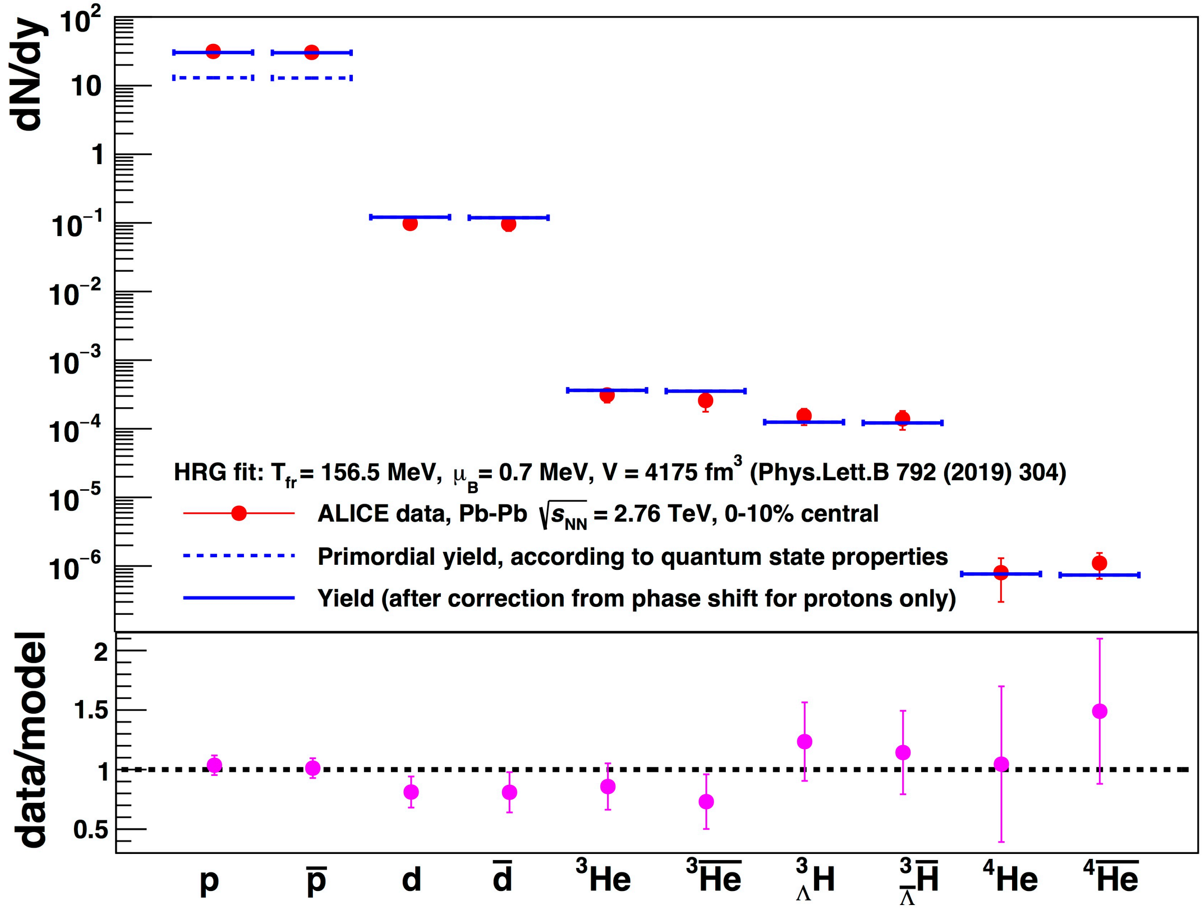


Fig. 1. Chemical freezeout lines in the temperature density plane (phase diagram) together with Mott lines for light clusters. The coexistence regions for the nuclear gas-liquid transition and for two examples of the hadron-quark matter transition are shown as grey shaded regions together with their critical endpoints. For details, see text.

D. Blaschke, G. Ropke, Yu. Ivanov, M. Kozhevnikova, and S. Liebing,
The XVIII International Conference on Strangeness in Quark Matter (SQM 2019)

Cluster formation at LHC/CERN

ALICE@LHC



Excellent description of data by the statistical model (chemical equilibrium)

T = 156 MeV

Beth-Uhlenbeck formula for interaction with further particles:

B. Doenigus, G.R., D. Blaschke, Phys. Rev. C **106**, 044908 (2022)

Outline/Conclusions

1. Equation of state and virial expansions in plasmas
   Virial coefficients for the uniform electron gas, interpolation formulas: PIMC

2. Electrical conductivity: virial expansion for hydrogen plasmas
   Inclusion of electron-electron collisions: DFT-MD, PIMC

3. Beth-Uhlenbeck formula for the second virial coefficient
   In-medium Schroedinger equation for nuclear matter: Few-body problem

4. Cross-over Bose Einstein condensate – BCS pairing
   Quartetting in low-density nuclear matter, Hoyle state and α decay: AMD

5. Cluster Beth-Uhlenbeck formula, continuum correlations
   Cluster-mean field, composition, freeze-out concept: self-consistent cluster

Thanks

to M. Bethkenhagen, D. Blaschke, T. Dornheim, Y. Funaki,
H. Horiuchi, W.-D. Kraeft, B. Mahato, Zh. Moldabekov, J. Natowitz,
H. Pais, R. Redmer, H. Reinholz, Z. Ren, M. Schoerner,
P. Schuck, A. Tohsaki, S. Typel, J. Vorberger,
H. Wolter, C. Xu, S. Yang,T. Yamada, B. Zhou
for collaboration

to you

for attention

D.G.

Electron-electron collisions and transport coefficients

Lorentz limit: electron-electron collisions not included in DFT-MD simulations.

slope from quantum Lenard-Balescu: dynamical screening by electrons

$$\lim_{T\to\infty} \tilde{\rho}_2(T) = \tilde{\rho}_2^{\mathrm{QLB}} = 0.4917$$

slope: fit gives 0.9886, static screening by electrons and ions:

$$\lim_{T\to\infty} \tilde{\rho}_2(T) = \frac{\pi^{3/2}}{24\sqrt{2}} \left[\frac{11}{2} - 3C + \ln\left(\frac{3}{2}\pi^2\right)\right] = 1.06036$$

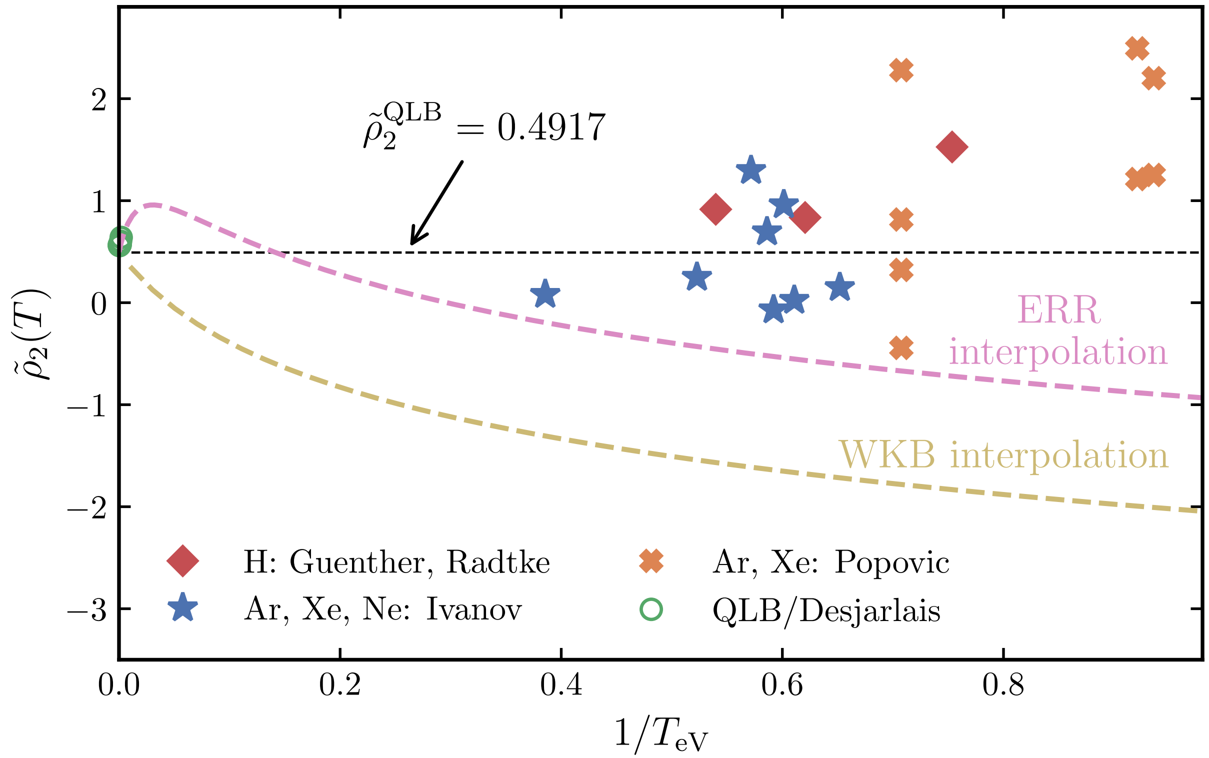
PIMC simulations can solve the problem of the contribution of e – e collisions

Higher order correlation functions?

Electronic transport coefficients from density functional theory across the plasma plane
M. French, G. R., M. Schörner, M. Bethkenhagen, M. P. Desjarlais, R. Redmer
Phys. Rev. E **105**, 065204 (2022)

Experiments

second virial coefficient: $\tilde{\rho}_2^{\rm eff}(n,T) = \frac{32405.4}{\sigma(n,T)[\Omega{\rm m}]}\left(\frac{T}{\rm eV}\right)^{3/2} - 0.846024\ \ln\left(\frac{\Theta}{\Gamma}\right)$



densities high,
temperatures low:

partially ionized plasmas;
ionization degree,
electron-atom collisions

Interpolation formulas, ERR: A Esser, R. Redmer, G. R., Contrib. Plasma Phys. **43,** 33 (2003).

WKB: $$\tilde{\rho}_2(T_{\rm eV}) \approx 0.4917 + 0.846 \ln\left[\frac{1 + 8.492/T_{\rm eV}}{1 + 25.83/T_{\rm eV} + 167.2/T_{\rm eV}^2}\right]$$

G. R., M. Schoerner, M. Bethkenhagen, R. Redmer, , Phys. Rev. E **104,** 045204 (2021) Supplemental material

nucleon-nucleon interaction potential

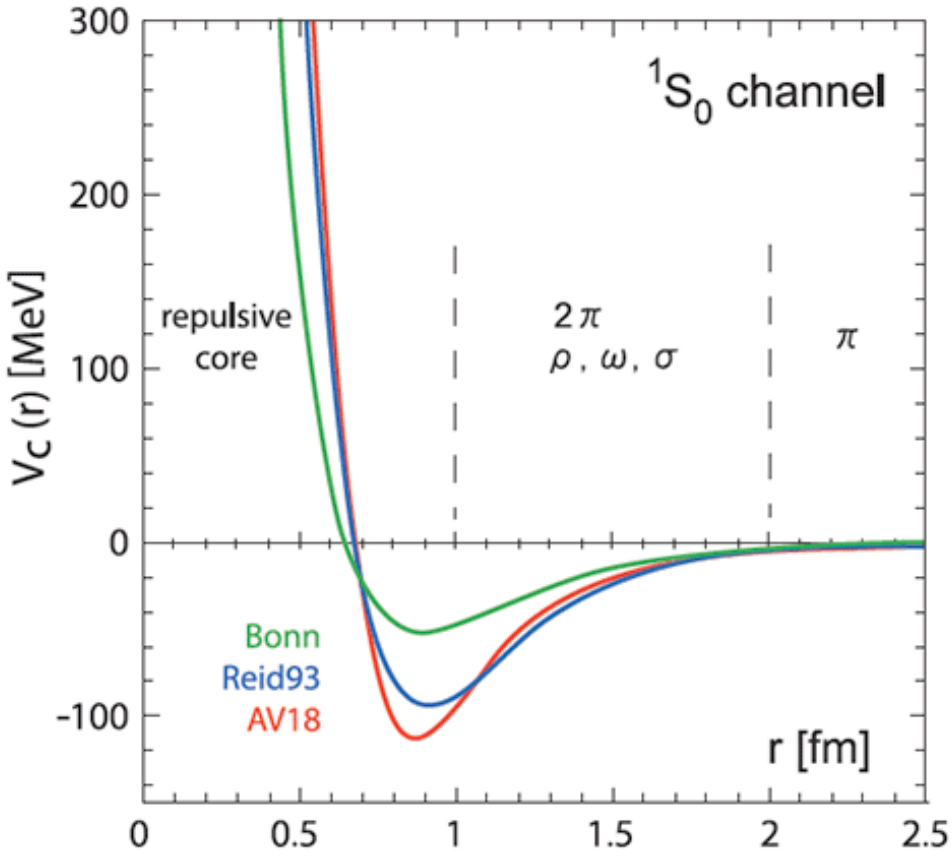
- Effective potentials
  (like atom-atom potential)
  binding energies, scattering

- non-local, energy-dependent?
  QCD?

- microscopic calculations
  (AMD, FMD)

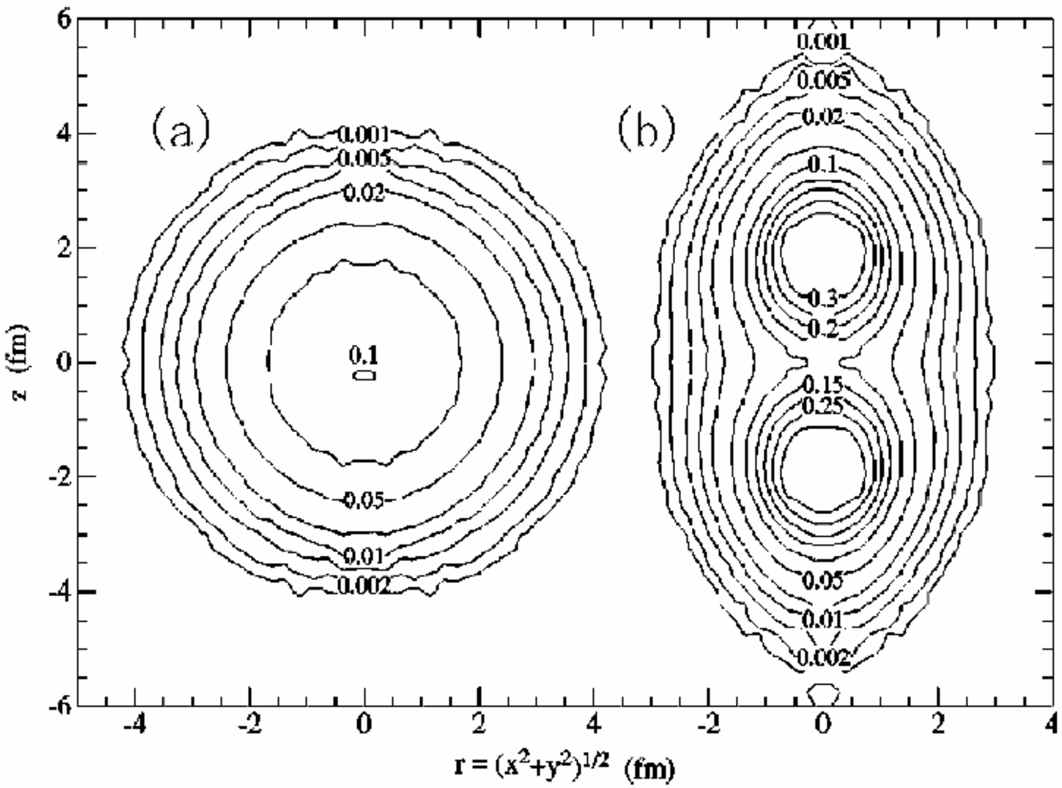
- single-particle descriptions:
  Thomas-Fermi approximation
  shell model
  density functional theory (DFT)

- correlations, clustering
  low-density nα nuclei, Volkov



5. Nuclear structure and reactions

α cluster structure of ^{8}Be

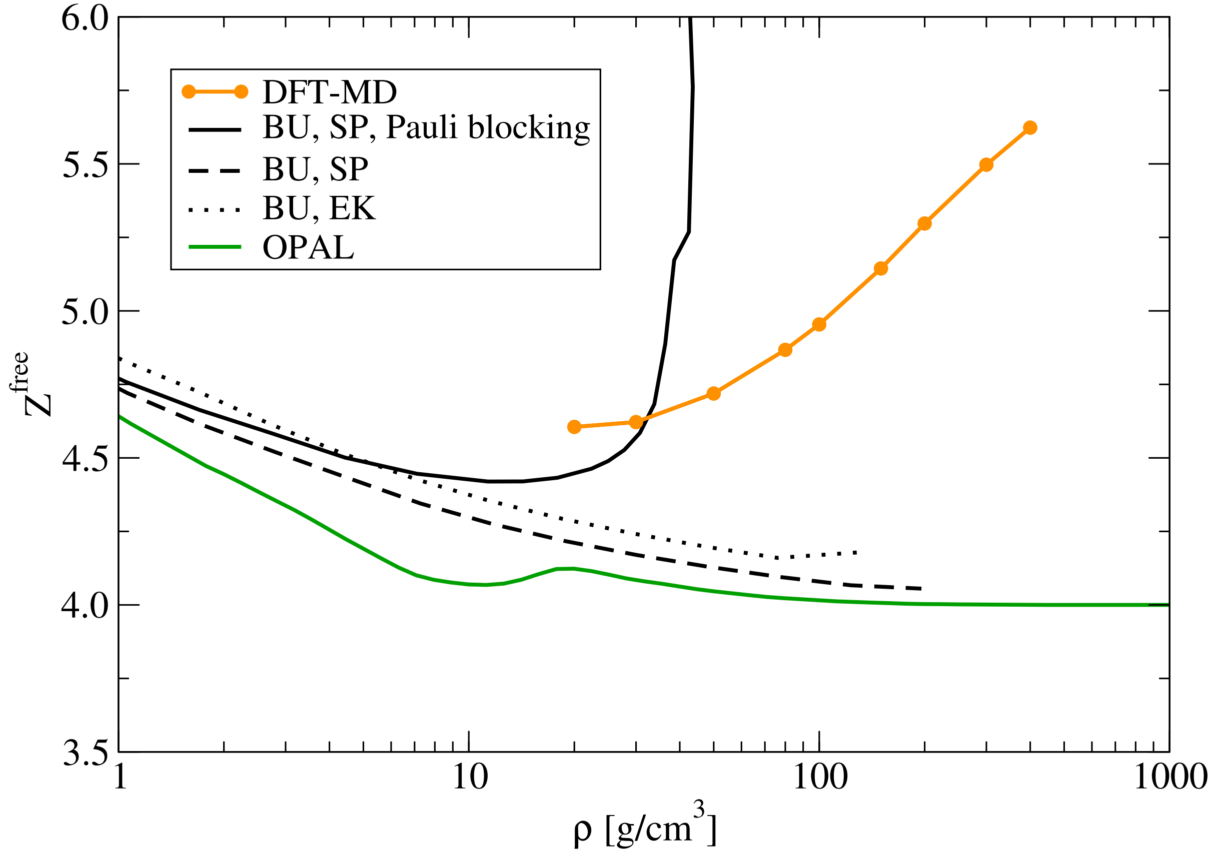


R.B. Wiringa et al., PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for ^{8}Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Ionization degree for carbon

T = 100 eV



Ionization degree of carbon Z^{free} derived from DFT- MD simulations (orange line) compared to predictions of OPAL (green line) and Beth-Uhlenbeck (BU) calculations (black lines).

BU results incorporate the EK and SP models, respectively.

Solid line takes into account Pauli blocking effects in addition.

M. Bethkenhagen et al., Phys. Rev. Research 2, 023260 (2020)

3. Clusters in an external potential

c. o. m. coordinate R, relative coordinates s_j $\Psi(\mathbf{R},\mathbf{s}_j) = \varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})\,\Phi(\mathbf{R})$

normalization $\int dR\,|\Phi(\mathbf{R})|^2 = 1$ $\int ds_j|\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})|^2 = 1$

N. Gidopoulos, E. Gross (2014)

Wave equation for the c.o.m. motion

$$-\frac{\hbar^2}{2Am}\nabla_R^2\Phi(\mathbf{R}) - \frac{\hbar^2}{Am}\int ds_j\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})[\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})][\nabla_R\Phi(\mathbf{R})]$$
$$-\frac{\hbar^2}{2Am}\int ds_j\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})[\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})]\Phi(\mathbf{R}) + \int dR'\,W(\mathbf{R},\mathbf{R}')\Phi(\mathbf{R}') = E\,\Phi(\mathbf{R})$$

c.o.m. effective potential

$$W(\mathbf{R},\mathbf{R}') = \int ds_j\,ds_j'\,\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})\,\left[T[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}_j') + V(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}_j')\right]\,\varphi^{\rm intr}(\mathbf{s}_j',\mathbf{R}')$$

Wave equation for the intrinsic motion

$$-\frac{\hbar^2}{Am}\Phi^*(\mathbf{R})[\nabla_R\Phi(\mathbf{R})][\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})] - \frac{\hbar^2}{2Am}|\Phi(\mathbf{R})|^2\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$
$$+\int dR'\,ds_j'\,\Phi^*(\mathbf{R})\,\left[T[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}_j') + V(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}_j')\right]\,\Phi(\mathbf{R}')\varphi^{\rm intr}(\mathbf{s}_j',\mathbf{R}') = F(\mathbf{R})\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$

G. R. et al., PRC **90**, 034304 (2014)

Quartet wave function

Four-particle wave equation in position space representation

$$[E_4 - \hat{h}_1 - \hat{h}_2 - \hat{h}_3 - \hat{h}_4]\Psi_4(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4) = \int d^3\mathbf{r}'_1\, d^3\mathbf{r}'_2 \langle \mathbf{r}_1\mathbf{r}_2|B\, V_{N-N}|\mathbf{r}'_1\mathbf{r}'_2\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}'_2\mathbf{r}_3\mathbf{r}_4)$$

$$+ \int d^3\mathbf{r}'_1\;\; d^3\mathbf{r}'_3 \langle \mathbf{r}_1\mathbf{r}_3|B\, V_{N-N}|\mathbf{r}'_1\mathbf{r}'_3\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}_2\mathbf{r}'_3\mathbf{r}_4) + \text{four further permutations.}$$

Single-nucleon Hamiltonian $\hat{h} = \frac{\hbar^2 p^2}{2m} + [1 - \sum_i^{\text{occ.}} |n\rangle\langle n|]V^{\text{mf}}(r)$

Pauli blocking B $B(1,2) = [1 - f_1(\hat{h}_1) - f_2(\hat{h}_2)]$

Local density approximation: momentum representation, no coupled gradient terms, Thomas-Fermi

Intrinsic motion: in-medium interaction

c.o.m. effective potential $W(\mathbf{R}) = W^{\text{ext}}(\mathbf{R}) + W^{\text{intr}}(\mathbf{R})$ $W^{\text{ext}}(\mathbf{R}) = W^{\text{mf}}(\mathbf{R}) = V^{\text{Coul}}_{\alpha-\text{O}}(R) + V^{\text{N-N}}_{\alpha-\text{O}}(R)$

$W^{\text{intr}}(\mathbf{R}) = 4E_F[n_B(\mathbf{R})],\qquad E_F(n_B) = (\hbar^2/2m)(3\pi^2 n_B/2)^{2/3}.$

$W^{\text{intr}}(\mathbf{R}) = -B_\alpha + W^{\text{Pauli}}[n_B(\mathbf{R})],\qquad n_B \le n_{\text{crit}}$

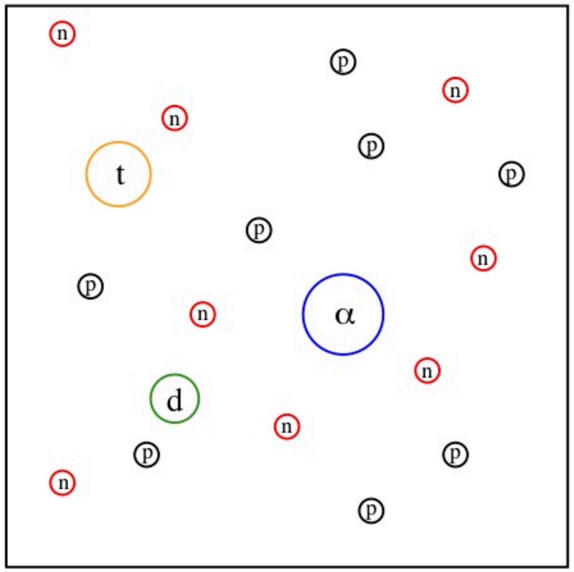
$$W^{\text{Pauli}}(n_B) \approx 4515.9\,\text{MeV fm}^3 n_B - 100935\,\text{MeV fm}^6 n_B^2 + 1202538\,\text{MeV fm}^9 n_B^3$$

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

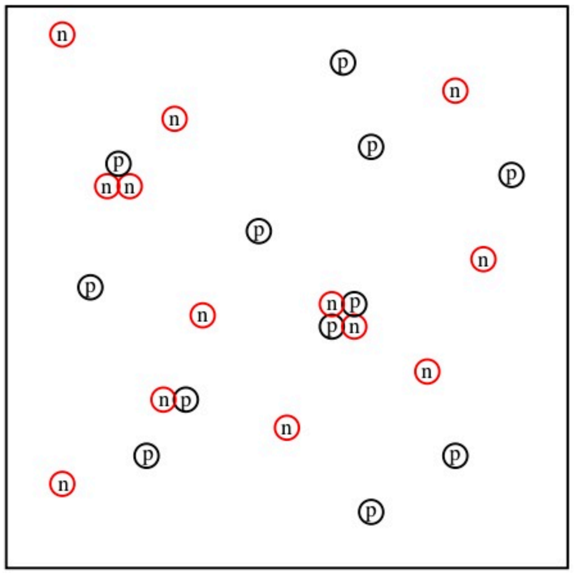


Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

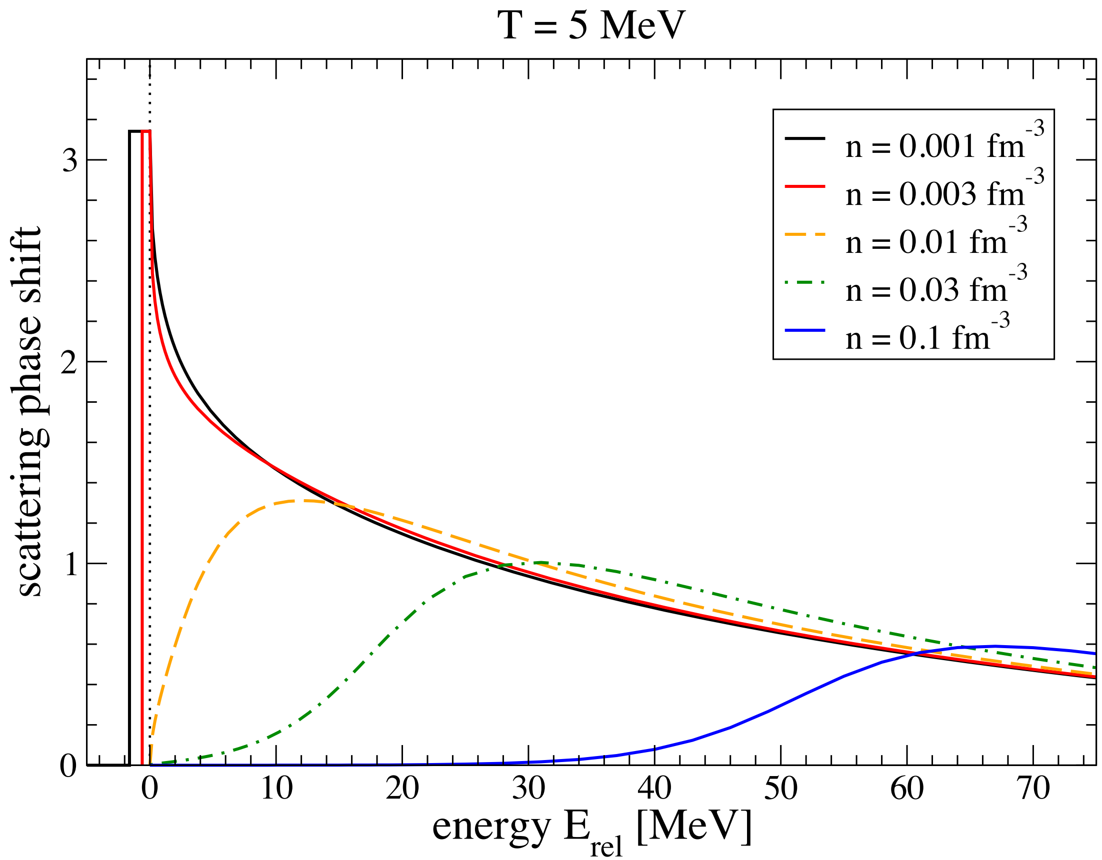
and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

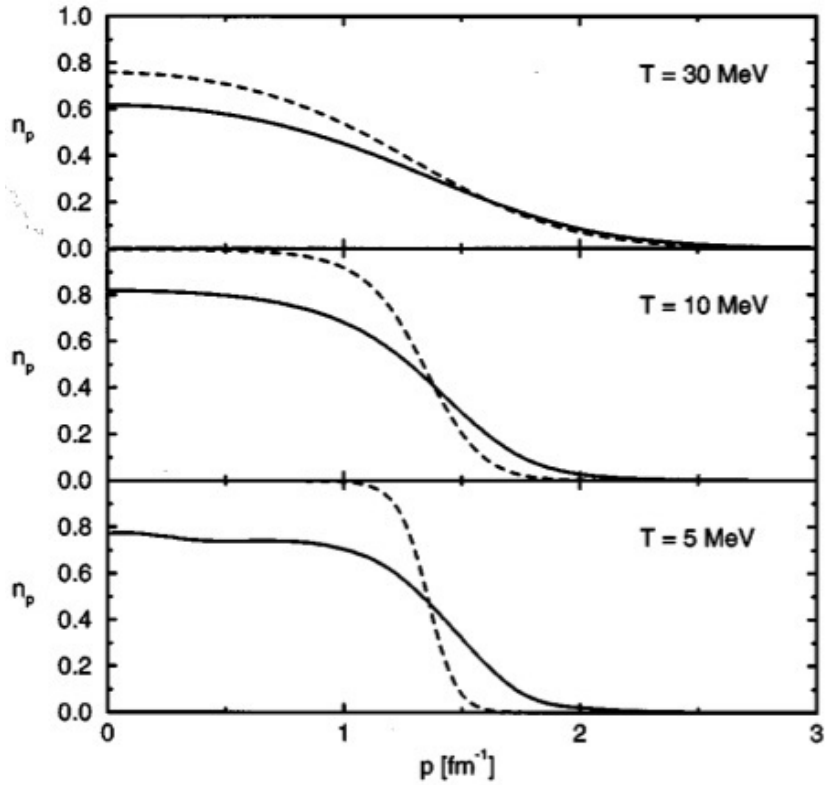
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Single nucleon distribution function

Dependence on temperature



saturation density

Alm et al., PRC 53, 2181 (1996)

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ideal mixture of all bound states
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Chemical equilibrium of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

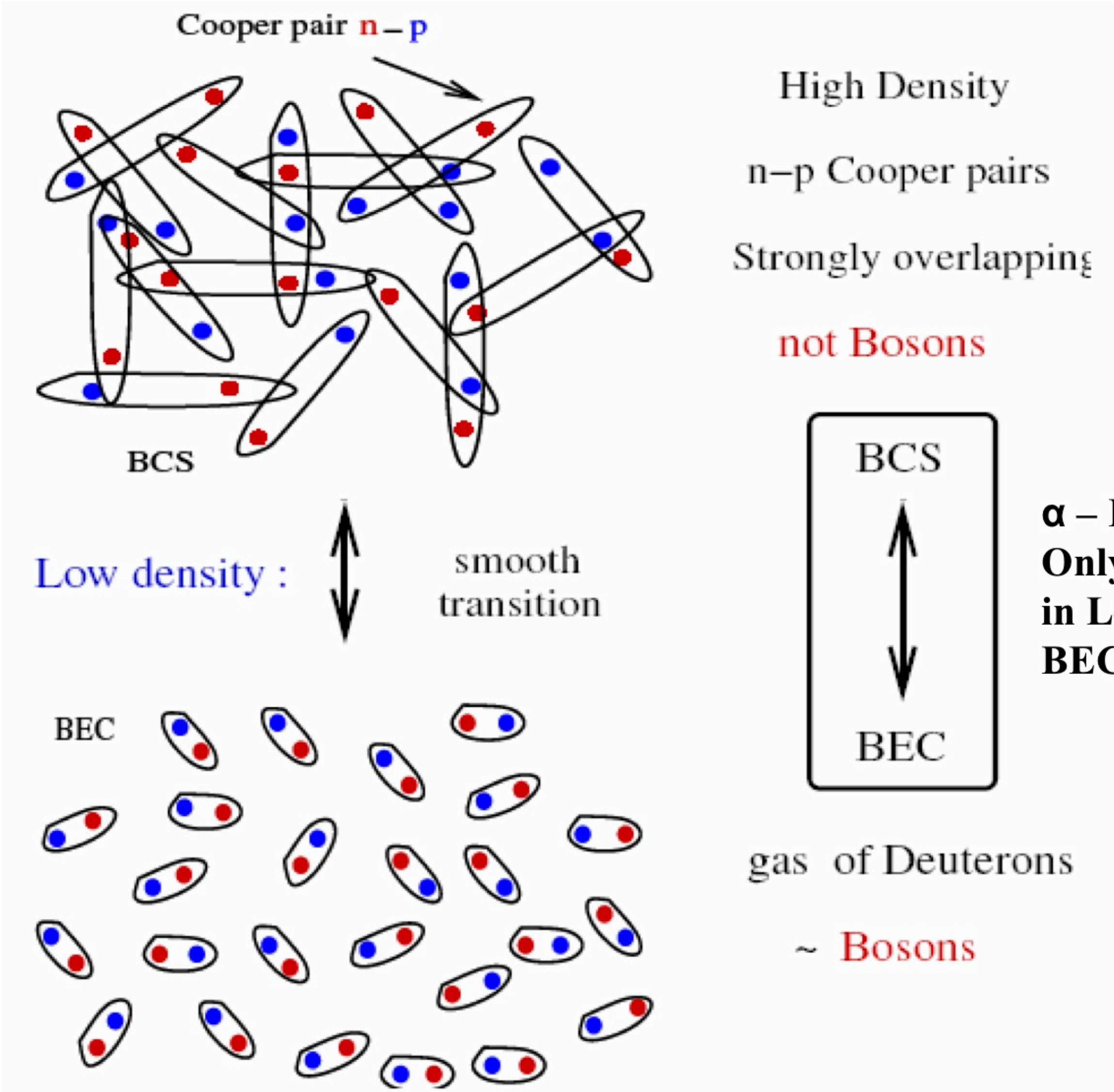
Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

Crossover from BEC to BCS pairing

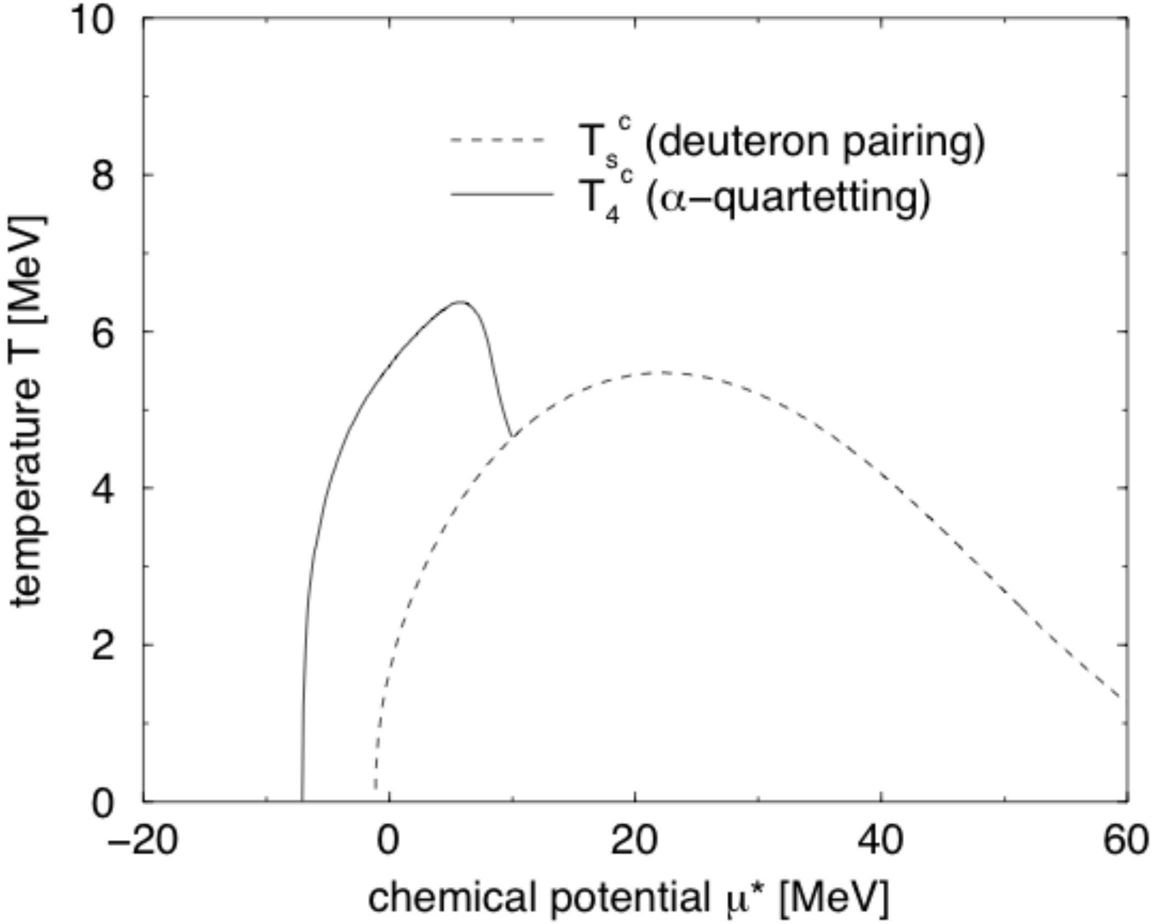


α – Particles
Only Exist
in Low Density
BEC Phase

P. Schuck

α-cluster-condensation

(quartetting)



G.R., A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

Suppresion of condensate fraction

- Alpha-alpha interaction (Ali/ Bodmer),

  no Pauli blocking:
- Variational calculation (Clark/ Jastrow approach

  to the alpha-particle condensate amplitude)

  (crosses)
- First order approximation

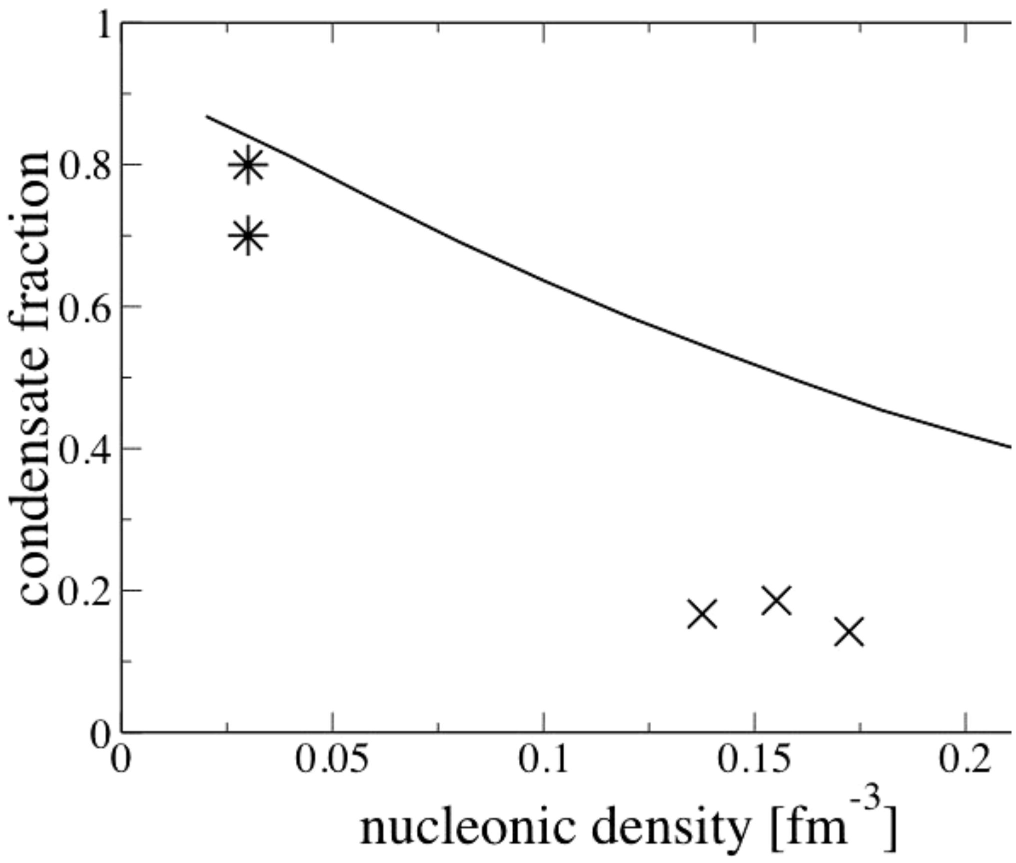
  (full line)

- Yamada/Schuck's

  result for condensate

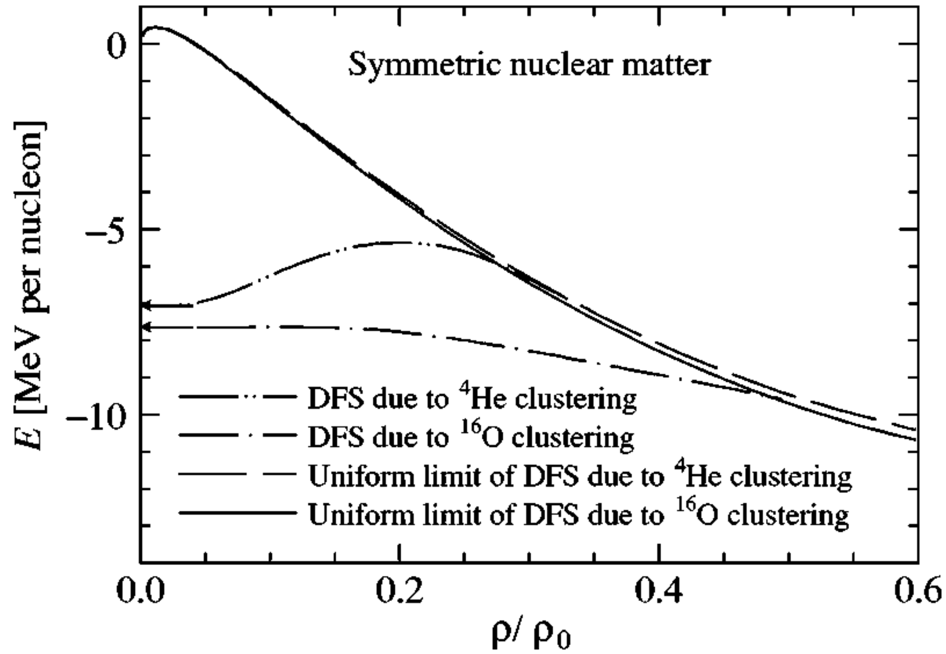
  in C12 - O2+

  (stars)



Y. Funaki et al., PRC **77**, 064312 (08)

Clustering phenomena in nuclear matter below the saturation density



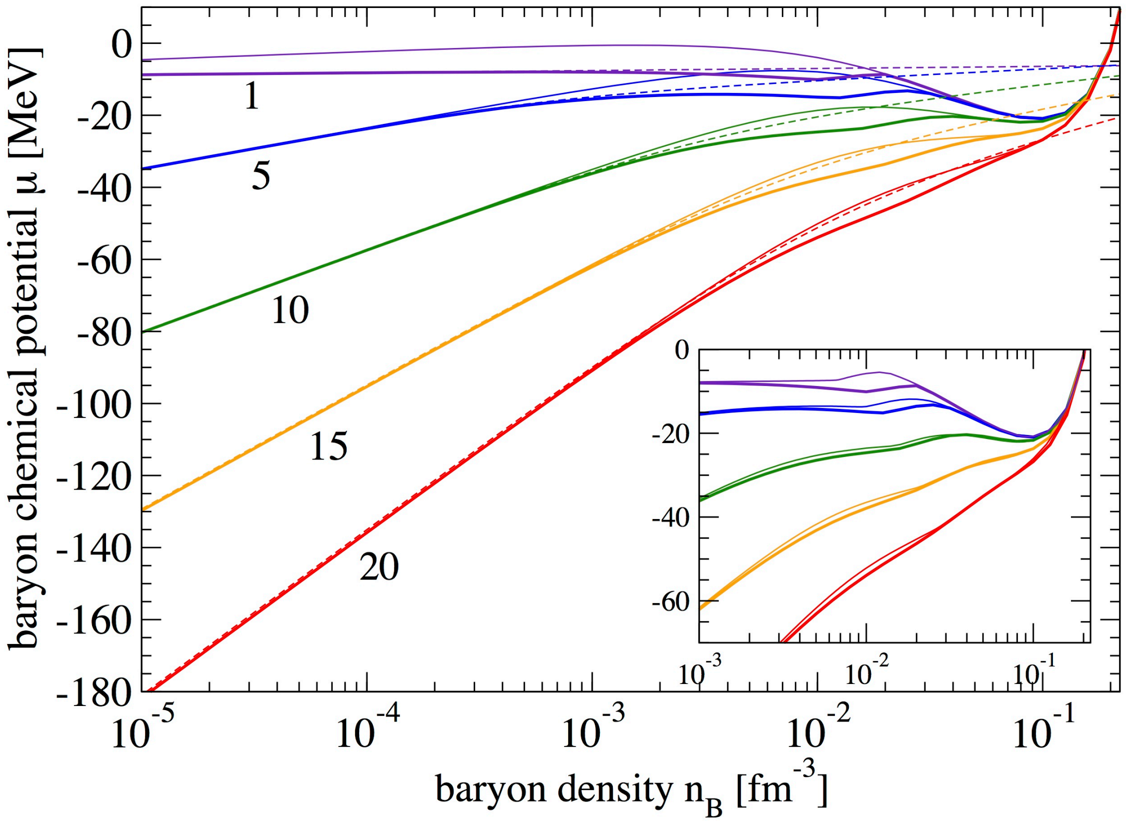
Overhauser: density-fluctuated states (DFSs) instead of plane-wave states.

Energy curves of DFSs due to α and ^{16}O clustering in the symmetric nuclear matter. The density of matter is normalized by the saturation density of the uniform matter.

Hiroki Takemoto et al.,

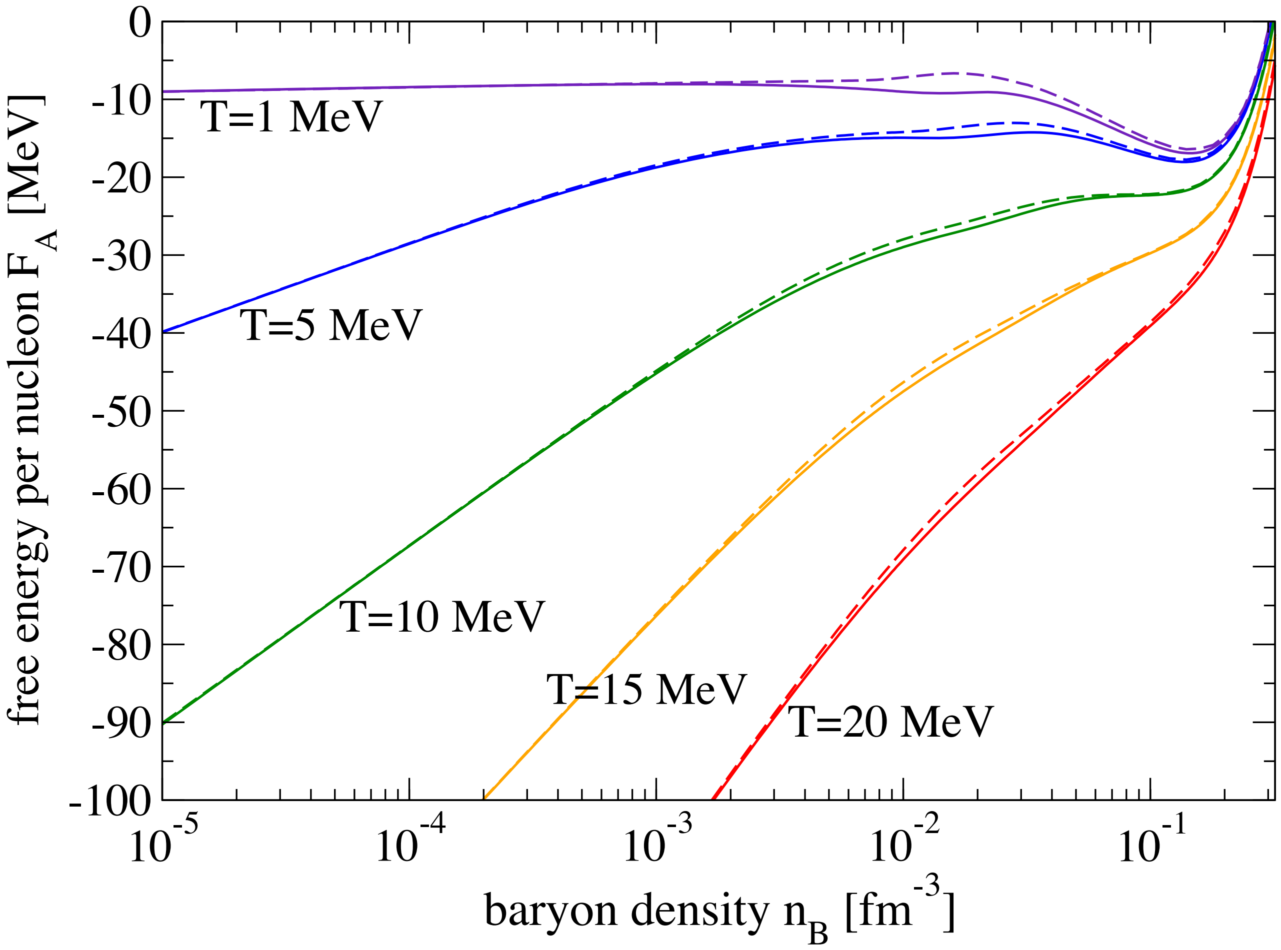
PR C **69**, 035802 (2004)

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

G. R., PRC **92**, 054001 (2015)